Dynamics of coreless vortices and rotation-induced dissipation peak in superfluid films on rotating porous substrates

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We analyze the dynamics of three-dimensional (3D) coreless vortices in superfluid films covering porous substrates. The 3D vortex dynamics is derived from the two-dimensional (2D) dynamics of the film. The motion of a 3D vortex is a sequence of jumps between neighboring substrate cells, which can be described, nevertheless, in terms of quasicontinuous motion with average vortex velocity. The vortex velocity is derived from the dissociation rate of vortex-antivortex pairs in a 2D film, which was developed in the past on the basis of the Kosterlitz-Thouless theory. The theory explains the rotation-induced dissipation peak in torsion-oscillator experiments on ^{4}He films on rotating porous substrates and can be used in the analysis of other phenomena related to vortex motion in films on porous substrates.

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I. INTRODUCTION

Superfluid ^{4}He films adsorbed in porous media are an actual topic in physics of superfluidity.[1] Studying this system gives a unique possibility to investigate the interplay between two-dimensional (2D) and three-dimensional (3D) physics, especially the character of the transition to the superfluid state. On one side, torsion-oscillator experiments reveal the dissipation peak near the temperature of the superfluid onset T_c, which is predicted by the dynamical theory of vortex-antivortex pairs[2] based on the Kosterlitz-Thouless theory for 2D films.[3] On another side, they found that in films on porous substrates, the superfluid density critical index $\sim 2/3$ (Ref. 1) of the 3D system and the sharp cusp of the specific heat[4] at T_c are similar to those near the λ transition of the bulk ^{4}He.

An important insight into the physics of superfluid films in porous media is provided by torsion-oscillator experiments with rotating substrates. The porous substrate is usually modeled with a "jungle-gym" structure:[5–8] a 3D cubic lattice of intersecting cylinders of diameter a with period l (Fig. 1). Multiple connectivity of superfluid films in porous media allows a variety of vortex configurations, and probably most important from them is a coreless or pore 3D vortex, which is just a flow around the vortex pores having nonzero circulation. Due to the presence of a new type of topological defects, one could expect an essential difference in the response between the plane film and the porous-medium film under rotation. This expectation was confirmed by torsional-oscillator experiments, which revealed a rotation-induced peak in dissipation (inverse quality factor) as a function of temperature. The additional peak was shifted from the stationary (static) peak that was observed without rotation.[9,10] Double-peak structure essentially differs from the case of the plane film, where the only effect of rotation was to broaden the stationary peak.[11,12]

A semiempirical interpretation of the rotation-induced dissipation peak was suggested in Ref. 10. It is clear that the rotation can affect dissipation via rotation contribution to the velocity field. So it is a nonlinear correction to the response. Instead of the derivation of such a correction from the theory, the authors of Ref. 10 used the data on the nonlinear response taken from the independent experiment on large-amplitude torsion oscillations. This provided a qualitative explanation of the rotation peak, and even of some quantitative features of it, but could not pretend to be a full theory of the effect. The present work suggests really a theory of the rotation peak deriving the parameters of the peak from the parameters of the film and the substrate. The key role in our scenario is played by the 3D coreless vortices. We derived the dynamics of these vortices from the dynamics of 2D films adsorbed in porous media. It was suggested[10] that the motion of the coreless 3D vortices occurs in a creeping manner by jumping from cell to cell. These jumps are related to the dissociation of the vortex-antivortex pair on one side of a rod separating different pores, with subsequent annihilation of the pair on another side of a rod. The result of the jump is a shift of velocity circulation to a neighboring pore. Though our analysis was focused on the application to the torsion-oscillator experiments, it is valid for a description of the 3D vortex motion in many other cases, at least in those where the vortex moves to distances much larger than the average period of the substrate structure. An important example is steady vortex motion in zero-frequency experiments.

In Sec. II, we describe how the dynamics of 3D coreless vortices is connected with the dynamics of 2D films covering

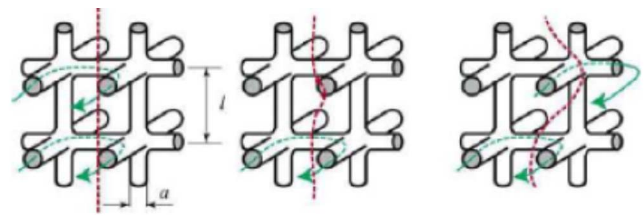

FIG. 1. (Color online) Vortex creep in the jungle gym structure: the coreless vortex (dashed line) crosses a cylinder between two cells via dissociation of the vortex-antivortex pair. The arrowed curved lines show circulation around the cells where the vortex line is located.

