Negative hopping magnetoresistance and dimensional crossover in lightly doped cuprate superconductors

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We show that, due to the weak ferromagnetism of $La_{2-x}Sr_xCuO_4$, an external magnetic field leads to a dimensional crossover from two to three dimensions for the in-plane transport. The crossover results in an increase of the hole's localization length and hence in a dramatic negative magnetoresistance in the variable range hopping regime. This mechanism quantitatively explains puzzling experimental data on the negative magnetoresistance in the Néel phase of $La_{2-x}Sr_xCuO_4$.

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I. INTRODUCTION

The physics of the high-temperature superconducting oxides is determined by the interplay between the charge and spin degrees of freedom, ultimately responsible for the superconductivity itself. A variety of interesting phenomena exists already at low doping when the oxide layers are insulating. In $La_{2-x}Sr_xCuO_4$ (LSCO), the insulating (spin-glass) region corresponds to doping $x<0.055$, with incommensurate magnetism which exists down to the boundary with the antiferromagnetic phase (at $x=0.02$), and even inside the Néel region ($x<0.02$).[1] A popular point of view favors an explanation of the incommensurate magnetism based on the tendency of the holes to form "stripes."[2] However, experimental data on variable range hopping (VRH) (see the review in Ref. 3) unambiguously indicate localization of holes for $x<0.055$ and therefore support an approach based on a purely magnetic scenario, where a spiral distortion of the spin background is generated by localized holes. The corresponding theory explains quantitatively the variety of magnetic and transport data in LSCO.[4–9]

Magnetic phenomena in the low-doping region reflect, in addition to the Heisenberg exchange, the presence of anisotropies in the spin-spin interactions, such as Dzyaloshinsky-Moriya (DM) and XY terms. In the present paper, we consider the Néel phase, $x<0.02$. In this phase, the anisotropies confine the spins to the (ab) plane and fix the direction of the Néel vector to the $\hat{b}$-orthorhombic axis. Moreover, the DM interaction induces a small out-of-plane spin component that is ferromagnetic in the plane (weak ferromagnetism) but staggered in the out-of-plane $\hat{c}$ direction. This component can be easily influenced by an external magnetic field applied in different directions, as it has been recently addressed both experimentally[10–14] and theoretically.[15,16] For example, a perpendicular field ($H\parallel\hat{c}$) can cause an alignment of the out-of-plane moments via a spin-flop transition at a critical field H_f, determined by the competition between the DM and interlayer Heisenberg exchange (typically $H_f\approx 5-7$ T).[11,13,14] Perhaps most intriguingly, the in-plane resistivity (along with the $\hat{c}$-axis resistivity) decreases by as much as 50% across such a transition.[10,11] The magnitude of the magnetoresistance (MR) shows a rapid increase only below $\approx$50 K (Ref. 11) where LSCO exhibits VRH conduction.[3,17] This implies that the MR is accumulated mostly in transitions between localized states. Therefore, it is very natural to assume that the large negative MR is due to an increase of the hole's localization length, as it was suggested in the first experimental paper.[10] From theoretical viewpoint, the problem is why the localization length increases at the spin-flop transition. The first model for the localization length increase, invoking a three-dimensional (3D) VRH mechanism, was proposed in Ref. 18. However, it is clear now that except for ultralow temperatures (that we estimate to be below ~50 mK), the VRH conduction at zero magnetic field is dominated by two-dimensional (2D) physics.[3,17] Because of this, the 3D picture is not able to describe the most recent and detailed MR data, as we discuss below. Experiments are performed typically in the temperature range of a few kelvin and higher where the out-of-plane resistivity anisotropy is large, $\rho_c/\rho_{ab}\sim 10^2-10^3$.[11] While we ultimately expect that at $T\to 0$ VRH will become 3D, in the temperature range of experimental interest, the 2D mechanism is the relevant one, as is clear from the analysis of the 2D-3D crossover temperature and the fits of the hopping conductivity presented in the next section.

