Electron heating in metallic resistors at sub-Kelvin temperature

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In the presence of Joule heating, the electronic temperature in a metallic resistor placed at sub-Kelvin temperatures can significantly exceed the phonon temperature. Electron cooling proceeds mainly through two processes: electronic diffusion to and from the connecting wires and electron-phonon coupling. The goal of this paper is to present a general solution of the problem in a form that can easily be used in practical situations. As an application, we compute two quantities that depend on the electronic temperature profile: the second and the third cumulant of the current noise at zero frequency, as a function of the voltage across the resistor. We also consider time-dependent heating, an issue relevant for experiments in which current pulses are used, for instance, in time-resolved calorimetry experiments.

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I. MOTIVATIONS AND OUTLINE

When performing electrical measurements, the signal to noise ratio can usually be improved by simply increasing the currents or voltages. In low-temperature experiments, this procedure is problematic because of Joule heating, which can affect the temperature of the circuit under investigation or the temperature of resistors on bias lines, leading to excess noise. Particularly critical is the situation in the sub-Kelvin range, because the temperature of the electrons decouples from the lattice temperature.[1–4] A very conservative, widespread rule of thumb among experimentalists is that the voltage V across the small conductors should not exceed $k_BT_{\rm ph}/e$, with $T_{\rm ph}$ the lattice (phonon) temperature. In contrast, macroscopic components, like commercial resistors, are believed to be immune to electron heating. In fact, the first rule is severe, and the second assertion is often incorrect. The goal of this paper is to provide the experimental physicist with easy evaluation tools of heating effects in order to optimize experiments.

The important parameters are the voltage V, the resistance R, the lattice temperature $T_{\rm ph}$, the resistor volume Ω, and a parameter Σ that describes electron-phonon coupling. The first step is to calculate the characteristic temperature T_Σ, which is the temperature that the electrons would reach if cooling would occur only through the coupling to a bath of zero-temperature phonons:

$$T_\Sigma = \left(\frac{V^2}{\Sigma\Omega R}\right)^{1/5}. \qquad (1)$$

The average electron temperature can then be directly read from Fig. 2, the central result of this work, in which the voltage V, the average temperature $T_{\rm av}$, and the lattice temperature $T_{\rm ph}$ are all expressed in units of T_Σ. In Sec. II A, we explain how this result is obtained and give analytical expressions in various limits. The results are used to calculate the second and third cumulants of the current noise produced by the resistor (Sec. II B, Fig. 4). Numerical applications are carried out explicitly in Sec. II C, showing, in particular, that heating in commercial resistors can be important. In Sec. III, we address time-dependent situations and calculate how fast electrons heat up in a resistor when a current is applied and how fast they cool down when the current is switched off. For small voltages ($eV \ll k_BT_\Sigma$), the variations of temperature in both transients is exponential, with the diffusion time τ_D across the whole conductor as a characteristic time (see Fig. 5). In the opposite limit ($eV \gg k_BT_\Sigma$), heating is exponential, but cooling proceeds very slowly, with a power law dependence (see Fig. 6). The time scale is the electron-phonon scattering time $\tau_{\text{e-ph}}$ at temperature T_Σ, defined by Eq. (17). As a numerical application, we consider in Sec. III C a situation where repeated current pulses are applied to a resistor, and compute the time dependence of the electron temperature (Fig. 7).

II. STATIONARY SITUATIONS

A. Solution of the heat equation

When a voltage V is applied to a two-terminal resistor (see top of Fig. 1), the Joule power V^2/R is delivered to the electrons. This power can dissipate by two mechanisms: the first one, which dominates at room temperature or in macroscopic resistors, is phonon emission. It follows, at temperatures well below the Debye temperature, a $T^n(x)-T^n_{\rm ph}$ dependence, with $4 \leq n \leq 6$, $T(x)$ the local electron temperature, and $T_{\rm ph}$ the phonon temperature.[5] The second mechanism is the simple diffusion of the energetic electrons out of the resistor. The energy is then dissipated in the connecting leads, which are, in typical situations, large and have low resistance. The balance between the Joule power and the two cooling mechanisms can be expressed in the form of a heat equation[6]

$$\frac{d}{dx}\left[\frac{L_oT(x)}{R}\frac{d}{dx}T(x)\right] = -\frac{V^2}{R} + \Sigma\Omega[T^5(x) - T^5_{\rm ph}], \qquad (2)$$

with x the position along the resistor in reduced units (x runs from 0 to 1), $L_o = \pi^2k_B^2/3e^2$ the Lorenz number, Ω the resistor

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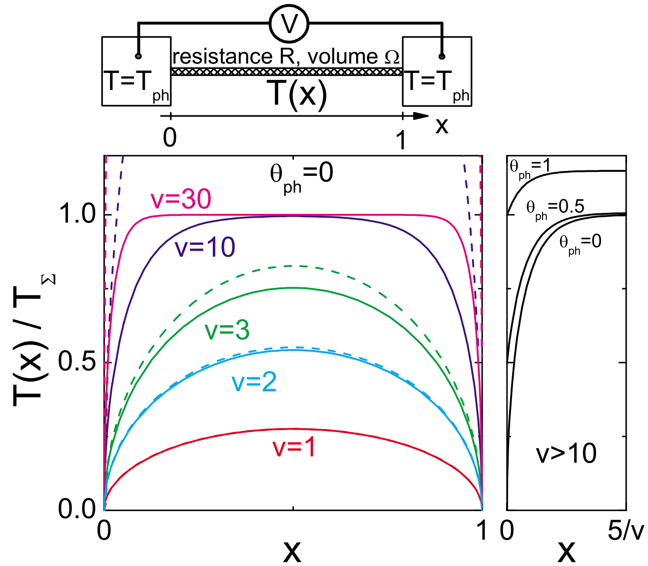

FIG. 1. (Color online) Top: Resistor biased by a voltage V and placed between two connecting wires in which the electron temperature T and the phonon temperature T_{ph} are equal. Left panel: (solid lines) temperature profile in the resistor for different values of $v=eV/k_BT_\Sigma$ with $T_\Sigma=(V^2/\Sigma\Omega R)^{1/5}$; (dashed lines) temperature profile expected when phonon cooling is neglected [Eq. (3)]. Right panel: Temperature profile near the ends of the resistor for $v>10$ and $\theta_{ph}=T_{ph}/T_\Sigma=0$, 0.5, and 1.

volume, and Σ the electron-phonon coupling constant (typically $\Sigma \simeq 2\ \mathrm{nW}/\mu\mathrm{m}^3/\mathrm{K}^5$ for good metals[7]). The left-hand side of Eq. (2) accounts for heat transport by electron diffusion, which is expressed by the Wiedemann-Franz law, stating that the electron thermal conductivity is proportional to the product of the electrical conductivity and the electron temperature. The following assumptions have been made in writing Eq. (2):

(1) The electron temperature $T(x)$ is assumed to be well defined locally, i.e., the local electron energy distribution function is a Fermi function. This requires that the thermalization of electrons among themselves (e.g., by Coulomb interaction) occurs faster than the diffusion of electrons across the resistor,[8] a condition usually obeyed except for short wires (length $\lesssim 50\ \mu$m) made of very pure materials.[9]

(2) The last term of the equation, which describes cooling by phonons, assumes that the lattice temperature T_{ph} does not depend on the local electron temperature $T(x)$. Corrections due to the Kapitza resistance between the phonons of the resistive film and the substrate could, in principle, be included,[3,10] but their contribution is not essential in practice.