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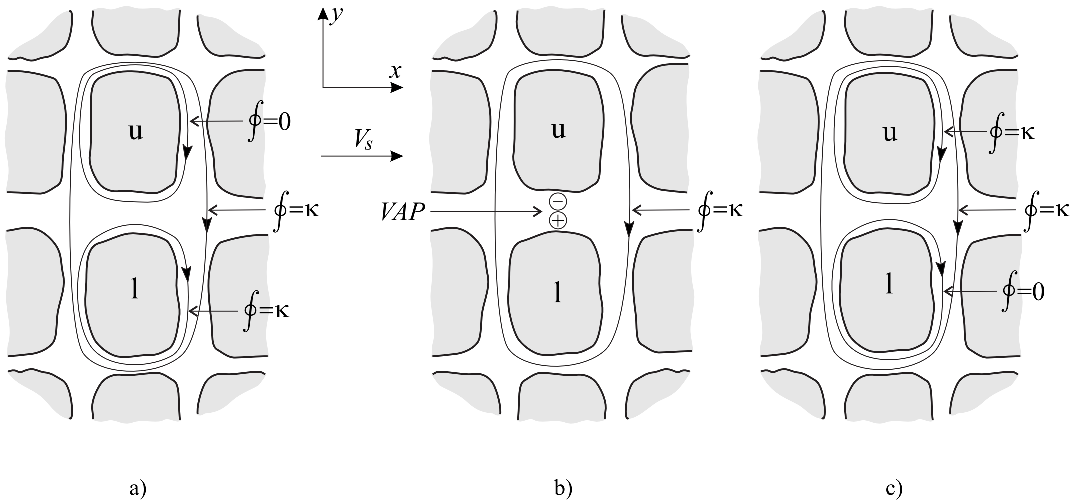

FIG. 2. Vortex creep. The superfluid moves with velocity $\mathbf{V}_s$ along the axis x, and the vortex creeps along the axis y. (a) The state before the jump. There is circulation κ around the lower cell (l) as shown in the picture. The same circulation exists around any path inclusive cell (l). The circulation around two cells (l) and (u) is shown. The coreless vortex is located at the cell (l). (b) The vortex on the way between cells (l) and (u). There is the vortex-antivortex pair (VAP) in the film. Circulations around the paths inclusive only of cell (l) or (u) are not defined, while for the path around the two cells, circulation is equal to κ as before. (c) The state after the jump. The coreless vortex is located at the cell (u). No circulation around the cell (l) anymore, but there is circulation around the cell (u), or around the two of them.

the substrate. This provides a bridge between the effective-continuous-medium 3D description and dynamics of 2D films. Section III reviews the theory of dissociation of vortex-antivortex pairs on the basis of the Kosterlitz-Thouless theory. In Sec. IV, the pair-dissociation rate is analyzed near the critical temperature, where there is an analytical solution of Kosterlitz's recursion equations. The theory is applied to the analysis of the torsion-oscillation experiments in Sec. V. The rotation-induced dissipation peak directly follows from the theory, and its position, shape, and even height are in good agreement with the experiment. Section VI is devoted to conclusions.

II. CORELESS VORTICES: FROM TWO-DIMENSIONAL TO THREE-DIMENSIONAL VORTEX DYNAMICS

In a continuous medium, a vortex is a topological defect with nonzero circulation around the vortex axis. There is an area around the vortex line with a suppressed order parameter, which is called vortex core. However, the topology of a porous medium allows the circulation of superfluid velocity around a pore *without suppression of the order parameter anywhere inside the superfluid film*. This leads to the concept of a coreless vortex. The vortex "line" in this case is not a line at all; this is a chain of the jungle-gym structure cells with nonzero circulation around them. Schematically, the coreless vortex is depicted in Fig. 1.

