

Energy bands of quantum dot arrays

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The energy bands are derived for a simple model of quantum dot arrays. The periodic array of quantum dots is linked by conducting atoms. The electrons in the conducting linkers are described by a tight-binding model. The electrons in the quantum dots are strong correlated, and the correlation is described by a Hubbard term. The electrons on the quantum dots are allowed to fluctuate through hopping to and from the conducting atoms. We solve the energy band for conducting electrons with perturbation method for the case that the quantum dots are in a spin singlet state.

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I. INTRODUCTION

Modern fabrication techniques permit the growth of many new materials. Of present interest are crystalline arrays of quantum dots connected by conducting chains of linker atoms.¹⁻⁴ The linkers are conducting atoms or molecules. The quantum dots are clusters of atoms or a molecule. Figure 1 shows a possible arrangement of such a material. The energy bands of these new crystals can be tuned by the structure and should have many industrial uses. They are expected to have large electrical conductivity due to the conducting atoms, low thermal conductivity due to the heavy mass of quantum dots, and high thermoelectric coefficient due to the high density of states near the chemical potential.⁵ These properties add up to a large value of the figure of merit, which is essential for application in solid state refrigerators. They can also be single electron transistors,⁶ or good optical materials. In optical physics, they are called metamaterials, but this name has not caught on in other fields. The experimental study of such materials is underway.⁴

In order to calculate the properties of such materials, we need to know the energy states of electrons that can move through the crystal. There have been several calculations of band structures of quantum dot crystals.⁷ They have treated the states of the quantum dot as a one-electron problem while ignoring any correlation. On the other hand, many other calculations note that electron transmission through a quantum dot is a problem with strong correlation.⁸⁻¹⁴ Here, we calculate the energy bands of conducting electrons through these metamaterials while treating the quantum dots as a strongly correlated system. Every quantum dot is treated as an Anderson impurity as in the periodic Anderson model (PAM).^{6,15,16}

The PAM has been well studied in the literature.¹⁷⁻²⁴ Our model is very different than the usual periodic Anderson model because we have conducting atoms between the correlated sites. In the usual form of the PAM, the conducting electrons, and f electrons, have parallel channels of conduction. The conduction electrons have a self-energy term which is usually called something like

$$\Delta(E) = V^2 \sum_k \frac{1}{E - E_k + i\eta}. \quad (1)$$

The imaginary part of this self-energy is $\Delta_2 = -\pi V^2 \rho(E)$, where $\rho(E)$ is the density of states. This damping imparts an

energy width to the various electron energy states. Our model does not have this term. The correlated states are not in a side channel but are blocking the main channel. This makes the physics of our model very different from that found in the usual PAM.

We convinced ourselves that an exact solution, in one dimension, using the Bethe ansatz, is not possible. So we have proceeded to use a perturbation method. Our method is first described in detail in one dimension, and then, it easily expanded to two and three dimensions. For simplicity, the spin singlet state for quantum dots is assumed. Then, each quantum dot is in the same quantum state, except for fluctuations, which does result in energy bands. Other ground state spin configurations for the quantum dot are the subject of future studies. With this assumption, the ground state for