In the present work, we demonstrate that the large MR arises from a change of the effective dimensionality of the VRH mechanism with applied field. We support our conclusions by detailed comparison with recent experiments on magnetotransport which can be described by our theory with excellent accuracy. The main idea of the present work is that a dimensional crossover (2D$\to$3D) occurs at the spin flop, and this is conceptually and quantitatively different from the 3D picture of Ref. 18. In particular, in our approach, the increase of the MR (and the localization length) is not simply due to the change of the out-of-plane effective mass as in Ref. 18 but rather arises from a change in the shape of the (localized) wave functions across the spin flop. In the temperature regime that we keep in mind, 1 K and higher, the change of the out-of-plane effective mass is a small, secondary effect (which can manifest itself only at ultralow temperatures where the full 3D VRH mechanism is responsible for transport). We show that the alignment of the weak ferromagnetic moments in neighboring planes with the field allows the interlayer hopping of localized holes, which, in turn, leads to an increase of the hole's in-plane hopping probability and thus negative MR. The presence of an interlayer hopping channel across the spin flop was already identified in Ref. 18; however, our analysis differs in the effects this additional channel can produce in VRH conduction. By investigating the evolution of the hole bound state as a function of magnetic field and temperature, we find that in various regimes, different numbers of layers are involved in transport. In the experimentally relevant temperature range, the hopping turns out to be quasi-two-dimensional, leading to a negative MR in very good agreement with the most recent experiments.[11,12]

The paper is organized as follows. In Sec. II, we analyze the effect of the magnetic field on the dispersion of the localized holes through the interlayer hopping. In Sec. III, we present a detailed analysis of the change of the hole's wave function due to the modified dispersion. In Secs. IV and V, we then use the wave functions to calculate the magnetoresistance for out-of-plane and in-plane magnetic fields and compare with experiment. Sec. VI contains our conclusions.

II. INTERPLANE HOPPING AND SPIN-FLOP TRANSITION FOR MAGNETIC FIELD $H \| \hat{c}$

First, we briefly summarize previous results related to the structure of the hole's 2D bound state at zero field. As a starting point, we consider the hole dynamics in the antiferromagnetic background within the framework of the $t-t'-t''-J$ model.[6] In the absence of the Coulomb potential $V(r)$ of the Sr ion, a hole resides, in momentum space, near the nodal points $(\pm\pi/2,\pm\pi/2)$ and has dispersion $\epsilon_{\mathbf{k}} \approx \frac{\beta_1}{2}k_1^2+\frac{\beta_2}{2}k_2^2$, where $\beta_1\approx\beta_2=\beta\equiv m_{\|}^{-1}\approx 2J$ is the inverse 2D effective mass appropriate for LSCO[6] ($m_{\|}\approx 2m_e$ in absolute units). We measure energies in units of $J=130$ meV, and the lattice spacing is set to unity. Due to $V(r)$, the hole is localized, and its wave function has the form $\psi(r)\sim e^{-\kappa_0 r}$, corresponding to binding energy $\epsilon_0=\beta\kappa_0^2/2$. Here, the inverse (2D) localization radius for LSCO is $\kappa_0\sim 0.3-0.4$,[6] giving $\epsilon_0\approx 10$ meV. On a perfect square lattice, the bound state is fourfold degenerate: the hole can reside on either up or down sublattices (pseudospin), and it can reside in either of the two pockets $(\pi/2,\pm\pi/2)$ (flavor). The orthorhombic distortion lifts the flavor degeneracy due to the presence of diagonal next-nearest neighbor hopping t'. Hence, the holes occupy only the pocket $(\pi/2,-\pi/2)$.[8] This is also consistent with the fact that the spin structure becomes incommensurate along the $\hat{b}$-orthorhombic direction, as seen in neutron scattering for $x<0.055$.[1,8]

Now, let us consider the correction to the 2D dispersion $\delta\epsilon_{\mathbf{k}}$ arising from the interlayer hopping $t_{\perp}$.