This coreless structure of a vortex rules out the usual type of vortex motion in a continuous medium simply because the coreless vortex has no continuous coordinate: its position is discrete and is determined by a cell with nonzero circulation around it. The only way for the vortex to move is to jump from cell to cell. They call such a type of vortex motion *vortex creep*. However, any jump is, in fact, a process in time (whatever short) during which the vortex line inevitably crosses a rod covered by a superfluid film. So during the jump, the "coreless" vortex does have cores: at the place where it enters the rod and at the place where it comes out from the rod (Fig. 1). These two cores together form a 2D vortex-antivortex pair, which should grow, dissociate, and eventually annihilate on the other side of the rod. This process leads to a discrete shift of the vortex line to a neighboring cell. For better illustration, a two-dimensional picture of this process is shown in Fig. 2. It shows two cells: upper (u) and lower (l). Before the process [Fig. 2(a)], there is circulation $\kappa = h/m_{\mathrm{He}}$ around the lower cell, as shown in the figure. Provided that there is no other coreless vortices nearby, the same circulation exists around any path inclusive of cell (l), as shown for the path around the two cells. The transient process of the "jump" from cell (l) to cell (u) is shown in Fig. 2(b). The vortex-antivortex pair is present in the film between two pores. This makes circulation around any of the two cells undefined: it depends on whether the path goes outside or inside the pair. Only circulation around the two cells together remains equal to κ. Figure 2(c) shows the state after the process: the coreless vortex is now located in cell (u). Though this scenario is shown in the plane picture, it is directly applicable to the 3D jungle-gym structure with the 2D vortex and antivortex moving around a cylindric rod.

In reality, creation and dissociation of vortex-antivortex pairs is a stochastic process, which is determined by the average dissociation rate $R(T, v_s)$ (number of dissociation events per second and per unit area of a film), which depends on temperature T and, most important for us, on average superfluid velocity v_s in the film. In our case, the superfluid velocity field consists of two parts: $\mathbf{v}_s = \mathbf{v}_c + \mathbf{V}_s$, where $\mathbf{v}_c$ originates from a circular flow around the vortex line at rest and $\mathbf{V}_s$ is a transport superfluid velocity with respect to a moving substrate. In Fig. 2, the velocity $\mathbf{V}_s$ is directed along the axis x. So it is added to the circular velocity $\mathbf{v}_c$ above the cell with circulation κ and is subtracted from $\mathbf{v}_c$ below this cell. Therefore, the jumps of the vortex line up and down are unbalanced and result in some average drift (creep) of the vortex line up the picture in Fig. 1. This jumplike process can, nevertheless, be described with an average vortex velocity determined as

$$V_L = Al[R(v_c+V_s,T)-R(v_c-V_s,T)] \approx 2Al\frac{\partial R}{\partial v_s}V_s, \quad (1)$$

where l is the period of the jungle-gym structure, and A is the area of the rod separating neighboring pores. According to this scenario of vortex motion, the vortex velocity $\mathbf{V}_L$ is strictly normal to the velocity $\mathbf{V}_s$.

Let us compare relation (1) with the general relation connecting the vortex velocity of the vortex line with the normal and superfluid velocities:[13]

$$\mathbf{V}_L = \mathbf{V}_s + \alpha\hat{z}\times(\mathbf{V}_n-\mathbf{V}_s) - \alpha'(\mathbf{V}_n-\mathbf{V}_s). \quad (2)$$

In the problem under consideration, we can assume that the normal component is clamped to the porous-glass substrate oscillating with the velocity $\mathbf{V}_g$. So their velocities coincide: $\mathbf{V}_n=\mathbf{V}_g$. In fact, Eq. (1) is written for the system moving with the substrate, where $\mathbf{V}_n=\mathbf{V}_g=0$. Eventually, Eqs. (1) and (2) agree if

$$\alpha = 2Al\frac{\partial R}{\partial v_s}, \quad \alpha' = -1. \quad (3)$$

Note that the condition $1+\alpha'=0$ providing that vortices move normally to the superfluid motion means that the absence of the "effective" Magnus force,[14] which is defined as the term $\propto[\hat{z}\times\mathbf{V}_L]$, is the balance of forces on the vortex. In superconductors, this leads to the total absence of the Hall effect. Absence of the effective Magnus force is typical for lattice systems,[15] in contrast to uniform continuous media with Galilean invariance. The crossover between these two cases was recently studied by numerically solving the Gross-Pitaevskii equation.[16]

Let us discuss one important question related to the energy of 3D coreless vortices. The energy of vortex line per unit length (tension) can be estimated in the usual manner with substitution such that superfluid density is effective 3D density ρ_{s3} (see Sec. V) and the low cutoff in logarithm is the size of pore l:

$$E_v = \frac{\rho_{s3}\kappa^2}{4\pi}\ln\left(\frac{L}{l}\right),$$

where L is external scale, e.g., interline space or radius of curvature of line, etc. Usually, this quantity is of several sizes of the period of the pore lattice. Therefore, $\ln(L/l)$ is of order unity; one can say that the 3D coreless vortex is a low-energy structure in comparison with the usual vortex, which has a large energy due to the low cutoff in logarithm which is the core radius a_0. During motion, the vortex line can increase its length, accordingly increasing rotational energy. In the usual case of the bulk helium, increase of energy appears due to the work done by friction force between vortices and normal component. In the case of the coreless vortices, the situation is more subtle. After dissociation of the pair, the vortex and antivortex run around the rod, separating the pore. This process occurs due to friction between 2D vortices and the normal component clamped to the porous-glass substrate. External flow executes the work to support running away of 2D vortices. Due to this work, redistribution on superfluid current occurs, corresponding to jump of the 3D coreless vortex in the new cell. In other words, increase of the vortex energy is realized at the expense of external superfluid flow.

