

Spin polarization in biased Rashba-Dresselhaus two-dimensional electron systems

P. Kleinert

Paul-Drude-Institut für Festkörperelektronik, Hausvogteiplatz 5-7, 10117 Berlin, Germany

V. V. Bryksin

A. F. Ioffe Physical Technical Institute, Politekhmicheskaya 26, 194021 St. Petersburg, Russia

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Based on spin-charge-coupled drift-diffusion equations, which are derived from kinetic equations for the spin-density matrix in a rigorous manner, the electric-field-induced nonequilibrium spin polarization is treated for a two-dimensional electron gas with both Rashba and Dresselhaus spin-orbit coupling. Most emphasis is put on consideration of the field-mediated spin dynamics for a model with equal Rashba and Dresselhaus coupling constants, in which the spin relaxation is strongly suppressed. Weakly damped electric-field-induced spin excitations are identified, which recall space-charge waves in crystals.

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I. INTRODUCTION

Spin-dependent transport phenomena are of great interest to both basic research and device applications. In particular, semiconductor-based spin electronics has been the subject of numerous investigations. In this field, spin rather than charge is exploited for signal processing. The spin-orbit interaction (SOI) opens the possibility of manipulating the spin of carriers using purely electrical means. Unfortunately, the very same SOI has the undesired effect of causing spin relaxation due to precession in a wave-vector-dependent effective magnetic field, which is traced back to the SOI. For a quantum well grown on a [001] substrate, the Dyakonov-Perel spin relaxation¹ is the dominant effect. In general, however, spin relaxation depends on the details of the band structure and the relevant scattering mechanisms (see Ref. 2 and references therein). For an asymmetric quantum well, bulk-inversion asymmetry and structure-inversion asymmetry give rise to Dresselhaus and Rashba spin-orbit contributions to the Hamiltonian, respectively. The interplay between the linear Rashba and Dresselhaus terms causes a spin relaxation anisotropy that has been treated in a number of theoretical works.³⁻⁷ For a quantum well grown along the [001] direction, the main axes of the spin-relaxation-time tensor are given by [110] and [1 $\bar{1}$ 0]. Most interesting is the observation that, under idealized conditions and when the linear Rashba and Dresselhaus terms have equal strength, the relaxation of spin oriented along the [110] axis is totally suppressed.³ In this particular case, a conserved quantity exists, which hinders spin randomization.^{5,6} This remarkable behavior of the spin-relaxation time led to the proposal of a nonballistic spin field-effect transistor.⁵ Other studies of the combined Rashba-Dresselhaus model referred to the spin- and charge-Hall effects.⁸⁻¹³

Recently, the field received a fresh impetus by the identification of an exact SU(2) symmetry of the model, when reaching the condition of equal Rashba and Dresselhaus coupling strengths.^{14,15} From a theoretical point of view, an equivalent system is the Dresselhaus [110] model. The revealed symmetry gives rise to a massless mode with infinite lifetime at nonzero wave vector. Qualitative features of the

associated spin pattern have been experimentally confirmed¹⁶ by optical techniques that probe spin relaxation rates. Furthermore, it has been predicted^{17,18} that coherent spatial oscillations of the spin polarization develop in such a system under appropriate injection conditions.

In this paper, we extend these interesting studies by treating spin effects under the influence of an applied in-plane electric field. For the combined Rashba-Dresselhaus model with a SOI that is linear in \mathbf{k} , spin-charge-coupled drift-diffusion equations are derived in a systematic manner. Results are presented and discussed for the special model with equal Rashba and Dresselhaus coupling strengths.