(3) The heat power transferred to phonons was taken proportional to $T^n(x)-T_{ph}^n$ with $n=5$. Theoretically, it is predicted that the exponent n, which is related to an E^{2-n} dependence of the electron-phonon scattering rate with electron energy E, can range from 4 to 6, depending on the relative sizes of the thermal phonon wavelength and the electron mean free path, on the dimensionality of the phonon system, and on the dynamics of impurities.[5] In most experiments, values close to $n=5$ have been found (see discussions in Refs. 5, 7, and 11), therefore, our choice. The calculations

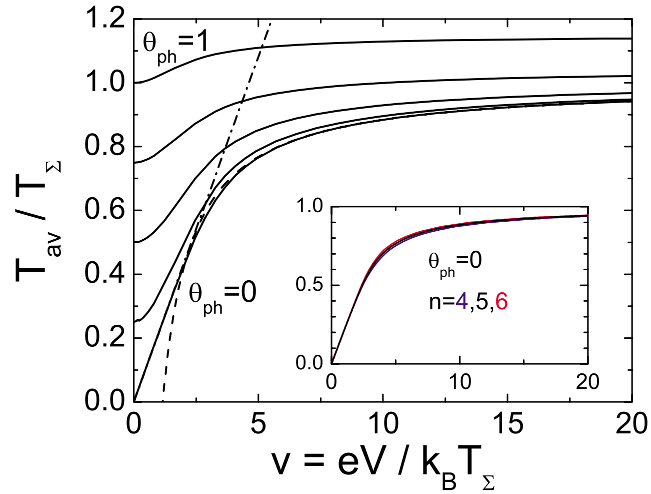

FIG. 2. (Color online) Average temperature T_{av} in units of T_Σ, as a function of $v=eV/k_BT_\Sigma$, for $\theta_{ph}=T_{ph}/T_\Sigma=0$, 0.25, 0.5, 0.75, and 1 (from bottom to top). The dotted line corresponds to the low-v approximation $T_{av}=\frac{\sqrt{3}}{8}\frac{eV}{k_B}$; the dashed-dotted line to the large-v approximation $T_{av}=1-1.16/v$. Inset: At $\theta_{ph}=T_{ph}/T_\Sigma=0$, comparison of the evolution of the average temperature with v for various exponents n of the temperature in the expression of the heat flow through electron-phonon coupling.

can, however, be easily extended to other values of n, and the results presented in Fig. 2 apply, with another definition of T_Σ, as discussed in the following.

(4) Radiative cooling,[11] which has a negligible effect in resistors connected to large, nonsuperconducting contacts,[12] is neglected.

The heat equation (2) has to be solved with boundary conditions for $T(x)$ at $x=0$ and $x=1$. When the connecting wires to the resistor have low resistance and are very large compared to the resistor, as is the case for macroscopic resistors made of thin and narrow metallic stripes of metal, one can assume $T(0)=T(1)=T_{ph}$. This simple hypothesis will be made in the following. For on-chip thin-film resistors, heating of the contact pads themselves may, however, not be negligible.[13]

Before a general solution of Eq. (2) is presented, we recall simple limits. The so-called interacting hot-electron limit[6] is obtained by neglecting phonon cooling:

$$T(x)=\sqrt{T_{ph}^2+\frac{3}{\pi^2}x(1-x)\left(\frac{eV}{k_B}\right)^2} \qquad (3)$$

(see left panel of Fig. 1, dashed lines). For $T_{ph}=0$, the maximal temperature is $T(\frac{1}{2})=(\sqrt{3}/2\pi)eV/k_B\simeq 0.28eV/k_B$, and the average temperature is $T_{av}=\int_0^1 T(x)dx=(\sqrt{3}/8)eV/k_B \simeq 0.22eV/k_B$. Electron-phonon coupling further reduces the temperature, so that this is an upper bound on the average electron temperature, which numerically reads, keeping[14] now T_{ph},

$$\frac{T_{\rm av}}{T_{\rm ph}} \leqslant \sqrt{1+\left(0.22\frac{eV}{k_B T_{\rm ph}}\right)^2}. \tag{4}$$

In particular, the rule of thumb $eV = k_B T_{\rm ph}$ corresponds to a 2.5% average overheating of the electrons. Numerically, one obtains the equivalent expression

$$T_{\rm av}(\text{mK}) \leqslant \sqrt{T_{\rm ph}^2(\text{mK}) + [2.5V(\mu\text{V})]^2}. \tag{5}$$

In the opposite limit where cooling by diffusion can be neglected, the electron temperature is homogeneous and equal to[3]

$$T = (T_{\rm ph}^5 + T_\Sigma^5)^{1/5}. \tag{6}$$

In the following, we call this limit "the fully thermalized regime." In the limit $T_{\rm ph}=0$, the temperature grows as $V^{2/5}$ (see Fig. 3).

In intermediate regimes, the temperature profile is obtained by solving numerically the heat equation. For generality, it is convenient to rewrite it in reduced units. One possibility is to take as a reference the "crossover temperature"[4] $T_{\rm co} = (\Sigma\Omega Re^2/k_B^2)^{-1/3}$, which is the energy scale for which phonon cooling and diffusion cooling are equally important. This temperature is an intrinsic quantity for the resistor, which, in particular, does not depend on V or $T_{\rm ph}$. However, it does not correspond to the electron temperature in any limit, therefore, we prefer to take as a reference T_Σ, keeping in mind that it depends on V. Defining $\theta(x) = T(x)/T_\Sigma, v = eV/k_B T_\Sigma [= (eV/k_B T_{\rm co})^{3/5}]$, and $\theta_{\rm ph} = T_{\rm ph}/T_\Sigma (\propto V^{-2/5})$, Eq. (2) reads

$$\frac{d^2}{dx^2}\theta^2(x) = \frac{6}{\pi^2}v^2[\theta^5(x) - \theta_{\rm ph}^5 - 1]. \tag{7}$$