So the creep of coreless vortices, which is realized via sequences of discrete jumps from cell to cell, can be described in the terms usually used for 3D vortices moving in a continuous medium. Still, the parameters of this 3D "effective-medium" description must be determined within the theory of 2D films covering the multiconnected substrate. The crucial parameter to be determined is the derivative dR/dv_s of the dissociation rate R of vortex-antivortex pairs in the 2D film.

III. RATE OF PAIR DISSOCIATION

The thermally activated dissociation of vortex-antivortex pair was analyzed by Ambegaokar *et al.*[2] (see also references to later works in Ref. 17) on the basis of the Kosterlitz-Thouless theory. They considered superfluid films on plane substrates, while in our case, films cover cylindrical surfaces. However, as we shall see below, the relevant scale (size of the pair at the saddle point) is small compared to the substrate curvature and the curvature may be ignored.

The pair dissociation is accompanied by overcoming the potential barrier. The barrier corresponds to the saddle point of the vortex-pair energy as a function of the radius vector $\mathbf{r}$ connecting vortex with antivortex,

$$U(\mathbf{r},\mathbf{v}_s) = \frac{\rho_{s0}\kappa^2}{2\pi}\int_{a_0}^{r}\frac{d^2\mathbf{r}}{\tilde{\epsilon}(r)r} - \rho_{s0}\kappa\mathbf{r}\cdot(\mathbf{v}_s\times\hat{z}). \quad (4)$$

Here, ρ_{s0} is the bare superfluid density, $\tilde{\epsilon}$ is Kosterlitz's static scale-dependent dielectric constant determined from the integral equation

$$\frac{1}{\tilde{\epsilon}(r)} - 1 = -\frac{\pi\rho_{s0}\kappa^2}{T}y_0^2\int_{r_0}^{r}dr\frac{r^3}{r_0^4}\exp\left[-\frac{\rho_{s0}\kappa^2}{2\pi T}\int_{r_0}^{r}\frac{dr}{\epsilon(r)r}\right], \quad (5)$$

r_0 is the core radius of the 2D vortex, $y_0=e^{-E_0/T}$, and E_0 is the energy of the vortex core. The dielectric constant $\tilde{\epsilon}(r)$ takes into account screening of interaction between a vortex and an antivortex at distance r by pairs of smaller size. In the limit $r\to\infty$, the dielectric constant determines the ratio of the bare and renormalized superfluid densities: $\rho_s=\rho_{s0}/\tilde{\epsilon}(\infty)$.

The integral equation (5) can be reduced to Kosterlitz's recursion equations:

$$\frac{dK(l)}{dl} = -4\pi^3y^2K^2, \quad \frac{dy^2(l)}{dl} = (4-2\pi K)y^2. \quad (6)$$

Here, $l=\ln(r/r_0)$, the dielectric constant is replaced by the ratio $\tilde{\epsilon}(l)=K(0)/K(l)$, where $K(0)=\rho_{s0}\kappa^2/4\pi^2k_BT$ is the bare Kosterlitz-Thouless coupling constant related to the bare superfluid density ρ_{s0}, and

$$y(l) = y_0 \exp\left[2l - \pi \int_0^l K(l')dl'\right] \tag{7}$$

is the rescaled activity.