II. SPIN-CHARGE-COUPLED DRIFT-DIFFUSION EQUATIONS

The effective Hamiltonian of our approach,

$$H_0 = \frac{\hbar^2 \mathbf{k}^2}{2m} + \alpha(k_y \sigma_x - k_x \sigma_y) + \beta(k_x \sigma_x - k_y \sigma_y), \quad (1)$$

includes the Rashba spin-orbit term, which is due to the inversion asymmetry of the confining potential of the quantum well. In addition, there is the Dresselhaus coupling, which is present in semiconductors lacking bulk-inversion symmetry. The model Hamiltonian refers to a two-dimensional semiconductor nanostructure grown along the [001] direction. In Eq. (1), \mathbf{k} , m , and σ_i ($i=x,y,z$) denote the in-plane wave vector, the effective electron mass, and the usual Pauli matrices, respectively. α and β are the strengths of the Rashba and Dresselhaus spin-orbit couplings. Our total Hamiltonian encompasses also contributions stemming from the short-range spin-independent elastic scattering on impurities and the in-plane electric field $\mathbf{E}=(E_x, E_y, 0)$. Its explicit form together with related kinetic equations for the spin-density matrix has been published recently.¹⁹ It should be noted that, for the combined Rashba-Dresselhaus model, the field-induced spin polarization depends on the orientation of the in-plane electric field.^{12,13}

The main quantity for the theoretical analysis of spin-related phenomena is the spin-density matrix $f_{\lambda}^{\Lambda}(\mathbf{k}, \mathbf{k}', t)$,

which is calculated from quantum kinetic equations.²⁰ From this set of equations, spin-charge-coupled drift-diffusion equations are derived for the physical components $\bar{f} = \text{Tr} \hat{f}$ and $\bar{f} = \text{Tr} \sigma \hat{f}$, which are integrated over the polar angle of \mathbf{k} (denoted by the overbar). In the case of weak SOI ($\alpha k_F \tau / \hbar$, $\beta k_F \tau / \hbar \ll 1$, with k_F and τ being the Fermi wave vector and elastic scattering time, respectively), the following ansatz is justified:²¹

$$\begin{aligned} \bar{f}(\mathbf{k}, \mathbf{q}, t) &= -F(\mathbf{q}, t) \frac{dn(\varepsilon_{\mathbf{k}})/d\varepsilon_{\mathbf{k}}}{dn/d\varepsilon_F}, \\ \bar{f}(\mathbf{k}, \mathbf{q}, t) &= -F(\mathbf{q}, t) \frac{dn(\varepsilon_{\mathbf{k}})/d\varepsilon_{\mathbf{k}}}{dn/d\varepsilon_F}, \end{aligned} \quad (2)$$

where $n(\varepsilon_{\mathbf{k}})$ denotes the Fermi function and $\varepsilon_{\mathbf{k}} = \hbar^2 \mathbf{k}^2 / (2m)$. Furthermore, we introduced the electron density $n = \int d\varepsilon \rho(\varepsilon) n(\varepsilon)$, with $\rho(\varepsilon)$ being the density of states of the two-dimensional electron gas. Applying this straightforward calculational scheme,²¹ spin-charge-coupled drift-diffusion equations are obtained, which read in spatial coordinates r_j

$$\frac{\partial F_i}{\partial t} + \frac{\partial J_{ij}}{\partial r_j} + M_{ij} F_j = \frac{2m\tau}{\hbar^3 \tau_s} L_i F, \quad (3)$$

with the expression for the spin flux

$$\begin{aligned} J_{ij} &= \left(\mu E_j - D \frac{\partial}{\partial r_j} \right) F_i + \frac{4Dm}{\hbar^2} \varepsilon_{ikl} Q_{kj} F_l - \frac{4mD\tau}{\hbar^3} (\alpha^2 - \beta^2) \\ &\quad \times \left(\frac{2m}{\hbar^2} Q_{ij} - \varepsilon_{ijk} \frac{\mu E_k}{2D} \right) F. \end{aligned} \quad (4)$$

