Paramagnonlike excitations and spin diffusion in magnetic resonance studies of copper oxide superconductors

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The relaxation function theory for a doped two-dimensional Heisenberg antiferromagnetic system in the paramagnetic state for all wave vectors through the Brillouin zone is presented in view of the low frequency response of high-T_c copper oxide superconductors. We deduced the regions of long lifetime $[T \lesssim 400(1-4x)\ \mathrm{K}]$ and "overdamped" $[T \gtrsim 700(1-4x)\ \mathrm{K}]$ paramagnonlike excitations in the temperature (T)-doping index (x) phase diagram from plane oxygen nuclear spin-lattice relaxation rate $^{17}(1/T_1)$ data right up to optimally doped $La_{2-x}Sr_xCuO_4$, thus providing the regimes for the spin-wave concept and the overdamped mode.

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I. INTRODUCTION

Plane copper oxide high-temperature superconductors (high T_c) are the *doped* $S=1/2$ two-dimensional Heisenberg antiferromagnetic (2DHAF) systems.[1] In the carrier-free regime, the elementary excitations are spin waves,[2,3] magnons in the quasiparticle language, a concept widely known and thoroughly investigated in the past.[4] Therefore, it is tempting to consider the doped 2DHAF systems in terms of magnonlike excitations (strictly speaking, the paramagnon, a notation used for spin fluctuations in the representation of *damped* spin waves) and the magnon lifetime is characterized by the damping of spin waves. Then, the questions arise: What happens with spin waves when we dope the system? What are the elementary excitations—are they still paramagnonlike? The motion of charge carriers even in the optimally doped (maximum T_c) high T_c is known to take place in the presence of strong AF fluctuations[5] and spin waves in 2DHAF systems persist even without long range order in the paramagnetic state,[2–6] so the questions make sense, but, finally, what can one say about the *lifetime* of these excitations when we dope the system?

From the experimental point of view, the spin-wave-like features have been revealed[7] by neutron scattering (NS) even in the nearly optimally doped $YBa_2Cu_3O_{6.85}$ and, contrary to the predictions within the weak coupling theory,[8] no isotope effect on the "resonance peak" (RP) frequency has been observed[9] in $YBa_2Cu_3O_{6.89}$. The RP phenomenon disappears in the overdoped phase,[10] thus raising questions about its appearance within the weak coupling theories,[11] and AF spin excitations disappear in overdoped $La_{1.7}Sr_{0.3}CuO_4$, leading to the conclusion that "the AF spin correlations in superconducting samples must be vestiges of the parent insulator."[12] Moreover, NS data[13,14] in underdoped $YBa_2Cu_3O_{6.35}$ with $T_c=18$ K show the *commensurate* AF short range order, no well-defined resonant mode, and similarities between the low-energy magnetic excitations in $YBa_2Cu_3O_{6.35}$ and carrier-free insulating 2DHAF, e.g., $YBa_2Cu_3O_{6.15}$ and La_2CuO_4.[15] The spin diffusive contribution $^{17}(1/T_1)_{diff}$ to the plane oxygen nuclear spin-lattice relaxation rate $^{17}(1/T_1)$ cannot be excluded solely by the failure to detect[16] the changes in $^{17}(1/T_1)$ by varying the nuclear magnetic resonance (NMR) frequency ω since $^{17}(1/T_1)_{diff}$ varies rather weakly[17] with ω.

In this paper, we use the Mori-Zwanzig projection operator procedure[18,19] and thus we are unprejudiced regarding the role of $q \approx 0$ (spin diffusion) and $Q \approx (\pi,\pi)$ wave vectors in the imaginary part of the dynamic spin susceptibility $\chi''(\mathbf{q},\omega)$ of doped 2DHAF system, which is especially important for $^{17}(1/T_1)$. Spin diffusion is a spatial smoothing of the heterogeneous spin polarization in a system of localized magnetic moments and, in the presence of strong damping (short magnon lifetime), the spin dynamics changes from wavelike to diffusive. The approximations we use for the relaxation function are within the Markovian approximation and "by itself the Markovian situation can be valid even in the absence of any picture of the system in terms of well-defined excitations."[6] We will emphasize the spin-wave-like features in $\chi''(\mathbf{q},\omega)$ of copper oxide high T_c and extract the lifetime of spin-wave-like excitations from $^{17}(1/T_1)$ data.