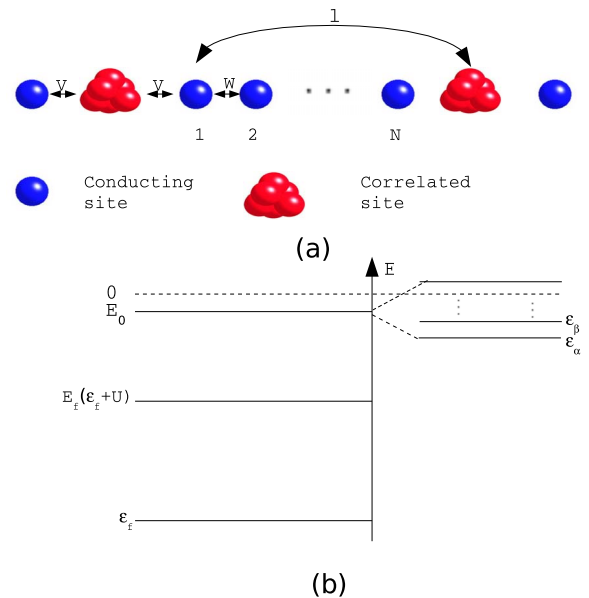


FIG. 1. (Color online) Model in one dimension. (a) shows the structure of the quantum dot array. l labels the l th unit cell. Other notations are explained in the text. (b) shows our assumptions of energy levels. The second electron in the quantum dot has an effective binding energy E_f , which is still much lower than the binding energy of conducting sites. This sets the quantum dots in spin singlet states. We concentrate our study on this case throughout this paper. Notations are explained in the text.

quantum dots is determined, and the spin of conducting electron is irrelevant. The energy band is found to depend on the number of conducting linkers between the quantum dots N , the coupling between linkers and quantum dots V , and the interaction inside of the quantum dot U .

The paper is organized as follows. The model for one dimensional (1D) quantum dot arrays is described in Sec. II. The energy bands in 1D are solved in Sec. III. Higher dimensions are discussed in Sec. IV. A discussion is presented in Sec. V.

II. DESCRIPTION OF THE MODEL

The energy bands are first solved in one dimension. Figure 1 illustrates the basic geometry. We consider a chain of atoms with n unit cells. Every unit cell has N conducting sites and one correlated site. Electrons in the conducting sites are described by a tight-binding model with transfer term W .

We assume that the ground state is a spin singlet of two electrons. They are denoted as f electrons, in agreement with the vast literature on heavy fermions. The choice of spin singlet means that every correlated site has the same ground state, and there is no need to worry about magnetic ordering. We restrict our calculation to a restricted Hilbert space, in which every correlated site can have one, two, or three such f electrons. Periodic boundary condition is also applied.

The Hamiltonian is

$$H = E_0 \sum_{l,j,s} C_{l,j,s}^\dagger C_{l,j,s} + W \sum_{l,s} \left(\sum_{j=1}^{N-1} C_{l,j+1,s}^\dagger C_{l,j,s} + \text{H.c.} \right) + \sum_l \left(\varepsilon_f \sum_s n_{l,s} + n_{l\uparrow} n_{l\downarrow} U \right) + V \sum_{l,s} [f_{l,s}^\dagger (C_{l+1,1,s} + C_{l,N,s}) + \text{H.c.}], \quad (2)$$

$$n_{l,s} = f_{l,s}^\dagger f_{l,s}, \quad (3)$$

where $l=1-n$ labels the period, $j=1-N$ labels the sites along the conducting chain, and s labels the spin. $C_{l,i,s}$ is the destruction operator for electron with spin s at conducting sites (l,i) . E_0 is the binding energy for the conducting electrons. With a wave vector transform,²⁵ we can diagonalize the conducting chain in each period,

$$\phi_j(\alpha) = \sqrt{\frac{2}{N+1}} \sin(k_\alpha j), \quad k_\alpha = \frac{\pi\alpha}{N+1}, \quad (4)$$

where j labels the conducting site locations, ranging from 1 to N , and α labels the wave vectors, ranging from 1 to N .

$$C_{l,j,s} = \sum_{\alpha=1}^N C_{l,\alpha,s} \phi_j(\alpha), \quad (5)$$

$$C_{l,\alpha,s} = \sum_{j=1}^N C_{l,j,s} \phi_j(\alpha), \quad (6)$$

$$H = \sum_{\alpha,l,s} \varepsilon_\alpha C_{l,\alpha,s}^\dagger C_{l,\alpha,s} + \sum_l \left\{ \sum_s \varepsilon_f n_{l,s} + n_{l\uparrow} n_{l\downarrow} U \right\} + V \sum_{l,\alpha,s} \{ f_{l,s}^\dagger [C_{l+1,\alpha,s} \phi_1(\alpha) + C_{l,\alpha,s} \phi_N(\alpha)] + \text{H.c.} \}, \quad (7)$$

$$\varepsilon_\alpha = E_0 + 2W \cos(k_\alpha). \quad (8)$$

Typically, the quantum dots are connected by short molecules, so that N is a small integer such as 1, 2, or 3. Then, the conducting electrons do not form a continuous band in energy states, between the dots, but have a few discrete energy values.

