Collective excitations in a magnetically doped quantized Hall ferromagnet

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(Received 4 April 2007; revised manuscript received 13 July 2007; published 8 November 2007)

A theory of collective states in a magnetically quantized two-dimensional electron gas (2DEG) with half-filled Landau level (quantized Hall ferromagnet) in the presence of magnetic 3*d* impurities is developed. The spectrum of bound and delocalized spin excitons as well as the renormalization of Zeeman splitting of the impurity 3*d* levels due to the indirect exchange interaction with the 2DEG are studied for the specific case of *n*-type GaAs doped with Mn, where the Landé *g* factors of impurity and 2DEG have opposite signs. If the sign of the 2DEG *g* factor is changed due to external influences, then impurity related transitions to new ground state phases, presenting various spin-flip and skyrmionlike textures, are possible. Conditions for the existence of these phases are discussed.

DOI: 10.1103/PhysRevB.76.205309 PACS number(s): 73.21.Fg, 73.43.Lp, 73.43.Nq, 78.67.De

I. INTRODUCTION

In a strong magnetic field, the two-dimensional electron states in semiconductor heterostructures[1] transform into Landau states with a completely discrete energy spectrum. This diamagnetically quantized two-dimensional electron gas (2DEG) possesses many remarkable features including quantum Hall effect.[2] The role of impurities in the thermodynamic, optical, and transport properties of 2DEG is extremely important. Among many facets of this problem, we choose for discussion in this paper the formation of impurity related collective excitations in a magnetically doped quantized 2DEG in the case of odd integer filling factor $\nu=2n+1$. In a pristine state, 2DEG with odd ν is in a quantized Hall ferromagnet (QHF) regime with nondegenerate ground state characterized by the total spin quantum number $S=N_\phi/2$ and maximum spin projection $S_z=S$. (N_ϕ is the magnetic-flux-quanta number.) Different branches of the excitons are well distinguishable among the low-energy excitations. They are classified as spin waves (spin excitons), magnetoplasmons, or multiexciton states, depending on the spin and orbital quantum numbers.[3–10] Besides, low-lying collective half-integer-spin fermionic states (trions, skyrmions, etc.) may be formed in a QHF under certain circumstances.[11–18] Magnetic impurities are characterized by their own spectrum of *local* spin excitations, and one can anticipate a strong interplay between local and collective excitations in a magnetically doped QHF.

It is known that the influence of impurities on the discrete spectrum of quantized Landau electrons in a 2DEG has many specific features. Even such a basic property as the interaction of a 2DEG with neutral short-range impurities is far from being trivial.[19–21] Only the Landau states with a finite probability density on the scatterer locations interact with impurities. This means that the whole set of Landau states breaks down into two groups: the major part of the Landau levels (LLs) is not affected by the impurity scattering, and the states having a nonzero scattering amplitude on an impurity form a separate system of bound Landau states in the energy gaps between the free LLs.

To be more specific, we consider a 2DEG formed in the *n*-type GaAs/GaAlAs heterostructures doped with transition metal (TM) impurities. The reason for this choice is that the technology of (Ga,Mn)As epilayers is well developed, and the QHF regime is achieved experimentally in GaAs based heterostructures. As a rule, transition metal ions substitute for the metallic component of the binary II-VI and III-V semiconductors.[22–24] The influence of isolated TM impurities on the spectrum of the Landau states was investigated in Ref. 25. It was shown that the resonance scattering in the *d* channel is in many respects similar to that of the short-range impurity scattering in the *s* channel.[19–21] The symmetry selection rules for the resonance *d* waves in the cylindrical (symmetric) gauge pick up the Landau states with the orbital number $m=0$ (in the symmetric gauge). These states are the same states that are involved in the *s* scattering by the impurities with a short-range scattering potential.[21] Besides, this scattering is spin selective in magnetically quantized 2DEG.

It should be emphasized that in the problem under consideration, the criterion of isolated impurities acquires a specific feature. In fact, the Mn concentration range, where our theory is applicable, is limited from below by technological capabilities and from above by the obvious requirement that the impurity induced disorder *does not destroy collective excitonic states*. So, the relevant interval of bulk Mn concentrations is $10^{13}\ \mathrm{cm}^{-3}<n_{\mathrm{Mn}}\lesssim 10^{15}\ \mathrm{cm}^{-3}$. Here, the upper limit corresponds to the two-dimensional (2D) concentration of $10^9\ \mathrm{cm}^{-2}$, which in our case is actually well below the Landau band capacity N_ϕ at $B\sim 10$ T that is equal to the 2D electron number on the upper (half-filled) LL. One may expect that the above mentioned classification of excitonic states is valid only at the bulk concentration $n_{\mathrm{Mn}}\lesssim 10^{15}\ \mathrm{cm}^{-3}$, which is much less than in the materials used for the creation of dilute magnetic semiconductors.[26]

We calculate in this paper the spectra of bound and continuous collective excitations related to magnetic impurities. When studying the influence of magnetic impurities on the excitonic spectrum of 2DEG, a distinction between the negative and positive signs of the gyromagnetic ratio of 2DEG electrons $g_{2\mathrm{DEG}}$ should be also mentioned. It will be shown

that in the conventional situation of negative $g_{2\mathrm{DEG}}$, the interaction with magnetic impurity lowers the ground state energy due to the effective antiferromagnetic character of the effective indirect exchange. This results in the formation of a set of bound and delocalized collective excitations presenting combined modes classified by a change in the total spin number S_z. When $g_{2\mathrm{DEG}}>0$, so that the g factors of both subsystems (2DEG electrons and impurities) have the same sign, magnetic impurities may form bound states in the gap below the spin exciton continuum and even initiate a global reconstruction of the QHF ground state.

II. MODEL HAMILTONIAN

Following Ref. 23, we describe the electron scattering on a TM impurity in a semiconductor within the framework of the Anderson impurity model Hamiltonian[27] generalized for the case of multicharged impurity states in semiconductors.[28–30] According to this model, the principal source of magnetic interaction is the resonance scattering of conduction electrons on the d-electron levels of TM impurity in the presence of a strong on-site Coulomb interaction U. Due to this interaction, the local moment of TM impurity survives in the crystalline environment, and a "kinematic" indirect exchange interaction between the conduction and impurity electrons arises in the second order in the s-d hybridization parameter, even in the absence of a direct exchange.

The generic Hamiltonian describing the QHF regime in a magnetically doped semiconductor is

$$\hat{H}=\hat{H}_d+\hat{H}_s+\hat{H}_t+\hat{H}_{sd}. \tag{2.1}$$

Here, $\hat{H}_d=\Sigma_i\hat{H}_{di}$ describes the TM impurities on the sites i, $\hat{H}_s$ is related to the band electrons on the LLs, and $\hat{H}_t$ is responsible for hybridization between the impurity d electrons and Landau electrons. Eventually, it is this hybridization that generates coupling between collective modes in 2DEG and localized spin excitations on the impurity sites. In our extremely weak doping regime, both the direct and indirect interactions between magnetic impurities are negligible. Each magnetic scatterer may be considered independently, and it is convenient to choose the symmetric gauge $\mathbf{A}=(-\frac{B}{2}y,\frac{B}{2}x,0)$ with the quantum numbers $\lambda=(n,m)$ for the Landau electrons hybridized with the atomic d electrons centered around the site i, positioned in the center of coordinates. The Coulomb interaction is taken into account in the impurity and in the band electron subsystems. Besides, the direct Coulomb interaction between the d and s electrons described by the last term in the Hamiltonian (2.1) is added to the conventional impurity Hamiltonian (cf. Ref. 27) described by the first and third terms. All additional interactions will be discussed below in detail.

Substitutional Mn impurity in GaAs retains all its five d electrons due to a special stability of the half-filled $3d$ shell. In the p-type GaAs, the electrically neutral state of Mn in Ga position is $\mathrm{Mn}^{(3+)}(3d^5+\mathrm{hole})$, where the hole is bound on the relatively shallow acceptor level near the top of the valence band, whereas the occupied d-electron levels are deep in the valence band.[23,24,31,32] In the n-type heterostructures, these acceptor states are overcompensated, and the chemical potential is pinned on one of the lowest Landau levels in the conduction band. Since we are interested only in the low-energy excitations above the ground state of the n-type system, Mn impurities will be considered as the $\mathrm{Mn}^{(3+)}(3d^5)$ ions in the subsequent calculations.

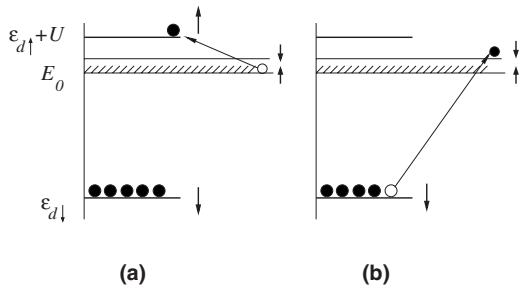

FIG. 1. Allen reactions that involve an additional (a) electron or (b) hole in the impurity site. $\epsilon_{d\downarrow}$ and $\varepsilon_{d\uparrow}+U$ are addition energies for the fifth and sixth electrons in the $3d$ shell of Mn ion in accordance with Eqs. (2.8). The ground state with the energy E_0 corresponds to the completely occupied lowest Landau subband. Spins of Mn $3d$ shell and occupied Landau subband are antiparallel because of the different signs of g factors for Mn and 2DEG in GaAs.

A. Single-orbital model: Spin-selective hybridization

One may significantly simplify the calculation of the spectrum of excitations by reducing the general Hamiltonian (2.1) to the form in which only the terms relevant to the calculation of the desired collective states are present. As a result of this simplification, outlined below in Sec. II B, one arrives at the *single-orbital, single Landau band hybridization Hamiltonian*, which explicitly takes into account the Hund rule governing the high-spin states $3d^5$ and $3d^6$ of the Mn $3d$ shell (the state $3d^4$ is proven to be irrelevant in our specific case of Mn in GaAs lattice, see Fig. 1). These impurity states are characterized by the maximum total spin quantum numbers $S^{(d)}=5/2$ at $3d^5$, and $S^{(d)}=2$ at $3d^6$, and the effective Hamiltonian $\hat{H}$ is defined in the charge sector $\{|d^5,N\rangle,|d^6,N-1\rangle\}$ of states with variable number N or $N-1$ of the electrons on the highest nth LL (of course, in our case, $N\approx N_\phi$). The Hamiltonian now reads

$$\hat{H}=\sum_\sigma \epsilon_{d\sigma}\hat{n}_{\gamma_0\sigma}+U\hat{n}_{\gamma_0\uparrow}\hat{n}_{\gamma_0\downarrow}+\sum_{m\sigma}\varepsilon_{n\sigma}a^\dagger_{nm\sigma}a_{nm\sigma}+\hat{H}'_{\mathrm{Coul}}+\hat{H}_t, \tag{2.2}$$

where the impurity Hamiltonian $\hat{H}_d$ of Eq. (2.1) is represented by the two first terms, in which $\hat{n}_{\gamma_0\sigma}=c^\dagger_{\gamma_0\sigma}c_{\gamma_0\sigma}$, and $c^\dagger_{\gamma_0\sigma}$ is the creation operator for the d electron at the orbital γ_0 with the spin z component σ. The notation γ_0 designates the only $3d$ orbital with the $Y_{02}\sim 3z^2-r^2$ symmetry, which effectively couples with the $m=0$ state of the LL.[25] The parameter U characterizes Coulomb and exchange interactions determining the addition energy for the transition $3d^5\rightarrow 3d^6$. The third term in Eq. (2.2) is the Hamiltonian of noninteracting Landau electrons, where $a^\dagger_{nm\sigma}$ is the creation operator

for the (n,m,σ) Landau state. Most of the interaction components are included in $\hat{H}'_{\mathrm{Coul}}$. This term does not include only the d-d interaction parametrized by U and the last term $\hat{H}_t$. The latter, generically, is also the part of the Coulomb interaction between impurity and Landau electrons which intermixes impurity and Landau orbitals. However, in our case, $\hat{H}_t$ acquires the form of a single-electron hybridization operator [see discussion after Eq. (2.13)],

$$\hat{H}_t = \sum_{\sigma} W_{n0} a^{\dagger}_{n0\sigma} c_{\gamma_0\sigma} + \mathrm{H.c.} \tag{2.3}$$

As was mentioned above, this operator is responsible for the resonance orbital-selective scatterings in QHF. It includes hybridization of the impurity electron with the 2DEG electrons within the nth LL. This means that only the influence of impurity on the intra-LL excitations (of the spin-wave type) is taken into account. The hybridization with the states with $n' \neq n$ describing the processes with energy change $\hbar\omega_c$ or higher is omitted.