The diffusion coefficient D and the mobility μ satisfy the Einstein relation $\mu = (eD/n) dn/d\varepsilon_F$. Other quantities that enter Eqs. (3) and (4) are defined by

$$\mathbf{L} = \frac{2m}{\hbar^2} \tau_s (-\alpha \mu E_y + \beta \mu E_x), (\alpha \mu E_x + \beta \mu E_y), 0),$$

$$\frac{1}{\tau_s} = 4D \frac{m^2}{\hbar^4} (\alpha^2 + \beta^2), \quad (5)$$

$$M_{ij} = \frac{A_{ij}}{\tau_s} + \frac{1}{\tau_s} \varepsilon_{ikj} L_k + \frac{2\alpha\beta}{(\alpha^2 + \beta^2) \tau_s} S_{ij}, \quad (6)$$

$$\vec{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \vec{Q} = \begin{pmatrix} \beta & \alpha & 0 \\ -\alpha & -\beta & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \vec{S} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (7)$$

ε_{ijk} is the totally antisymmetric tensor in three dimensions. The spin flux J_{ij} , which gives the i th component of the particle flow with spin polarization along the axis j ($i, j = x, y, z$), has a general form that is in accordance with symmetry requirements.²² Neglecting the spin-charge coupling and the influence of the electric field, we obtain the result

$$\frac{\partial F_i}{\partial t} = -\frac{m}{\hbar^2} Q_{jl} \varepsilon_{jik} J_{kl}, \quad J_{kl} = \frac{4Dm}{\hbar^2} \varepsilon_{kmn} Q_{ml} F_n, \quad (8)$$

in which the spin flux J_{kl} appears as a source of nonequilibrium spin polarization.^{22,23} The set of basic equations (3) and (4), which restore published results¹⁴ in the case of vanishing electric field, are solved and discussed in the Fourier space with respect to spatial coordinates. For the sake of a better readability of the paper, these equations are summarized in the Appendix.

III. RESULTS AND DISCUSSION

A number of quite interesting electric-field effects on the spin polarization occurring in systems with Rashba and Dresselhaus SOI can be studied on the basis of the coupled spin-charge drift-diffusion equations (3) and (4) [or (A1) and (A2)]. To keep the presentation transparent, we restrict ourselves to spatially infinite systems by omitting any boundary effects.

A. Charge current

The current density of charge carriers is defined by the time derivative of the dipole moment, which can be expressed by

$$\mathbf{j}(t) = -ie \nabla_{\mathbf{\kappa}} \frac{\partial}{\partial t} F(\mathbf{\kappa}, t) |_{\mathbf{\kappa}=\mathbf{0}}. \quad (9)$$

As we are mainly interested in the charge transport along the [110] and $[1\bar{1}0]$ directions, the spatial coordinates are rotated by introducing new wave vectors $\kappa_{\pm} = (\kappa_x \pm \kappa_y) / \sqrt{2}$. In these coordinates, the charge current along the κ_+ direction is given by

$$j_+(t) = -ie \frac{\partial^2}{\partial t \partial \kappa_+} F(\kappa_+, \kappa_- = 0, t) |_{\kappa_+ = 0}. \quad (10)$$

Taking into account Eq. (A1), we immediately obtain for the Laplace-transformed current density

$$j_+(s) = e\mu E_+ \frac{F}{s} + \frac{e}{\hbar} (\alpha + \beta) F_-(s) - \frac{2me\tau}{\hbar^3} (\alpha^2 - \beta^2) \mu E_- F_z(s), \quad (11)$$

where $E_{\pm} = (E_x \pm E_y) / \sqrt{2}$ and $F_{\pm} = (F_x \pm F_y) / \sqrt{2}$. The components of the density matrix, which enter this equation, refer to a spatially homogeneous system. The related ($\mathbf{\kappa} = \mathbf{0}$) drift-diffusion equations (A1) and (A2) are easily solved. Inserting the solution into Eq. (11) and taking into account only contributions linear in the electric field, we get

$$j_+(\omega) = e\mu E_+ F \left(1 + \frac{\alpha^2 \tau}{\hbar^2 D i \omega \tau_s - (1 - \gamma)^2 / (1 + \gamma^2)} \right), \quad (12)$$

with $\gamma = \beta / \alpha$. As the current is aligned along the electric-field direction, there is no Hall current, the appearance of which would require the treatment of the nonlinear field term in Eq. (11), which is of higher order in the spin-orbit coupling con-

stant. In Eq. (12), the pure charge current $e\mu E_+ F$ is complemented by a spin contribution, which exhibits a Drude-like frequency dependence governed by the spin relaxation time τ_s , which may be much larger than the elastic scattering time τ . Consequently, the frequency dispersion of the spin-mediated current contribution can set in at lower frequencies as in the ordinary Drude formula. The spin-induced current contribution disappears, when the Rashba and Dresselhaus coupling constants are equal ($\alpha=\beta$). For the Rashba model ($\beta=0$), Eq. (12) resembles the results derived previously for small polarons.²⁴