Our strategy for solving for the energy bands is to effectively replace the correlated site by a T matrix. To order $O(V^2)$, it can be replaced by a term in the Hamiltonian which connects conducting sites on each side of it,

$$H_T = \sum_{l,\alpha,\beta,s} T_{l,\alpha;l+1,\beta} C_{l,\alpha,s}^\dagger C_{l+1,\beta,s} + \text{H.c.}, \quad (9)$$

$$T_{l,\alpha;l+1,\beta} = V^2 \phi_1(\beta) \phi_N(\alpha) G_f. \quad (10)$$

The correlated site has been replaced by a term of the form $V^2 G_f$, where G_f is the Green's function of the correlated electrons. This type of replacement has been a standard mathematical trick since the early days of the Anderson model. The remaining question is what is the form of G_f for our model. That is the topic of the rest of the paper. In our model, where the ground state of the correlated site is a spin singlet, the transfer matrix is the same for each correlated site. Then, we can omit the site labels and call it $T_{\alpha,\beta}$. Once we introduce the transfer term $T_{l,\alpha;l+1,\beta}$, then it is easy to see that the conducting electrons have an energy band of the form

$$E(\alpha) = \varepsilon_\alpha + 2T_{\alpha,\alpha} \cos(k) + \Sigma_\alpha, \quad (11)$$

$$\Sigma_\alpha = \sum_{\beta \neq \alpha} \frac{T_{\alpha,\beta}^2}{\varepsilon_\alpha - \varepsilon_\beta} + O(T^4), \quad (12)$$

where $k = \pi m/n$ is the wave vector of the periodic array of n monomers. The wave vector q creates a true energy band.

Every correlated site is assumed to have the energy levels shown in Fig. 1. The correlated site has two f electrons in a spin singlet. The energy of this dot is $2\varepsilon_f + U$, which we write as $E_G = 2E_f - U$ and $E_f = \varepsilon_f + U$. The value of E_G may, or may not, be similar to the energy of the linker chains $\varepsilon_\alpha = E_0 + 2W \cos(k_\alpha)$. The bandwidth for the system is $V^2/(E_f - \varepsilon_\alpha)$, which we assume is small compared to W . All of these assumptions are possible in real materials, but they are chosen somewhat arbitrarily here. The intention is to keep the calculation simple. Later publications may consider more complex arrangements of energy levels.

III. PERTURBATION THEORY IN ONE DIMENSION

The Hubbard term is treated as a perturbation, in the manner of Refs. 22 and 23. The rest of the Hamiltonian is solved exactly. Such treatment is common for the Anderson model,

and has proved to be useful,^{24,26–29} especially for the asymmetric Anderson model.^{28,29}

We separate the Hamiltonian into two parts:

$$H = H_0 + V_H, \quad (13)$$

$$H_0 = \sum_{\ell} \left\{ E_f \sum_s n_{\ell,s} - U \langle n_{\ell,s} \rangle^2 + \sum_{\alpha,s} \varepsilon_{\alpha} C_{\ell,\alpha,s}^{\dagger} C_{\ell,\alpha,s} \right\} + V \sum_{\ell,\alpha,s} \{ f_{\ell,s}^{\dagger} [C_{\ell,\alpha,s} \phi_1(\alpha) + C_{\ell-1,\alpha,s} \phi_N(\alpha)] + \text{H.c.} \}, \quad (14)$$

$$E_f = \varepsilon_f + U \langle n_{\ell,\uparrow} \rangle, \quad (15)$$

$$V_H = U \sum_{\ell} (n_{\ell\uparrow} - \langle n_{\ell,s} \rangle)(n_{\ell\downarrow} - \langle n_{\ell,s} \rangle). \quad (16)$$