Solving the Fokker-Planck equation for the distribution of vortex-antivortex pairs, Ambegaokar *et al.*[2] obtained the following expression for the dissociation rate [see Eq. (4.6) in Ref. 2]:

$$R = \frac{2D}{r_s^4} y^2(l_s)\exp[2\pi K(l_s)]. \tag{8}$$

Here, D is the vortex diffusion coefficient, r_s is the saddle-point value of the pair size r, and $l_s = r_s/r_0$ is its logarithm. The saddle point for the effective potential (4) is reached when $\mathbf{r}$ is perpendicular to $\mathbf{v}_s$, and r_s satisfies the condition

$$\frac{\kappa}{2\pi\tilde{\epsilon}(r_s)r_s} = \frac{\kappa K(0)}{2\pi K(l_s)r_s} = v_s. \tag{9}$$

If the velocity v_s is small, the values of l_s and r_s are large, and this condition yields

$$r_s = \frac{\kappa K(0)}{2\pi K(\infty)v_s}. \tag{10}$$

In order to find the mutual friction parameter α from Eq. (3), we need the derivative of the dissociation rate with respect to the velocity v_s. Using Eqs. (6) and (9), we obtain

$$\frac{dR}{dv_s} = \frac{8\pi^2 DK(0)}{\kappa r_s^3} y^2(l_s)\exp[2\pi K(l_s)]. \tag{11}$$

In summary, Eqs. (1), (3), and (11), show how the creep motion of the coreless vortices in a porous medium is described in terms of parameters determining the motion of 3D quantum vortices in continuous media. This description can be used for various problems related to vortex motion in porous media.

Thus, the dissociation is a thermally activated process accompanied by overcoming the potential barrier of the height $\sim \rho_s \kappa^2 \ln\left(\frac{\kappa}{2\pi v_s a_0}\right)$, where ρ_s is the 2D film superfluid density. Under usual conditions, the logarithm is of order 10, $\ln\left(\frac{\kappa}{2\pi v_s a_0}\right) \sim 10$. On the other hand, since any jump shifts only a small element of the 3D line of length $\sim l$, it should be accompanied by random local lengthening of the 3D vortex line, which increases the barrier for the vortex creep. This increase of the barrier can be estimated via the 3D line tension as $\rho_{s3}\kappa^2 l$, where ρ_{s3} is the effective 3D superfluid mass density (see the end of Sec. II). This means that the line-tension barrier is by a factor $a/\left[l \ln\left(\frac{\kappa}{2\pi v_s a_0}\right)\right]$ smaller than the pair-dissociation barrier, and in the following analysis, the line-tension contribution to the activation barrier will be ignored.

IV. PAIR DISSOCIATION NEAR THE CRITICAL POINT

For understanding the nature of the rotation dissipation peak, we need to study the temperature dependence of the dissociation rate at temperatures close to the critical one. At these temperatures, Kosterlitz's recursion equations have an analytical solution.[2] One can introduce a small $x(l) = \pi[K(l) - K_c(\infty)]$, where $K_c(l)$ yields values of $K(l)$ at the critical point and, at $l \to \infty$, $K_c(\infty) = 2/\pi$. Then the recursion relations can be written as

$$\frac{dx(l)}{dl} = -(4\pi y)^2, \quad \frac{dy^2(l)}{dl} = -2xy^2. \tag{12}$$

Their solution is

$$\begin{aligned} x(l) &= x_\infty \coth\left(x_\infty l + \coth^{-1}\frac{x_0}{x_\infty}\right) \\ &= x_\infty \frac{x_0 \cosh(x_\infty l) + x_\infty \sinh(x_\infty l)}{x_0 \sinh(x_\infty l) + x_\infty \cosh(x_\infty l)}, \end{aligned} \tag{13}$$

$$\begin{aligned} 4\pi y(l) &= x_\infty \operatorname{csch}\left(x_\infty l + \coth^{-1}\frac{x_0}{x_\infty}\right) \\ &= \frac{4\pi y_0 x_\infty}{x_0 \sinh(x_\infty l) + x_\infty \cosh(x_\infty l)}. \end{aligned} \tag{14}$$

Here, $x_0 = x(0)$ and $x_\infty = x(\infty)$. The solution satisfies the condition

$$x_\infty^2 = x(l)^2 - [4\pi y(l)]^2 = x_0^2 - (4\pi y_0)^2. \tag{15}$$

At the critical point, $x_\infty = 0$ and Eqs. (13) and (14) become

$$x_c(l) \approx \frac{x_0}{1 + x_0 l}, \quad 4\pi y_c(l) \approx \frac{x_0}{1 + x_0 l}.$$

According to Eq. (15), at the critical point, the parameters x_0 and y_0 satisfy the relation

$$x_{0c} = \pi\left[K_c(0) - \frac{2}{\pi}\right] = (4\pi y_{0c})^2. \tag{16}$$

As usually assumed in the 2D vortex dynamics, the bare superfluid density does not vary near the critical point. Then, since $K(0) \propto 1/T$ and $y_0 = e^{-E_0/T}$, their dependence on the relative temperature $t = (T_c - T)/T_c$ is

$$K(0) \approx K_c(0)(1 + t), \quad y_0 \approx y_{0c}\left(1 - \frac{E_0}{T_c} t\right). \tag{17}$$

Then Eq. (15) yields that at $t > 0$, $x_\infty = 2b\sqrt{t}$, with $b = 16\pi e^{-E_0/T_c}[1 + 2\pi(1 + E_0/T_c)e^{-E_0/T_c}]$. This leads to the square-root cusp

$$\rho_s(T) = \rho_s(T_c)(1 + b\sqrt{t}) \tag{18}$$

in the critical behavior of the renormalized superfluid density, which was revealed by Nelson and Kosterlitz[18] with numerical calculations.