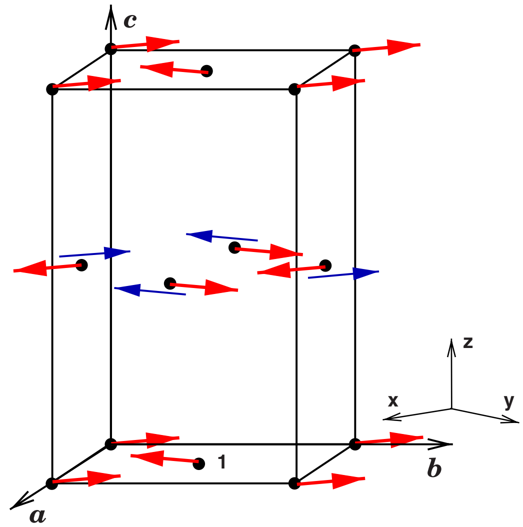

FIG. 1. (Color online) Spin structure of LSCO. Red arrows correspond to $H<H_f$; the blue arrows in the middle layer show the spin reversal for $H>H_f$.

Without the account of correlations, we have $\delta\epsilon_{\mathbf{k}}=-8t_{\perp}\cos(k_x/2)\cos(k_y/2)\cos(k_z)$, with $\mathbf{k}$-dependent $t_{\perp}$, $t_{\perp}=t_c\frac{1}{4}(\cos k_x-\cos k_y)$,[2] where $t_c\sim 50$ meV.[19] By averaging over the momentum distribution (k_x,k_y) in the 2D bound state, we find the effective value of interlayer hopping $t_{\perp}\to t_c\kappa_0^2/4\sim 1$ meV. Since this is a crude, order of magnitude estimate, below, we will use $t_{\perp}$ as a fitting parameter.

The *t-J* model correlations change $\delta\epsilon_{\mathbf{k}}$. First, the hopping matrix element should be replaced by $t_{\perp}\to Zt_{\perp}$, where $Z\approx 0.3$ is the quasiparticle residue. Second, direct hopping is allowed only between spins in the same sublattice. In LSCO, a spin in a given plane (e.g., spin "1" in Fig. 1) interacts with other four in the plane above (and below),[3] but at $H=0$ (or $H<H_f$), only out-of-plane hopping in the $\hat{b}$ direction is allowed, because this corresponds to ferromagnetic ordering of spins in neighboring planes (see Fig. 1). However, when a magnetic field is applied along the $\hat{c}$ axis, the spins in the next layer reverse their signs across the spin-flop transition at H_f, so that for $H>H_f$, only hopping in the $\hat{a}$ direction contributes. This is schematically shown in Fig. 1 and reflects in the effective dispersion,

$$\delta\epsilon_{\mathbf{k}}=-4Zt_{\perp}\cos(k_z)\cos\left(\frac{k_x\pm k_y}{2}\right),$$

$$\text{"}-\text{"}:H<H_f,\qquad \text{"}+\text{"}:H>H_f, \tag{1}$$

where we define the z direction $z\|\hat{c}$. Since in the occupied pocket $k_x\approx\pi/2$ and $k_y\approx-\pi/2$, Eq. (1) reads

$$\delta\epsilon_{\mathbf{k}}^{H<H_f}\approx 0,\quad H<H_f$$

$$\delta\epsilon_{\mathbf{k}}^{H>H_f}=-4Zt_{\perp}\cos(k_z),\quad H>H_f. \tag{2}$$

Thus, at $H<H_f$, there is no z dispersion, the coherent dynamics is purely 2D, and the VRH in-plane resistivity behaves as $\rho=\rho_0\exp(T_0/T)^{1/3}$.[20] Here, T_0 is strongly

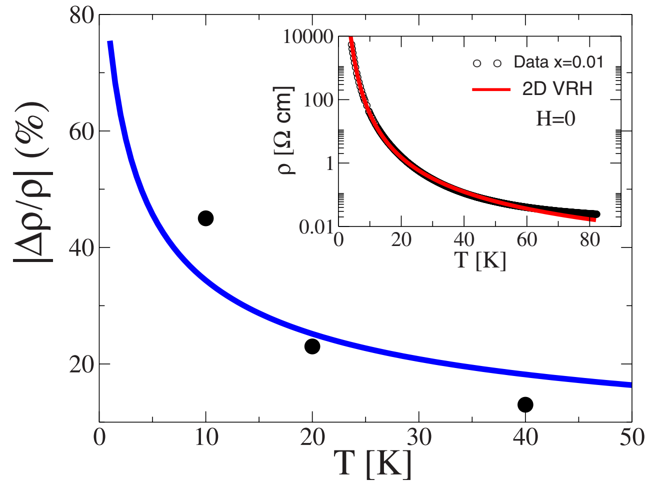

FIG. 2. (Color online) The MR jump $|\Delta\rho/\rho|$ (solid blue line) across the spin-flop transition for $H \| \hat{c}$. Here, $Zt_\perp/\epsilon_0=0.057$ and $T_0=3.6\times10^4$ K. The circles are data from Ref. 11. Inset: zero field in-plane resistivity fit to 2D VRH form (solid, red line) with $T_0=3.6\times10^4$ K.