In the absence of the interaction term $\hat{H}'_{\mathrm{Coul}}$, the Hamiltonian (2.2) acts in the subspace

$$|d^5, s; \mathrm{vac}\rangle, \quad \left| d^6, s+\frac{1}{2}; a_{n0\uparrow}|\mathrm{vac}\right\rangle,$$

and

$$|d^5, s+1; a^{\dagger}_{n0\downarrow} a_{n0\uparrow}|\mathrm{vac}\rangle, \tag{2.4}$$

where the fully polarized 2DEG without impurity is chosen to be the "vacuum" state $|\mathrm{vac}\rangle = |\uparrow,\uparrow,\ldots,\uparrow\rangle$. Therefore, $a^{\dagger}_{nm\uparrow}|\mathrm{vac}\rangle = a_{nm\downarrow}|\mathrm{vac}\rangle \equiv 0$. We represent the total spin component as $S_z = \frac{N_\phi}{2} + s$. Thus, we characterize the states in the set Eq. (2.4) by the quantum number S_z. It is important that only $S_z = S_z^{(s)} + S_z^{(d)}$ is an exact spin quantum number in our system. Separately, the Hamiltonian (2.2) commutes neither with the spin component $S_z^{(s)}$ of the LL electrons nor with the impurity spin component $S_z^{(d)}$. Equally, it does not commute with the total spin $\mathbf{S}^2$ and with the spins $(\mathbf{S}^{(s)})^2$ and $(\mathbf{S}^{(d)})^2$ (see Appendix B). The number s in the set Eq. (2.4) changes within the interval $-\frac{5}{2} \leqslant s < \frac{5}{2}$. It is convenient to choose the state $|d^5, \frac{5}{2}; \mathrm{vac}\rangle$ as a reference point ("global vacuum"). This state is not mixed with any other state of the system by the operator (2.3) so that it enters the set of eigenstates of the Hamiltonian (2.2), although at $g_{2\mathrm{DEG}} < 0$ it is one of the excited states of the system.

Within a given "triad" (2.4), i.e., at a given s, the operator (2.3) intermixes these basis states. The corresponding nondiagonal matrix elements are $\langle \mathrm{vac}; s, d^5 | \hat{H}_t | d^6, s+\frac{1}{2}; a_{n0\uparrow} | \mathrm{vac}\rangle$ and $\langle \mathrm{vac} | a^{\dagger}_{n0\uparrow}; s+\frac{1}{2}, d^6 | \hat{H}_t | d^5, s+1; a^{\dagger}_{n0\downarrow} a_{n0\uparrow} | \mathrm{vac}\rangle$, where the bra and ket vectors are appropriately normalized. Therefore, *for any given quantum number* $S_z = \frac{N_\phi}{2} + s$, the magnetic impurity scattering problem can be effectively described in terms of a *single-orbital* impurity model that involves only one or two $d\gamma_0$ electrons. The single-orbital basis

$$|s_-; \mathrm{vac}\rangle, \quad |s_0; a_{n0\uparrow}|\mathrm{vac}\rangle, \quad \text{and} \quad |s_+; a^{\dagger}_{n0\downarrow} a_{n0\uparrow}|\mathrm{vac}\rangle \tag{2.5}$$

arises instead of the original multielectron basis (2.4), where the indices $(-,\ 0,\ +)$ label the bare energies E_{s_-}, $E_{s_0} = E_{s_-} + U + \epsilon_{d\uparrow} - \varepsilon_{n\uparrow}$, and $E_{s_+} = E_{s_-} + (g_i - g_{2\mathrm{DEG}})\mu_B B$. Here, $g_i \mu_B B = \epsilon_{d\uparrow} - \epsilon_{d\downarrow}$ and $g_{2\mathrm{DEG}}\mu_B B = \varepsilon_{n\uparrow} - \varepsilon_{n\downarrow}$ are Zeeman energies for impurity and 2DEG, respectively. The two states $E_{s_\pm}$ form a Zeeman doublet for a given s, while the state E_{s0} becomes resonant with the LL continuum.

As a result of this mapping, where only one component γ_0 of the normalized multielectron states is responsible for the hybridization, the $\hat{H}_t$ operator can be redefined for each triad (2.5) as

$$\hat{\mathcal{H}}_t(S_z) = V[\beta_\uparrow(s) c^{\dagger}_{\uparrow} a_0 + \beta_\downarrow(s) c^{\dagger}_{\downarrow} b_0] + \mathrm{H.c.} \tag{2.6}$$

It becomes thereby *spin selective*. The shorthand notation $a_0 \equiv a_{n0\uparrow}$, $b_0 \equiv a_{n0\downarrow}$ is used here and below; $c^{\dagger}_{\uparrow/\downarrow}$ is the creation Fermi operator for the $s_\pm$ impurity states $|s_\pm; \mathrm{vac}\rangle = c^{\dagger}_{\uparrow/\downarrow}|\mathrm{vac}\rangle$ and $|s_0; \mathrm{vac}\rangle = c^{\dagger}_{\uparrow} c^{\dagger}_{\downarrow}|\mathrm{vac}\rangle$. The Clebsch-Gordan coefficients $\beta_\sigma(s)$ reflect the normalization of eigenvectors (2.4) by replacing them with normalized single-orbital basis (2.5). For $s = -5/2, -3/2, -1/2, 1/2$, and $3/2$, we have

$$\beta_\uparrow = \sqrt{\frac{1}{2} - \frac{s}{5}}, \quad \beta_\downarrow = \sqrt{\frac{7}{10} + \frac{s}{5}}. \tag{2.7}$$

The highest state in the bare Zeeman ladder $|(\frac{5}{2})_-; \mathrm{vac}\rangle \equiv |d^5, \frac{5}{2}; \mathrm{vac}\rangle$ remains nonhybridized.

Unlike the original Anderson model,[27] the mixing coefficient, $V \equiv W_{n0}$ in our particular case, arises as a nondiagonal component of the s-d Coulomb interaction (see Secs. II C and II D for further discussion).

B. Description of the employed simplifications

Here, we list the simplifications which have allowed us to reduce Eq. (2.1) to the Hamiltonian (2.2) with the following change $\hat{H}_t \to \hat{\mathcal{H}}_t$, and apply it to our system.

The first simplification exploits the fact that the characteristic Coulomb energy of Landau electrons $E_C = \alpha e^2/\kappa l_B$ is considered to be small in the QHF regime as compared to the cyclotron energy $\hbar\omega_c$. Here, α is the average form factor related to the finite thickness of the 2DEG $(0.3 \leqslant \alpha < 1)$. In the $E_C \ll \hbar\omega_c$ limit, one may neglect the LL mixing. Besides, it implies that, in our case, the energies of collective excitations are smaller than $\hbar\omega_c$.

The second simplification is related to the "deepness" of the $3d$ levels of a neutral Mn impurity relative to the bottom of the conduction band in GaAs. We know from the previous studies[25] that the scattering potential created by a TM impurity for the Landau electrons is generated by the s-d hybridization. It has a resonant character, and the spin-selective scattering becomes strong when one of the impurity $3d$ levels is close to the LLs of conduction electrons. The process of s-d hybridization may be represented by the so-called Allen reactions[23,24,33] (see Fig. 1)

$$3d^5 \to 3d^6 + h, \tag{2.8a}$$

$$3d^5 \rightarrow 3d^4 + e. \tag{2.8b}$$

The first of these reactions describes hopping of an electron from the filled Landau subband to the impurity d shell, whereas the second one means hopping of an electron from the d shell to a state in the empty Landau subband. It is known from the numerical calculations[32] that the addition energy for the sixth electron in the Mn $3d$ shell (e_-^{CFR} state in terms of Ref. 24) is in resonance with the states near the bottom of the GaAs conduction band. It really means that the values U and $\varepsilon_{d\uparrow}$ well compensate each other in the sum $\varepsilon_{d\uparrow}+U$. So, one may retain only the processes (2.8a) in $\hat{H}_t$ and neglect contributions of the $3d^5 \rightarrow 3d^4$ ionization.

The third major reduction of the Hamiltonian is the elimination of the impurity orbital degrees of freedom due to the selection rules for the s-d hybridization matrix elements.[25] This orbital selectivity arises, first, because of symmetry reasons since only electrons with equal axial m numbers in d and LL states are hybridized. Second, a precondition of the selectivity is related to the fact that the magnetic length l_B is much larger than the radius r_d of the $3d$-electron state (in the energy scale, this condition takes the form of inequality $U, \varepsilon_{d\sigma} \gg \hbar\omega_c$). The hybridization integral determined by the overlap of the d and Landau wave functions behaves as $\propto (r_d/l_B)^m \ll 1$ for $m \neq 0$. All resonance scattering (hybridization) amplitudes with $m \neq 0$ are, thus, negligibly small, and only the s-scattering term ($m=0$) can be retained in H_t. This explains why only one of the five $3d$ orbitals, namely, γ_0, is involved in the resonance interaction with the 2D Landau electrons.

C. Interaction Hamiltonian in excitonic representation

As was mentioned above, the states of the system are characterized by the total spin component S_z. For a given S_z, we may deduce the effective Hamiltonian

$$\hat{\mathcal{H}}(S_z) = \hat{H}_d + \hat{H}_1^{(s)} + \hat{\mathcal{H}}_t(S_z) + \hat{H}_{s\text{-}s} + \hat{H}_{s\text{-}d} \tag{2.9}$$

with the single-orbital impurity term $\hat{H}_d = \epsilon_{d\uparrow}\hat{n}_\uparrow + \epsilon_{d\downarrow}\hat{n}_\downarrow + U\hat{n}_\uparrow\hat{n}_\downarrow$ ($\hat{n}_\sigma = c_\sigma^\dagger c_\sigma$) and with the hybridization term determined by Eq. (2.6).

The remaining terms in the Hamiltonian (2.9) are defined within the framework of the single-LL approximation for the Landau electrons.[3–6,9–12,16,18,34–36] Although only the states with $m=0$ in the LL are involved in the resonance scattering, the complete basis for the description of collective excitations includes all m orbitals of the LL, and the corresponding Schrödinger field operators should be taken in the form

$$\hat{\Psi}_\uparrow(\mathbf{R}) = c_\uparrow\psi_d(\mathbf{R}) + \zeta_s(z)\sum_m a_m\varphi_m(\mathbf{r}),$$

$$\hat{\Psi}_\downarrow(\mathbf{R}) = c_\downarrow\psi_d(\mathbf{R}) + \zeta_s(z)\sum_m b_m\varphi_m(\mathbf{r}). \tag{2.10}$$

Here, the shorthand notation $a_m = a_{nm\uparrow}$, $b_m = a_{nm\downarrow}$ is used. $\mathbf{R} = (\mathbf{r}, z)$ is the three-dimensional (3D) coordinate with the reference point at the impurity site, $\zeta_s(z)$ is the size-quantized functions of s electrons in the layer, and φ_m is the wave function of the nth LL, where index m in the symmetric gauge changes within the interval $(-n, -n+1, \ldots, N_\phi - n - 1)$.