The notation is the same as in the prior section. Moreover, V_H is the perturbation. For a doubly occupied ground state,

$$\langle n_{\ell,\uparrow} \rangle = \langle n_{\ell,\downarrow} \rangle \approx 1. \quad (17)$$

We introduce the lattice transform for the n unit cells,

$$f_{\ell,s} = \frac{1}{\sqrt{n}} \sum_k f_{k,s} e^{ik\ell}, \quad (18)$$

$$C_{\ell,\alpha,s} = \frac{1}{\sqrt{n}} \sum_k C_{k,\alpha,s} e^{ik\ell}. \quad (19)$$

The Hamiltonian H_0 is now

$$H_0 = \sum_k \left\{ E_f \sum_s n_{k,s} + \sum_{\alpha,s} \varepsilon_{\alpha} C_{k,\alpha,s}^{\dagger} C_{k,\alpha,s} \right\} - U \langle n_{\ell,s} \rangle^2 + \sum_{k,\alpha,s} [V_{k\alpha} f_{k,s}^{\dagger} C_{k,\alpha,s} + \text{H.c.}], \quad (20)$$

$$V_{k\alpha} = V[\phi_1(\alpha) + \phi_N(\alpha)e^{-ik}] = V\phi_1(\alpha)[1 - (-1)^{\alpha} e^{-ik}], \quad (21)$$

$$|V_{k\alpha}|^2 = 2V^2 \phi_1^2(\alpha)[1 - (-1)^{\alpha} \cos(k)], \quad (22)$$

$$n_{k,s} = f_{k,s}^{\dagger} f_{k,s}. \quad (23)$$

The factor of $\cos(k)$ in $|V_{k\alpha}|^2$ provides the basic dispersion of the electron energy bands. The exact Green's function of the f electrons is³⁰

$$G_f(k, \omega) = \frac{1}{\omega - E_f - \tilde{\Sigma}_f(k, \omega)}, \quad (24)$$

$$\tilde{\Sigma}_f(k, \omega) = \sum_{\alpha} \frac{|V_{k\alpha}|^2}{\omega - \varepsilon_{\alpha} + i\eta}. \quad (25)$$

The f -electron Green's function has poles at the $N+1$ solutions to

$$E = E_f + \tilde{\Sigma}_f(E). \quad (26)$$

An approximate solution has an f -electron pole at

$$E_f(k) \approx E_f + \tilde{\Sigma}_f(E_f). \quad (27)$$

Another exact way to write Eq. (26) is

$$E = \varepsilon_{\alpha} + |V_{k\alpha}|^2 \tilde{G}_{f\alpha}(k, E), \quad (28)$$

$$\tilde{G}_{f\alpha}(k, E) = \frac{1}{E - E_f - \tilde{\Sigma}_{f\alpha}(E)}, \quad (29)$$

$$\tilde{\Sigma}_{f\alpha}(k, E) = \sum_{\beta \neq \alpha} \frac{|V_{k\beta}|^2}{E - \varepsilon_{\beta} + i\eta}. \quad (30)$$

The Green's function is

$$\mathcal{G}_{\alpha}(k, ik_n) = \frac{1}{ik_n - \varepsilon_{\alpha} - \Sigma_{\alpha}(k, ik_n)}, \quad (31)$$

$$\Sigma_{\alpha}(k, ik_n) = |V_{k\alpha}|^2 \tilde{\mathcal{G}}_{f\alpha}(k, ik_n), \quad (32)$$

$$\tilde{\mathcal{G}}_{f\alpha}(k, ik_n) = \frac{1}{ik_n - E_f - \tilde{\Sigma}_{f\alpha}(ik_n)}. \quad (33)$$

Approximate solutions of the energy bands from the linker states are

$$E_{\alpha}(k) \approx \varepsilon_{\alpha} + |V_{k\alpha}|^2 \tilde{G}_{f\alpha}(k, \varepsilon_{\alpha}). \quad (34)$$