The temperature profile being symmetric with respect to the middle of the wire, Eq. (7) needs to be solved for $0 < x < \frac{1}{2}$ with the boundary conditions $\theta(0) = \theta_{\rm ph}$ and $\theta'(\frac{1}{2}) = 0$. Instead of solving this nonlinear differential equation with boundary conditions specified at different points, it is convenient to rewrite it in the following integral form:

$$\frac{2\sqrt{3}v}{\pi\theta_m}x = \int_{\theta_{\rm ph}^2/\theta_m^2}^{\theta^2(x)/\theta_m^2} du\left[\frac{2}{7}\theta_m^5(u^{7/2}-1) - \lambda^5(u-1)\right]^{-1/2}, \tag{8}$$

with $\theta_m = \theta(\frac{1}{2})$ and $\lambda = (\theta_{\rm ph}^5 + 1)^{1/5}$. The value of θ_m is obtained by solving Eq. (8) for $x = 1/2$. In the left panel of Fig. 1, the temperature profile along the resistor is given for $v = 1$, 2, 3, 10, and 30, assuming $T_{\rm ph} = 0$. At $v \lesssim 1$, one recovers the result of Eq. (3), plotted as dashed lines: phonon cooling can be neglected. For $v \gtrsim 10$, it is an excellent approximation to take $\theta_m = \lambda$ in Eq. (8), a value that does not depend on v. $T(x)$ is then a function of xv only, which is essentially constant around the middle of the wire, whereas at a distance $5/v$ from the contacts, one obtains the profiles shown in the right panel of Fig. 1 for $\theta_{\rm ph} = 0$ (see Ref. 15), 0.5, and 1. The characteristic length over which the electron temperature varies from $T_{\rm ph}$ to T_Σ is, therefore, $L_\Sigma = L/v = \frac{\sqrt{3}}{8}L^{3/5}L_{\text{e-ph}}^{2/5}$, where the "electron-phonon length"

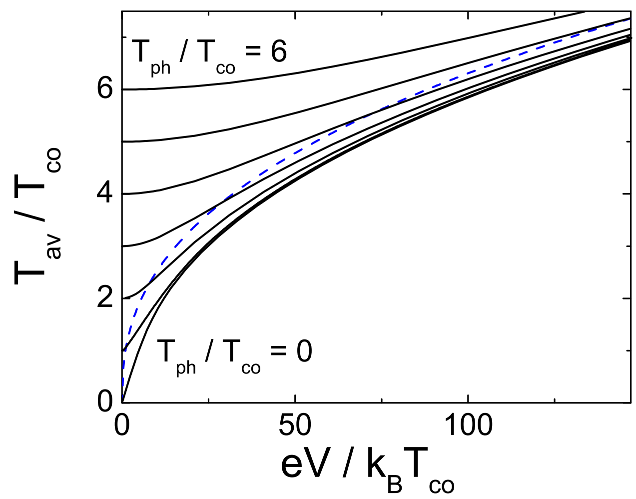

FIG. 3. (Color online) Average temperature $T_{\rm av}$ as a function of voltage V for various temperatures $T_{\rm ph}$, all in units of $T_{\rm co} = (\Sigma\Omega Re^2/k_B^2)^{-1/3}$. The value of $T_{\rm ph}/T_{\rm co}$ is given by the intersection of the curves with the vertical axis. The blue dashed line is the reference temperature T_Σ.

$$L_{\text{e-ph}} = \left(\frac{8k_B}{\sqrt{3}e}\right)^{5/2}(\rho\Sigma V^3)^{-1/2} \tag{9}$$

(ρ being the resistivity) is defined, following[16] Ref. 6, as the resistor length for which $4k_B T_\Sigma/R$ is equal to the current noise in the interacting hot-electron regime $\frac{\sqrt{3}}{2}eI$. This length is typically of the order of a few micrometers for voltages of the order of 1 mV.

From the complete temperature profile $\theta(x)$, the average electron temperature $T_{\rm av}$ is obtained using $T_{\rm av} = T_\Sigma \int_0^1 \theta(x)dx$. The central result of this work is the resulting plot, shown in Fig. 2, of the average temperature $T_{\rm av}$ as a function of the voltage V, both in units of T_Σ, for $T_{\rm ph}/T_\Sigma = 0$, 0.25, 0.5, 0.75, and 1. At $\theta_{\rm ph} = 0$ and $v \lesssim 2, T_{\rm av} \sim \frac{\sqrt{3}}{8}eV/k_B$ (dotted line), whereas for $v \gtrsim 4, T_{\rm av}/T_\Sigma \approx 1 - 1.16/v$ (dashed line). This $1/v$ dependence is due to the crossover regions of width $\sim 5/v$ at the resistor ends. Figure 2 can be directly used to read out the average electron temperature $T_{\rm av}$ for a given set of experimental parameters $(V, T_{\rm ph})$ after having computed T_Σ with Eq. (1). Interestingly, the corresponding curves for other exponents of the temperature in the last term of Eq. (2) ($n = 4$ or 6 instead of 5) are almost identical (see inset), and the same curves can be used to evaluate $T_{\rm av}$, however, with the generalized definition of the reference temperature $T_\Sigma = (V^2/\Sigma\Omega R)^{1/n}$.

However, because of the use of reduced units which depend on V, the v dependence of $T_{\rm av}$ at a fixed value of $\theta_{\rm ph}$ shown in Fig. 2 does *not* correspond to a situation in which V is changed at a fixed $T_{\rm ph}$, since $\theta_{\rm ph} \propto V^{-2/5}$. In order to visualize how temperature increases with V at a given phonon temperature, we plot in Fig. 3 the average temperature $T_{\rm av}(V)$ for various $T_{\rm ph}$, with V and $T_{\rm ph}$ given in units of[4] $T_{\rm co} = (\Sigma\Omega Re^2/k_B^2)^{-1/3}$, which is constant for a given resistor. The range in voltage V is the same as in Fig. 2. We have used the relations $v^{5/3}=eV/kT_{co}$ and $(T_{av}/T_\Sigma)v^{2/3}=T_{av}/T_{co}$.

B. Second and third cumulants of the current noise at zero frequency

The temperature profile can be used to evaluate the current noise properties of the resistor. We focus here on the second (S_2) and third (S_3) cumulants of noise at low frequencies ($\hbar\omega \ll eV$ for S_2 and $\hbar\omega \ll eV$, $\hbar/\tau_D$ for S_3,[17] with $\tau_D = L^2/D$ the diffusion time and D the diffusion constant):

$$S_2 = 2\iint dt\langle \delta I(0)\delta I(t)\rangle$$

and

$$S_3 = \iint dt_1 dt_2 \langle \delta I(0)\delta I(t_1)\delta I(t_2)\rangle,$$

with $\delta I(t)=I(t)-\langle I\rangle$. It has been shown that when phonon cooling can be disregarded and $eV \gg k_B T_{ph}$, S_2 and S_3 are proportional to the applied current:[6,13] $S_2=F_2\times 2eI$ and $S_3=F_3\times e^2 I$, with F_2 and F_3 the generalized "Fano factors." When, furthermore, the rate of electron-electron interaction is negligible compared to $1/\tau_D$, the distribution function is not a Fermi function, but a function with two steps,[8,18] and[18–20] $F_2=\frac{1}{3}\approx 0.33$ and $F_3=\frac{1}{15}\approx 0.067$. In the opposite limit where electron-electron interaction is strong, electrons thermalize locally to distribute in energy according to a Fermi function, and the temperature profile is given by Eq. (3). One then obtains[21–23] $F_2=\frac{\sqrt{3}}{4}\approx 0.43$ and[24] $F_3=\frac{8}{\pi^2}-\frac{9}{16}\approx 0.248$.