Using the above definitions and Eqs. (2.10) in the generic interaction operator

$$\hat{H}_{\mathrm{Coul}} = \frac{1}{2}\sum_{\sigma_1,\sigma_2=\uparrow,\downarrow}\int d^3R_1 d^3R_2 \hat{\Psi}^\dagger_{\sigma_2}(\mathbf{R}_2)\hat{\Psi}^\dagger_{\sigma_1}(\mathbf{R}_1)W(\mathbf{R}_1 - \mathbf{R}_2)\hat{\Psi}_{\sigma_1}(\mathbf{R}_1)\hat{\Psi}_{\sigma_2}(\mathbf{R}_2) \tag{2.11}$$

[where $W(\mathbf{R}) \approx e^2/\kappa R$ at $R \gg r_d$], one may rewrite the s-s and s-d Coulomb interactions in the *excitonic representation* (ER).[6,9,18] This actually means that after substitution of Eqs. (2.10) into formula (2.11), the latter can be expressed in terms of combinations of various components of the density-matrix operators. These are so-called ER operators presented in our case only by the intra-LL set, i.e., by the spin-exciton operators $\mathcal{Q}_{\mathbf{q}}^\dagger$, where an electron is promoted from one spin sublevel to another [see Refs. 6, 9, and 18, and Appendix A, where the necessary ER equations are given with reference to our case], and by operators $\mathcal{A}_{\mathbf{q}}^\dagger$ and $\mathcal{B}_{\mathbf{q}}^\dagger$ acting within the sublevels a or b (see ibidem). As a result, the Coulomb terms of Eq. (2.9) can be written only by means of the intrasublevel operators $\mathcal{A}_{\mathbf{q}}^\dagger$ and $\mathcal{B}_{\mathbf{q}}^\dagger$ [their definitions are given by Eq. (A4) in Appendix A],

$$\hat{H}_{s\text{-}s} = \frac{N_\phi}{2}\sum_{\mathbf{q}} W_{ss}(q)(\mathcal{A}_{\mathbf{q}}^\dagger\mathcal{A}_{\mathbf{q}} + 2\mathcal{A}_{\mathbf{q}}^\dagger\mathcal{B}_{\mathbf{q}} + \mathcal{B}_{\mathbf{q}}^\dagger\mathcal{B}_{\mathbf{q}}) - \frac{1}{2}(\mathcal{A}_0 + \mathcal{B}_0)\sum_{\mathbf{q}} W_{ss}(q), \tag{2.12}$$

$$\hat{H}_{s\text{-}d} = (\hat{n}_\uparrow + \hat{n}_\downarrow)\sum_{\mathbf{q}} W_{sd}(q)(\mathcal{A}_{\mathbf{q}} + \mathcal{B}_{\mathbf{q}}). \tag{2.13}$$

The Coulomb vertices are presented also in Appendix A [Eqs. (A12) and (A13)].

We neglect in Eq. (2.11) the direct exchange s-d interaction terms (see the next section). The mixing operator $\hat{\mathcal{H}}_t(S_z)$ in our model Hamiltonian (2.6) includes, in fact, off-diagonal interaction terms from $\hat{H}_{\mathrm{Coul}}$. Indeed, the Coulomb interaction described by the terms $\propto c_\downarrow^\dagger\hat{n}_\uparrow b_m + \mathrm{H.c.}$ and $\propto c_\uparrow^\dagger\hat{n}_\downarrow a_m + \mathrm{H.c.}$ induces transitions adding or removing one electron to the d center in accordance with the Allen reaction diagrams (2.8). These terms represent the s-d hybridization formally conditioned by the d-center occupation; however, since in our case the reaction (2.8b) is forbidden, they actually operate as $\propto c_\downarrow^\dagger b_m + \mathrm{H.c.}$ and $\propto c_\uparrow^\dagger a_m + \mathrm{H.c.}$ in Eq. (2.6), respectively. [In terms of the $d^5 \leftrightarrow d^6$ transitions, the hybridization is taken just in the form of Eq. (2.3).]

The single particle Hamiltonian for LL electrons may be also written in the ER representation,

$$\hat{H}_1^{(s)} = N_\phi[(\varepsilon_n - \varepsilon_Z/2)\mathcal{A}_0 + (\varepsilon_n + \varepsilon_Z/2)\mathcal{B}_0], \tag{2.14}$$

where $\varepsilon_Z = |g_{2\mathrm{DEG}}|\mu_B B$ and $\varepsilon_n = (n+1/2)\hbar\omega_c$.

D. Numerical estimates of the energy parameters

Before turning to our main task, i.e., to the calculation of excitation spectra, it is worthwhile to evaluate the character-

istic energy parameters related to this problem. We estimate the parameters of 2DEG in GaAs for the typical value $B=10$ T of magnetic field. In this field, $E_C \sim 5$ meV characterizes the Coulomb interaction (A13). Below, in our calculation, this value is mostly presented by the spin-exciton mass, which can be estimated empirically, i.e., the inverse mass is $1/M_x \sim 2$ meV in energy units. The LLs' spacing is $\hbar\omega_c \approx 16$ meV, and the Zeeman splitting between two Landau subbands is $\varepsilon_Z \approx 0.25$ meV (because $g_{2\mathrm{DEG}}=g_{\mathrm{GaAs}}\approx -0.44$). The Zeeman splitting for Mn ion is $g_i\mu_B B \approx 1.1$ meV (because $g_i=g_{\mathrm{Mn}}\approx 2.0$). The hybridization constant V and the repulsion U are the other important parameters characterizing the magnetic impurity. It is rather difficult to extract them from the available experimental data. We can only roughly estimate the energy U as a distance between the Mn-related peaks in the density of states of occupied and empty states in the spectrum of bulk (Ga,Mn)As, calculated while taking into account the electron-electron interaction.[32] Such an estimate gives $U \sim 4-4.5$ eV. From the same calculations, we estimate the energy difference

$$\Delta = \epsilon_{d\uparrow} + U - \varepsilon_n + \varepsilon_Z/2, \tag{2.15}$$

which determines the position of the Mn(d^6) electron level above the bottom of the Landau band [see Fig. 1(a)] as $\Delta \lesssim 0.1$ eV. In order to estimate the parameter V, one should recollect that the dominating contribution to the hybridization integral is given by the matrix elements of the Coulomb interaction, having the form $Vc_\uparrow^\dagger a_0 c_\downarrow^\dagger c_\downarrow$ (see above). This means that $V \sim U r_d^{3/2}\zeta(z_d)/l_B$. Estimating the radius of the ψ_d function as $r_d \sim 2$ Å, and $\zeta(z_d)\sim 0.15$ Å$^{-1/2}$ (for the impurity located in the vicinity of the quantum well bottom), one gets $V \sim 20$ meV. This gives $|V|^2/\Delta \sim 4-8$ meV for the relevant kinematic exchange parameter. At the same time, the direct exchange turns out to be insignificant. Indeed, one can estimate from Eq. (2.11) that the characteristic coupling constants for the terms $\propto c_\uparrow^\dagger c_\downarrow b_{m_1}^\dagger a_{m_2}$ and $c_\uparrow^\dagger c_\downarrow^\dagger b_{m_1} a_{m_2}$ are of the order of $U r_d^3 |\zeta(z_d)|^2/l_B^2$, being, therefore, by a factor $\sim \Delta/U$ smaller than $|V|^2/\Delta$.

III. COLLECTIVE SPIN-FLIP STATES: NEGATIVE $g_{2\mathrm{DEG}}$ FACTOR

The Coulomb interactions $\hat{H}_{s\text{-}s}$ and $\hat{H}_{s\text{-}d}$ admix the LL states with $m \neq 0$ to the three-state basis (2.5). Instead of triads (2.5), the basis

$$|s_-;\mathrm{vac}\rangle, \quad |s_0;a_m|\mathrm{vac}\rangle, \quad \text{and} \quad |s_+;\mathcal{Q}_{\mathbf{q}}^\dagger|\mathrm{vac}\rangle \tag{3.1}$$

contains spin-exciton continua $\mathcal{Q}_{\mathbf{q}}^\dagger|\mathrm{vac}\rangle$ attached to the spin-flipped impurity state s_+. [The definition of the spin-exciton creation operator is given by Eq. (A1).]

This set is complete only within the single-orbital approximation.[37] At a given s, it is convenient to take the energy $E_{0+}(s)$ of the state $|0\rangle=|s_+;\mathrm{vac}\rangle \equiv c_\uparrow^\dagger|\mathrm{vac}\rangle$ as the reference point, because this state is not affected by the hybridization within the framework of the single-orbital model. This energy is defined as $E_{0+}(s)=\langle \mathrm{vac}|;s+1,d^5|\hat{H}|d^5,s+1;|\mathrm{vac}\rangle$, where the Hamiltonian $\hat{H}$ is given by Eqs. (2.2) and (2.3). For a given $S_z=\frac{N_\phi}{2}+s$, we have $E_{0+}(s)=E_{\mathrm{vac}}-(\frac{5}{2}-s)g_i\mu_B B$, with E_{vac} defined as the energy of the global vacuum state $|d^5,5/2;|\mathrm{vac}\rangle$. One can check with the help of expressions (B1) and (B4) in Appendix B that the vectors $|s_0;a_m|\mathrm{vac}\rangle$ and $\mathcal{Q}_{\mathbf{q}}^\dagger|0\rangle$ at $\mathbf{q}\neq 0$ correspond to the definite total spin state with $S=S_z$, whereas $|s_-;|\mathrm{vac}\rangle$ and $\mathcal{Q}_0^\dagger|0\rangle$ are not characterized by any definite number S.[38]

A. Secular equation

Following the above discussion, the spin-flip operator may be represented in the form

$$\hat{X}^\dagger = c_\downarrow^\dagger c_\uparrow - \sum_m D_m c_\downarrow^\dagger a_m + \sum_{\mathbf{q}} f(\mathbf{q})\mathcal{Q}_{\mathbf{q}}^\dagger. \tag{3.2}$$

The normalizability condition $\langle X|X\rangle < \infty$ for the bound spin-exciton state $|X\rangle=X^\dagger|0\rangle$ then reads $\Sigma_m|D_m|^2+\Sigma_{\mathbf{q}}|f(\mathbf{q})|^2<\infty$, and the sum

$$N_b = \sum_{\mathbf{q}} |f(\mathbf{q})|^2 = \frac{N_\phi}{2\pi}\int d\mathbf{q}|f(\mathbf{q})|^2$$

presenting the contribution of continuous spin excitons into the norm $\langle X|X\rangle$ becomes thereby an essential characteristic of the spin-flip excitation. For the regular (normalizable) solutions, we expect $f(\mathbf{q}) \sim N_\phi^{-1/2}$. Besides, singular states for which the sum $\Sigma_{\mathbf{q}}|f(q)|^2$ diverges also exist. These states form a continuous spectrum of impurity-related spin excitons.

The coefficients D_m and $f(\mathbf{q})$ are determined from the equation

$$[\hat{\mathcal{H}},\hat{X}^\dagger]|0\rangle = E|X\rangle, \tag{3.3}$$

where the energy E is counted from $E_{0+(s)}$. Before turning to the computation, we specify the energy levels of the basis states (3.1) at $V=0$. The state $|s_-;\mathrm{vac}\rangle$ has the energy $E_{0-}(s)=E_{0+}(s)-g_i\mu_B B$. The doubly occupied impurity state $|d_0;a_m|\mathrm{vac}\rangle$ appears due to a charge transfer with the creation of a conventional "hole" in the LL. Its energy is $E_{d,m}(s)=E_{0+}+\mathcal{E}_{d,m}$, where

$$\mathcal{E}_{d,m} = \epsilon_{d\downarrow} + U + \varepsilon_Z/2 - \varepsilon_n + \epsilon_m + \mathcal{E}_\infty \tag{3.4}$$

[cf. Eq. (2.13)]. Here, $\mathcal{E}_\infty=(1/N_\phi)\Sigma_{\mathbf{q}}W_{ss}(q)$ [see Eq. (A11) for the definition of $\mathcal{E}_\infty$]. This term appears due to the global electroneutrality requirement when calculating the energy of the hole $a_m|\mathrm{vac}\rangle$.[4,5,9,11] The term $\epsilon_m=-(2/N_\phi)\Sigma_{\mathbf{q}}h_{m+n,m+n}(\mathbf{q})W_{sd}(q)$ is the Coulomb interaction energy of the hole $a_m|\mathrm{vac}\rangle$ with the doubly occupied d center [see Eq. (A2) for functions h_{ik}].

