Superfluid transition temperature of the boson-fermion model on a lattice

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The properties of a mixture of mutually interacting bound electron pairs and itinerant fermions (the boson-fermion model) on a lattice are further studied. We determine the superconducting critical temperature from a pseudogap phase by applying a self-consistent T-matrix approach, which includes the pairing fluctuations and the boson self-energy effects. The analysis is made for a three dimensional cubic lattice with tight-binding dispersion for electrons and for both standard bosons and the case of hard-core bosons. The results describe the BCS-Bose-Einstein condensation crossover with varying position of the bosonic (local pair) level and give a further insight into the nature of resonance superfluidity in the boson-fermion model.

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I. INTRODUCTION

The model of coexisting local pairs (LPs) and electrons (mixture of charged bosons and fermions) for nonconventional superconductors was proposed several years ago.[1–3] It has been demonstrated that in this model, due to the intersubsystem charge exchange coupling, a mechanism of superconductivity can be developed, which leads to the superconducting state involving both types of particles. The properties of a mixture of interacting bosons (bound fermion pairs) and itinerant electrons can show features which are intermediate between those of local pair superconductors and those of conventional BCS systems.[2–4] Such a two-component model is of relevance for high temperature superconductors (HTSs).[2–13] The boson-fermion system has also been widely adopted as a model of resonance superfluidity and BCS-Bose-Einstein condensation (BEC) crossover in ultracold fermionic atomic gases with a Feshbach resonance.[14–16]

In the context of HTS, the two-component boson-fermion (BF) model has been proposed on the phenomenological grounds or it has been derived as the effective low-energy model. In particular, it has been obtained from the generalized periodic Anderson model with on-site hybridization between wide- and narrow-band electrons, in which the narrow-band electrons are strongly coupled with the lattice deformation, and formation of polarons and LP (bipolarons) takes place.[2] These LPs are hard-core bosons made up of two tightly bound fermions. The boson-fermion model can also be justified as a low-energy model for hole pairing in strongly correlated systems, showing resonating valence bond plaquette states.[10] The BF scenario has been proposed in studies of superconductivity mechanism based on heterogeneity of the electronic structure of HTS in the pseudogap phase, both in the momentum space[7,9,12] and in the real space.[8,11] The effective boson-fermion model on the cluster lattice has been recently derived from the inhomogeneous (checkerboard) Hubbard model.[11]

In the atomic Fermi gases near the Feschbach resonance, the BF scenario describes a macroscopic coherence between the atom pairs and molecules, which is controlled by the applied magnetic field.[14–16] Here, the standard bosons are envisioned to be the "closed channel" bosons associated with a Feshbach resonance. Thus, the model considered contains rich physics with important applications for both fields: HTS and ultracold atomic gases. Recently, we have studied a generalization of the model to the case of anisotropic pairings of d-wave symmetry or extended s-wave type.[12,13]

In this paper, we discuss the pseudogap behavior and present the evaluation of the superconducting transition temperature from a pseudogap phase by going beyond the BCS-mean-field approximation (MFA). In our analysis, we have applied a generalized T-matrix approach adapted to a two-component boson-fermion model. Our approach is an extension of the pairing fluctuation theory of the BCS-BEC crossover developed previously for one-component fermion systems with attractive interaction.[16–18] (For a review of the self-consistent T-matrix approach, see Ref. 16.) In Sec. II, we give equations determining T_c from the pseudogap phase for the boson-fermion model in the case of standard bosons, while those for the hard-core bosons are given in Sec. III. The numerical results, for a three dimensional (3D) cubic lattice assuming the tight-binding dispersion for fermions, are presented and discussed in Sec. IV.