In the presence of strong phonon cooling ($v \gg 1$), the electron temperature becomes homogeneous, at a value T_Σ smaller than eV/k_B. It is then expected[24] that $F_2, F_3 \to 0$.

For intermediate coupling to phonons, F_2 and F_3 depend on the voltage across the resistor. Their values are obtained from the full solution of the heat equation: the second cumulant is given by a Johnson-Nyquist-like formula[6] $S_2=4k_B T_{av}/R$, in which the noise temperature is the average electron temperature T_{av}, yielding $F_2=2k_B T_{av}/eV=2\theta_{av}/v$. This formula can be understood as resulting from the added Johnson-Nyquist noise of small sections of the resistor, each at a temperature $T(x)$. The decay of F_2 at large V was discussed in Ref. 21, and the complete crossover was calculated in Ref. 25 by numerical integration of Eq. (2). In turn, F_3 is given by (see Appendix)

$$F_3 = \frac{36}{\pi^2}\int_0^1 dxdy \frac{1}{\theta(x)} G_1(\theta,x,y)\{\theta(y)-2\theta_{av}\}, \qquad (10)$$

where $G_1(\theta,x,y)$ is the Green's function such that $\left[\nabla^2 + \frac{15}{\pi^2}v^2\theta^3(x)\right]G_1(\theta,x,y)=\delta(x-y)$ and $G_1(\theta,0,y)=G_1(\theta,x,0)=0$. The calculation of F_3 is detailed in the Appendix. Figure 4 shows the voltage dependence of F_2 (blue line) and F_3 (red line) as a function of v (bottom axis) and $L/L_{e\text{-}ph}=(\sqrt{3}v/8)^{5/2}$ (top axis) for $T_{ph}=0$. Also shown with a dashed line is the curve obtained for F_2 when electron diffusion is neglected,[21] using Eq. (6) (dashed line), which gives $F_2=2/v \propto V^{-3/5}$. In turn, at large voltages, $F_3 \propto v^{-2} \propto V^{-6/5}$. If one considers a situation where the resistor length L is varied at constant current, then $F_2 \propto 1/L$ and $F_3 \propto 1/L^2$. The decay to zero of F_2 and F_3 from the interacting hot-electron values ($\frac{\sqrt{3}}{4}$ and $\frac{8}{\pi^2}-\frac{9}{16}$) is, therefore, very slow, as already pointed out in Refs. 24 and 25. The non-Gaussian character of the current noise, evidenced by $F_3 \neq 0$, is washed out at $L/L_{e\text{-}ph} \gtrsim 10$.

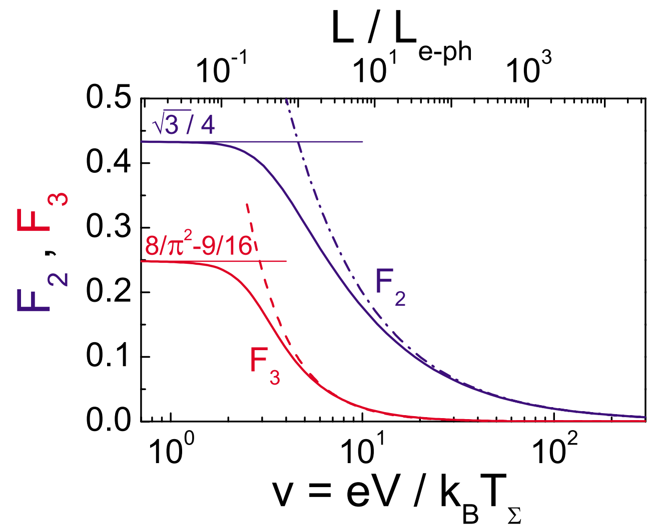

FIG. 4. (Color online) Solid lines: Fano factors $F_2=S_2/2eI$ (blue) and $F_3=S_3/e^2I$ (red) for the zero-frequency second and third cumulants of noise, from the hot-electron limit to the fully thermalized regime. Dashed-dotted line: Asymptotic dependence of F_2 neglecting electron diffusion, $F_2=2/v$. The top axis is resistor length over $L_{e\text{-}ph}$ [see Eq. (9)].

C. Examples

We consider here a few cases illustrating the use of the results given in the preceding sections. As a first example, we consider a 10-μm-long, 100-nm-wide, and 5-nm-thick Cr resistor with resistance $R=25$ kΩ like those used in Ref. 26, biased at $V=1$ mV and placed at $T_{ph}=20$ mK. Assuming $\Sigma_{Cr}=2$ nW/μm^3/K^5, the characteristic temperature is $T_\Sigma=1.3$ K and the voltage and phonon temperature in reduced units are $v=eV/k_B T_\Sigma \sim 9$ and $\theta_{ph}=0.015$. The noise temperature is directly read from Fig. 2: $T_{av}\sim 0.87 T_\Sigma \sim 1.15$ K. At this voltage, heating of the resistor is, thus, very important, an effect which hindered the authors of Ref. 26 from drawing clear-cut conclusions from Coulomb blockade measurements at finite voltage. Increasing the resistor volume Ω with "cooling fins" can help in decreasing electron heating,[3] but such a procedure is extremely inefficient since the characteristic temperature T_Σ decreases as $\Omega^{-1/5}$ only.

As a second application, we now consider a commercial macroscopic surface mount resistor with $R=500$ Ω. Such resistors, made of thin ($\sim$10 nm) NiCr films[27] with resistivity $\rho \sim 100$ $\mu\Omega$ cm and dimensions $\sim 1\times 0.2$ mm^2, were used as bias resistors in measurements of the state of superconducting Josephson Q-bits[28,29] performed at 15 mK, with a bias current $\sim$0.8 μA, resulting in a voltage $V \sim 400$ μV. The corresponding temperature scale $T_\Sigma \sim 150$ mK yields v

$=eV/k_BT_\Sigma \sim 30$ and $\theta_{\rm ph} \sim 0.1$, hence, from Fig. 2, $T_{\rm av} \sim T_\Sigma \sim 150$ mK. Even in such a macroscopic resistor, the volume is not sufficient to provide enough electron-phonon coupling, and heating is important. In the next section, we show how this heating is limited when pulses are used instead of static voltages.