Substituting operators (2.9) and (3.2) into Eq. (3.3), projecting the result onto the basis vectors (3.1) and using Eqs. (2.6), (A3), (2.12)–(2.14), and (A5)–(A10), we obtain a closed system of equations for the coefficients D_m and $f(\mathbf{q})$. This system defines the eigenvalues of Eq. (3.3) for a given s. The symmetry of the problem allows us to look for the solutions in the form $f(\mathbf{q})=f_m(q)e^{im\phi}$. Below, we limit ourselves to a study of the isotropic case of $m=0$. (Discussion of

the case $m \neq 0$ may be found in Ref. 35.) As a result, we get $D_m = D_0 \delta_{m,0}$, and our system for a given $S_z = \frac{N_\phi}{2} + s$ acquires the simple form

$$E + g_i \mu_B B = \beta_\uparrow(s) V^* D_0,$$

$$(E - \mathcal{E}_{d,0}) D_0 = \beta_\uparrow(s) V - \beta_\downarrow(s) V N_\phi^{-1/2} \sum_{\mathbf{q}} h_{nn}^*(q) f(q),$$

$$(E - \varepsilon_Z - \mathcal{E}_q) f(q) = -N_\phi^{-1/2} h_{nn}(q) \beta_\downarrow(s) V^* D_0. \quad (3.5)$$

The energy of the free exciton state $\mathcal{Q}_{\mathbf{q}}^\dagger |0\rangle$ is $\varepsilon_Z + \mathcal{E}_q$ [see Eq. (A11)].

The collective states localized around a magnetic impurity are described by solutions of Eq. (3.5) outside the free spin-wave band (i.e., in the energy interval $E < \varepsilon_Z$ or $E > \varepsilon_Z + \mathcal{E}_\infty$). The corresponding eigenfunctions are characterized by the regular envelope function $f_0(q)$. We arrive then at the secular equation

$$\frac{\beta_\downarrow^2(s)}{N_\phi} \sum_{\mathbf{q}} \frac{|h_{nn}(q)|^2}{E - \varepsilon_Z - \mathcal{E}_q} + \frac{\beta_\uparrow^2(s)}{E + g_i \mu_B B} = \frac{E - \mathcal{E}_{d,0}}{|V|^2} \quad (3.6)$$

for the energy E. The first term in the left-hand side (lhs) of Eq. (3.6), including the sum of spin-exciton propagators, presents the self-energy, which usually arises in the Schrödinger or Lippmann-Schwinger equation describing the perturbation introduced by a short-range potential into the continuous spectrum. The prototype of this term in the theory of magnetic defects is the self-energy for localized spin waves in the Heisenberg ferromagnet with a single substitution impurity.[39] Specific features of our model are manifested by the energy dependence of impurity-related processes. First, instead of a constant term (inverse impurity potential) in the right-hand side (rhs) of Eq. (3.6), we have the inverse resonance potential $|V|^2/(E - \mathcal{E}_{d,0})$, which stems from the hybridization between LLs and the $3d$ level of impurity electron.[28,29] Second, an additional term describing an impurity spin-flip process in terms of the single-orbital model arises in the lhs of Eq. (3.6).

B. Spectrum of the localized states

First, we carry out a simple study considering solutions of Eq. (3.6) in the absence of an exciton band, i.e., by formally substituting $\mathcal{E}_q = 0$ into Eq. (3.6). (This is instructive in order to classify the bound collective states.) We obtain then a simple algebraic equation with two roots. When solving this equation, we use the sum rule $\Sigma_{\mathbf{q}} |h_{nn}(q)|^2 = N_\phi$ and neglect the energy dependence in the rhs due to the condition $\mathcal{E}_{d,0} \approx \Delta \gg E$. Each doublet is bound to its own reference energy $E_{0+}(s)$ in accordance with the corresponding spin component $S_z^{(d)} = s + 1$ of the $Mn^{(+2)}$ ion. Due to the kinematic exchange (second order spin-flip processes), each state in the Zeeman grid [lower root of Eq. (3.6)] acquires a partner state [upper root of Eq. (3.6)], except for the highest level with $s = 5/2$ which remains intact, because the spin-flip processes are kinematically forbidden for this state. The level splitting is illustrated by the scheme in Fig. 2. We see that the kinematic exchange makes the Zeeman states of impurity ion nonequidistant, and an additional multiplet of excited states arises as a prototype of the bound spin excitons.

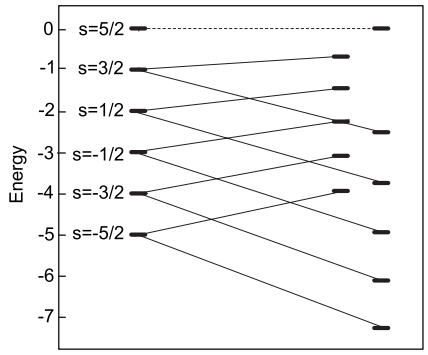

FIG. 2. A scheme of the $Mn^{(+2)}$ Zeeman level splitting due to the kinematic exchange in the absence of exciton dispersion. The bare Zeeman ladder is shown on the left. Five of six levels in this grid are shifted down (extreme right column), whereas the $s = 5/2$ level remains not renormalized. Each of these five levels has its high-energy counterpart. The energy is measured in the $g_i \mu_B B$ units. The following values of the input parameters are chosen: $\varepsilon_Z = 0.2$, $|V|^2/\Delta = 2$, and $\mathcal{E}_{d,0} = \Delta \gg 1$. The factors $\beta_\downarrow^2(s)$ and $\beta_\uparrow^2(s)$ are presented by Eq. (2.7).

Having this classification in mind, we turn to calculating the bound exciton states for a finite dispersion of the free spin waves. According to the estimates of the model parameters presented in Sec. II C, we solve Eq. (3.6) for the realistic conditions $E_C \gtrsim g_i \mu_B B \gg \varepsilon_Z$, whereas the ratio between the energies E_C and $|V|^2/\Delta$ may be arbitrary.

All the generic features of impurity-related states may be seen in the case of a unit filling where $n = 0$ ($\nu = 1$), and we study this situation in detail. The solutions we are looking for are localized in the energy interval $|E - \mathcal{E}_{d,0}| \approx \Delta$, so one can neglect the energy dependence in the rhs of Eq. (3.6). A graphical solution of Eq. (3.6) is schematically shown in Fig. 3.

Two intersection points labeled $E_0^{(s)}$ and $E_x^{(s)}$ correspond to two discrete solutions. Just as in Fig. 2, this pair of solutions

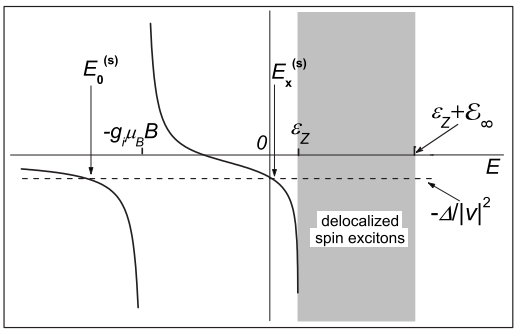

FIG. 3. Graphical solution of the secular equation. The lhs and rhs of Eq. (3.6) are shown as functions of the argument E by solid and dashed lines, respectively (E dependence in the rhs is neglected). The filled area indicates possible values of the lhs because it belongs to the interval of E where the sum in Eq. (3.6) becomes indefinite.

arises at any s except for $s=5/2$. The lower solution with the energy $E_0^{(s)}$ is the state of the $Mn^{(+2)}$ ion with the spin component $\langle \hat{S}_z^{(d)} \rangle \approx s$ shifted downwards from the value $E_{\rm vac}+(s-\frac{5}{2})g_i\mu_B B$ by the effective exchange interaction with the spin-wave continuum (in this case, $D_0 > N_b$). The upper solution corresponds to the spin-flipped state of the $Mn^{(+2)}$ ion with $\langle \hat{S}_z^{(d)} \rangle \approx s+1$ dressed with the spin-wave localized on the impurity. In this case, $\langle \hat{S}_z^{(s)} \rangle \approx \frac{N_\phi}{2}-1$ and $N_b > D_0$. This bound exciton state, described semiphenomenologically in Ref. 35, is shallow compared with the main characteristic energy parameter E_C. Like in many other impurity-related states in 2DEG,[21,25,40] its energy is confined within the interval $-g_i\mu_B B < E_x^{(s)} < \varepsilon_Z$ in the logarithmic vicinity of the bottom of the delocalized spin-exciton band. Due to this fact, one may find the level position analytically. Using the quadratic approximation for the exciton dispersion law $\mathcal{E}_q = q^2/2M_x$ and turning from summation to integration in the lhs of Eq. (3.6), one has

$$\frac{\beta_\downarrow^2}{N_\phi}\sum_{\mathbf{q}\neq 0}\frac{|h_{00}(q)|^2}{E-\varepsilon_Z-\mathcal{E}_q} \approx \beta_\downarrow^2 M_x \ln[\gamma M_x(|\varepsilon_Z - E|)]. \quad (3.7)$$

Here, M_x, is the spin-exciton mass defined as $1/M_x = \int_0^\infty dpp^3 v_{ss}(p)e^{-p^2/2}/2 \sim E_C$ [see Eqs. (A11) and (A13); the $l_B=1$ unit is used] and $\gamma=1.781\ldots$. Then the binding energy

$$E_x^{(s)} = \varepsilon_Z - \frac{1}{\gamma M_x}\exp\left(-\frac{\beta_\uparrow^2}{\beta_\downarrow^2 M_x g_i\mu_B B} - \frac{\Delta}{\beta_\downarrow^2 M_x |V|^2}\right) \quad (3.8)$$

is found from Eq. (3.7). This result is valid provided at least one of the two inequalities, $\beta_\downarrow^2 M_x g_i\mu_B B \ll \beta_\uparrow^2$ or $\beta_\downarrow^2 M_x |V|^2/\Delta \ll 1$, holds, which is not too strict a requirement due to the exponential smallness of the second term in the rhs of Eq. (3.8).

The asymptotic value of the lower state $E_0^{(s)}$ is also easily found. In the case of strong hybridization $|V|^2/\Delta \gg E_C$, one gets $E_0^{(s)} \approx -g_i\mu_B B - 5|V|^2/6\Delta$. In this asymptotic limit, the excitation energy does not depend on s. In the opposite limit $|V|^2/\Delta \ll E_C$, we have $E_0^{(s)} \approx -g_i\mu_B B - \beta_\uparrow^2(s)|V|^2/\Delta$.

In the intermediate region $|V|^2/\Delta \sim E_C$, Eq. (3.6) for $E_0^{(s)}$ can be solved numerically. It is convenient to rewrite this equation in the dimensionless form

$$\beta_\downarrow^2(s)\int_0^\infty \frac{e^{-q^2/2}qdq}{F^{(s)}-\xi e(q)} + \frac{\beta_\uparrow^2(s)}{F^{(s)}+g} + 1 = 0, \quad (3.9)$$

where $\xi = \Delta/M_x|V|^2$ is the ratio of the characteristic Coulomb energy in the Landau band and the characteristic kinematic exchange energy. The relevant energy parameters in Eq. (3.6) are redefined as $E=(|V|^2/\Delta)F^{(s)}(\xi)$, $\varepsilon_Z+\mathcal{E}_q = M_x^{-1}e(q)$, and $g_i\mu_B B = (|V|^2/\Delta)g$. Then the system of localized levels $\tilde{E}_{0,x}^{(s)}$ counted from the global vacuum energy is described by the set of equations

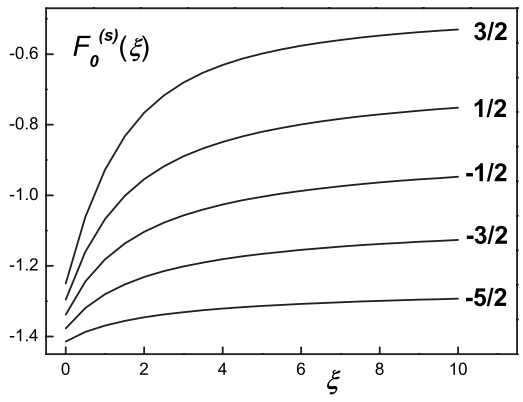

FIG. 4. The lower root of Eq. (3.9) with $g=0.25$. The numbers s are indicated near the curves. See text for further details.

$$\tilde{E}_{0,x}^{(s)} = -g_i\mu_B B(3/2-s) + F_{0,x}^{(s)}(\xi)|V|^2/\Delta, \quad (3.10)$$

with $s=-5/2$, $-3/2$, $-1/2$, $1/2$, and $3/2$. The family of lower roots $F_0^{(s)}(\xi)$ of Eq. (3.9), changing smoothly from $-\beta_\uparrow^2 - g$ at $\xi=\infty$ to approximately $-6/5-5g\beta_\uparrow^2/6$ at $\xi=0$, describes the renormalization of the Zeeman grid of impurity spin-flipped states due to the kinematic exchange with LL continuum. To illustrate this dependence, we have found the solution of Eq. (3.10) for $g=0.25$, neglecting ε_Z and modeling the spin-exciton dispersion by the function $e(q)=2-2e^{-q^2/4}I_0(q^2/4)$, which corresponds to the ideal 2D case.[3–5] (At the same time, the parameter M_x may be considered as an empirical value.) The results of this calculation are presented in Fig. 4.