II. PAIRING FLUCTUATIONS: EQUATIONS FOR T_c FROM THE PSEUDOGAP STATE

We consider the boson-fermion model with s-wave pairing on the lattice (bosons without hard core) given by the following Hamiltonian, written in the momentum space:

$$\mathcal{H} = \sum_{\mathbf{k}\sigma} (\varepsilon_{\mathbf{k}} - \mu) c^{\dagger}_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{q}} (E^0_{\mathbf{q}} + 2\Delta_0 - 2\mu) b^{\dagger}_{\mathbf{q}} b_{\mathbf{q}} - \frac{U}{N} \sum_{\mathbf{k},\mathbf{k}',\mathbf{q}} c^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\uparrow} c^{\dagger}_{-\mathbf{k}+\mathbf{q}/2,\downarrow} c_{-\mathbf{k}'+\mathbf{q}/2,\downarrow} c_{\mathbf{k}'+\mathbf{q}/2,\uparrow} + \frac{I}{\sqrt{N}} \sum_{\mathbf{k},\mathbf{q}} (b^{\dagger}_{\mathbf{q}} c_{-\mathbf{k}+\mathbf{q}/2,\downarrow} c_{\mathbf{k}+\mathbf{q}/2,\uparrow} + c^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\uparrow} c^{\dagger}_{-\mathbf{k}+\mathbf{q}/2,\downarrow} b_{\mathbf{q}}). \quad (2.1)$$

Here, $c^{\dagger}_{\mathbf{k}\sigma}, c_{\mathbf{k}\sigma}$ are the fermion operators and $b_{\mathbf{q}}, b^{\dagger}_{\mathbf{q}}$ represent the boson operators satisfying the standard commutation rules. $\varepsilon_{\mathbf{k}}$ is the electron band energy and $E^0_{\mathbf{q}}$ is the boson kinetic energy, and they are defined on the hypercubic lattice. $E^0_0=0$. $2\Delta_0$ is the bottom of the boson band and μ is the chemical potential. I is the intersubsystem coupling constant. U is the direct (nonresonant) interaction between fermions. The total number of particles per site is $n=n_F+2n_B$, where $n_F=\frac{1}{N}\Sigma_{\mathbf{k}\sigma}\langle c^\dagger_{\mathbf{k}\sigma}c_{\mathbf{k}\sigma}\rangle$ is the electron concentration, $n_B=\frac{1}{N}\Sigma_{\mathbf{q}}\langle b^\dagger_{\mathbf{q}}b_{\mathbf{q}}\rangle$ the boson concentration, and N the number of lattice sites.

If $I=0$, we have practically two independent subsystems and they can undergo a transition at T_{BCS} (for weak attraction U) and T_{BEC}. For $I\neq 0$, there will be one common phase transition to the superfluid state.

In the following, we give only the final equations determining the critical temperature (T_c) resulting from the T-matrix theory applied to the coupled boson-fermion system.[19] They are for $I\neq 0$,

$$1=\left[U+\frac{I^2}{2(\Delta_0-\mu)}\right]\frac{1}{N}\sum_{\mathbf{k}}\frac{\tanh(\beta_c E_{\mathbf{k}}/2)}{2E_{\mathbf{k}}}, \qquad (2.2)$$

$$\Delta^2_{pg}=\frac{I^2_{eff}}{1+I^2_{eff}A'_0}\frac{1}{N}\sum_{\mathbf{q}\neq 0}b(\tilde{\Omega}_{\mathbf{q}}), \quad \tilde{\Omega}_{\mathbf{q}}=\frac{E^0_{\mathbf{q}}+I^2_{eff}A'_0\Omega_{\mathbf{q}}}{1+I^2_{eff}A'_0}, \qquad (2.3)$$

$$n=n_F+2n_B=\frac{1}{N}\sum_{\mathbf{k}}\left[1-\frac{\bar{\varepsilon}_{\mathbf{k}}}{E_{\mathbf{k}}}\tanh(\beta_c E_{\mathbf{k}}/2)\right]+2n_B, \qquad (2.4)$$

$$I^2_{eff}=[2(\Delta_0-\mu)U+I^2]^2/I^2, \quad n_B=\Delta^2_{pg}/I^2_{eff}, \qquad (2.5)$$

$E_k=\sqrt{\bar{\varepsilon}^2_{\mathbf{k}}+\Delta^2_{pg}}$, $\bar{\varepsilon}_{\mathbf{k}}=\varepsilon_{\mathbf{k}}-\mu$, $b(x)=1/[\exp(\beta_c x)-1]$ is the Bose function, and $\beta_c=1/k_BT_c$.