All of these solutions have the feature that they have replaced the correlated site with a hopping term of the form $O(V^2G)$. They all have band dispersion: they all have the factor of $\cos(k)$. This dispersion does not contribute to damping. The Green's function in Eq. (24) is a series of poles. The Green's function and the spectral function³⁰ have the forms

$$G_f(k, E) = \sum_{j=1}^{N+1} \frac{r_j}{E - E_j(k) + i\eta}, \quad (35)$$

$$A_f(k, E) = -2 \text{Im}\{G_f(k, E)\} = 2\pi \sum_{j=1}^{N+1} r_j \delta[E - E_j(k)], \quad (36)$$

$$r_j = \frac{\prod_{a=1}^N (E_j - \varepsilon_a)}{\prod_{\ell=1, \ell \neq j}^{N+1} (E_j - E_{\ell})}, \quad (37)$$

where r_j is the residue of the pole. The collective states $E_{\alpha}(k), E_f(k)$ depend on wave vector, but the wave vector does not give scattering. The local operators do have damping

$$\langle n_{\ell,s} \rangle = \frac{1}{n} \sum_k \langle n_{k,s} \rangle = \frac{1}{n} \sum_k \int \frac{dE}{2\pi} A_f(k, E) n_F(E) \quad (38a)$$

$$= \int_{-\pi}^{\pi} \frac{dk}{2\pi} \sum_j r_j n_F[E_j(k)], \quad (38b)$$

where $n_F(E)$ is the Fermion occupation factor.

Next, we consider the changes in this expression due to the Hubbard term V_H . We define $\langle n_{\ell,s} \rangle$ as being evaluated

using the exact eigenstates of H_0 . In that case, the first-order perturbation $\langle V_H \rangle = 0$. The first contribution is due to second-order perturbation theory. It is introduced into the theory as an additional self-energy in \tilde{G} ,

$$\tilde{G}_{f\alpha}(k, ik_n) = \frac{1}{ik_n - E_f - \tilde{\Sigma}_{f\alpha}(k, ik_n) - \Sigma_U(k, ik_n)}, \quad (39)$$

$$\Sigma_U(k, ik_n) = \frac{U^2}{n^2} \sum_{pq} S(ik_n; k, p, q), \quad (40)$$

$$S(ik_n; k, p, q) = \frac{1}{\beta^2} \sum_{iq, ip} \mathcal{G}_f(k+q, ik+iq) \mathcal{G}_f(p, ip) \times \mathcal{G}_f(p+q, ip+iq). \quad (41)$$

We use the form in Eq. (35) as a summation over poles,

$$S(ik_n; k, p, q) = \sum_{j\ell n} r_j r_\ell r_n \frac{n_n(n_j - n_\ell) + n_\ell(1 - n_j)}{ik_n - E_n(k+q) - E_j(p) + E_\ell(p+q)}, \quad (42)$$

$$n_j = n_F[E_j(p)], \quad n_\ell = n_F[E_\ell(p+q)], \quad n_n = n_F[E_n(k+q)]. \quad (43)$$

In evaluating this expression, we assume the energy level scheme shown in Fig. 1. The f level is lower in energy than the states from the linker molecules. Then, $r_f \sim 1$, while $r_j \sim V^2/(E_f - E_0)^2$ for the other bands. We then consider possible values for $r_j r_\ell r_n$:

(i) If all three (j, ℓ, n) are in the f state, the factor r_f^3 has its largest value. However, all occupation numbers $n_f \sim 1$, so the numerator is zero.

(ii) the next largest value of $r_j r_\ell r_n$ occurs when any two of them are in the f level. The only case that has a nonzero numerator is when $j=n=f$ and $\ell \neq f$. This gives a contribution,

$$S(ik_n; k, p, q) = r_f^2 \sum_{\ell \neq f} r_\ell \frac{1 - n_\ell}{ik_n - E_f(k+q) - E_f(p) + E_\ell(p+q)}. \quad (44)$$

Since we are concerned with the self-energy of conduction electrons, the energy denominator (with $ik_n \rightarrow \varepsilon_\alpha$) is $O(E_0 - E_f)$. The whole term is of order

$$\Sigma_U \sim O\left(\frac{U^2 V^2}{(E_0 - E_f)^3}\right), \quad (45)$$

which we assume is small. It also does not seem very dependent on the wave vector of the conduction band. The main wave vector dependence is through the numerator $|V_{k\alpha}|^2$.