C. Delocalized impurity-related excitations

We conclude this section by a brief discussion of the delocalized states (free spin waves distorted by the resonance magnetic impurity scattering). These states are described by the functions $f(q)$ with a divergent norm in the expansion (3.2). The secular equation for these states cannot be presented in the form (3.6), but there are solutions satisfying Eqs. (3.5) at any energy within the spin-exciton band, $\varepsilon_Z < E < \varepsilon_Z + \mathcal{E}_\infty$. These states are the "counterparts" of the levels $E_x^{(s)}$ in the spin-wave continuum. Let $q_0(E)$ be a root of equation $\varepsilon_Z + \mathcal{E}_{q_0} = E$. Substituting

$$f(q) = C\frac{\sqrt{2\pi}}{4q_0}\delta_{|\mathbf{q}|,q_0} + u(q)(1-\delta_{|\mathbf{q}|,q_0}) \quad (3.11)$$

into Eqs. (3.5), one gets three equations for the coefficients D_0, C, and $u(q)$. Turning from summation to integration and using the rule $\Sigma_{\mathbf{q}}\delta_{|\mathbf{q}|,q_0} = 2q_0L/\pi$ ($L^2 = 2\pi N_\phi$ being the 2DEG area), one finds the coefficient $u(q)$ from the equation $\beta_\uparrow^2(E-\varepsilon_Z-\mathcal{E}_q)u(q) = -\beta_\downarrow^2 N_\phi^{-1/2}h_{nn}(q)(g_i\mu_B B+E)$. Then equation

$$Ce^{-q_0^2/4} = 1 + \frac{(E + g_i\mu_B B)}{\beta_\uparrow^2}\left(\beta_\downarrow^2 \int_0^\infty \frac{qdq|h_{nn}|^2}{E - \varepsilon_Z - \mathcal{E}_q} + \frac{\mathcal{E}_{d,0} - E}{|V|^2}\right) \tag{3.12}$$

for the spectrum is derived from Eq. (3.5) in the thermodynamic limit ($L, N_\phi \to \infty$). It can be readily seen that the norm of the function (3.11) diverges as $\Sigma_{\mathbf{q}}|u(q)|^2 \sim N_\phi$.

IV. POSITIVE g_{2DEG} FACTOR: PINNING OF THE QUANTIZED HALL FERROMAGNET SPIN

Experimentally, the magnitude of the g_{2DEG} factor in GaAs/Al_xGa_{1-x}As structures can be altered gradually by changing pressure or by varying Al content (x). It can be made very small and even change its sign.[14] The value of g_{2DEG} may be effectively reduced also due to optical orientation of nuclear spins, changing the electron Zeeman splitting (Overhauser shift).[13,17] In this section, we discuss the impurity-related reconstruction of the ground state and the spectrum of spin-flip excitations at small but positive values of g_{2DEG}. It will be shown below that even a minute amount of magnetic impurities can drastically influence the QHF state.

Keeping the previous notations, it is now convenient to redirect the magnetization axis ($\hat{z} \to -\hat{z}$), i.e., to make formal transformation $g_i \to -g_i$ instead of changing the sign of g_{2DEG}. It is clear that at least in the absence of the s-d hybridization, the global vacuum state $|d^5, 5/2; |\text{vac}\rangle$ serves as the ground state, and all the spin flips cost positive energy. The localized states can still be found from Eq. (3.6) with redefined Zeeman energies, $g_i\mu_B B \to -g_i\mu_B B$ and $\varepsilon_Z \to \varepsilon_Z^*$. The latter parameter actually takes the values $\varepsilon_Z^* = g_{2DEG}^*\mu_B B \gtrsim 0.1$ K. Making change $g \to -g$ in Eq. (3.9) we denote the lower root of this new equation as $F_{\underline{x}}^{(s)}$. This root corresponds to the energy of the localized spin exciton with changed impurity spin projection, $\delta S_z^{(d)} \approx 3/2 - s$, where $s = \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}$, and $-\frac{5}{2}$. The total spin component is $S_z = -\frac{N_\phi}{2} - s$ (when presenting results, we return to the "normal" coordinate system where $\hat{z}$ is directed along $\vec{B}$), and the energy counted off the global vacuum level is given by the formula

$$\tilde{E}_{\underline{x}}^{(s)} = g_i\mu_B B(3/2 - s) + F_{\underline{x}}^{(s)}(\xi)|V|^2/\Delta. \tag{4.1}$$

(It should be noted that now the new global vacuum is really below the old one by the energy $5g_i\mu_B B$.) Functions $F_{\underline{x}}^{(s)}(\xi)$ are presented in Fig. 5.

Other roots of the secular equation belong to the continuous spectrum. These states may be analyzed following the approach described in Sec. III C. The special "resonance" solution of Eq. (3.12) with g_i substituted for $-g_i$ arises, in this case, at $E = g_i\mu_B B > \varepsilon_Z^*$. Then $u(q) = 0$ and the norm $\langle X|X\rangle$ diverges not as $\sim N_\phi$ but as $L \sim N_\phi^{1/2}$ (see discussion in the next section). As a function of s, the delocalized resonance states form a set of equidistant levels

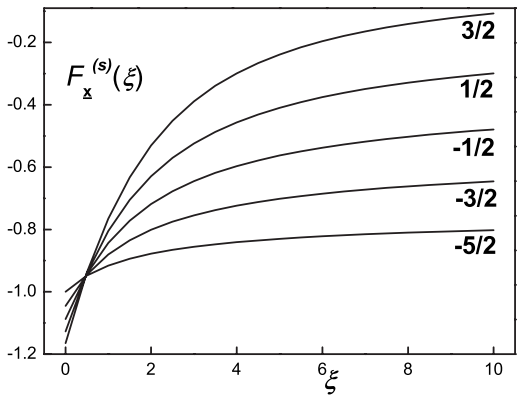

FIG. 5. The lower root of Eq. (3.9) with negative parameter g. Calculation is performed for $g = -0.25$ and $e(q) = 2 - 2e^{-q^2/4}I_0(q^2/4)$. The values of s are indicated near the curves.

$$\tilde{E}_{\text{res}}^{(s)} = g_i\mu_B B\left(\frac{5}{2} - s\right) \tag{4.2}$$

(again the energy of the global vacuum is taken as the reference level).

When looking for the $\underline{x}$-type solutions at $E < \varepsilon_Z^*$ but $|E| \ll 1/M_x$, one may use Eq. (3.7). Then one obtains for the localized spin-exciton energy[41] $E_{\underline{x}}^{(s)} = |V|^2 F_{\underline{x}}^{(s)}(\xi)/\Delta$ the following equation:

$$E_{\underline{x}}^{(s)} \approx \varepsilon_Z^* - \frac{1}{\gamma M_x}\exp\left(\frac{\beta_\uparrow^2}{\beta_\downarrow^2 M_x g_i\mu_B B} - \frac{\Delta}{\beta_\downarrow^2 M_x |V|^2}\right) \tag{4.3}$$

instead of Eq. (3.8). Here, $s = \frac{3}{2}$ has to be taken for the excitation from the ground state, then $\beta_\uparrow^2 = \frac{1}{5}$ and $\beta_\downarrow = 1$. The exponentially small energy $E_{\underline{x}}^{(3/2)}$ thus corresponds to the formation of a bound spin exciton of large radius. However, for sufficiently small ε_Z^* (or for a strong enough kinematic exchange), the energy $E_{\underline{x}}^{(3/2)}$ becomes negative, which means that an instability of the global vacuum $|d^5, 5/2; |\text{vac}\rangle$ is considered as the QHF ground state. This instability appears provided

$$\xi < \xi_{c1}, \tag{4.4}$$

where ξ_{c1} is determined by the equation

$$|V|^2 F_{\underline{x}}^{(3/2)}(\xi_{c1})/\Delta + \varepsilon_Z^* = 0. \tag{4.5}$$

The question that arises is whether the condition (4.4) means the global reconstruction of the ground state and the appearance of a new state with *many* spin excitons bound to the magnetic impurity. To clarify this point, we discuss the limiting situation where $\varepsilon_Z^* \to 0$ but still $N_\phi \varepsilon_Z^* \to \infty$. Then the ground state at any ξ is no longer the global vacuum because the creation of one spin exciton bound to the impurity lowers the energy of the system. The corresponding energy gain compared to the global vacuum is presented as $|V|^2 G_1(\xi)/\Delta$. [The subscript "1" corresponds to one bound exciton; specifically, we have $G_1(\xi) = F_{\underline{x}}^{(3/2)}$.] To answer the question, one should consider the situation with K captured spin excitons

(then $S_z=K-\frac{N_\phi+5}{2}$) and calculate the proper value $G_K(\xi)$ at arbitrary K. The latter is determined by the competition between the antiferromagnetic kinematic exchange, which forces 2DEG spins to reorient in the direction opposite to the impurity spin, and the Coulomb-exchange energy appearing due to 2DEG inhomogeneity in a cluster of K spin excitons bound to the impurity. This inhomogeneity energy is measured in $1/M_x$ units. The calculation of G_K at $K\gtrsim 1$ (but $K\neq 1$) is beyond the abilities of our present approach, but we can consider the case of $K\gg 1$ and find the conditions under which such a massive pinning of 2DEG spins in the vicinity of the impurity turns out more advantageous than the binding of single spin exciton (i.e., $G_\infty > G_1$).

A. Skyrmionic states created by magnetic impurities

The state with $K\gg 1$ can be described as a collective topological defect (skyrmion) pinned to a magnetic impurity.[36] A smooth inhomogeneity in the system of spins may be presented as a continuous rotation in the 3D space. If one characterizes the local direction of the spins by a unit vector $\vec{n}(\mathbf{r})$ with components $n_x=\sin\theta\cos\varphi$, $n_y=\sin\theta\sin\varphi$, and $n_z=\cos\theta$ (φ and θ are the two first Eulerian angles), then the conditions $\theta|_{\mathbf{r}=0}=0$ and $\theta|_{\mathbf{r}=\infty}=\pi$ inevitably result in the appearance of the topological invariant $q_T=\int d\mathbf{r}\rho(\mathbf{r})$, where the density

$$\rho(\mathbf{r})=\frac{1}{4\pi}\vec{n}\cdot(\partial_x\vec{n})\times(\partial_y\vec{n}) \tag{4.6}$$

is a vortex characteristic of the spatial twist. The value q_T has to be a nonzero integer number.[42] Its physical meaning is the number of excessive ($q_T<0$) or deficient ($q_T>0$) electrons in the system,[11,12,16,18] i.e., $q_T=N_\phi-N$. In a perfect 2DEG and at nearly zero Zeeman gap ($\varepsilon_Z^*\to 0$), such a weakly inhomogeneous skyrmion state has the energy

$$\mathcal{E}_{\mathrm{sk}}=\frac{3}{4}\mathcal{E}_\infty q_T+\frac{1}{2M_x}(|q_T|-q_T). \tag{4.7}$$

This result is valid within the single Landau level approximation (see, e.g., Ref. 18). It is enough to consider the case $q_T=\pm 1$, because any state with $|q_T|>1$ is merely a combination of "singly charged" skyrmions. Due to the hybridization with the impurity, the skyrmionic state gains a negative kinematic exchange energy. The latter has to be taken into account in combination with the Coulomb-exchange energy (4.7) and with the finite positive Zeeman energy at $g^*_{\mathrm{2DEG}}>0$,

$$E_Z=\varepsilon_Z^* K \quad \text{where} \quad K=\frac{1}{4\pi l_B^2}\int(1+\cos\theta)d\mathbf{r} \tag{4.8}$$

(in the clean 2DEG, the skyrmion energy is given by $\mathcal{E}_{\mathrm{sk}}+E_Z$).