Equation (2.2) is obtained from the Thouless criterion of the T-matrix $\Gamma(\mathbf{q},\Omega)$ divergence at $\mathbf{q}=\Omega=0$. Simultaneously, the bosonic Green function diverges, indicating a common transition in the boson-fermion system.[19,20] Equation (2.3) is for the pseudogap parameter $\Delta_{pg}=\Delta_{pg}(T_c)$, which is the real quantity, and Eq. (2.4) is the particle number equation. The effective boson dispersion is given by $\tilde{\Omega}_{\mathbf{q}}$ [Eq. (2.3)], in which $\Omega_{\mathbf{q}}$ describes propagating long-lived finite $\mathbf{q}$ pairs of c electrons, and this spectrum is determined self-consistently.

$$\Omega_{\mathbf{q}}=\frac{1}{A'_0}\frac{1}{N}\sum_{\mathbf{k}}\left[\frac{f(E_{\mathbf{k}})+f(\bar{\varepsilon}_{\mathbf{q}-\mathbf{k}})-1}{\bar{\varepsilon}_{\mathbf{q}-\mathbf{k}}+E_{\mathbf{k}}}u^2_{\mathbf{k}}+\frac{f(\bar{\varepsilon}_{\mathbf{q}-\mathbf{k}})-f(E_{\mathbf{k}})}{\bar{\varepsilon}_{\mathbf{q}-\mathbf{k}}-E_{\mathbf{k}}}v^2_{\mathbf{k}}+\frac{1-2f(E_{\mathbf{k}})}{2E_{\mathbf{k}}}\right], \qquad (2.6)$$

$$A'_0=\frac{1}{2\Delta^2_{pg}}\left[n_F-\frac{1}{N}\sum_{\mathbf{k}}2f(\bar{\varepsilon}_{\mathbf{k}})\right], \qquad (2.7)$$

$u^2_{\mathbf{k}}=1-v^2_{\mathbf{k}}=\frac{1}{2}\left(1+\frac{\bar{\varepsilon}_{\mathbf{k}}}{E_{\mathbf{k}}}\right)$, and $f(x)=1/[\exp(\beta x)+1]$ is the Fermi function.

In the long wave limit, $\Omega_{\mathbf{q}}$ has the following form [taken in Eq. (2.3)]: $\Omega_{\mathbf{q}}=C\mathbf{q}^2=\frac{\mathbf{q}^2}{2M^\star}$, where $M^\star$ is the effective mass. We note that there are two types of (hybridized) bosonic contributions to Δ_{pg} [Eq. (2.3)]: i.e., that coming from the long-lived finite $\mathbf{q}$ pairs of c electrons, which due to the intersubsystem coupling (and the direct interaction U) give rise to the bosonic dispersion, and from the direct bosonic hopping $E^0_{\mathbf{q}}$.

We should add that the number of bosons n_B is determined from the boson Green function and has the form

$$n_B=\frac{1}{1+K^2_{eff}A'_0}\frac{1}{N}\sum_{\mathbf{q}}b\left(\frac{E^0_{\mathbf{q}}+K^2_{eff}A'_0\Omega_{\mathbf{q}}}{1+K^2_{eff}A'_0}\right),$$

$$K^2_{eff}=I^2/[1-U\Pi(0)]^2, \qquad (2.8)$$

where $\Pi(0)=\frac{1}{N}\Sigma_{\mathbf{k}}\frac{\tanh(\beta_c E_{\mathbf{k}}/2)}{2E_{\mathbf{k}}}$ and $\Omega_{\mathbf{q}}$ and A'_0 are given by Eqs. (2.6) and (2.7), respectively. With the use of Eq. (2.2) and the definition of I^2_{eff}, one has $K^2_{eff}=I^2_{eff}$, hence; the relation $\Delta^2_{pg}=I^2_{eff}n_B$ at T_c. In comparison with the BCS-MFA for T_c, the self-consistent T-matrix approach to the boson-fermion model includes pairing fluctuations and takes into account the boson self-energy effect.