II. BASIC RELATIONS

We start from the t-J Hamiltonian[20] known as the minimal model for the electronic properties of high-T_c cuprates,

$$H_{t\text{-}J} = \sum_{i,j,\sigma} t_{ij} X_i^{\sigma 0} X_j^{0\sigma} + J \sum_{i>j} \left(\mathbf{S}_i \mathbf{S}_j - \tfrac{1}{4} n_i n_j \right), \qquad (1)$$

written in terms of the Hubbard operators $X_i^{\sigma 0}$ that create an electron with spin σ at site i, and $\mathbf{S}_i$ are spin-1/2 operators. Here, the hopping integral $t_{ij}=t$ between the nearest neighbors (NNs) describes the motion of electrons causing a change in their spins, and $J=0.12$ eV is the NN AF coupling constant. The spin and density operators are defined as follows: $S_i^{\sigma}=X_i^{\sigma\tilde{\sigma}}$, $S_i^z=(1/2)\Sigma_\sigma \sigma X_i^{\sigma\sigma}$, and $n_i=\Sigma_\sigma X_i^{\sigma\sigma}$ $(\sigma=-\tilde{\sigma})$, with the standard normalization $X_i^{00}+X_i^{++}+X_i^{--}=1$.

We formulate our study of the spin fluctuations following Mori,[18] who showed its efficiency for both the classical (and essential equivalence to the Brownian motion) and quantum (e.g., Heisenberg systems of arbitrary dimension) many body systems.[6] The time evolution of a dynamical variable $S_{\mathbf{k}}^z(\tau)$, say, is given by the equation of motion,

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$$\dot{S}^z_{\mathbf{k}}(\tau) \equiv \frac{dS^z_{\mathbf{k}}(\tau)}{d\tau} = i\mathcal{L}S^z_{\mathbf{k}}(\tau) \to [H_{t-J}, S^z_{\mathbf{k}}(\tau)], \tag{2}$$

where the Liouville operator $\mathcal{L}$ in the quantal case represents the commutator with the Hamiltonian. The projection of the vector $S^z_{\mathbf{k}}(\tau)$ onto the $S^z_{\mathbf{k}} \equiv S^z_{\mathbf{k}}(\tau=0)$ axis, $\mathcal{P}_0 S^z_{\mathbf{k}}(\tau) = \mathcal{R}(\mathbf{k},\tau)S^z_{\mathbf{k}}$, defines the linear projection Hermitian operator $\mathcal{P}_0$. One may separate $S^z_{\mathbf{k}}(\tau)$ into the projective and vertical components $S^z_{\mathbf{k}}(\tau) = \mathcal{R}(\mathbf{k},\tau)S^z_{\mathbf{k}} + (1-\mathcal{P}_0)S^z_{\mathbf{k}}(\tau)$ with respect to the $S^z_{\mathbf{k}}$ axis, where $\mathcal{R}(\mathbf{k},\tau) \equiv (S^z_{\mathbf{k}}(\tau), (S^z_{-\mathbf{k}})^*)(S^z_{\mathbf{k}}, (S^z_{-\mathbf{k}})^*)^{-1}$ is the relaxation function in the inner-product notation $(S^z_{\mathbf{k}}(\tau), (S^z_{-\mathbf{k}})^*) \equiv k_B T \int_0^{1/k_B T} d\varrho \langle e^{\varrho H} S^z_{\mathbf{k}}(\tau) e^{-\varrho H} (S^z_{-\mathbf{k}})^* \rangle$, and the angular brackets denote the thermal average.

One may construct a continued fraction representation for the Laplace transform of the relaxation function, for which Lovesey and Meserve[21] (see also Ref. 6) used a three pole approximation, $\mathcal{R}^L(\mathbf{k},s) = \int_0^\infty d\tau e^{-s\tau}\mathcal{R}(\mathbf{k},\tau) \approx 1/\{s + \Delta^2_{1\mathbf{k}}/[s + \Delta^2_{2\mathbf{k}}/(s + 1/\tau_{\mathbf{k}})]\}$, with a cutoff characteristic time $\tau_{\mathbf{k}} = \sqrt{2}/(\pi\Delta^2_{2\mathbf{k}})$, by arguing that $S^z_{\mathbf{k}}(\tau)$ fluctuations are weakly affected by the higher order random forces. For the relaxation shape function $\mathcal{F}(\mathbf{k},\omega) = \mathrm{Re}[\mathcal{R}^L(\mathbf{k}, i\omega)]/\pi$, this gives