Using all these relations together with the assumption that v_s is so low that at the saddle point $l_s \gg x_{0c}$, we obtain the linear dependence of the dissociation rate on t:

$$\frac{dR}{dv_s} = \frac{DK_c(0)}{2\kappa r_s^3 l_s^2} e^4 (1 - \gamma t), \tag{19}$$

where

$$\gamma = 2b^2 l_s + \frac{4b^2}{3} - 1.$$

V. TORSIONAL OSCILLATIONS OF ROTATING POROUS SUBSTRATE AND COMPARISON WITH THE OBSERVED ROTATION DISSIPATION PEAK

Let us now apply the theory to the oscillatory motion of the substrate superimposed on its steady rotation with the angular velocity $\mathbf{\Omega}$. So the substrate velocity field is $[\mathbf{\Omega} \times \mathbf{r}] + \mathbf{V}_g$. Only the oscillatory component $\mathbf{V}_g \propto e^{-i\omega t}$ is important for us. Using the Euler equation

$$\frac{\partial \mathbf{V}_s}{\partial t} + [2\mathbf{\Omega} \times \mathbf{V}_L] = -\nabla \mu \tag{20}$$

and Eq. (2) with $\alpha' = -1$, one obtains for an oscillatory component of the velocities (the chemical potential μ is not relevant for the azimuthal motion)

$$\mathbf{V}_s = \frac{2\Omega\alpha}{i\omega + 2\Omega\alpha} \mathbf{V}_g. \tag{21}$$

This relation determines the drag of the superfluid component by the oscillating substrate. Analyzing now the balance of forces for the torsional resonator, as was done many times in the past, one obtains the following contribution to the inverse quality factor of the torsion oscillator for the slow rotation $\Omega\alpha \ll \omega$:

$$\Delta Q^{-1} = \frac{V\rho_{s3}}{M} \frac{2\Omega\alpha}{\omega}, \tag{22}$$

where M is the total mass of the torsional oscillator, V is the total 3D volume (including pores), and ρ_{s3} is the effective 3D superfluid mass density in the porous-glass substrate of the volume V, which is connected with the 2D superfluid density ρ_s of the film by the relation

$$\rho_{s3} = \frac{\rho_s A_{tot}}{V}. \tag{23}$$

Here, A_{tot} is the total area of the film. For the jungle-gym structure with cell size l and the rod diameter a,

$$\frac{A_{tot}}{V} \approx \frac{\pi a}{l^2}. \tag{24}$$

On the other hand, in the theory of 2D superfluid films starting from Ref. 2, they describe the drag of the superfluid component introducing the "dynamical dielectric constant" $\epsilon(\omega)$ (one should not mix it up with Kosterlitz's static dielectric constant $\tilde{\epsilon}$ introduced in Sec. III):

$$\mathbf{V}_s = \left[1 - \frac{1}{\epsilon(\omega)}\right] \mathbf{V}_g. \tag{25}$$

Then

$$\Delta Q^{-1} = -\frac{V\rho_{s3}}{M} \operatorname{Im} \frac{1}{\epsilon} = -\frac{A_{tot}\rho_s}{M} \operatorname{Im} \frac{1}{\epsilon}, \tag{26}$$

with the imaginary part of the inverse dielectric constant equal to

$$\operatorname{Im} \frac{1}{\epsilon} = -\frac{2\Omega\alpha}{\omega} = -\frac{4Al\Omega}{\omega} \frac{\partial R}{\partial v_s}. \tag{27}$$

Substituting Eqs. (18) and (19) into Eqs. (26) and (27), we see that the square-root cusp in the superfluid density is crucial for its temperature dependence in the critical area and for the existence of the rotation dissipation peak:

$$\Delta Q^{-1} = \frac{4Al\Omega}{\omega} \frac{A_{tot}}{M} \rho_s \frac{\partial R}{\partial v_s} = \Delta Q^{-1}(T_c)(1 + b\sqrt{t} - \gamma t). \tag{28}$$

The factor $\gamma \propto l_c$ is expected to be large, but the square-root cusp is more essential at small t, and the inverse quality factor has a maximum at $t = b^2/4\gamma^2$ rather close to the critical point.