B. Out-of-plane spin polarization for $\alpha=\beta$

As another application of the spin-charge drift-diffusion equations (A1) and (A2), we treat the evolution of an initial spin lattice produced at $t=0$ along the κ_+ direction,

$$f_{z0}(\kappa_+) = \frac{1}{2}f_{z0}\delta(\kappa_+ - \kappa_0) + \frac{1}{2}f_{z0}^*\delta(\kappa_+ + \kappa_0), \quad f_{z0} = |f_{z0}|e^{i\varphi}, \quad (13)$$

with κ_0 being a given wave vector. Restricting ourselves to the special case $\alpha=\beta$, we obtain the exact solution

$$F_z(\kappa_+, s) = f_{z0}(\kappa_+) \frac{\sigma}{\sigma^2 + 2\Omega_{\kappa_+}^2}, \quad (14)$$

with the effective Laplace variable

$$\sigma = s - i\mu E_+ \kappa_+ + D\kappa_+^2 + 2/\tau_s \quad (15)$$

and the frequency

$$\Omega_{\kappa_+} = 2\sqrt{2} \frac{m\alpha}{\hbar^2} (\mu E_+ + 2iD\kappa_+). \quad (16)$$

The inverse Laplace and Fourier transformations of Eq. (14) are easily calculated, and we obtain for the asymptotic dynamics at large times

$$F_z(\kappa_+, \omega) = G \frac{(i\Omega + 2/\tau_{s+})(i\Omega + 2/\tau_{s-})}{(i\Omega + 2/\tau_{s+})(i\Omega + 2/\tau_{s-})(i\Omega + 2/\tau_s) + T_+^2(i\Omega + 2/\tau_{s-}) + T_-^2(i\Omega + 2/\tau_{s+})}, \quad (20)$$

where the shorthand notations

$$i\Omega = i\omega - i\mu\kappa_+ E_+ + D\kappa_+^2, \quad T_+ = 2K_+(\mu E_+ + 2iD\kappa_+),$$

$$T_- = 2K_-\mu E_-, \quad K_{\pm} = (\alpha \pm \beta) \frac{m}{\hbar^2} \quad (21)$$

were used. This solution becomes more transparent for the special case $\alpha=\beta$, where we obtain

$$F_z(r_+, t) = \frac{|f_{z0}|}{2} \exp[-D(\kappa_0 - 2K)^2 t] \times \cos[\kappa_0 r_+ + \varphi + \mu E_+(\kappa_0 - 2K)t]. \quad (17)$$

For $\kappa_0=2K$, the original static spin lattice survives and retains its shape in the steady state. Under this excitation condition, a long-lived spin pattern is expected to occur. If κ_0 slightly deviates from $2K$, the electric field drives a propagating spin wave, the amplitude of which diminishes with time exponentially. The frequency of this wave is given by $\mu E_+(\kappa_0 - 2K)$.

C. Field-induced spin accumulation and Hanle effect

In this section, the electric-field-induced out-of-plane spin accumulation is studied for a different initial spin preparation. First, we focus on the contribution that appears in a homogeneous electron gas ($\kappa=0$). From Eqs. (A7)–(A10) given in the Appendix, we obtain the steady-state solution

$$F_z = -4\alpha\beta \frac{m\tau}{D\hbar^3} \frac{\mu^2(E_x^2 - E_y^2)}{2/\tau_s + (\mu E)^2/D} F, \quad (18)$$