III. TIME-DEPENDENT SITUATIONS: SWITCHING ON AND OFF JOULE HEATING

The case of a constant voltage V across the resistor, which was investigated above, can be extended to the case of slowly varying voltages directly. However, when V changes on time scales shorter than the diffusion time or the electron-phonon scattering time[30] (see below), the previous results cannot be used to calculate instantaneous temperatures. These issues are solved by adding to the heat equation (2) a time-dependent term $dQ/dt = C_e dT/dt$, with $C_e = \gamma T \Omega$ the electronic heat capacity, $\gamma = (\pi^2/3)k_B^2 \nu_F$ (from Fermi liquid theory), and ν_F the density of states at Fermi energy (spin degeneracy included). When $V(t) = Vf(t)$, the time-dependent heat equation can then be rewritten, in reduced units, as

$$\frac{\partial \theta^2}{\partial \tau} = \frac{\partial^2 \theta^2}{\partial x^2} - \frac{6}{\pi^2} v^2 [\theta^5 - \theta_{\rm ph}^5 - f^2(\tau)], \qquad (11)$$

where $\tau = t/\tau_D$ is the reduced time, with $\tau_D = L^2/D$ the diffusion time. Note that the reference temperature T_Σ used to define $\theta(x) = T(x)/T_\Sigma$ is calculated with the voltage scale V, not with the time-dependent value $V(t)$. In the following, we treat more explicitly two situations: how a resistor heats up when the voltage is applied at $t=0$, i.e., $f(\tau) = H(\tau)$, and how a resistor cools down when the voltage is set to zero at $t=0$, i.e., $f(\tau) = 1 - H(\tau)$. Here, $H(\tau)$ is the Heaviside function (0 for $\tau < 0$, 1 for $\tau > 0$). These situations also allow one to describe experiments in which current or voltage pulses are used like, for example, measuring the switching rate of Josephson junctions.[28] Understanding how the pulse characteristics can reduce the noise in such measurements is, therefore, important to design the readout of superconducting Q-bits.

When a voltage is applied, the linear drop of the electrical potential, which results from the collective charge modes, is established after a RC time, where the capacitance C is the capacitance of the wire to ground. This time is generally much shorter than the time necessary to build up the temperature profile, which involves diffusion of individual electrons. Hence, we consider here that Joule heating is homogeneous as soon as a voltage is applied. When $v \lesssim 1$, the temperature profile is entirely determined by the temperature at the ends of the resistor, therefore a steady-state regime is reached only when the electrons have diffused across the whole resistor and the characteristic time is the diffusion time τ_D. If $v \gg 1$, the transient is shorter because, apart from very close to the ends, the temperature is mostly determined by a local equilibrium between Joule heating and phonon emission. We now treat quantitatively these two limits.

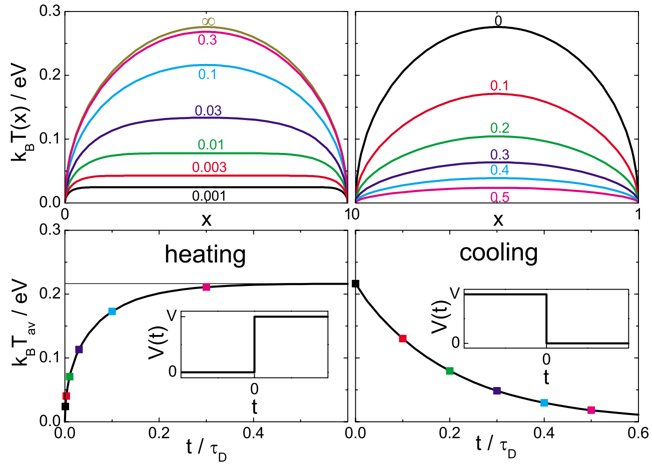

FIG. 5. (Color online) Time evolution of the temperature profile (top panels) and of the average temperature $T_{\rm av}$ (bottom panels) in the limit $v \ll 1$, for $T_{\rm ph} = 0$. Left panels: Heating sequence of the resistor when $V(t)/V = H(t)$ (as shown in the inset). Right panels: Cooling sequence of the resistor when $V(t)/V = 1 - H(t)$ (as shown in the inset). The profiles are plotted at various values of t/τ_D, with $\tau_D = L^2/D$ the diffusion time. The colors of the solid curves in the top panels correspond to those of the square symbols in the bottom panels. The curves in the bottom panels cannot be distinguished from $k_B T_{\rm av}/eV \simeq \frac{\sqrt{3}}{8}\sqrt{1-\exp(-10\tau)}$ and $\frac{\sqrt{3}}{8}\exp(-5\tau)$, respectively.

A. Small v limit

If $v \lesssim 1$, $\theta(x,t) - \theta_{\rm ph} \ll 1$ even when the stationary regime is reached, and Eq. (11) reduces to

$$\frac{\partial \theta^2}{\partial \tau} = \frac{\partial^2 \theta^2}{\partial x^2} + \frac{6}{\pi^2} v^2 f^2(\tau). \qquad (12)$$

As shown in Sec. II A, the proper energy scale when $v \lesssim 1$ is eV, and the solution of Eq. (12) that satisfies the boundary conditions $T(0,\tau) = T(1,\tau) = T_{\rm ph}$ reads

$$\left[\frac{k_B T(x,\tau)}{eV}\right]^2 = \left(\frac{k_B T_{\rm ph}}{eV}\right)^2 + \sum_{k\ \rm odd} a_k(\tau)\sin(\pi k x), \qquad (13)$$

with $a_k(\tau)$ solution of

$$\frac{da_k(\tau)}{d\tau} + \pi^2 k^2 a_k(\tau) = \frac{24}{\pi^3 k} f^2(\tau). \qquad (14)$$

In the case where $f(\tau) = H(\tau)$, heating is then given by $a_k(\tau) = (24/\pi^5 k^3)(1 - e^{-\pi^2 k^2 \tau})$, and in the case $f(\tau) = 1 - H(\tau)$, cooling from the profile (3) follows $a_k(\tau) = (24/\pi^5 k^3) e^{-\pi^2 k^2 \tau}$. Corresponding temperature profiles at various times are plotted in the top panels of Fig. 5 assuming $T_{\rm ph} = 0$, whereas the time evolution of the average temperature $T_{\rm av}$ is plotted in the bottom panels. At very short times, the average temperature grows as $k_B T_{\rm av}/eV = \sqrt{6\tau}/\pi$. At $\tau \gtrsim 0.01$, a better approximation is

$$\frac{k_B T_{av}}{eV} \simeq \frac{\sqrt{3}}{8}\sqrt{1-\exp(-10\tau)}, \tag{15}$$

which cannot be distinguished from the exact solution in Fig. 5. At $\tau \simeq 0.5$, the asymptotical temperature profile given by Eq. (2) is essentially established. Conversely, after the voltage is turned off, the temperature decay is well approximated (within the linewidth in Fig. 5) by

$$\frac{k_B T_{av}}{eV} \simeq \frac{\sqrt{3}}{8}\exp(-5\tau). \tag{16}$$

Hence, in the low-voltage regime, heating and cooling occur exponentially and the time scale is the diffusion time τ_D. In the example of the 500 Ω commercial resistor in Sec. II B, $\tau_D \sim 3$ ms. For metallic wires made of pure materials[27] with an elastic mean free path of the order of 40 cm^2/s, $\tau_D \sim 20$ ns for a length $L \sim 20$ μm.