$$\mathcal{F}(\mathbf{k},\omega) = \frac{\tau_{\mathbf{k}}\Delta^2_{1\mathbf{k}}\Delta^2_{2\mathbf{k}}/\pi}{[\omega\tau_{\mathbf{k}}(\omega^2 - \Delta^2_{1\mathbf{k}} - \Delta^2_{2\mathbf{k}})]^2 + (\omega^2 - \Delta^2_{1\mathbf{k}})^2}, \tag{3}$$

where $\Delta^2_{1\mathbf{k}}$ and $\Delta^2_{2\mathbf{k}}$ are related to the frequency moments, $\langle\omega^n_{\mathbf{k}}\rangle = \int_{-\infty}^{\infty} d\omega \omega^n \mathcal{F}(\mathbf{k},\omega) = (1/i^n)[d^n\mathcal{R}(\mathbf{k},\tau)/d\tau^n]_{\tau=0}$, of $\mathcal{R}(\mathbf{k},\tau)$ as $\Delta^2_{1\mathbf{k}} = \langle\omega^2_{\mathbf{k}}\rangle$, $\Delta^2_{2\mathbf{k}} = (\langle\omega^4_{\mathbf{k}}\rangle/\langle\omega^2_{\mathbf{k}}\rangle) - \langle\omega^2_{\mathbf{k}}\rangle$ for $\tau \gtrsim \tau_{\mathbf{k}}$. Note that $\mathcal{F}(\mathbf{k},\omega)$ is normalized to unity $\int_{-\infty}^{\infty} d\omega \mathcal{F}(\mathbf{k},\omega) = 1$ and is even in both $\mathbf{k}$ and ω. The expression for the second moment, $\langle\omega^2_{\mathbf{k}}\rangle = i\langle[\dot{S}^z_{\mathbf{k}}, S^z_{-\mathbf{k}}]\rangle/\chi(\mathbf{k}) = -(8Jc_1 - 4t_{eff}T_1)(1-\gamma_{\mathbf{k}})/\chi(\mathbf{k})$, is compact, while $\langle\omega^4_{\mathbf{k}}\rangle = i\langle[\ddot{S}^z_{\mathbf{k}}, \dot{S}^z_{-\mathbf{k}}]\rangle/\chi(\mathbf{k})$ is cumbersome and is not reproduced here (see Ref. 17 for details).

The static spin susceptibility has been *derived* within the *t-J* model in the overall temperature and doping range,[22]

$$\chi(\mathbf{k}) = \frac{4|c_1|}{Jg_-(g_+ + \gamma_{\mathbf{k}})}, \tag{4}$$

and has the same structure as in the isotropic spin-wave theory.[23] The parameter g_+ is related to the correlation length ξ via the expression $\xi/a = 1/(2\sqrt{g_+ - 1})$, where $a = 3.8$ Å is a lattice unit. The transfer amplitude between the NN is given by $T_1 \equiv -(1/4)\Sigma_\rho \langle X^{\sigma 0}_i X^{0\sigma}_{i+\rho}\rangle = p\Sigma_{\mathbf{k}}\gamma_{\mathbf{k}} f^h_{\mathbf{k}}$, where the index ρ runs over NN, $\gamma_{\mathbf{k}} = (1/4)\Sigma_\rho \exp(i\mathbf{k}\rho) = (1/2)(\cos k_x a + \cos k_y a)$, and $f^h_{\mathbf{k}} = [\exp(-E_{\mathbf{k}} + \mu)/k_B T + 1]^{-1}$ is the Fermi function of holes. The number of *extra* holes, due to doping δ per one plane Cu^{2+}, can be identified with the Sr content x in $La_{2-x}Sr_xCuO_4$. The chemical potential μ is related to δ by $\delta = p\Sigma_{\mathbf{k}} f^h_{\mathbf{k}}$, where $p = (1+\delta)/2$. The excitation spectrum of holes is given by $E_{\mathbf{k}} = 4t_{eff}\gamma_{\mathbf{k}}$, where the hoppings t are affected by electronic and AF spin-spin correlations c_1, resulting in effective values,[20,24,25] for which we set $t_{eff} = \delta J/0.2$ in order to match the insulator-metal transition.