which applies for a semiconductor with different Rashba and Dresselhaus SOI constants ($\alpha \neq \beta$). This field-mediated spin accumulation depends on the orientation of the electric field within the plane and has the character of a second-order field effect. Spin accumulation, which is proportional to the electric field, occurs only in the plane.¹² The quadratic field effect in Eq. (18) disappears for the pure Rashba ($\beta=0$) and Dresselhaus ($\alpha=0$) systems as well as under the condition $|E_x|=|E_y|$. For sufficiently high electric fields [$2/\tau_s \ll (\mu E)^2/D$], the out-of-plane spin polarization approaches the constant field-independent value $F_z = -4\alpha\beta m\tau \cos(2\varphi)F/\hbar^3$, with φ being the polar angle of the electric field.

In addition to this field contribution of a homogeneous system, we study the response to a permanent harmonic spin generation of the form

$$G(r_+, t) = G e^{i\omega t + i\kappa_+ r_+}. \quad (19)$$

Disregarding the spin-charge coupling, the drift-diffusion equations (A8)–(A10) are analytically solved by

$$F_z(\kappa_+, \omega) = G \frac{i\omega - i\mu\kappa_+ E_+ + D\kappa_+^2 + 2/\tau_s}{N_+ N_-},$$

$$N_{\pm} = i\omega - i\mu E_+(\kappa_+ \pm 2K) + D(\kappa_+ \pm 2K)^2. \quad (22)$$

The dispersion relation of eigenmodes $\omega = \mu E_+(\kappa_+ - 2K)$ is derived from the denominator N_- . This mode has the char-

acter of free carrier oscillations complemented by a spin part that gives rise to a soft mode at $\kappa_+ = 2K$.

Further information about the solution in Eq. (20) is obtained for $\alpha \approx \beta$ and $\Omega \ll 1/\tau_s$. Two limits can be distinguished in this case. Under the condition $\Omega \ll 1/\tau_{s-}$, which applies to the steady state, Eq. (20) simplifies to

$$F_z = \frac{G}{2/\tau_s + (\mu E)^2/D}, \quad (23)$$

which can be interpreted as the electric-field analogy of the Hanle effect.^{21,25} Another result is obtained in the opposite case $1/\tau_{s-} \ll \Omega$, when the out-of-plane spin polarization depends on the orientation of the electric field,

$$F_z = \frac{G}{2/\tau_s + [\mu(E_x + E_y)]^2/(2D)}. \quad (24)$$

This solution dictates the behavior of the spin polarization in the limit $\alpha \rightarrow \beta$. Both results agree for the special field configuration $E_x = E_y = E/\sqrt{2}$.

D. Spin remagnetization waves for $\alpha = \beta$

Finally, we treat the relaxation of an initial homogeneous spin moment \mathbf{F}_0 . In this case, we use $\boldsymbol{\kappa} = \mathbf{0}$ in our basic equations so that spin and charge degrees of freedom decouple from each other. Most interesting results are expected when the Rashba and Dresselhaus SOI couplings coincide ($\alpha = \beta$). The set of equations (A8)–(A10) is easily solved for the Laplace-transformed functions. The result,

$$F_x(t) = \frac{F_{x0} - F_{y0}}{2} + e^{-2t/\tau_s} \left(\frac{F_{x0} + F_{y0}}{2} \cos(\Omega_E t) - \frac{F_{z0}}{\sqrt{2}} \sin(\Omega_E t) \right), \quad (25)$$

$$F_y(t) = -\frac{F_{x0} - F_{y0}}{2} + e^{-2t/\tau_s} \left(\frac{F_{x0} + F_{y0}}{2} \cos(\Omega_E t) - \frac{F_{z0}}{\sqrt{2}} \sin(\Omega_E t) \right), \quad (26)$$

$$F_z(t) = e^{-2t/\tau_s} \left(\frac{F_{x0} + F_{y0}}{\sqrt{2}} \sin(\Omega_E t) + F_{z0} \cos(\Omega_E t) \right), \quad (27)$$

describes spin rotations with the frequency $\Omega_E = \sqrt{2}K\mu(E_x + E_y)$. Such rotations of the magnetic moment were studied previously both for hopping of small polarons^{24,26} and for extended states in a two-dimensional electron gas.^{27,28} In analogy to space-charge waves, these eigenmodes are called spin-remagnetization waves. Typically, the amplitudes of these excitations exponentially decrease with increasing time. It is a peculiarity of the special Rashba-Dresselhaus model that the in-plane magnetic moment is conserved: $F_x(t) - F_y(t) = F_{x0} - F_{y0}$. This observation provides a further example of the occurrence of undamped spin excitations,

whenever the coupling constants α and β are equal.