B. Large v limit

If $v \gg 1$, it was shown in Sec. II A that the temperature becomes almost homogeneous in the wire. The relevant time scale is then the electron-phonon scattering time[11,31,34] at the characteristic temperature T_Σ:

$$\tau_{e\text{-}ph}(T_\Sigma) = \frac{\gamma}{\Sigma T_\Sigma^3} = \frac{\pi^2}{3}\frac{L_\Sigma^2}{D}, \tag{17}$$

with L_Σ the characteristic length for the variation of $T(x)$ introduced in Sec. II A. Numerically, $\gamma/\Sigma \approx 0.03$ μs K^3. Using $\frac{6}{\pi^2}v^2\tau = 2\tau^*$ with

$$\tau^* = t/\tau_{e\text{-}ph}(T_\Sigma), \tag{18}$$

Eq. (11) reduces to

$$\frac{\partial\theta^2}{\partial\tau^*} = -2[\theta^5 - \theta_{ph}^5 - f^2(\tau^*)], \tag{19}$$

which for $\theta_{ph} = f = 0$ is simply equivalent to

$$\frac{\partial T}{\partial t} = -\frac{T}{\tau_{e\text{-}ph}(T)}, \tag{20}$$

expressing that the instantaneous decay rate of T is exponential with a characteristic time $\tau_{e\text{-}ph}(T)$. Equation (19) yields

$$\int_{\theta^2(0)}^{\theta^2(\tau^*)} \frac{dw}{f^2(\tau^*) + \theta_{ph}^5 - w^{5/2}} = 2\tau^*. \tag{21}$$

When $f(\tau) = 1 - H(\tau)$ and $T_{ph} = 0$, the temperature decay from T_Σ has a simple form:

$$\theta(\tau^*) = (1 + 3\tau^*)^{-1/3}. \tag{22}$$

This temperature decay, which was directly measured in Ref. 11, follows a power law only, so that it takes a very long time to recover the base temperature after the voltage is set to 0, which is due to the divergence of $\tau_{e\text{-}ph}(T)$ when $T \to 0$. The results of Eq. (21) with $f=1$ (heating) and $f=0$ (cooling) are plotted in Fig. 6 in the case $T_{ph}=0$, with linear (top) and logarithmic (bottom) time scales. The temperature rise is

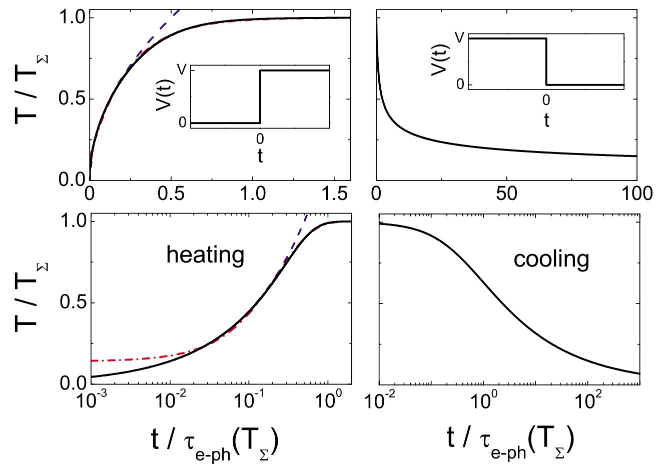

FIG. 6. (Color online) Evolution of the temperature with time in linear (top) and logarithmic (bottom) scales in the limit $v \gg 1$ for $T_{ph}=0$. Left panels: Heating sequence of the resistor when $V(t)/V = H(t)$ (as shown in the inset). Right panels: Cooling sequence of the resistor when $V(t)/V = 1 - H(t)$ (as shown in the inset). Times are given in units of the electron-phonon time at temperature T_Σ: $\tau_{e\text{-}ph}(T_\Sigma) = \gamma/\Sigma T_\Sigma^3$. The blue dashed line is $\sqrt{2\tau^*}$ and the red dashed-dotted line is $1 - 0.86\exp(-4.2\tau^*)$, with $\tau^* = t/\tau_{e\text{-}ph}(T_\Sigma)$.

well approximated by $\theta(\tau^*) \approx \sqrt{2\tau^*}$ when $\tau^* \lesssim 0.2$ (dashed line) and $\theta(\tau^*) \approx 1 - 0.86\exp(-4.2\tau^*)$ when $\tau^* \gtrsim 0.2$ (dotted line). More generally, when $\tau^* \ll 1$, for $T_{ph} \neq 0$,

$$\theta(\tau^*) \approx \sqrt{\theta^2(0) + 2[1 + \theta_{ph}^5 - \theta^5(0)]\tau^*}. \tag{23}$$

Even though $v \gg 1$, we now estimate cooling by electron diffusion to the connecting leads. Starting from a constant temperature T_0, cooling by diffusion follows Eq. (13) with $a_k(\tau) = \left(\frac{k_B T_0}{eV}\right)^2 \frac{4}{\pi}\frac{e^{-\pi^2 k^2 \tau}}{k}$ and $T_{av}/T_0 \sim \exp(-5\tau)$. Because of this exponential dependence, to be compared with the power law (22), diffusion can contribute to cooling when t becomes comparable to τ_D.

C. Numerical application

In experiments where the voltage is applied in repeated pulses, heating is reduced and the temperature oscillates in time. To illustrate this effect, we reconsider the second example of Sec. II C, but we now assume that the voltage is applied during short pulses of duration $t_p = 0.1$ μs, repeated every period $t_r = 20$ μs (which corresponds to actual experimental conditions in Refs. 28 and 29). We now show that despite the short duty cycle $d = t_p/t_r = 0.005$, heating is not negligible. In our example, $v \sim 30$, therefore the relevant time scale when V is applied is $\tau_{e\text{-}ph}(T_\Sigma) = \gamma/\Sigma T_\Sigma^3 \sim 10$ μs. Equation (23) with $\theta_{ph} = 15$ mK$/T_\Sigma = 0.1$ gives $T(t_p) = 0.17 T_\Sigma = 25$ mK, indicating slight heating by the first pulse. The resistance then cools down during a time t_r before the next pulse is applied, following Eq. (21), to $T = 0.169 T_\Sigma$, hardly less than at the end of the first pulse.[35] The temperature rises further during the next pulses until steady oscillations are established. The full time evolution of T shown in Fig. 7 is obtained by iterating Eq. (21). At each pulse, the