It is of interest to remark that for $I=0$, but $U\neq 0$, the pseudogap equation has the simple form

$$\Delta^2_{pg}=\frac{1}{A'_0}\frac{1}{N}\sum_{\mathbf{q}\neq 0}b(\Omega_{\mathbf{q}}), \qquad (2.9)$$

with $\Omega_{\mathbf{q}}$ and A'_0 given by Eqs. (2.6) and (2.7) and has been obtained for the fermion system with attractive interaction in Refs. 16 and 18. In such a case, the pseudogap is caused by the long-lived finite $\mathbf{q}$ pairs of electrons which can exist due to the direct attraction U.

III. HARD-CORE BOSON-FERMION MODEL

The case of hard-core boson-fermion model and s-wave pairing is described by the following Hamiltonian:[2]

$$H=\sum_{i,j,\sigma}{}' t_{ij}c^\dagger_{i\sigma}c_{j\sigma}+(D-\mu)\sum_{i,\sigma}c^\dagger_{i\sigma}c_{i\sigma}+\sum_i(2\Delta_0-2\mu)\hat{b}^\dagger_i\hat{b}_i-\sum_{i,j}{}' J_{ij}\hat{b}^\dagger_i\hat{b}_j+I\sum_i(\hat{b}^\dagger_i c_{i\downarrow}c_{i\uparrow}+c^\dagger_{i\uparrow}c^\dagger_{i\downarrow}\hat{b}_i). \qquad (3.1)$$

The hard-core boson operators $\hat{b}_i,\hat{b}^\dagger_j$ obey the Pauli spin-$\frac{1}{2}$ commutation relations: $[\hat{b}_i,\hat{b}^\dagger_j]=(1-2\hat{n}_i)\delta_{ij}$, $[\hat{b}_i,\hat{b}_j]=0$, $(\hat{b}^\dagger_i)^2=(\hat{b}_i)^2=0$, $\hat{b}^\dagger_i\hat{b}_i+\hat{b}_i\hat{b}^\dagger_i=1$, and $\hat{n}_i=\hat{b}^\dagger_i\hat{b}_i$. These operators are commuting for different sites but obey the anticommutation rules on the same lattice site. The hard-core condition allows the only single-boson occupancy of a given lattice site. J_{ij} is the direct bosonic hopping. Here, $D=zt$, where t is the nearest neighbor hopping parameter of c electrons and z is the coordination number of the underlying lattice, the primed sum excludes terms with $i=j$. I is the on-site boson-fermion coupling; Δ_0 is the position of the bare LP level with respect to the bottom of the electronic band. The total number of particles per site is given by $n=n_F+2n_B$, where $n_F=\frac{1}{N}\Sigma_{i\sigma}\langle c^\dagger_{i\sigma}c_{i\sigma}\rangle$ and $n_B=\frac{1}{N}\Sigma_i\langle\hat{b}^\dagger_i\hat{b}_i\rangle$. As in Sec. II, we apply the T-matrix approach for calculation of the critical temperature, however, with modification of the normal state boson Green function because of the hard core.[19,12]

The Thouless criterion yields for T_c,

$$1 = \left[J_0 + \frac{I^2}{N} \sum_{\mathbf{k}} \frac{\tanh(\beta_c E_{\mathbf{k}}/2)}{2E_{\mathbf{k}}} \right] \frac{1-2n_B}{2(\Delta_0-\mu)}, \tag{3.2}$$

where $E_{\mathbf{k}}=\sqrt{(\varepsilon_{\mathbf{k}}-\mu)^2+\Delta_{pg}^2}$, $\varepsilon_{\mathbf{k}}=D-t\Sigma_{\boldsymbol{\delta}}\exp(i\mathbf{k}\cdot\boldsymbol{\delta})$, where $\boldsymbol{\delta}$ is the vector linking the nearest neighbor sites.