One can conclude from symmetry considerations that the impurity is located at the center of the topological defect. Then additional pinning energy may be found by means of the conventional energy minimization procedure where the Euler angles are used as variational parameters. This energy is the difference between the energy of the global vacuum state with a distant skyrmion and the ground state energy calculated in the presence of a magnetic impurity at the center of the topological defect (cf. Ref. 36, where a similar procedure was elaborated in the limit of potential impurity scattering). Namely, to calculate the contribution of magnetic impurity at $K\gg 1$, one should consider a domain around an impurity which is small in comparison with a characteristic area of the skyrmion, but contains a large enough number of spin-flipped LL electrons involved in the formation of pinned topological defect. Then the situation becomes similar to that considered in Sec. III: *s-d* hybridization of the impurity electron with the $m=0$ electron in this domain generates the kinematic exchange in accordance with Fig. 1, and leads to the reconstruction of the spectrum in accordance with Eq. (3.6). The shift of the energy with respect to the global vacuum is given by the value $5g_i\mu_B B+\tilde{E}_0^{(-5/2)}$, where $\tilde{E}_0^{(-5/2)}$ is determined by Eq. (3.10).[43] Hence, we obtain that the pinning energy is $E_{\mathrm{sk,pin}}=-g_i\mu_B B-F_0^{(-5/2)}(\xi)|V|^2/\Delta$, where $F_0^{(-5/2)}$ is shown in Fig. 4. In the limit of strong pinning ($E_{\mathrm{sk,pin}}\gg\mathcal{E}_{\mathrm{sk}}$) and "frozen" impurity spin ($g\gg 1$), this result agrees with the pinning energy found earlier.[36]

The energy $E_{\mathrm{sk,pin}}$ is calculated in the leading approximation, which does not depend on the charge q_T. However, it is instructive to obtain the correction related to the inhomogeneity of the texture. It is known[16,18] that the density (4.6) may be interpreted in terms of an effective magnetic field appearing in the Schrödinger equation due to this inhomogeneity. One may introduce the renormalized magnetic length $l_B\to\tilde{l}_B$ as

$$\frac{1}{\tilde{l}_B^2}=\frac{1}{l_B^2}-2\pi\rho(\mathbf{r}). \tag{4.9}$$

Taking into account that $|V|^2/\Delta\sim 1/l_B^2$ and $\xi\sim l_B$, and rewriting Eqs. (4.6) and (4.9) in terms of the Euler angles,[42] one finds the correction to pinning energy due to the finite radius R^* of the skyrmion core (see Ref. 36 for a detailed calculation). The corrected energy is determined by the value $\rho(0)$ and has the form

$$E_{\mathrm{sk,pin}}^{(q_T)}=-g_i\mu_B B-\frac{|V|^2}{\Delta}\left[F_0^{(-5/2)}(\xi)-q_T\left(\frac{l_B}{R^*}\right)^2\times\left(2F_0^{(-5/2)}-\xi\frac{dF_0^{(-5/2)}}{d\xi}\right)\right],\quad q_T=\pm 1. \tag{4.10}$$

It is assumed here that $g\ll 1$.

The skyrmion core radius R^* is found by considering the competition between the Zeeman energy (4.8) and the energy of Coulomb repulsion.[11,44] Generally speaking, in our case, in order to find R^*, we should include the energy $E_{\mathrm{sk,pin}}$ in the minimization procedure. However, this correction only insignificantly influences the result due to the condition $R^*\gg l_B$ and because of the rather strong *e-e* interaction resulting in the skyrmion formation. Using the realistic estimate for the kinematic exchange energy $|V|^2/\Delta\lesssim E_C$, the minimization yields the same formula

$$E_Z = \frac{\varepsilon_Z^*}{2}\left(\frac{R^*}{l_B}\right)^2 \ln\left(\frac{l_B^2 E_C}{\varepsilon_Z^* R^{*2}}\right) \tag{4.11}$$

as in the case of "free" skyrmions,[44] where $R^{*3} = 9\pi^2 l_B^2 e^2/[64\varepsilon_Z^* \kappa \ln(E_C/\varepsilon_Z^*)]$. The number of spin-flipped electrons turns out to be rather large

$$K = \frac{1}{96}\left(\frac{9\pi^2 e^2}{\kappa \varepsilon_Z^* l_B}\right)^{2/3}\left[\ln\left(\frac{E_C}{\varepsilon_Z^*}\right)\right]^{1/3} \sim 10-20 \quad (\text{if } \varepsilon_Z^* \sim 0.1 \text{ K}). \tag{4.12}$$

We first consider the regime where there are no skyrmions in the clean system, but these collective excitations could be created due to strong enough kinematic exchange interaction between the LL electrons and magnetic impurities. This is the situation where the inequality (4.4) is valid, and besides, the condition $|N-N_\phi| \ll N_i$ is realized, where N_i is the number of impurities. The electroneutrality of the system requires that the topological defects are created as skyrmion-antiskyrmion pairs. Two impurities are able to create a skyrmion-antiskyrmion pair provided the pinning energy $E_{\text{sk,pin}}^{(+)} + E_{\text{sk,pin}}^{(-)}$ exceeds the energy increase due to the skyrmion-antiskyrmion gap. The latter, in accordance with Eq. (4.7), includes the Coulomb-exchange part equal to M_x^{-1} and twice the Zeeman energy [Eqs. (4.8) and (4.12)]. In addition, the energy of a skyrmion and an antiskyrmion pinned by two neighboring magnetic impurities has to be lower than the double energy of a pinned spin exciton. Thus, the condition $G_\infty < G_1$ for the creation of a pinned skyrmion-antiskyrmion pair can be rewritten in the form

$$\xi < \xi_{c,\infty}, \tag{4.13}$$

where the critical value $\xi_{c,\infty}$ can be obtained with the help of Eq. (4.10):

$$g + F_0^{(-5/2)}(\xi_{c,\infty}) + \xi_{c,\infty}/2 + E_Z \Delta/|V|^2 = F_{\underline{x}}^{(3/2)}(\xi_{c,\infty}). \tag{4.14}$$

Under the condition (4.13), an impurity acquires the localized magnetic moment $K \sim B^{-1/3}$ antiparallel to its own moment and exceeding it (when, e.g., $K > 5/2$ in the GaAs:Mn case).

B. Phase diagram of quantized Hall ferromagnet ground state at $g_{2\text{DEG}}^* > 0$

There are two critical transitions in our problem: first, the global vacuum is destroyed when ξ becomes less than ξ_{c1} and single spin-flip exciton appears (this state may be characterized as a "local pinning"); second, the massive pinning of 2DEG spins takes place when ξ reaches the value $\xi_{c,\infty}$. However, this scenario is somewhat changed if one takes into account finite ratios N_i/N_ϕ. Indeed, up to this point, we have supposed that the Zeeman energy $\varepsilon_Z^* N_\phi$ corresponding to the "global flip" of all 2DEG spins is larger than any contribution to the QHF energy due to the magnetic impurities. This global spin flip actually represents the spin configuration treated as the ground state in the previous section. When counted from the global vacuum, its energy per impurity is $E_{\text{sf}} = g_i \mu_B B + (N_\phi \varepsilon_Z^*/N_i) + |V|^2 F_0^{(-5/2)}(\xi)/\Delta$. Negative E_{sf} means that available magnetic impurities are able to polarize completely all 2DEG electrons even at positive $g_{2\text{DEG}}^*$. In agreement with the above discussion, one can conclude that such a complete polarization takes place when

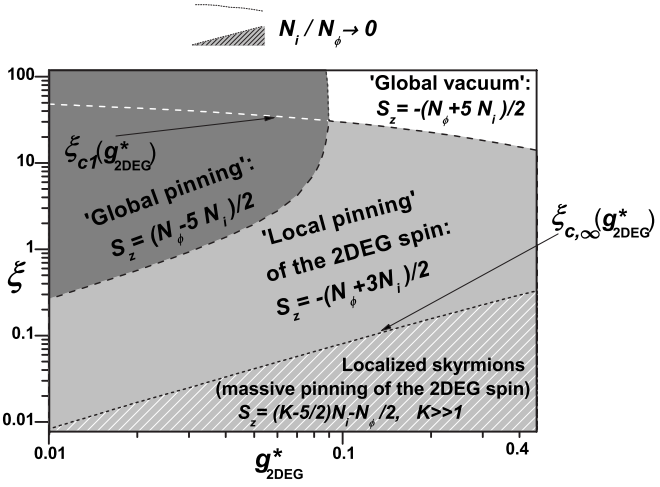

FIG. 6. Phase diagram illustrating the reconstruction of the QHF ground state at $g_{2\text{DEG}}^* > 0$ for two cases: isolated impurity $N_i/N_\phi \to 0$ (see elucidating legend above the main picture) and finite impurity concentration $N_i/N_\phi = 0.01$. The calculation was carried out for the Zeeman parameters $g = g_i \mu_B B \Delta/|V|^2 = 0.25$ and $\varepsilon_Z^* = 0.05(g_{2\text{DEG}}^*/0.44)|V|^2/\Delta$, and for the spin-exciton dispersion equal to $\xi e(q) = 0.05(g_{2\text{DEG}}^*/0.44) + 2\xi[1 - e^{-q^2/4} I_0(q^2/4)]$ in $|V|^2/\Delta$ units. Comments in the figure refer to the $N_i/N_\phi = 0.01$ case. See text for other details.

$$E_{\text{sf}}(\xi) < E_{\min}(\xi), \tag{4.15}$$

where

$$E_{\min} = \min\{0,\ |V|^2 F_{\underline{x}}^{(3/2)}(\xi)/\Delta + \varepsilon_Z^*, \\ g_i \mu_B B + |V|^2 F_0^{(-5/2)}(\xi)/\Delta + (1/2) M_x^{-1} + E_Z\}.$$

The phase diagram of our system at zero temperature is determined by the interplay between Zeeman splitting, Coulomb interaction, and kinematic impurity exchange energy, and controlled by the impurity concentration. These factors are characterized by the dimensionless parameters $g_{2\text{DEG}}^*$, ξ, and N_i/N_ϕ. One can construct this diagram by employing the inequalities (4.4), (4.13), and (4.15). The results for both cases of infinitely small and finite ratio N_i/N_ϕ are presented in Fig. 6 in the $(g_{2\text{DEG}}^*, \xi)$ coordinates. We expressed the $e-e$ interaction values entering the skyrmion Zeeman energy [Eqs. (4.11) and (4.12)] in terms of the parameter M_x: $E_C = M_x^{-1}$ and $e^2/\kappa l_B = M_x^{-1}(8/\pi)^{1/2}$. The phase diagram for the $N_i/N_\phi \to 0$ case is explicated by the legend above the main picture. The $\xi = \xi_{c1}(g_{2\text{DEG}}^*)$ curve in Fig. 6 separates states with unbroken global vacuum (the area above this line) and states of local pinning, where each impurity is dressed by one bound spin exciton. The dotted line $[\xi_{c,\infty}(g_{2\text{DEG}}^*)$ curve] separates the state with local pinning and the state of massive spin reversal ($K \gg 1$) determined by the pinned skyrmions (hatched area below this line).

In the more realistic case of $N_i/N_\phi=0.01$, the curves $\xi_{c1}(g^*_{2\mathrm{DEG}})$ and $\xi_{c,\infty}(g^*_{2\mathrm{DEG}})$ formally remain the same since the parameter N_i/N_ϕ does not enter Eqs. (4.5) and (4.14). However, in this case, an essential part of the $(g^*_{2\mathrm{DEG}},\xi)$ area belongs to states where the 2DEG spins are globally polarized in the $\vec{B}$ direction in spite of positive $g^*_{2\mathrm{DEG}}$. This dark-gray area presents solutions of inequality (4.15). Unbroken global vacuum occupies only the blank sector in the upper right corner of the phase diagram. At large ξ but fixed $|V|^2/\Delta$, the line separating the blank and dark-gray sectors tends to $g^*_{2\mathrm{DEG}}=0.088$, which corresponds to a value $\varepsilon^*_Z=N_i|V|^2/N_\phi\Delta$ being the result of the $E_{\mathrm{sf}}(\xi\to\infty)=0$ equation. At the same time, if the $\xi\to\infty$ limit is realized owing to vanishing V, then both systems of the impurities and of the 2DEG become independent, and at any positive $g^*_{2\mathrm{DEG}}$, the global vacuum presents certainly the ground state. The light-gray area below the $\xi=\xi_{c1}(g^*_{2\mathrm{DEG}})$ line, but above the dotted line, corresponds to the singly spin-flip states with one exciton bound to an impurity. The hatched light-gray domain below the dotted line corresponds to the state with the localized skyrmions created by strong kinematic exchange [Eq. (4.13)]. In our specific case of the $N_i/N_\phi=0.01$ ratio, the dark-gray sector is not contiguous to this skyrmionic region. The total QHF spin S_z in various states of the phase diagram is indicated in the picture.