TABLE I. The calculated values in the $T \to 0$ limit of NN AF spin-spin correlation function $c_1 = (1/4)\Sigma_\rho \langle S^z_i S^z_{i+\rho}\rangle$, the parameter g_-, the spin stiffness constant ρ_S using the expressions and the procedure as described in Refs. 22 and 24, the calculated spin diffusion constant D following Ref. 17 together with the values of Korringa-type contribution constant K_K, and the spin-wave-like damping renormalization constant Γ_r, as extracted from comparison with $^{17}(1/T_1)$ NMR data.

x	c_1	g_-	$2\pi\rho_S/J$	D/Ja^2	K_K (s K)$^{-1}$	Γ_r (K^{-3})
0.025	−0.1133	4.102	0.36	2.60	0.023	4.1×10^{-9}
0.035	−0.1115	4.060	0.35	2.54	0.024	5.5×10^{-9}
0.05	−0.1018	3.827	0.285	2.47	0.051	7.5×10^{-9}
0.115	−0.0758	3.252	0.2	3.51	0.147	32×10^{-9}
0.15	−0.0617	2.947	0.13	3.81	0.215	41×10^{-9}

For low-temperature behavior, we use the expression, resulting in effective correlation length ξ_{eff}, given by[17,24,26]

$$\xi^{-1}_{eff} = \xi^{-1}_0 + \xi^{-1}. \tag{5}$$

Here, ξ is affected by doped holes, in contrast with the empirical equation of Keimer *et al.*,[26] where ξ is given by the Hasenfratz-Niedermayer formula, and there was no influence of the hole subsystem on ξ. Thus, from now on, we replace ξ by ξ_{eff}. For doped systems, we use the explicit expression[22] for ξ, which is much more complicated compared with the simple relation $\xi/a \simeq (J\sqrt{g_-}/k_B T)\exp(2\pi\rho_S/k_B T)$, valid for carrier-free or lightly doped systems.[17,22] In the best fit of ξ_{eff} to experimental data,[26,27] the relation $\xi_0 = a/n_\xi x$ is most suited[11,17] which one may attribute to the stripe picture, where $n_\xi = 2$ for $x \le 0.05$ and $n_\xi = 1$ near optimal ($x \approx 0.15$) doping. The results of the calculations are summarized in Table I. We consider here the case of $La_{2-x}Sr_xCuO_4$ with the simplest crystalline structure for brevity and luck to thorough experimental data set.

III. PLANE OXYGEN NUCLEAR SPIN-LATTICE RELAXATION

The plane oxygen nuclear spin-lattice relaxation rate $^{17}(1/T_1)$ has three contributions,

$$^{17}(1/T_1) = {}^{17}(1/T_1)_{sw} + {}^{17}(1/T_1)_{Korr} + {}^{17}(1/T_1)_{diff}. \tag{6}$$

The contribution from spin-wave-like excitations is given by

$$^{17}(1/T_1)_{sw} = \frac{2k_B T}{\omega_0} \sum_{|\mathbf{k}|>1/\xi_{eff}} {}^{17}F(\mathbf{k})^2 \chi''_L(\mathbf{k},\omega_0), \tag{7}$$

where $\omega_0 = 2\pi \times 52$ MHz $\simeq 2.15\times10^{-4}$ meV ($\ll T, J$) is the measured NMR frequency at 9 T. The quantization axis is along the crystal c axis and the wave vector dependent hyperfine form factor for plane ^{17}O sites is given by $^{17}F(\mathbf{k})^2 = 2C^2(1+\gamma_{\mathbf{k}})$, with $C = 2.8\times10^{-7}$ eV.[28]

The contribution from itinerant holes, of Korringa type, $^{17}(1/T_1)_{Korr} = K_K T$, should grow with doping x and will be the adjustable parameter. The contribution from spin diffu-

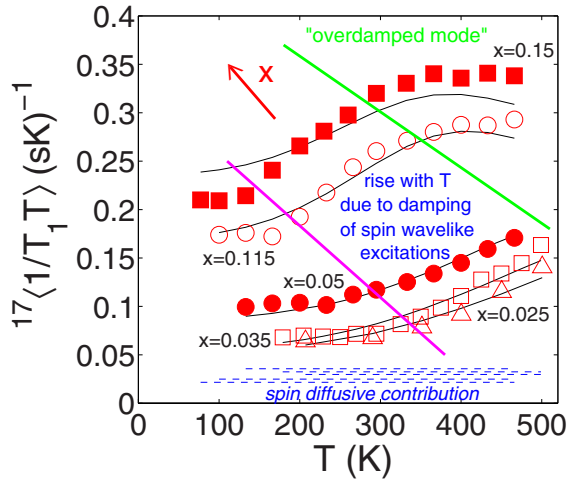

FIG. 1. (Color online) Temperature and doping behavior of plane oxygen $^{17}(1/T_1T)$ for $La_{2-x}Sr_xCuO_4$. Dashed lines show the spin diffusive contribution. Thin solid curves are the fits by Eq. (6) to NMR data (Refs. 16 and 32), as described in the text. Thick solid lines separate schematically the regions of lightly damped (low T region), damped, and overdamped magnonlike excitations.