doping and sample dependent, and the data at zero field, $x=0.01$, can be fitted by $\rho_0=8\times10^{-6}\ \Omega$ cm and $T_0=3.6\times10^4$ K with astonishing accuracy in the range of 4 K $<T<$ 50 K, as shown in Fig. 2 (inset).

Observe that a more accurate estimate of the z dynamics below the spin flop can be obtained by expanding the in-plane dispersion $\epsilon_{\mathbf{k}}+\delta\epsilon_{\mathbf{k}}$ around $(\pi/2,-\pi/2)$ and minimizing the resulting quadratic form. This gives a nonzero dispersion in the z direction, but the corresponding effective mass is huge, $M_\perp=\beta/[8(Zt_\perp)^2]\sim10^4 m_e$. Consequently, we find that at temperatures below $T^*\sim2^{-4}\kappa^{-3}(m_\|/M_\perp)^{3/2}T_0 \sim 50$ mK, the VRH is ultimately three dimensional, but this regime is irrelevant to present experiments.

III. EVOLUTION OF THE HOLE BOUND STATE ACROSS THE SPIN-FLOP TRANSITION

To qualitatively understand the effect of the change of dispersion [Eq. (2)] at $H>H_f$ on the in-plane hole dynamics, we first estimate the change in the hole binding energy. From Eq. (2), after the spin flop, the edge of the continuum $(k_z=0)$ decreases by $4Zt_\perp$, while the absolute energy of the bound state, to second order in the small parameter $Zt_\perp/\epsilon_0\ll1$, decreases only by the amount of $\Delta E\sim(Zt_\perp)^2/\epsilon_0\ll t_\perp$. The binding energy, which is the magnitude of the difference between the absolute energy and the continuum limit, changes after the flop as

$$\epsilon_0\to\epsilon\approx\epsilon_0-4Zt_\perp+(Zt_\perp)^2/\epsilon_0\approx\epsilon_0-4Zt_\perp. \quad (3)$$

Within the VRH picture, the conduction is proportional to the hole's hopping probability, which decays exponentially away from the donor site due to the hole localization. Thus, the decrease of the hole binding energy [Eq. (3)] across the spin-flop signals an increase of the localization length and, in turn, an increase of the VRH conductivity. This is our central idea that explains the negative MR.

To make this argument more quantitative, we need to compute the change in the hole's hopping probability across the spin-flop transition. Let us enumerate planes by the index n and assume for simplicity that Sr produces a *local* potential $V(r)$ that acts only in the plane $n=0$. For a shallow level, the exact form of the local potential is not important,[21] and for simplicity, we will use the δ-function approximation: $V(r)=-g/r\,\delta(r-r_0)$, where r_0 is assumed to be smaller than the localization length, $r_0\ll1/\kappa_0$. The bound state is described by the wave function $\psi_n(r)$ that depends on both n and r. Before the spin flop, $H<H_f$, it obeys the Schrödinger equation: $[-\frac{\beta}{2}\Delta_r-\delta_{n0}\frac{g}{r}\delta(r-r_0)]\psi_n(r)=E_0\psi_n(r)$, where $E_0=-\epsilon_0=-\beta\kappa_0^2/2$. The solution for $r<r_0$ is given by $I_0(\kappa_0 r)$, and for $r>r_0$, it reads

$$\psi_n=0,\quad n\neq0,$$

$$\psi_0=\frac{\kappa_0}{\sqrt{\pi}}K_0(\kappa_0 r)\sim\sqrt{\frac{\kappa_0}{2r}}e^{-\kappa_0 r},\quad \kappa_0 r\gg1,$$