Now we discuss the regime where free skyrmions are already available in the system because the number of electrons well deviates from the quantum flux number. Namely, we consider that $|N-N_\phi|>N_i$ (although still $|N-N_\phi|\ll N_\phi$). In this case "excessive" skyrmions may be bound to an impurity. The result depends on the QHF phase. In the globally pinned phase (dark-gray area), the binding is impossible since the effective interaction between the impurity and the skyrmion is repulsive. In the state of local pinning (light-gray unhatched domain), the binding also does not occur. Indeed, the binding energy would be equal to $E_{\mathrm{sk,pin}}$ (4.10), but due to the condition (4.4), this value is smaller than the spin-exciton delocalization energy $-|V|^2F_{\mathrm{x}}^{(3/2)}/\Delta$. At the global vacuum (blank sector), the binding takes place and the binding energy is equal to the pinning energy (4.10). Certainly, the binding takes place in the skyrmionic ground state (light-gray hatched sector). However, in contrast to the $|N-N_\phi|<N_i$ case, now all N_i impurities bind skyrmions of the same charge q_T, where $q_T=\pm1$, if, correspondingly, $N\lesseqgtr N_\phi$.

To conclude this section, it is worthy to note that we have only considered the situation where the $g^*_{2\mathrm{DEG}}>0$ ground state is realized in the most symmetric phases when the pinned spin K is equal to 0, 1, or $K\gg\infty$. As has been seen, there are only two critical parameters ξ_{c1} and $\xi_{c,\infty}$ in this case. However, one might suppose that transition from the local pinning $(K=1)$ to the skyrmionic phase of massive pinning would proceed more smoothly with diminishing parameter ξ. Namely, below the $\xi_{c1}(g^*_{2\mathrm{DEG}})$ curve, there should be a critical value $\xi=\xi_{c2}(g^*_{2\mathrm{DEG}})$ at which the transition $K=1\to K=2$ occurs. This value would be the root of equation $G_1(\xi_{c2})=G_2(\xi_{c2})$. The next critical point would correspond to the $K=2\to K=3$ transition and so on. This sequence of values $\xi_{c1}>\xi_{c2}>\ldots\xi_{cK}>\ldots$ where $G_{K-1}(\xi_{cK})=G_K(\xi_{cK})$ should condense in the vicinity of $\xi_{c,\infty}$. Actually, this means that the light-gray unhatched domain in Fig. 6 would present not only the singly spin-flip 2DEG state but a set of states with $K=1,2,3,\ldots$, spin excitons localized at the impurity where K is growing with diminishing ξ. In reality, for a finite ε^*_Z, K reaches the value given by Eq. (4.8) at $\xi=\xi_{c,\infty}$. Such a "stratification" of the light-gray unhatched sector would be the only qualitative change of the phase picture of Fig. 6. Quantitative changes would be presented by appropriate shifts of the $\xi_{c,\infty}$ curve and of the boundary between the dark-gray and light-gray areas. However, it is clear that these shifts would not be significant. The corresponding crossover parameters ξ would at least remain of the same order as the ones calculated with the help of Eqs. (4.14) and (4.15).

V. DISCUSSION

We have found that the interplay between the kinematic impurity exchange and the Coulomb interaction in 2DEG results in the appearance of bound exciton states and in the renormalizaton of impurity spin states, including the reconstruction of the QHF ground state at $g^*_{2\mathrm{DEG}}>0$.

Among the available experimental techniques, the inelastic light scattering (ILS) method seems to be the most useful method for an experimental study of the 2DEG spectra (see Refs. 7, 10, and 45, and references therein). However, this tool has some special features, and it is helpful to discuss our results from this point of view. Let us first consider the $g_{2\mathrm{DEG}}<0$ case. When measuring the energy from the ground state level $\tilde{E}_0^{(-5/2)}$, where the impurity has the maximum spin projection [see Eq. (3.10)], one obtains ten levels of the localized excitations ΔE related to the spin changes $\delta S_z=0,1,\ldots,5$,

$$\Delta E_{\delta S_z,t}=g_i\mu_B B\delta S_z+(|V|^2/\Delta)[F_t^{(s)}(\xi)-F_0^{(-5/2)}(\xi)]\quad(t=0,x),\tag{5.1}$$

where $s=\delta S_z-5/2$, and the index t labels the type of the excited state [in Eq. (5.1), it is considered that $F_t^{(5/2)}\equiv0$]. Within the scope of the experiment where only the $|\delta S_z|\lesseqgtr2$ excitations seem to be observable as ILS peaks, we plot in Fig. 7 these five levels calculated with the help of Eq. (3.9) for the parameters $g=0.25$ and $\varepsilon_Z=0.05|V|^2/\Delta$. This calculation is done for the sake of demonstration, with the function $\mathcal{E}_q=2M_x^{-1}[1-e^{-q^2/4}I_0(q^2/4)]$ and the fitting parameter M_x used to describe the spin-wave dispersion. In the available wide quantum wells, the inverse spin-exciton mass is relatively small.[46] Hence, the values $\xi<1$ seem to be experimentally relevant, and the evolution of nonequidistant excitations $\Delta E_{\delta S_z,x}$ as a function of ξ (and, therefore, of B) should be observable in this interval.

The nonlocalized states discussed in Sec. III C actually present a transformation of the x-type excitations when the spin exciton is detached from the impurity and falls in the spin-wave continuum. The bottoms of continuous bands are shown as filled areas in Fig. 7. The band edges are higher than the $\Delta E_{\delta S_z,x}$ curves by the quantity $-(|V|^2/\Delta)F_x^{[\delta S_z-(5/2)]}(\xi)$ [see Eqs. (3.8) and (3.10)], and therefore, the latter may be treated as the spin-exciton binding

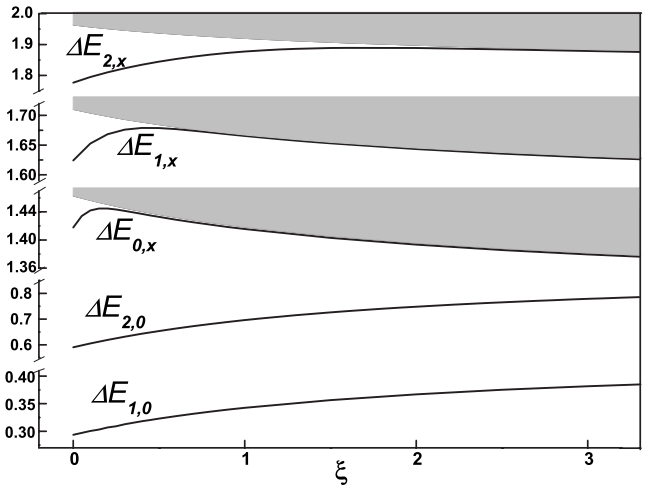

FIG. 7. The case of $g_{2\mathrm{DEG}}<0$. Energies of the excitations measured from the ground state are plotted in units of $|V|^2/\Delta$ as functions of $\xi=\Delta/M_x|V|^2$. Filled areas show energies of the delocalized spin excitons. See text for details.

energy. However, it seems difficult to observe these states in the ILS spectra because of comparatively small oscillator strengths, specifically due to the divergence of the envelope function $f(q)$.

Similar ILS picture should also take place for $g^*_{2\mathrm{DEG}}>0$ in the phase of the 2DEG global pinning (dark-gray area in Fig. 6). In the skyrmionic phase (light-gray hatched sector), there are intraimpurity ILS transitions determined by Eq. (5.1). Besides, two other types of resonances are expected: the first is the skyrmion delocalization with $\delta S_z=0$ and with excitation energy equal to $E_{\mathrm{sk,pin}}$ (4.10); another one is the transition $\delta S_z=-1$, where the delocalized skyrmion leaves the impurity with the bound spin exciton. In the latter case, the transition energy is $E_{\mathrm{sk,pin}}-|V|^2F_{\underline{x}}^{(3/2)}+\varepsilon_Z^*$.

In the global vacuum and local-pinning states (blank and light-gray unhatched domains), the ILS spectrum is determined by transitions between levels (4.1) and (4.2), so that, e.g., the ILS transitions to the localized states from the global vacuum are determined by the energies $\Delta E_{\delta S_z}=\varepsilon_Z^*+g_i\mu_B B(\delta S_z-1)+|V|^2F_{\underline{x}}^{(5/2-\delta S_z)}/\Delta$ and correspond to nonzero spin change $\delta S_z=1,2(3,4,5)$. At the same quantum numbers δS_z, there should also be resonance features related to the impurity spin rotation, which cost the energy $\Delta E_{\delta S_z,\mathrm{res}}=g_i\mu_B B\delta S_z$. These resonances are, in fact, transitions to the continuous spectrum, which should be noticeable on the background of free spin wave contribution ($\varepsilon_Z^*<\Delta E<\mathcal{E}_\infty$) [see the comment above Eq. (4.2)].[47] The ILS spectrum of excitations from the local-pinning ground state is presented: first, by the $\delta S_z=-1$ transition to the global vacuum (this energy is equal to $\Delta E_{-1}=-|V|^2F_{\underline{x}}^{(3/2)}/\Delta-\varepsilon_Z^*$); second, by the $\delta S_z=1,2(3,4)$ transitions to the localized spin-flip states with energies $\Delta E_{\delta S_z}=g_i\mu_B B\delta S_z+|V|^2[F_{\underline{x}}^{(3/2-\delta S_z)}-F_{\underline{x}}^{(3/2)}]/\Delta$; and third, by the $\delta S_z=0,1,2(3,4)$ transitions to the resonance states in the continuous spectrum with transition energies $\Delta E_{\delta S_z,\mathrm{res}}=g_i\mu_B B(\delta S_z+1)-|V|^2F_{\underline{x}}^{(3/2)}/\Delta-\varepsilon_Z^*$.

Finally, we note that currently there are several possibilities for the experimental study of skyrmionlike textures (e.g., see Refs. 13, 14, 17, and 48). However, for any method, one of the most serious obstacles impeding the observation of spin-flip phases is the very narrow interval in the vicinity of the $g^*_{2\mathrm{DEG}}=0$ factor, where the spin-flip reconstruction of the ground state or skyrmionlike excitations are possible. From this point of view, the minor magnetic doping would become an additional fine tuning tool allowing one to change the balance between E_Z and E_C, and to influence the skyrmion formation.

ACKNOWLEDGMENTS

The work was mainly done during the authors' stay at MPIPKS, Dresden. S.D. thanks the hospitality of the Grenoble High Magnetic Field Laboratory and the Abdus Salam International Center of Theoretical Physics (Trieste), where part of this work was also carried out, and acknowledges support of the Russian Foundation for Basic Research. V.F. and K.K. acknowledge partial support of the Israel Science Foundation, Grant No. 0603214212.

APPENDIX A: EXCITONIC REPRESENTATION

The excitonic representation is a convenient tool for a description of electron-hole states in a 2DEG multiply degenerate in m. When acting on the vacuum state $|\mathrm{vac}\rangle$ (in our case, this vacuum is defined in Sec. II A), the exciton creation operators form a system of basis states diagonalizing the Hamiltonian, including a considerable part of the Coulomb interaction. Due to translational invariance of a clean 2DEG, these exciton states are classified by the 2D momentum $\mathbf{q}$ and the degeneracy turns out to be lifted. The exciton operators for a single LL were first introduced in Ref. 6. The commutation rules for the same case of single LL were found in Ref. 49 (see also Ref. 9, and references therein).