IV. SUMMARY

Both for the Rashba-Dresselhaus model with equal coupling constants and for the Dresselhaus [110] model, it has recently been realized that the relaxation of spin oriented along the [110] axis is totally suppressed. As the spin lifetime becomes arbitrarily long within these models, the existence of a persistent spin grating has been predicted. Recent experiments^{16,29,30} indeed revealed evidence supporting this interesting peculiarity of slow spin relaxation rates. The extreme suppression of spin relaxation in the special theoretical models for semiconductor nanostructures with SOI is due to a spin symmetry that gives rise to a soft mode at $\kappa = 2K$. This weakly damped eigenmode leads also to interesting phenomena in the biased spin-orbit-coupled system that was studied in this paper. Based on rigorous spin-charge-coupled drift-diffusion equations for the components of the spin-density matrix, a number of field-mediated spin effects were considered.

(i) A Drude-like spin-induced contribution was identified in the longitudinal charge current, that disappears in the special Rashba-Dresselhaus model with equal coupling strengths. The frequency position and the width of the resonance are determined by the spin relaxation time and the coupling constants α and β .

(ii) A persistent spin pattern created at $\kappa = 2K$ is not destroyed by the electric field. However, under the condition of slightly off resonance, the field forces the pattern to move.

(iii) In a homogeneous electron gas with Rashba and Dresselhaus SOI, an out-of-plane spin polarization develops, which has a nonlinear field character. In addition, long-lived remagnetization waves can be excited, whose frequency is given by $\omega = \mu E_+(\kappa_+ - 2K)$. These eigenmodes can be studied in a similar manner as space-charge waves in crystals. In addition, the electric-field analogy of the Hanle effect changes its character at $\alpha \approx \beta$.

(iv) The in-plane spin polarization of the special Rashba-Dresselhaus model with $\alpha = \beta$ is conserved and the spins rotate due to the electric field with the frequency $\Omega_E = \sqrt{2}K\mu(E_x + E_y)$.

The experimental confirmation of the field-mediated spin effects predicted here would stimulate further progress in both basic research and technological innovations.

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APPENDIX: FOURIER-TRANSFORMED DRIFT-DIFFUSION EQUATIONS

The Fourier-transformed version of our basic spin-charge-coupled drift-diffusion equations (3) and (4) has the form

$$\left(\frac{\partial}{\partial t} - i\mu\mathbf{E}\boldsymbol{\kappa} + D\kappa^2\right)\mathbf{F} + i\boldsymbol{\omega}_{\boldsymbol{\kappa}} \cdot \mathbf{F} - \frac{2im\tau}{\hbar^3}(\alpha^2 - \beta^2)[(\boldsymbol{\kappa} \times \mu\mathbf{E}) \cdot \mathbf{F}] = 0, \quad (\text{A1})$$

$$\left(\frac{\partial}{\partial t} - i\mu\mathbf{E}\boldsymbol{\kappa} + D\kappa^2 + \frac{\vec{T}}{\tau_s}\right)\mathbf{F} - (\mathbf{H} \times \mathbf{F}) - \frac{i\chi}{\hbar}(\boldsymbol{\kappa} \times \mu\mathbf{E})\mathbf{F} + \chi\mathbf{H}\mathbf{F} = 0, \quad (\text{A2})$$

with the abbreviations

$$H_x = \frac{2m}{\hbar^2}[\alpha(\mu E_y + 2iD\kappa_y) + \beta(\mu E_x + 2iD\kappa_x)], \quad (\text{A3})$$