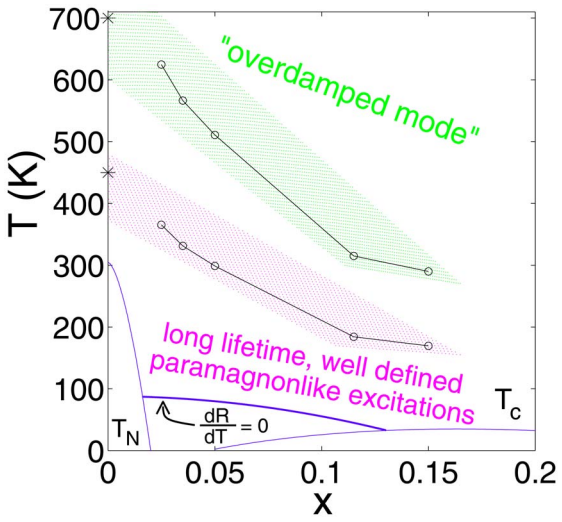

FIG. 2. (Color online) $La_{2-x}Sr_xCuO_4$ phase diagram. Solid curves show the Néel temperature (T_N) and the superconducting dome (T_c), and $dR/dT=0$ curve indicates the gradual crossover from insulating to metallic behavior from resistivity measurements (Ref. 1). Lower and upper shaded lines with circles have been extracted from $^{17}(1/T_1T)$ data shown in Fig. 1 with the conditions $\Gamma_rT^3=0.2$ and $\Gamma_rT^3=1$, respectively, where Γ_r is given in Table I. These shaded lines with circles may be approximated by $T\approx 400(1-4x)$ and $T\approx 700(1-4x)$, respectively, and separate the regions of lightly damped (long lifetime, well defined), damped, and overdamped paramagnonlike excitations. The asterisks mark approximately the corresponding temperatures as extracted from the data analysis (Ref. 16) for $Sr_2CuO_2Cl_2$.

sion (small wave vectors $\mathbf{k}<1/\xi_{eff}$) may be calculated from general physical grounds, namely, the linear response theory, hydrodynamics, and fluctuation-dissipation theorem[29] (see also Ref. 17),

$$^{17}(1/T_1)_{diff}=\frac{^{17}F(0)^2k_BTa^2\chi(\mathbf{k}=0)}{\pi\hbar D}\Lambda, \tag{8}$$

where $\Lambda=[1/(4\pi)]\ln[1+D^2/(\omega_0^2\xi_{eff}^4)]$ and the calculated values of spin diffusion constant, D, are given in Table I.

Since the relaxation function can be understood within the spin-wave framework,[6] the temperature and doping dependence of the *damping* of the spin-wave-like excitations may be studied further. The spin-wave-like dispersion, renormalized by interactions, is given by the relaxation function[6]

$$\omega_{\mathbf{k}}^{sw}=2\int_0^\infty d\omega\omega\mathcal{F}(\mathbf{k},\omega), \tag{9}$$

where the integration over ω in Eq. (9) has been performed analytically and exactly. We assume the Lorentzian form of the imaginary part of the dynamic spin susceptibility,

$$\chi_L''(\mathbf{k},\omega)=\frac{\chi(\mathbf{k})\omega\Gamma_{\mathbf{k}}}{[\omega-\omega_{\mathbf{k}}^{sw}]^2+\Gamma_{\mathbf{k}}^2}+\frac{\chi(\mathbf{k})\omega\Gamma_{\mathbf{k}}}{[\omega+\omega_{\mathbf{k}}^{sw}]^2+\Gamma_{\mathbf{k}}^2}, \tag{10}$$

for $\mathbf{k}$ around the AF wave vector (π,π). We accept to the leading order the cubic temperature dependence[30,31] for the damping of spin-wave-like excitations $\Gamma_{\mathbf{k}}=\Gamma_rT^3\eta_{\mathbf{k}}$, where the wave vector dependence is given by $\eta_{\mathbf{k}}=\sqrt{\langle\omega_{\mathbf{k}}^2\rangle-(\omega_{\mathbf{k}}^{sw})^2}$.