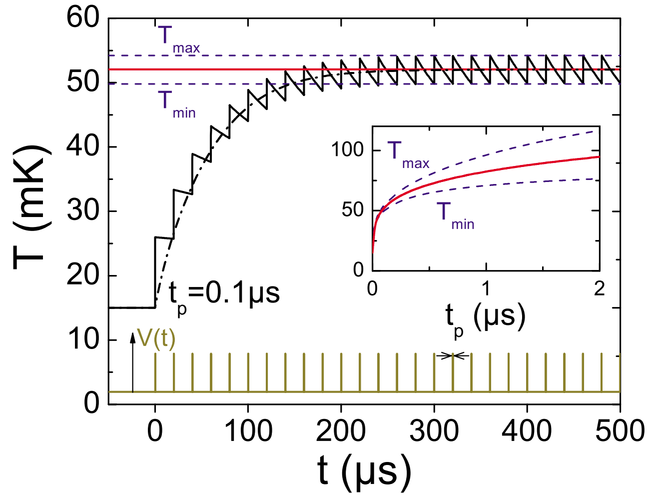

FIG. 7. (Color online) Main panel: Time dependence of the temperature of a commercial macroscopic surface mount 500 Ω resistor (see text) heated by voltage pulses (bottom curve) of length $t_p=0.1$ μs applied every 20 μs. The dashed-dotted line is the predicted heating with the average Joule power. Inset: (blue dashed lines) minimal and maximal temperatures reached in the stationary regime as a function of the pulse length t_p. The red solid line is the temperature evaluated with the average Joule power.

temperature rise gets smaller than the preceding pulse because the starting temperature is larger and the heat transfer to phonons becomes more efficient. For the same reason, the cooling between the pulses gets more and more efficient. At $t \gtrsim 250$ μs, a stationary regime is reached, with the reduced temperature oscillating between $\theta_{\rm min}$ and $\theta_{\rm max}$ such that $\int_{\theta_{\rm min}^2}^{\theta_{\rm max}^2} dw/(1+\theta_{\rm ph}^5-w^{5/2})=2\tau_p^*$ and $\int_{\theta_{\rm max}^2}^{\theta_{\rm min}^2} dw/(\theta_{\rm ph}^5-w^{5/2})=2(\tau_r^*-\tau_p^*)$. One obtains $\theta_{\rm min}=0.33$ ($T_{\rm min}=49$ mK) and $\theta_{\rm max}=0.36$ ($T_{\rm max}=54$ mK). The amplitude of the oscillations $\Delta\theta=\theta_{\rm max}-\theta_{\rm min}$ is, therefore, very small. However, it increases with t_p, as shown in the inset of Fig. 7, and can become sizable.

The main features of the time evolution of the temperature can be calculated more simply from the average Joule power dV^2/R. From Sec. II, the characteristic temperature is then $T_\Sigma^{\rm eff}=52$ mK, which fits with the average temperature in the stationary regime of the pulse sequence. According to Sec. III B, this temperature is reached in a time $\tau_{\text{e-ph}}(T_\Sigma^{\rm eff}) \sim 220$ μs. The rise of temperature with time calculated with Eq. (21), in which all quantities (θ,τ^*) are calculated using $T_\Sigma^{\rm eff}$, is shown as a dotted line in Fig. 7 and reproduces well the overall behavior. The amplitude $\Delta\theta$ of the temperature oscillations can be evaluated using Eq. (23) under the assumption that the starting temperature is $T_\Sigma^{\rm eff}$, which is a good approximation for oscillations of small amplitude, and considering that the voltage V is always present during the pulse of duration t_p, i.e., with $\theta(0)=T_\Sigma^{\rm eff}/T_\Sigma=d^{1/5}$ and $\tau^*=t_p/\tau_{\text{e-ph}}(T_\Sigma)$. If $\theta_{\rm ph} \ll 1$ and $d \ll 1$, one obtains $\Delta\theta \approx \theta(0)\tau^*$, an approximation only 20% larger than the exact result in the worst case of the inset of Fig. 7 ($t_p=2$ μs).

IV. SUMMARY

The solution of the heat equation in a resistor is determined by a characteristic temperature $T_\Sigma=(V^2/\Sigma\Omega R)^{1/5}$. If $v=eV/k_BT_\Sigma \lesssim 1$, cooling by phonons is negligible, $T(x)$ is given by Eq. (3), and the average temperature $T_{\rm av}=(\sqrt{3}/8)\times(eV/k_B)$. If $v \gtrsim 10$, the temperature is $(T_\Sigma^5+T_{\rm ph}^5)^{1/5}$ except at distances shorter than $\sim 5L/v$ from the ends. At $v \gtrsim 4$ and $T_{\rm ph}=0$, the average temperature is $T_{\rm av} \approx T_\Sigma(1-1.16/v)$. Using these results, we have calculated the decay of the Fano factors F_2 and F_3 relative to the second and third cumulants of current fluctuations with the resistor length L. We have also addressed time-dependent situations to describe the heating and cooling of resistors. If $v \lesssim 1$, the characteristic time scale is the diffusion time τ_D, heating follows $k_BT_{\rm av}(t) \simeq \frac{\sqrt{3}}{8}eV\sqrt{1-\exp(-10\tau)}$, and cooling follows $k_BT_{\rm av}(t) \simeq \frac{\sqrt{3}}{8}eV\exp(-5\tau)$, with $\tau=t/\tau_D$. If $v \gg 1$, the instantaneous relaxation time is $\tau_{\text{e-ph}}(T)=\gamma/\Sigma T^3$. Heating from $T(0)$ to T_Σ is achieved in a time $\tau_{\text{e-ph}}(T_\Sigma)$, following $\theta(\tau^*) \approx \sqrt{\theta(0)^2+2[1+\theta_{\rm ph}^5-\theta(0)^5]\tau^*}$ at short times and $\theta(\tau^*) \approx 1-0.86\exp(-4.2\tau^*)$ at long times, with $\theta=T/T_\Sigma$ and $\tau^*=t/\tau_{\text{e-ph}}(T_\Sigma)$. Cooling from a temperature T_0 occurs very slowly, along a power law: at $T_{\rm ph}=0$, $T(t)/T_0=[1+3t/\tau_{\text{e-ph}}(T_0)]^{-1/3}$.

Finally, we recall that the actual temperature can be higher than the predictions made here for at least two reasons. First, the electronic temperature can be larger than $T_{\rm ph}$ in the connecting wires because of their finite resistivity[13,21] or because of imperfect thermalization to the cryogenic unit. Second, we have neglected the Kapitza resistance,[10] due to which the phonon temperature inside the resistor can differ from the bath temperature $T_{\rm ph}$. However, this latest effect is relatively less important in very thin resistors because the ratio of the heat flow from electrons to resistor phonons to the heat flow from resistor phonons to substrate is proportional to the film thickness.[3]

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APPENDIX: CALCULATION OF THE THIRD CUMULANTS OF CURRENT IN PRESENCE OF THE ELECTRON-PHONON SCATTERING

The calculation of the third cumulant of current in the presence of electron-phonon scattering is an extension of the expressions of Pilgram *et al.*[17] The third cumulant at zero frequency is expressed as a function of the correlator between temperature and current fluctuations:

$$S_3 = \frac{6k_B}{R}\int_0^1 dx\langle \delta T(x)\delta I\rangle. \qquad \text{(A1)}$$