Equation for the pseudogap parameter at T_c has the form

$$\Delta_{pg}^2 = \frac{g^2}{1+g^2A_0'} \frac{1}{N} \sum_{\mathbf{q}\neq 0} b\left(\frac{\tilde{E}_{\mathbf{q}}^0 + g^2A_0'\Omega_{\mathbf{q}}}{1+g^2A_0'} \right), \tag{3.3}$$

where now $\tilde{E}_{\mathbf{q}}^0=(J_0-J_{\mathbf{q}})(1-2n_B)$ and $g^2=I^2(1-2n_B)$. $J_{\mathbf{q}}$ is the Fourier transform of J_{ij}. Moreover, one finds that $n_B=\Delta_{pg}^2/I^2$. In Eq. (3.3), the pair dispersion $\Omega_{\mathbf{q}}$ and A_0' are the same as given previously [Eqs. (2.6) and (2.7)]. Equations (3.2) and (3.3) are solved together with the particle number condition $n=n_F+2n_B$, where n_F is given as in Eq. (2.4). If one sets $\Delta_{pg}=0$ and $1-2n_B=\tanh[\beta_c(\Delta_0-\mu)]$ in Eq. (3.2), then it reduces to the BCS-MFA result for T_c.[2,12] We should add that in strictly two dimensional (2D) system, the presented method yields $T_c=0$, in agreement with the Mermin-Wagner-Hohenberg theorem.

IV. RESULTS

The numerical results presented in Figs. 1 and 2 are for a 3D simple cubic (sc) lattice, assuming the tight-binding dispersion for fermions and bosons of the following form: $\varepsilon_{\mathbf{k}}=D(1-\gamma_{\mathbf{k}})$, $J_{\mathbf{q}}=J_0\gamma_{\mathbf{q}}$, $J_0=zJ$, $\gamma_{\mathbf{k}}=[\cos(k_x)+\cos(k_y)+\cos(k_z)]/3$, and $z=6$. For the free bosonic dispersion, we take $E_{\mathbf{q}}^0=J_0-J_{\mathbf{q}}$. In contrast to the continuum case, the momentum summations are restricted to the first Brillouin zone.

Figure 1 shows the evolution of T_c, the chemical potential at T_c, the pseudogap parameter at T_c, as well as n_F and n_B with the position of bosonic level Δ_0, for the boson-fermion model [Eq. (2.1)]. Bosons are without hard core, but the direct boson hopping and interaction between fermions are included. We set here $E_{\mathbf{q}}^0\simeq J\mathbf{q}^2$ and $J/t=1/2$, which correspond to $m_B=2m_F$, where $m_B=1/(2J)$, $m_F=1/(2t)$ are effective masses of bosons and fermions on the lattice, respectively ($\hbar=a=1$, where a the lattice spacing).

The superfluid transition changes smoothly from BCS-like to BEC-like when fluctuations associated with Cooper pairs are included. One notices that the stable undamped bosons exist if the renormalized [due to boson self-energy $\Sigma_B(q)$] threshold energy is negative; i.e., $2\Delta^*=2\Delta_0+\Sigma_B(q)<0$. In this regime, the bosons practically cannot decay, the chemical potential is negative, and the transition temperature approaches the BEC temperature for free bosons with decreasing Δ_0. Moreover, the strong effective pairing interaction binds fermions into the preformed pairs. However, if $\Delta^*>0$, the interchange boson-pair of fermions (c-electrons) process is responsible for the resonance sperfluidity and for the enhancement of T_c. The regime of resonance superfluidity is additionally characterized by a pseudogap [(PG) region in Fig. 1]. Finally, in the BCS-like regime, dominated by fermions, the decay (bosons into two fermions) rate is low, as is n_B, and the T_c approaches the BCS-MFA result. In the latter regime, the chemical potential approaches the Fermi energy, and the pseudogap becomes very small. The weak (to moderate) direct attractive interaction U expands the BCS-like regime, but the repulsive U shrinks it.