$$H<H_f, \quad (4)$$

where $I_0(r)$ and $K_0(r)$ are the modified Bessel functions of the first and second kinds respectively. For the inverse localization length, we obtain $\kappa_0=\frac{2e^{-\gamma}}{r_0}\exp[-\beta/(2g)]$, where $\gamma=0.577$ is Euler's constant. However, the exact dependence of κ_0 on the parameters of the potential is not important, since the energy scale that κ_0 determines (the binding energy $\epsilon_0=\beta\kappa_0^2/2$), was extracted from experiment in earlier work, $\epsilon_0\approx10$ meV (see Sec. II).

For $H>H_f$, the Schrödinger equation becomes

$$\left(-\frac{\beta}{2}\Delta_r+\delta_{n0}V(r)\right)\psi_n-2Zt_\perp[\psi_{n+1}+\psi_{n-1}]=E\psi_n, \quad (5)$$

with $V(r)=-\frac{g}{r}\delta(r-r_0)$ and $E=-\beta\kappa^2/2$. After the Fourier transform, $\psi_n(r)=\Sigma_p\phi_p(r)e^{ipn}$, we obtain

$$-\frac{\beta}{2}\Delta_r\phi_p(r)+V(r)\psi_0(r)=(E+4Zt_\perp\cos p)\phi_p(r). \quad (6)$$

By solving this equation, we find

$$\kappa/\kappa_0\approx1+2(Zt_\perp/\epsilon_0)^2,$$

$$\phi_p(r)=\frac{\kappa_0}{\sqrt{\pi}}K_0[r\sqrt{\kappa^2-(8Zt_\perp/\beta)\cos p}]. \quad (7)$$

Thus, as we have already pointed out before Eq. (3), after the spin flop, the absolute energy E is shifted only in the second order in $Zt_\perp/\epsilon_0$, and hence this shift can be neglected. Consequently, below, we set $\kappa=\kappa_0$. However, in contrast to Eq. (4), the wave function,

$$\psi_n(r)=\frac{\kappa_0}{\sqrt{\pi}}\int\frac{dp}{2\pi}e^{ipn}K_0[\kappa_0 r\sqrt{1-(4Zt_\perp/\epsilon_0)\cos p}],$$

$$H>H_f, \quad (8)$$

does not have a simple exponential decay.

IV. MAGNETORESISTANCE ACROSS THE SPIN-FLOP TRANSITION FOR FIELD $H \| \hat{c}$

To evaluate the MR across the spin flop, we compute now the change of the hole's probability to propagate at the (large) VRH distance R_T: $\kappa_0 R_T = \frac{1}{3}(\frac{T_0}{T})^{1/3}$.[20] We can then identify two large-distance regimes for the wave function [Eq. (8)]: (1) very large distances, $\kappa_0 r \gg \epsilon_0/(2Zt_\perp)$ and (2) intermediate large distances, $\epsilon_0/(2Zt_\perp) \gg \kappa_0 r \gg 1$.

In the first regime, the integral in [Eq. (8)] can be evaluated using the saddle-point approximation, and the wave function is spread over many transverse channels, $n \sim \sqrt{4\kappa_0 r Z t_\perp/\epsilon_0} \gg 1$. The probability density at distance r, $P(r,t_\perp) = \Sigma_n |\psi_n(r)|^2$, is

$$P(r,t_\perp) = \frac{\kappa_0}{2r} \frac{1}{\sqrt{8\pi(Zt_\perp/\epsilon_0)\kappa_0 r}} e^{-2\kappa_0 r\sqrt{1-(4Zt_\perp/\epsilon_0)}},$$
$$\text{for } \kappa_0 r \gg \epsilon_0/(2Zt_\perp). \quad (9)$$

In this regime, the wave function has a pure exponential decay with a localization length corresponding to the shift of the binding energy given in Eq. (3). We then find that the ratio of conductivities after ($H > H_f$) and before ($H < H_f$) the flop is

$$\frac{\sigma_{H>H_f}}{\sigma_{H<H_f}} = \frac{P(R_T,t_\perp)}{P(R_T,t_\perp=0)} \sim \exp\left\{\frac{4Zt_\perp}{3\epsilon_0}\left(\frac{T_0}{T}\right)^{1/3}\right\}. \quad (10)$$