IV. RESULTS AND DISCUSSION

Figure 1 shows the plane oxygen $^{17}(1/T_1)$ fitted by Eq. (6) with two adjustable parameters, K_K and Γ_r, which values are given in Table I. The quality of the fit is very good, which we treat as the validity of our theory. The importance of $^{17}(1/T_1)_{Korr}$ and $^{17}(1/T_1)_{diff}$ in plane oxygen $^{17}(1/T_1)$, in contrast[17,24] to plane copper $^{63}(1/T_1)$, is due to the filtering of the (π,π) contribution by the plane oxygen hyperfine form factor. Obviously, in the absence of $^{17}(1/T_1)_{Korr}$ and $^{17}(1/T_1)_{diff}$, it is hard to explain the measured $^{17}(1/T_1)$ at large x with any form of the damping function. In general, the damping grows with doping x, as it should. It should be emphasized that the increase of $^{17}(1/T_1)$ with temperature is caused by the increase of the damping $\Gamma_{\mathbf{k}}$. The low T region of lightly damped paramagnonlike excitations, where the data may be explained by the theory that neglects the damping,[17] is quantified through the relation $\Gamma_rT^3<0.2$.

Figure 2 shows the temperature (T)-doping index (x) phase diagram with the spin-wave-like damping regimes deduced from plane oxygen nuclear spin-lattice relaxation rate $^{17}(1/T_1)$ data. It is tempting to speculate that the doping and temperature behavior of these curves resemble the characteristic "pseudogap" temperatures. The form of $\chi_L''(\mathbf{k},\omega)$ in Eq. (10) gives the *commensurate* response at low ω and the incommensurate response at high ω in agreement with NS studies in the lightly doped regime.[1,13,14] Very recently, Stock *et al.*[13,14] reported the evidence for spin waves from NS studies of underdoped $YBa_2Cu_3O_{6.35}$ with $T_c=18$ K, where the magnetic excitations are very similar to that of carrier-free 2DHAF systems. These observations, together with the undoubtful evidence for the disappearance of AF spin excita-

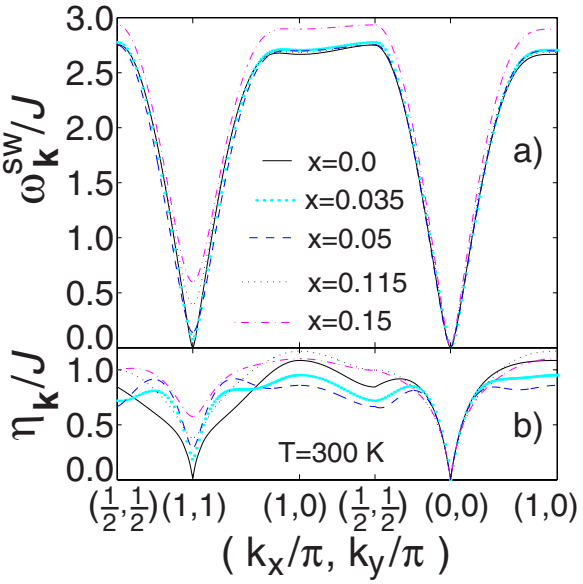

FIG. 3. (Color online) (a) Spin-wave-like dispersion $\omega^{sw}_{\mathbf{k}}$ and (b) damping function $\eta_{\mathbf{k}}$ along the $(\frac{1}{2},\frac{1}{2})$-(1,1)-(1,0)-$(\frac{1}{2},\frac{1}{2})$-(0,0)-(1,0) route in the Brillouin zone at various dopings.

tions in the overdoped regime,[12] show that AF spin excitations in the overall doping range of copper oxide high T_c emanate from those of the parent insulator, i.e., the spin waves.