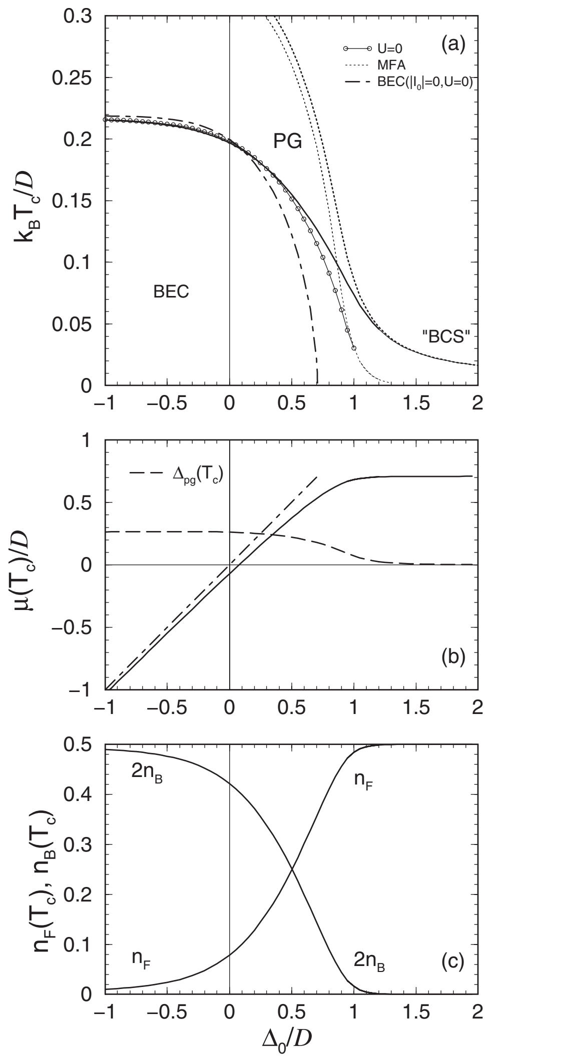

FIG. 1. Self-consistent T-matrix results for the boson-fermion model on the lattice, the bosons without hard core. $n=0.5$, $|I_0|/D=0.5$, $(I=-|I_0|)$, $U/D=0.25$, $D=6t$, and $J/t=0.5$. Panel (a): superfluid transition temperature from T matrix. T_c vs Δ_0 is shown by the solid line; the line with circles is for $U=0$. The dotted lines indicate the BCS-MFA transition temperatures. The dot-dashed line shows the BEC temperature in the absence of interactions. Panel (b): chemical potential (solid line) and pseudogap parameter at T_c (in D-units) (dashed line) vs Δ_0. The dot-dashed line is the chemical potential for BEC transition in the absence of interactions ($2\mu=2\Delta_0$). Panel (c): n_F and n_B vs Δ_0 at T_c.

Figure 2 presents the evaluation of T_c and the phase diagram for the hard-core boson-fermion model [Eq. (3.1)], without direct boson hopping. Here, the hard-core bosons are initially incoherent, and the interchange process gives rise to boson itinerancy and common superconducting transition. Except for the c-electron regime, the calculated T_cs are much lower as compared to BCS-MFA results, and if $J=0$, T_c is strongly depressed as soon as Δ_0 is close to the bottom of the electronic band. Let us add that the almost vanishing asymp-

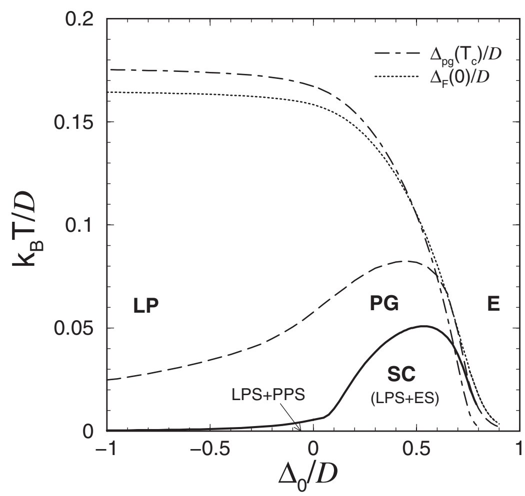

FIG. 2. Phase diagram of the hard-core boson-fermion model as a function of Δ_0/D for s-wave pairing and sc lattice. $n=0.25$, $|I_0|/D=0.5$, $D=6t$, and $J=0$. The transition temperature derived within the T-matrix approach is shown by the solid line. The dashed line indicates the BCS-MFA transition temperature. The dot-dashed line and dotted line indicate the pseudogap and the zero temperature fermionic gap in the superconducting state, respectively. LP—normal state of predominantly LPs, SC—superconducting state (LPS+ES), E—electronic metal, and PG—pseudogap region. LPS+PPS indicates the region where the superconductivity results from both LPs and preformed pairs of c electrons.