Here, $P(R_T,t_\perp=0) = |\psi_0|^2$, with ψ_0 from Eq. (4). This result is valid when $\kappa_0 R_T \sim (T_0/T)^{1/3} \gg \epsilon_0/(2Zt_\perp)$, which happens, in practice, when the temperature is below 1 K. In this case, $\sigma_{H>H_f}/\sigma_{H<H_f} \gg 1$, i.e., the corresponding MR is large: $|\Delta\rho/\rho \equiv (\rho_{H>H_f}/\rho_{H<H_f}) - 1| \approx 1$, and a full 2D→3D crossover is expected in the spin flop. Notice that the MR is always negative, $\Delta\rho/\rho < 0$.

In the intermediate large-distance regime, $\epsilon_0/(2Zt_\perp) \gg \kappa_0 r \gg 1$, the integral in Eq. (8) can be evaluated by direct expansion in powers of $\kappa_0 r Z t_\perp/\epsilon_0$. Only the layers $n = 0, \pm 1$ contribute in this case, $P(r,t_\perp) = \Sigma_{n=0,\pm 1}|\psi_n|^2$, leading to

$$P(r,t_\perp) = \frac{\kappa_0}{2r} e^{-2\kappa_0 r}\left(1 + 4(\kappa_0 r)^2 \frac{(Zt_\perp)^2}{\epsilon_0^2}\right),$$
$$\text{for } \epsilon_0/(2Zt_\perp) \gg \kappa_0 r \gg 1. \quad (11)$$

From Eq. (11), we obtain ($r \to R_T$) across the spin flop,

$$\frac{\sigma_{H>H_f}}{\sigma_{H<H_f}} = 1 + \frac{4}{9}\left(\frac{T_0}{T}\right)^{2/3} \frac{(Zt_\perp)^2}{\epsilon_0^2}. \quad (12)$$

This formula corresponds to a crossover from a pure 2D case to an "intermediate dimension" (three transverse channels), and it is justified when $(\sigma_{H>H_f}/\sigma_{H<H_f}) - 1 \ll 1$. We have also performed a full numerical evaluation of $P(r,t_\perp)$ and of the MR with $\psi_n(r)$ from Eq. (8), since the above considerations are based on asymptotic behavior. The exact numerical form was used in both Figs. 2 and 3 below. We have found quite clearly that indeed in the temperature range where experimental data are available, $T > 10$ K, the intermediate asymptotic formula [Eq. (12)] is the relevant one, since the MR is still relatively small there, e.g., $|\Delta\rho/\rho| \approx 0.35$ for $T = 10$ K. However, as the temperature is lowered, $T < 10$ K, the MR increases and the system enters a crossover region between the asymptotic formulas [Eqs. (12) and (10)], which requires the use of the exact wave functions. The calculated MR is plotted in Fig. 2 and we observe that the fitted value $Zt_\perp/\epsilon_0 = 0.057$ agrees quite well with the estimate $Zt_\perp/\epsilon_0 \sim 0.03$ from band structure calculations.[19]

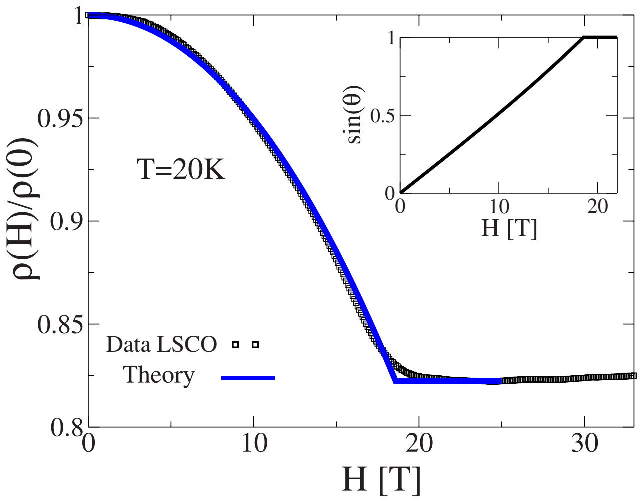

FIG. 3. (Color online) The MR $\rho(H)/\rho(0)$ (solid, blue line) for in-plane field $H \| \hat{b}$, with $Zt_\perp/\epsilon_0 = 0.047$. The squares are data from Ref. 12. Inset: evolution of the angle [Eq. (13)] with field.