(iii) The largest terms are when $j=\ell$, so the polarization occurs within the same band. If this band is completely full of electrons, so $n_j=n_\ell$ equals one, then the above expression is zero. It is also zero if the band is empty. Since our interest is in conducting systems, we assume that there is one band that is partially filled. We call this band $j=\ell=c$ for conduction band.

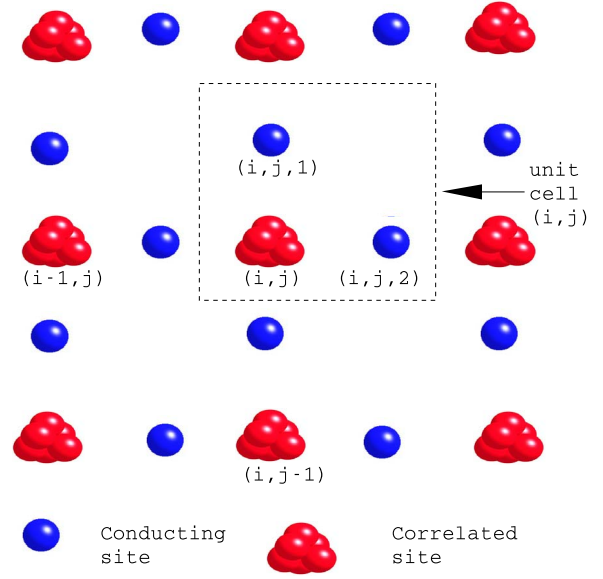


FIG. 2. (Color online) Square lattice. The plot inside the rectangle is unit cell (i, j) of the square lattice. $(i, j, 1)$ is the conducting site at the top of that unit cell. $(i, j, 2)$ is the conducting site at the right of that unit cell. Similar labeling is used for other unit cells. The unit cell $(i-1, j)$ and $(i, j-1)$ are also shown in the plot.

$$\Pi_n^{(1)}(ik_n) = \int \frac{dp}{2\pi} \frac{n_F[E_c(p)] - n_F[E_c(p+q)]}{ik_n - E_n(k+q) - E_c(p) + E_c(p+q)}, \quad (46)$$

$$\Pi_n^{(2)}(ik_n) = \int \frac{dp}{2\pi} \frac{n_F[E_c(p+q)]\{1 - n_F[E_c(p)]\}}{ik_n - E_n(k+q) - E_c(p) + E_c(p+q)}, \quad (47)$$

$$\Sigma_U(k, ik_n) = U^2 r_c^2 \int \frac{dq}{2\pi} \sum_n r_n \{n_F[E_n(k+q)]\} [\Pi^{(1)} + \Pi^{(2)}]. \quad (48)$$

In order for perturbation theory to be converging, this self-energy should be relatively small. Its size depends on the residues r_c^2 . They will depend on the particular band structure. For example, where is the energy level of the correlated site. This self-energy also depends on wave vector k , and it contributes to the energy band dispersion.

IV. HIGHER DIMENSIONS

Our model is easily extended to higher dimensions. First, consider two dimensions. We take the crystal structure of the conducting planes of the cuprate superconductors. The correlated sites are in a square lattice, and there is one bridging atom between them ($N=1$). This structure is shown in Fig. 2. The Hamiltonian is

$$\begin{aligned}
H = E_0 \sum_{i,j,s} [C_{i,j,1,s}^\dagger C_{i,j,1,s} + C_{i,j,2,s}^\dagger C_{i,j,2,s}] \\
+ \varepsilon_f \sum_{i,j,s} n_{i,j,s} + U \sum_{i,j} n_{i,j,\uparrow} n_{i,j,\downarrow} + V \sum_{i,j,s} [f_{i,j,s}^\dagger (C_{i,j,1,s} + C_{i,j,2,s}) \\
+ C_{i,j-1,1,s} + C_{i-1,j,2,s}] + \text{H.c.}, \quad (49)
\end{aligned}$$