To calculate the integrand, we start from the stochastic diffusion equation for the fluctuations δf of the electron energy distribution function

$$\left(\frac{\partial}{\partial t}-\frac{1}{\tau_D}\frac{\partial^2}{\partial x^2}\right)\delta f-\delta I_{ee}-\delta I_{\text{e-ph}} =-e\delta\dot{\phi}\frac{\partial f}{\partial \varepsilon}-\frac{1}{L}\frac{\partial}{\partial x}\delta F^{imp}-\delta F^{ee}, \tag{A2}$$

with δI_{ee} the linearized electron-electron diffusion integral, $\delta I_{\text{e-ph}}$ the linearized electron-phonon diffusion integral, and δF^{imp} and δF^{ee} random extraneous sources associated with electron-impurity and electron-electron scattering. The correlation function of extraneous sources is

$$\langle \delta F^{imp}(\varepsilon,x)\delta F^{imp}(\varepsilon',x')\rangle_\omega =2\frac{D}{\nu_F\Omega}\delta(x-x')\delta(\varepsilon-\varepsilon')f(\varepsilon,x)[1-f(\varepsilon,x)]. \tag{A3}$$

The energy distribution function is assumed to have a Fermi shape with coordinate dependent temperature $T(x)$ and electrical potential $\phi(x)$:

$$f(\varepsilon,x)=\left\{1+\exp\left[\frac{\varepsilon-e\phi(x)}{k_BT(x)}\right]\right\}^{-1}. \tag{A4}$$

To derive the correlator $\langle \delta T_e(x)\delta I\rangle_\omega$, Eq. (A2) is multiplied by ε and integrated over energy,[21] assuming that the rate of energy dissipation associated with electron-phonon scattering is of the form

$$\nu_F\int d\varepsilon\varepsilon I_{\text{e-ph}}=\Sigma[T^5(x)-T^5_{\text{ph}}]. \tag{A5}$$

The electron-electron collision integral and the associated extraneous source drop out because of energy conservation, and one obtains

$$\left(\frac{\partial}{\partial t}-\frac{1}{\tau_D}\frac{\partial^2}{\partial x^2}\right)(e^2L_oT\delta T)+5\nu_F^{-1}\Sigma T^4\delta T-\frac{1}{\tau_D}\frac{\partial^2}{\partial x^2}(e^2\phi\delta\phi) =-\frac{1}{L}\int d\varepsilon\varepsilon\frac{\partial}{\partial x}\delta F^{imp}. \tag{A6}$$

Now we multiply Eq. (A6) and the equation for the fluctuations of the total current, which in the low-frequency limit reads[18]

$$\delta I=\frac{e\nu_F\Omega}{L}\int d\varepsilon\int dx\,\delta F^{imp}. \tag{A7}$$

Upon averaging, it gives in the low-frequency limit

$$\frac{\partial^2}{\partial x^2}[L_oT\langle\delta T(x)\delta I\rangle_\omega]-L_o\alpha T^4\langle\delta T(x)\delta I\rangle_\omega =-\frac{\partial^2}{\partial x^2}[\phi\langle\delta\phi(x)\delta I\rangle_\omega]+\frac{2}{e}\frac{\partial}{\partial x}\int d\varepsilon\varepsilon f(1-f), \tag{A8}$$

with $\alpha=5\Sigma\Omega R/L_o$. The right-hand side of this equation was calculated in Ref. 17. The solution of this equation may be written in a symbolic form as

$$\langle\delta T(x)\delta I\rangle_\omega=\frac{2k_B}{L_oT}\left(\frac{\partial^2}{\partial x^2}-\alpha T^3\right)^{-1}\left\{\frac{\partial(\phi T)}{\partial x}-\frac{\partial^2}{\partial x^2}\left[\phi\left(\frac{\partial^2}{\partial x^2}\right)^{-1}\frac{\partial T}{\partial x}\right]\right\}, \tag{A9}$$

where the symbol $(\partial^2/\partial x^2-f)^{-1}$ is the Green's function $G(x,y)$ such that $(\partial^2/\partial x^2-f)G(x,y)=\delta(x-y)$ and $G(0,y)=G(x,0)=G(1,y)=G(x,1)=0$. Using $\phi=-Vx$, the expression in brackets greatly simplifies:

$$\left\{\frac{\partial(\phi T)}{\partial x}-\frac{\partial^2}{\partial x^2}\left[\phi\left(\frac{\partial^2}{\partial x^2}\right)^{-1}\frac{\partial T}{\partial x}\right]\right\}=V(T-2T_{\text{av}}). \tag{A10}$$

To calculate the third cumulant of the current, one has to solve Eq. (A9) and substitute the solution into Eq. (A1). The generalized Fano factor $F_3=S_3/e^2I$ is then

$$F_3=\frac{36}{\pi^2}\int_0^1dx\frac{1}{T(x)}\left[\frac{\partial^2}{\partial x^2}-\alpha T^3(x)\right]^{-1}\{T-2T_{\text{av}}\} \tag{A11a}$$

$$=\frac{36}{\pi^2}\int_0^1dxdy\frac{1}{\theta(x)}G_1(\theta,x,y)\{\theta(y)-2\theta_{\text{av}}\}, \tag{A11b}$$

with $G_1(\theta,x,y)$ the Green's function such that

$$\left[\frac{\partial^2}{\partial x^2}-\frac{15}{\pi^2}v^2\theta^3(x)\right]G_1(\theta,x,y)=\delta(x-y), \tag{A12}$$

which can be calculated from[24]

$$G_0(x,y)=\left(\frac{\partial^2}{\partial x^2}\right)^{-1}=\min(x,y)[\max(x,y)-1] \tag{A13}$$

using

$$G_1=\left[1-\frac{15}{\pi^2}v^2G_0\theta^3(x)\right]^{-1}G_0. \tag{A14}$$

In practice, we performed this calculation by discretization of the coordinates and matrix inversion: the resistor is cut into N pieces of length $\varepsilon=1/N$ and the function $G_0(x,y)$ is represented with a matrix G^0 such that

$$G^0_{ij}=-\frac{\varepsilon}{N}\min(i,j)[\max(i,j)-1] \tag{A15}$$

$(0\leqslant i,j\leqslant N)$. The term $\frac{15}{\pi^2}v^2G_0\theta^3(x)$ is represented by the matrix F built on the calculated temperature profile $\theta(x)$ using

$$F_{ij}=G^0_{ij}\frac{15}{\pi^2}v^2\theta^3(j\varepsilon). \tag{A16}$$

We then invert the matrix A with $A_{ij}=\frac{1}{\varepsilon}\delta_{ij}-F_{ij}$ and compute $G^1=A^{-1}G^0$. Finally,

$$F_3=\frac{36}{\pi^2}\varepsilon\sum_{i,j}\frac{G^1_{ij}\{-\theta(j\varepsilon)+2\theta_{\text{av}}\}}{\theta(i)}. \tag{A17}$$

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[15] At $T_{ph}=0$ and $xv \ll 1$, $\frac{T(x)}{T_\Sigma} \simeq \sqrt{axv}$, with $a=\frac{2}{\pi}\sqrt{\frac{15}{7}} \simeq 0.93$.
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[35] Cooling by diffusion is here ineffective because of the too long diffusion time $\tau_D \sim 2$ ms.