The linear approximation used for derivation of Eqs. (13) and (14) is more or less truthful only for $|T - T_c|$ not exceeding 0.005 K. Though the dissipation maximum at $T \approx 0.6234$ K is in this interval (see below), the low temperature side of the dissipation peak is not, and the peak width cannot be determined using the analytical formulas (13) and (14). Thus, though the analytical expression (28) qualitatively explains the observed dissipation peak itself, it is not sufficient for its quantitative description. A more accurate quantitative analysis required numerical calculations.

Let us now gather from Refs. 9 and 10 all quantitative data needed for comparison with the theory. The porous-glass substrate can be modeled with the jungle-gym structure with the diameter of rods $a \approx 1$ μm and the structure period $l \approx 2.5$ μm. Then the circulation velocity around the pore is estimated as $v_s = \kappa/4l \approx 1$ cm/s. According to Ref. 10, the transition temperature is $T_c \approx 0.628$ K. The areal density of superfluid component at the critical temperature (jump of density) can be evaluated from the Kosterlitz-Thouless relation

$$\rho_s = \frac{8\pi k_B T_c}{\kappa^2} = 2.1791 \times 10^{-9} \text{ g/cm}^2.$$

We also need the bare areal superfluid density ρ_{s0}. It can be calculated from the dependence of the transition temperature on the thickness: According to Ref. 9, the coverage of the substrate is about 33×10^{-10} mol/cm^2. Then $\rho_{s0} \approx 1.32 \times 10^{-8}$ g/cm^2, and the bare Kosterlitz -Thouless coupling constant $K(0)$ is

$$K(0) = \frac{\rho_{s0}\kappa^2}{4\pi^2 k_B T} \approx 3.8.$$

Quantity b entering relations (13) and (14) can be obtained from the width ~ 0.02 K of the experimental curve describing the static dissipation peak of paper.[10] It yields b about 3.6975. Furthermore, the diffusion coefficient is $D = 2.8 \times 10^{-7}$ cm^2/s, and the core radius of vortices on the film can be estimated as $a_0 \approx 25 \times 10^{-8}$ cm.

We can now find the pair size r_s at the saddle point from Eq. (10): $r_s \approx 2.98 \times 10^{-5}$ cm. This corresponds to $l_s = \ln(r_s/r_0) = 4.78$. On one hand, these values are large enough in order to justify our assumption of large l_s compared to x_{0c}.

On the other hand, we see that $r_s \ll a$, which justifies our consideration of vortex-antivortex unbinding using the theory for plane films.

We also need the value of $A_{tot}\rho_s(T)/M$ in Eq. (26). This is the ratio of the temperature dependent superfluid mass to the mass of the empty cell. According to Ref. 9, it is approximately equal to 10^{-5}. For the determination of the temperature dependent superfluid mass $\rho_s(T)$, we have built the extrapolating function using the experimental results obtained from measuring the shift of the oscillation period (see Fig. 2 from Ref. 10).

In order to calculate $K(l,T)$ and $y(l,T)$ numerically, we use procedure proposed in Ref. 12 (see Appendix A there). In the temperature interval from T_c to the temperature $T \approx 0.6234$ K corresponding to the dissipation maximum, the analytical expressions based on the linear approximation are more or less truthful. However, in the low temperature region, $T < 0.6234$ K, we performed the numerical calculation using the MATHEMATICA program. Following Ref. 12, for any temperature $|T-T_c|$ and corresponding x_0, we choose a value l_0 so that $x_\infty l_0 = \pi/2$. Under this choice, both $y(l_0,T)$ and deviation of $K(l_0,T)$ from $2/\pi$ are rather small and the analytical expressions (13) and (14) are still good enough for the evaluation of $K(l,T)$ and $y(l,T)$. Further, we take $K(l_0,T)$ and $y(l_0,T)$ as initial conditions for numerical integration (with respect to variable l) of the recursive Kosterlitz-Thouless relations (6). In this way we are able to restore $K(l_s,T)$ and $y(l_s,T)$ at saddle point l_s and, further, to find $\partial R/\partial v_s$ and eventually ΔQ^{-1} in the whole temperature interval $T < T_c$.