where the sites are labeled in Fig. 2. Site (i, j) is a correlated site, $(i, j, 1)$ is the linker in the $+y$ direction, and $(i, j, 2)$ is the linker site in the $+x$ direction. We follow the same general steps, for one dimension, and write the Hamiltonian as $H_0 + V_H$, where V_H is the Hubbard term. Then, a two dimensional lattice transform gives

$$\begin{aligned}
H_0 = \sum_{\mathbf{k},s} \{E_f f_{\mathbf{k},s}^\dagger f_{\mathbf{k},s} + E_0 (C_{\mathbf{k},1,s}^\dagger C_{\mathbf{k},1,s} + C_{\mathbf{k},2,s}^\dagger C_{\mathbf{k},2,s}) \\
+ 2V f_{\mathbf{k},s}^\dagger [C_{\mathbf{k},1,s} \cos(k_y/2) + C_{\mathbf{k},2,s} \cos(k_x/2)] + \text{H.c.}\}. \quad (50)
\end{aligned}$$

The Green's function for the correlated site is

$$\mathcal{G}_f(\mathbf{k}, ik_n) = \frac{1}{ik_n - E_f - \Sigma_f(\mathbf{k}, ik_n)}, \quad (51)$$

$$\Sigma_f(\mathbf{k}, ik_n) = \frac{2V^2}{ik_n - E_0} [2 + \cos(k_x) + \cos(k_y)]. \quad (52)$$

In this case, the collective states, and residues, are

$$\begin{aligned}
E_\pm(\mathbf{k}) = \frac{1}{2} [E_0 + E_f \\
\pm \sqrt{(E_0 - E_f)^2 + 8V^2 [2 + \cos(k_x) + \cos(k_y)]}], \quad (53)
\end{aligned}$$

$$r_\pm = \frac{1}{2} \left[1 \pm \frac{E_f - E_0}{\sqrt{(E_0 - E_f)^2 + 8V^2 [2 + \cos(k_x) + \cos(k_y)]}} \right]. \quad (54)$$

The two dimensional band structure is evident. The Hamiltonian H_0 also has a dispersionless eigenvalue $E_3 = E_0$.

The extension to three dimensions yields a dispersion of

$$\begin{aligned}
2[\cos^2(k_x/2) + \cos^2(k_y/2) + \cos^2(k_z/2)] \\
= 3 + \cos(k_x) + \cos(k_y) + \cos(k_z).
\end{aligned}$$

Similar tight-binding dispersion relations can be found for other crystal lattices.

V. DISCUSSION

We have derived the energy band structure of a periodic system of quantum dots, or molecules, linked by conducting chains of atoms. The quantum dots are regarded as highly correlated. We took the Hubbard model to describe the correlated sites. We assumed that it had a ground state of a spin singlet, in order that all correlated sites have the same ground state. The basic theoretical step is to replace the correlated sites by a transfer term,

$$T \approx V^2 G_f, \quad (55)$$

where V is the hybridization matrix element and G_f is the Green's function for the electrons on the correlated site. We then constructed an accurate approximation to this Green's function.

The interesting feature of this method is that the bands have only a small amount of dispersion and very little damping. This feature is different from that found in solutions of the periodic Anderson model. However, our model is different, so it should have different solutions. Without the correlated sites, the molecular chains linking them are of finite size, so the system is not conducting. They can only conduct by having electrons hop on and hop off of the correlated sites.

We neglect the correlation between correlated sites. The ground state of each correlated site is nonmagnetic. We also showed that our method is easily extended to two and three dimensions.

There are an unlimited number of possible systems of quantum dots linked by various molecules. Here, we have taken a typical system and shown some of its properties. Other arrangements of quantum dots, and linkers, may have quite different energy level arrangements. Those cases will be treated as they are created by experimentalists. If the quantum dot has an odd number of electrons in its ground state, then the dot will have a magnetic moment. There will be a Kondo resonance at low temperatures. This is an interesting case, which will be treated later.

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