tote to T_c in the deep LP (BEC) regime in Fig. 2 compared to the finite asymptote in Fig. 1(a) is a consequence of turning off the direct boson hopping.[13] In the pseudogap region, the electronic spectrum is gapped, and the pseudogap parameter at T_c for $\Delta^*>0$ essentially measures a mean square amplitude of the pairing field (of the c electrons). The values of the pseudogap parameter at T_c, beyond the c regime, are comparable to the zero temperature superconducting gap values in the fermionic spectrum $[\Delta_F(0)]$ computed in the BCS-MFA (see Ref. 13). Almost a constant difference between $\Delta_{pg}(T_c)$ given by $|I|\sqrt{n_B(T_c)}$ and $\Delta_F(0)$ in the LP regime [where $\Delta_F(0)\to|I|\sqrt{n_B(0)[1-n_B(0)]}$] partially reflects the approximate treatment of the hard-core effects by the present T-matrix approach. This difference becomes smaller with lower n_B. In addition, when the (renormalized) LP level reaches and goes below the bottom of the electronic band, the concentration of c electrons is small, and strong attractive interaction gives rise to the formation of bound c-electron pairs.[12,13] In this regime, a superconducting state is formed by two kinds of (hybridized) bosons: preformed c-electron pairs and LPs (LPS+PPS region in Fig. 2).

In the self-consistent T-matrix approach, the fluctuations of the order parameter are included at the Gaussian level. Nevertheless, it is interesting to observe that the phase diagram, shown in Fig. 2, displays similar regimes as that determined in Refs. 12 and 13 from the BCS and Kosterlitz-Thouless theories in 2D. We also remark that for $J_0=0$, in both cases, the shapes of T_c vs Δ_0 are qualitatively similar. As we proceed from the regime of predominantly c electrons ($n_F\gg n_B$) to that of predominantly LPs ($n_F\ll n_B$) with decreasing Δ_0, T_c at first sharply increases, then it goes through a maximum inside the mixed regime and is suppressed when the (renormalized) LP level reaches the bottom of the c band and the system enters the LP regime.

The two versions of the boson-fermion model analyzed in this paper can also be considered as particular cases of a more general coupled boson-fermion-Hubbard model with s-wave pairing of the form

$$\tilde{\mathcal{H}}=\mathcal{H}_F+\mathcal{H}_B+H_1, \tag{4.1}$$

$$\mathcal{H}_F=\sum_{i,j,\sigma}{}' t_{ij}c^\dagger_{i\sigma}c_{j\sigma}+(D-\mu)\sum_{i,\sigma}c^\dagger_{i\sigma}c_{i\sigma}+U_F\sum_i c^\dagger_{i\uparrow}c_{i\uparrow}c^\dagger_{i\downarrow}c_{i\downarrow}, \tag{4.2}$$

$$\mathcal{H}_B=\sum_i(2\Delta_0-2\mu)b^\dagger_ib_i-\sum_{i,j}{}' J_{ij}b^\dagger_ib_j+U_B\sum_i(b^\dagger_i)^2(b_i)^2, \tag{4.3}$$

$$H_1=I\sum_i(b^\dagger_ic_{i\downarrow}c_{i\uparrow}+c^\dagger_{i\uparrow}c^\dagger_{i\downarrow}b_i), \tag{4.4}$$

and $n=n_F+2n_B$. The bosonic part $\mathcal{H}_B$ [Eq. (4.3)] is given by the boson Hubbard model with the on-site repulsion U_B and fermionic part $\mathcal{H}_F$ [Eq. (4.2)] by the Hubbard model with the on-site interaction U_F. If $U_B=0$, we obtain the boson-fermion model [Eq. (2.1)], where $\Delta_0\to\Delta_0-J_0/2$ and $U_F=-U$. If $U_B\to\infty$, one gets the case of hard-core bosons (or pseudospins), i.e., for $U_F=0$ the model [Eq. (3.1)], which we analyzed.

In conclusion, by using the self-consistent T-matrix approach, we have presented the results for the superfluid transition temperature and the phase diagram of the boson-fermion model on the lattice. The results go beyond the mean-field theory and describe the BCS-BEC crossover with varying position of the LP level. The region of the resonance superfluidity is preceded by the pseudogap due to pairing correlations. An extended version of this work[19] will be published elsewhere.

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