In Fig. 3, we plot the theoretical $\Delta Q^{-1}(T)$ (dashed line) together with the experimental data of Fukuda *et al.*[10] The observed rotation-induced dissipation peak (the left peak) was scaled by the angular velocity Ω, reducing it to the value measured at $\Omega = 6.28$ rad/s. The plots for different values of Ω collapse on the same curve (see Fig. 2 in Ref. 10), which proves the linear dependence of rotation-induced dissipation on Ω. For a better comparison of the peak shape, we fit the theoretical height of the peak to the experimental one. However, in fact, the values of $\Delta Q^{-1}(T)$ at the maxima do not differ essentially: At the angular velocity $\Omega = 6.28$ rad/s, they are 3.54×10^{-8} in theory and 2.4×10^{-8} in experiment (i.e., about 70% from the theoretical value). Keeping in mind that nice agreement in the peak shape and position was achieved without any additional fitting, the agreement is really satisfactory.

We conclude the quantitative analysis of the torsional-oscillation experiment with the estimation of the creep velocity V_L of the pore vortex using Eq. (1). For $V_s \approx 1$ cm/s, which is typical in the torsional-oscillation experiments, we get $V_L \approx 0.7$ cm/s. For the frequency $\nu = 477$ Hz in the experiment, this corresponds to vortex displacements about 1.5×10^{-3} cm, which is about six structure periods. This looks rather satisfactory for our scenario, reducing sequences of jumps between neighboring cells to quasicontinuous vortex motion.

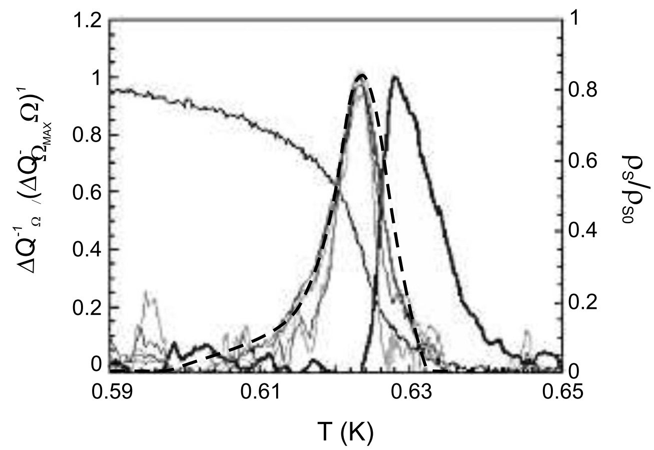

FIG. 3. Comparison of the experimental and theoretical inverse quality factors of torsional oscillations. The right peak is the static peak, which is present without rotation. The group of peaks on the left is the rotation-induced ones obtained experimentally for various Ω and scaled (divided) by Ω. The fact that all these curves collapse well on the same curve proves linearity on Ω. The dashed line shows the theoretical inverse quality factor. The vertical scale of it was fitted so that the height of the theoretical peak coincided with the experimental rotation peak, but the width and the position of the theoretical peak were calculated without any fitting. The theoretical curve ends with the critical point. The experimental superfluid density scaled by its zero temperature value is also displayed (the left vertical axis).

VI. CONCLUSION

We suggested the theory of motion of 3D coreless vortices through a 2D film covering a porous substrate. The dynamics of 3D vortices is derived from the dynamics of vortex-antivortex pairs in the 2D film. The 3D vortices move by jumping from cell to cell of the substrate structure, but it can be described in terms of the average vortex velocity like vortex motion in continuous media. Frequency of jumps, and vortex velocity correspondingly, is determined by the dissociation rate of vortex-antivortex pairs, which is known from the dynamics of 2D superfluid films based on the Kosterlitz-Thouless theory. We calculated the dissipation intensity and its temperature dependence analytically and numerically. The theory is compared with the experiments on torsional oscillations of ^{4}He superfluid film adsorbed on a rotating porous-glass substrate. We explain the second (rotation-induced) dissipation peak on the temperature dependence, which was revealed in these experiments. Quantitative comparison between theory and experiment looks satisfactory, especially for the shape and the position of the peak.

Though we focused on the application of our theory to torsional-oscillation experiments in ^{4}He films on porous-glass substrates, we believe that the theory has a much wider area of possible applications. The concept of quasicontinuous vortex motion in porous media with parameters determined from the dynamics of 2D superfluid films should be applicable not only to straight vortices induced by steady rotation. For example, one may apply the theory also to vortex rings and to the vortex tangle if they can be created in porous media. The theory can also be used for the analysis of steady vortex motion in various situations, e.g., in the process of heat transfer.

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