V. MAGNETORESISTANCE FOR IN-PLANE FIELD $H \| \hat{b}$

In this case, due to the alignment of the DM-induced moments with the magnetic field, the spins rotate in the $\hat{b}$-$\hat{c}$ plane (in opposite directions on the two sublattices)[10,12,13,15,16,22] and align completely along $\hat{c}$ at a field H_{c2}. Thus, once we introduce the angle $\theta(H)$ that the spins make with the $\hat{b}$ direction, our previous calculations leading to Eq. (12) remain valid with the replacement $t_\perp \to t_\perp \sin\theta$. Observe that while for the undoped LCO an intermediate in-plane spin flop is expected at $H_{c1} < H_{c2}$,[15,16] in LSCO, the doped holes contribute to enhance the $\hat{b}$-axis spin susceptibility and then to confine the spins in the $\hat{b}$-$\hat{c}$ plane, as it has been discussed recently in Ref. 22. We can then write the field dependence of $\sin\theta$ between $H = 0$ and $H = H_{c2}$ as

$$\sin\theta(H) = \frac{HD/\sigma_0}{\Delta_{out}^2 + 4\eta - (1 - x\chi_{imp})H^2}. \quad (13)$$

In the above expression, D is the DM anisotropy, $\eta = 2JJ_\perp$ is the interlayer exchange, Δ_{out} is the out-of-plane (or XY) anisotropy gap, σ_0 is the staggered order parameter, x is the doping, and χ_{imp} is a dimensionless measure of the hole-induced spin susceptibility. Here, H is measured in units of $g_s^b \mu_B H$, with $g_s^b = 2.1$ and μ_B is the Bohr magneton. The parameter values can be extracted from the experiments,[15,22] and for $x=0.01$, we take $D=2.16$ meV, $\eta=1$ (meV),[2] $\Delta_{out}=3.2$ meV, $\chi_{imp}=80$, and $\sigma_0=0.36$, which gives $H_{c2}\approx 19$ T.[12] Using $Zt_\perp/\epsilon_0=0.047$ as the only fitting parameter, we find a remarkable agreement with the experimental data of Ref. 12 at $T=20$ K, as shown in Fig. 3.

We also note that in oxygen-doped compounds, the contribution of the localized holes to the longitudinal susceptibility is much smaller, with $\chi_{imp}\sim 1$.[22] As a consequence, one expects to observe at a field $H_{c1}\sim 10$ T an intermediate flop which reflects in a kink in both the sin $\theta(H)$ curve and in the MR curve, as it has been measured indeed in the earlier transport measurements in La_2CuO_{4+y}.[10]

VI. DISCUSSION AND CONCLUSIONS

In conclusion, we have shown that in the strongly localized VRH regime for $T<50$ K, the in-plane MR is sensitive to the interlayer hopping $t_\perp$ because an external magnetic field effectively changes the dimensionality of the problem, making the hopping quasi-2D. The MR reflects the physics of the spin flop and is always negative; its value as well as temperature and field dependence are in excellent quantitative agreement with recent experiments in LSCO at $x=0.01$.[11,12] Orbital effects, typically causing positive MR,[20] are negligible at this small doping because the hole's localization length $1/\kappa\sim 2-3$ is much smaller than the magnetic length at any reasonable fields.

Finally, we comment that unlike LSCO where the role of magnetic anisotropies is very well established, in insulating $YBa_2Cu_3O_{6+x}$ (YBCO) such anisotropies are expected to be very weak, which is related to the absence of strong localization in this material.[23] Spin-related effects in the in-plane MR are absent for field in the $\hat{c}$ direction, while the in-plane field MR remains small and appears to be due to the dynamics of holes that are very weakly influenced by disorder.[24] The complete understanding of these phenomena in YBCO remains an open issue although it is clear that magnetotransport is not dominated by the local spin physics.

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