The wave vector dependence of spin-wave-like dispersion $\omega^{sw}_{\mathbf{k}}$ and damping $\eta_{\mathbf{k}}$ is shown in Figs. 3(a) and 3(b), respectively, for various doping levels. Both $\omega^{sw}_{\mathbf{k}}$ and $\eta_{\mathbf{k}}$ show negligible temperature dependence below $T<J/2$ except the region around (π,π). For $x=0$, our calculated magnon energy, $\omega^{sw}_{\mathbf{k}}$, at $(\pi,0)$ is 3% lower than at $(\pi/2,\pi/2)$ in qualitative agreement with Monte Carlo simulations and series expansion calculations[33] and is similar to that in $Sr_2Cu_3O_4Cl_2$.[34] However, it is a bit different from NS data in La_2CuO_4.[15] The dispersion of $\omega^{sw}_{\mathbf{k}}$ remains approximately the same and the wave vector dependence of the damping function, $\eta_{\mathbf{k}}$, in contrast, possesses significant changes with doping. We emphasize that below $T\approx 400(1-4x)$ K, where $\Gamma_r T^3<0.2$, the damping $\Gamma_{\mathbf{k}}$ is much smaller compared with $\omega^{sw}_{\mathbf{k}}$; thus, the spin-wave-like excitations are indeed well defined (long lifetime). Above $T\approx 700(1-4x)$ K, where $\Gamma_r T^3>1$, the damping $\Gamma_{\mathbf{k}}$ becomes compatible with $\omega^{sw}_{\mathbf{k}}$ and grows further with T and x, thus providing the overdamped mode region.[35,36]

It should be mentioned that the so-called anomalous "1/8" doping problem can be viewed as a consequence of the particular parameter set at $x\simeq 1/8$ within an extended t-J model with the Coulomb repulsion between the NN and taking into account the polarization of NN copper spins around copper-oxygen singlet that gives the in-phase domain structure for the "stripe" picture.[37] The in-phase domain is favorable, also in view of the experimental data,[38,39] compared with the antiphase domain model.[40] This particular case is beyond our present consideration because of its narrowness.

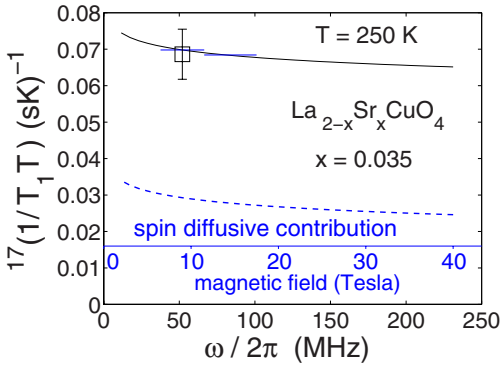

FIG. 4. (Color online) The calculated field (frequency) dependence of $^{17}(1/T_1T)$ (solid curve). The dashed curve shows the spin diffusive contribution. The error bar shows the size of the symbol (error bar) in Fig. 1. Short solid horizontal lines indicate the calculated values of $^{17}(1/T_1T)$ at the fields 9 and 14.1 T.

Figure 4 and Eq. (8) show that in doped 2DHAF system, the spin diffusive contribution $^{17}(1/T_1)_{diff}$ scales with the NMR frequency ω as $\ln(\text{const}\, J/\omega)$, which is very weak in view of the extraordinary large superexchange coupling constant J. We argue that at low doping level ($x\approx 0.03$ per Cu site in $La_{2-x}Sr_xCuO_4$) and temperature $T<300$ K, $^{17}(1/T_1)$ is strongly affected by the $^{17}(1/T_1)_{diff}$ luck of the filtering of (π,π) contribution by the oxygen form factor. Only a new accurate NMR experiment at very low and very high magnetic fields may uncover the $^{17}(1/T_1)_{diff}$ contribution to $^{17}(1/T_1)$.

V. CONCLUSION

We applied the relaxation function theory for doped 2DHAF system in the paramagnetic state and deduced the lifetime of spin-wave-like excitations, its evolution with doping, from plane oxygen $^{17}(1/T_1)$ NMR data in the underdoped high-T_c layered copper oxides. It is shown that the spin-wave-like theory is able to reproduce the main features of low frequency spin dynamics in the normal state of high-T_c cuprates, as observed experimentally. We identified the regions of long lifetime $[T\lesssim 400(1-4x)$ K] and overdamped $[T\gtrsim 700(1-4x)$ K] paramagnonlike excitations in the temperature (T)-doping index (x) phase diagram right up to optimally doped $La_{2-x}Sr_xCuO_4$. The results indicate that spin-wave-like excitations are indeed a good description of the quasiparticle excitations even for *strongly doped* high-T_c layered cuprates at low temperatures, $T\lesssim 400(1-4x)$ K.

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