$$H_y = -\frac{2m}{\hbar^2}[\alpha(\mu E_x + 2iD\kappa_x) + \beta(\mu E_y + 2iD\kappa_y)], \quad H_z = 0, \quad (\text{A4})$$

$$\vec{T} = \begin{pmatrix} 1 & \frac{2\alpha\beta}{\alpha^2 + \beta^2} & 0 \\ \frac{2\alpha\beta}{\alpha^2 + \beta^2} & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}, \quad \chi = \frac{2m\tau}{\hbar^3}(\alpha^2 - \beta^2). \quad (\text{A5})$$

For the Rashba model ($\beta=0$), these equations agree with results derived previously.²¹

Let us treat these equations in another representation characterized by the components

$$\kappa_{\pm} = \frac{\kappa_x \pm \kappa_y}{\sqrt{2}}, \quad F_{\pm} = \frac{F_x \pm F_y}{\sqrt{2}}. \quad (\text{A6})$$

For excitations of the spin polarization that propagate exclusively along the κ_+ direction ($\kappa_-=0$), Eqs. (A1) and (A2) are written in the form

$$\left(\frac{\partial}{\partial t} - i\mu E_+\kappa_+ + D\kappa_+^2\right)F - \frac{i}{\hbar}(\alpha + \beta)\kappa_+F_- + \frac{2im\tau}{\hbar^3}(\alpha^2 - \beta^2)\mu E_-\kappa_+F_z = 0, \quad (\text{A7})$$

$$\left(\frac{\partial}{\partial t} - i\mu E_+\kappa_+ + D\kappa_+^2 + \frac{2}{\tau_{s+}}\right)F_+ + \frac{2m}{\hbar^2}(\alpha + \beta)(\mu E_+ + 2iD\kappa_+)F_z - \frac{4m^2\tau}{\hbar^5}(\alpha - \beta)^2(\alpha + \beta)\mu E_-F = 0, \quad (\text{A8})$$

$$\left(\frac{\partial}{\partial t} - i\mu E_+\kappa_+ + D\kappa_+^2 + \frac{2}{\tau_{s-}}\right)F_- + \frac{2m}{\hbar^2}(\alpha - \beta)\mu E_-F_z + \frac{4m^2\tau}{\hbar^5}(\alpha + \beta)^2(\alpha - \beta)(\mu E_+ + 2iD\kappa_+)F = 0, \quad (\text{A9})$$

$$\left(\frac{\partial}{\partial t} - i\mu E_+\kappa_+ + D\kappa_+^2 + \frac{2}{\tau_s}\right)F_z + \frac{2im\tau}{\hbar^3}(\alpha^2 - \beta^2)\mu E_-\kappa_+F - \frac{2m}{\hbar^2}(\alpha + \beta)(\mu E_+ + 2iD\kappa_+)F_+ - \frac{2m}{\hbar^2}(\alpha - \beta)\mu E_-F_- = 0, \quad (\text{A10})$$

with $E_{\pm} = (E_x \pm E_y)/\sqrt{2}$ and

$$\frac{2}{\tau_{s+}} = \frac{(\alpha + \beta)^2}{\tau_s(\alpha^2 + \beta^2)}, \quad \frac{2}{\tau_{s-}} = \frac{(\alpha - \beta)^2}{\tau_s(\alpha^2 + \beta^2)}. \quad (\text{A11})$$

For the particular case $\alpha=\beta$, the coupling between spin and charge degrees of freedom disappears and the secular equation for the spin components gives the following dispersion relations for eigenmodes of the biased spin system:

$$\omega_1 = -\mu E_+\kappa_+ - iD\kappa_+^2,$$

$$\omega_{2,3} = -(\kappa_+ \pm 2K)[\mu E_+ + iD(\kappa_+ \pm 2K)], \quad (\text{A12})$$

with $K=2m\alpha/\hbar^2$. This result implies that there appears a field-induced undamped soft mode at $\kappa_+=2K$, which reflects the presence of the spin rotation symmetry.¹⁴

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