Unlike previous papers, where the ER technique was developed for the Landau gauge, we use the symmetric gauge for bare one-electron states. In the *symmetric gauge*, the spin-exciton creation operator is expressed in terms of the a_m and b_m Fermi operators,

$$\mathcal{Q}^\dagger_{\mathbf{q}}=N_\phi^{-1/2}\sum_{m,m'=0}^{N_\phi-1}h^*_{mm'}(\mathbf{q})b^\dagger_{m-n}a_{m'-n} \tag{A1}$$

(cf. the definition based on the Landau gauge[6,9,18,36]). In this expression,

$$h_{mk}(\mathbf{q})=(m!/k!)^{1/2}(q_-)^{k-m}L_m^{k-m}(q^2/2)e^{-q^2/4} \tag{A2}$$

are the building block functions used in the ER technique, $q_-=iqe^{-i\varphi}/\sqrt{2}\equiv i(q_x-iq_y)/\sqrt{2}$, and L_m^{k-m} are the Laguerre polynomials. Here and below, all lengths are measured in the magnetic length $l_B=1$ unit. The spin-exciton states are orthogonal and normalized,

$$\langle\mathrm{vac}|\mathcal{Q}_{\mathbf{q}_1}\mathcal{Q}^\dagger_{\mathbf{q}_2}|\mathrm{vac}\rangle=\delta_{\mathbf{q}_1,\mathbf{q}_2}. \tag{A3}$$

The operators (A1) together with the intrasublevel operators

$$\mathcal{A}_{\mathbf{q}}^{\dagger}=\frac{1}{N_{\phi}}\sum_{m,m'=0}^{N_{\phi}-1}h_{mm'}^{*}(\mathbf{q})a_{m-n}^{\dagger}a_{m'-n}$$

and

$$\mathcal{B}_{\mathbf{q}}^{\dagger}=\frac{1}{N_{\phi}}\sum_{m,m'=0}^{N_{\phi}-1}h_{mm'}^{*}(\mathbf{q})b_{m-n}^{\dagger}b_{m'-n} \tag{A4}$$

form a closed Lie algebra. In order to check it, we first obtain the commutation relations

$$[\mathcal{Q}_{\mathbf{q}}^{\dagger},a_{m}^{\dagger}]=N_{\phi}^{-1/2}\sum_{k=0}^{N_{\phi}-1}h_{m+n,k}^{*}(\mathbf{q})b_{k-n}^{\dagger},$$

$$[\mathcal{Q}_{\mathbf{q}}^{\dagger},b_{m}]=-N_{\phi}^{-1/2}\sum_{k=0}^{N_{\phi}-1}h_{k,m+n}^{*}(\mathbf{q})a_{k-n}, \tag{A5}$$

$$[\mathcal{A}_{\mathbf{q}},a_{m}]=-\frac{1}{N_{\phi}}\sum_{k=0}^{N_{\phi}-1}h_{m+n,k}(\mathbf{q})a_{k-n},$$

$$[\mathcal{B}_{\mathbf{q}},b_{m}^{\dagger}]=\frac{1}{N_{\phi}}\sum_{k=0}^{N_{\phi}-1}h_{k,m+n}(\mathbf{q})b_{k-n}^{\dagger}, \tag{A6}$$

and

$$[\mathcal{Q}_{\mathbf{q}},a_{m}^{\dagger}]=[\mathcal{Q}_{\mathbf{q}},b_{m}]=[\mathcal{A}_{\mathbf{q}},b_{m}]=[\mathcal{A}_{\mathbf{q}},b_{m}^{\dagger}]=[\mathcal{B}_{\mathbf{q}},a_{m}]$$
$$=[\mathcal{B}_{\mathbf{q}},a_{m}^{\dagger}]\equiv 0. \tag{A7}$$

As a result, we see that operators (A1) and (A4) really form a closed algebra with the commutation relations[9,18,49]

$$[\mathcal{Q}_{\mathbf{q}_1},\mathcal{Q}_{\mathbf{q}_2}^{+}]=e^{i(\mathbf{q}_1\times\mathbf{q}_2)_z/2}\mathcal{A}_{\mathbf{q}_1-\mathbf{q}_2}-e^{-i(\mathbf{q}_1\times\mathbf{q}_2)_z/2}\mathcal{B}_{\mathbf{q}_1-\mathbf{q}_2},$$

$$e^{-i(\mathbf{q}_1\times\mathbf{q}_2)_z/2}[\mathcal{A}_{\mathbf{q}_1}^{\dagger},\mathcal{Q}_{\mathbf{q}_2}^{\dagger}]=-e^{i(\mathbf{q}_1\times\mathbf{q}_2)_z/2}[\mathcal{B}_{\mathbf{q}_1}^{\dagger},\mathcal{Q}_{\mathbf{q}_2}^{\dagger}]$$
$$=-N_{\phi}^{-1}\mathcal{Q}_{\mathbf{q}_1+\mathbf{q}_2}^{\dagger},$$

$$[\mathcal{A}_{\mathbf{q}_1}^{\dagger},\mathcal{A}_{\mathbf{q}_2}^{\dagger}]=\frac{2i}{N_{\phi}}\sin\left[\frac{(\mathbf{q}_1\times\mathbf{q}_2)_z}{2}\right]\mathcal{A}_{\mathbf{q}_1+\mathbf{q}_2}^{\dagger},$$

$$[\mathcal{B}_{\mathbf{q}_1}^{\dagger},\mathcal{B}_{\mathbf{q}_2}^{\dagger}]=\frac{2i}{N_{\phi}}\sin\left[\frac{(\mathbf{q}_1\times\mathbf{q}_2)_z}{2}\right]\mathcal{B}_{\mathbf{q}_1+\mathbf{q}_2}^{\dagger}. \tag{A8}$$

Acting on the vacuum state, the intrasublevel operators result in

$$\mathcal{A}_{\mathbf{q}}^{\dagger}|\mathrm{vac}\rangle=\delta_{\mathbf{q},0}\quad\text{and}\quad\mathcal{B}_{\mathbf{q}}^{\dagger}|\mathrm{vac}\rangle\equiv 0. \tag{A9}$$

The excitonic basis $\mathcal{Q}_{\mathbf{q}}^{\dagger}|\mathrm{vac}\rangle$ determine the set of eigenstates of a clean 2DEG,

$$[(\hat{H}_1^{(s)}+\hat{H}_{s-s}),\mathcal{Q}_{\mathbf{q}}^{\dagger}]|\mathrm{vac}\rangle=(\varepsilon_Z+\mathcal{E}_q)\mathcal{Q}_{\mathbf{q}}^{\dagger}|\mathrm{vac}\rangle. \tag{A10}$$

Here, $\mathcal{E}_q$ stands for the Coulomb energy of the free spin wave defined by the equation[3–5]

$$\mathcal{E}_q=\frac{1}{N_{\phi}}\sum_{\mathbf{p}}W_{ss}(p)[1-e^{i(\mathbf{p}\times\mathbf{q})_z}]$$
$$\equiv\int_0^{\infty}dppv_{ss}(p)[h_{nn}(p)]^2[1-J_0(pq)], \tag{A11}$$

$J_0(pq)$ is the Bessel function.

The Coulomb vertices in the Hamiltonian (2.12) and (2.13) are given by the equations

$$W_{ss}(q)=v_{ss}(q)[h_{nn}(q)]^2,\quad W_{sd}(q)=v_{sd}(q)h_{nn}(q), \tag{A12}$$

where $2\pi v_{ss}(\mathbf{q})$ and $2\pi v_{sd}(\mathbf{q})$ are the 2D Fourier transforms of the average s-s and s-d interaction potentials. One can present them as[1]

$$v_{ss}(q)=\frac{e^2}{\kappa l_B q}\int\int dz_1dz_2e^{-q|z_1-z_2|}|\zeta(z_1)|^2|\zeta(z_2)|^2,$$

$$v_{sd}(q)=\frac{e^2}{\kappa l_B q}\int dze^{-q|z-z_d|}|\zeta(z)|^2. \tag{A13}$$

(The impurity site is assumed to be at the point $\mathbf{R}_d=\{0,0,z_d\}$.)

APPENDIX B: SPIN OPERATORS

Bound spin excitons are characterized by the spin numbers S_z and S^2. The corresponding operators have the form

$$\hat{S}_z=\hat{S}_z^{(s)}+\hat{S}_z^{(d)}, \tag{B1}$$

where

$$\hat{S}_z^{(s)}=\frac{N_{\phi}}{2}(\mathcal{A}_0-\mathcal{B}_0),\quad(\hat{\mathbf{S}}^{(s)})^2=N_{\phi}\mathcal{Q}_0^{\dagger}\mathcal{Q}_0+(\hat{S}_z^{(s)})^2+\hat{S}_z^{(s)} \tag{B2}$$

and

$$\hat{S}_z^{(d)}=\frac{1}{2}(\hat{n}_{\uparrow}-\hat{n}_{\downarrow}),\quad(\hat{\mathbf{S}}^{(d)})^2=\frac{3}{4}(\hat{n}_{\uparrow}+\hat{n}_{\downarrow})-\frac{3}{2}\hat{n}_{\uparrow}\hat{n}_{\downarrow} \tag{B3}$$

are the spin operators for 2DEG (in the excitonic representation) and for magnetic impurity (in terms of the single-orbital model), respectively. The total squared spin operator for the system is defined as

$$\hat{\mathbf{S}}^2=(\hat{\mathbf{S}}^{(s)})^2+2\hat{S}_z^{(s)}\hat{S}_z^{(d)}+N_{\phi}^{1/2}(\mathcal{Q}_0^{\dagger}c_{\uparrow}^{\dagger}c_{\downarrow}+c_{\downarrow}^{\dagger}c_{\uparrow}\mathcal{Q}_0)+(\hat{\mathbf{S}}^{(d)})^2. \tag{B4}$$

The operator $\hat{S}_z$ commutes with the Hamiltonian (2.9), while for $\hat{\mathbf{S}}^2$ one has

$$[\hat{\mathbf{S}}^2,\hat{\mathcal{H}}]\equiv N_{\phi}^{1/2}(g_i\mu_BB-\varepsilon_Z)c_{\downarrow}^{\dagger}c_{\uparrow}\mathcal{Q}_0+(\beta_{\uparrow}-\beta_{\downarrow})V[c_{\downarrow}^{\dagger}(\hat{n}_{\uparrow}b_0$$
$$+N_{\phi}^{1/2}\mathcal{Q}_0a_0)-c_{\uparrow}^{\dagger}(\hat{n}_{\downarrow}a_0+N_{\phi}^{1/2}\mathcal{Q}_0^{\dagger}b_0)]-\mathrm{H.c.} \tag{B5}$$

The difference between the g factors of the magnetic impurity and the host QHF, and the difference between the projection factors $\beta_{\uparrow}$ and $\beta_{\downarrow}$ measure the spin nonconservation.

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[37] If the interaction in the 2DEG is taken into account, the single-orbital model needs additional substantiation. In this case, the state with $S_z=\frac{N_\phi}{2}+s$ may be created, e.g., as a two-exciton vector $\mathcal{Q}^\dagger_{\mathbf{q}_1}\mathcal{Q}^\dagger_{\mathbf{q}_2}|d^5,s+2;|\mathrm{vac}\rangle$ which is admixed to the state $|d^5,s+1;\mathcal{Q}^\dagger_{\mathbf{q}}|\mathrm{vac}\rangle\equiv|s_+;\mathcal{Q}^\dagger_{\mathbf{q}}|\mathrm{vac}\rangle$. One may neglect this admixture because the biexciton binding energy $E_{\text{bi-x}}$ is much smaller than both $g_i\mu_B B$ and $|V|^2/\Delta$ values, whereas the single-exciton binding energy $\mathcal{E}_\infty$ (A11) is comparable with these two characteristic energies. It is known (Refs. 8, 9, and 15) that even in the ideal 2D case (i.e., at $\alpha=1$), $E_{\text{bi-x}}<0.05444E_C$ so that the inequality $E_{\text{bi-x}}\ll g_i\mu_B B$, $|V|^2/\Delta$ really takes place.
[38] In terms of the single-orbital model, the combinations $|s_-;\mathrm{vac}\rangle+N_\phi^{1/2}\mathcal{Q}_0^\dagger|0\rangle$ and $|s_-;\mathrm{vac}\rangle-N_\phi^{1/2}\mathcal{Q}_0^\dagger|0\rangle$ present pure spin states with definite numbers $S=S_z+1+O(1/N_\phi)$ and $S=S_z+O(1/N_\phi)$, respectively.
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