

## Renormalization of hole-hole interaction at decreasing Drude conductivity: Gated GaAs/In<sub>x</sub>Ga<sub>1-x</sub>As/GaAs heterostructures

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The diffusion contribution of the hole-hole interaction to the conductivity is analyzed in gated GaAs/In<sub>x</sub>Ga<sub>1-x</sub>As/GaAs heterostructures. We show that the change of the interaction correction to the conductivity with the decreasing Drude conductivity results both from the compensation of the singlet and triplet channels and from the arising prefactor  $\alpha_i < 1$  in the conventional expression for the interaction correction.

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### I. INTRODUCTION

The quantum corrections to the conductivity in disordered metals and doped semiconductors have been intensively studied since 1980.<sup>1</sup> Two mechanisms lead to these corrections: (i) the interference of the electron waves propagating in opposite directions along closed paths [weak localization (WL) correction]; (ii) electron-electron ( $e-e$ ) or hole-hole ( $h-h$ ) interaction.

The role of the  $e-e$  ( $h-h$ ) interaction has been a subject of theoretical<sup>1-9</sup> and experimental<sup>10-12</sup> studies for more than two decades. The new interest in the matter is associated with discussion of the nature of metalliclike temperature dependence of the conductivity observed at low temperature in some two dimensional (2D) systems, e.g., in  $n$ -Si metal-oxide-semiconductor field-effect transistor (MOSFET) and in dilute 2D hole gas in Al<sub>x</sub>Ga<sub>1-x</sub>As/GaAs and Ge<sub>1-x</sub>Si<sub>x</sub>/Ge structures (see Refs. 13–15 and references therein). As a rule such behavior is observed in low-density high-mobility structures which are featured by the relatively large value of gas parameter  $r_s = \sqrt{2}/(a_B k_F)$  characterizing the interaction strength and by the too high value of  $T\tau$ , where  $a_B$ ,  $k_F$ , and  $\tau$  are the Bohr radius, the Fermi quasimomentum, and the transport relaxation time, respectively (hereafter we set  $\hbar = 1$ ,  $k_B = 1$ ). The role of the interaction at  $r_s > 3-5$  (i.e., at strong interaction), and/or  $T\tau \geq 1$  (i.e., at intermediate and ballistic regimes) was theoretically studied in Refs. 2, 4, 5, and 16–20, the experimental situation was reviewed in Refs. 13–15.

It should be noted that the metalliclike behavior is observed when the conductivity is not too high, therefore the corrections can lead to an essential change of the conductivity with the temperature. The changing of the interaction correction at decreasing temperature and/or conductivity was theoretically studied in the framework of the theory of the renormalization group (RG) in Refs. 3, 6–8, 16, and 17. It has been shown that the correction renormalization depends on both the Drude conductivity and the Fermi liquid amplitude  $\gamma_2$  that controls the  $e-e$  interaction in the triplet channel. The contributions from singlet and triplet channels are opposite in sign favoring localization and antilocalization, respectively. In conventional conductors with high values of the Drude conductivity,  $\sigma_0 = \pi k_F l G_0 \gg G_0$  [where  $l$  is the mean free path and  $G_0 = e^2/(2\pi^2 \hbar)$ ], the initial value of the ampli-

tude  $\gamma_2$  is small, and the net effect is in favor of localization. At  $\sigma_0 \lesssim (10-15)G_0$  or in dilute systems, however, this amplitude may be enhanced due to  $e-e$  correlations and thus results in the metallic sign of  $d\sigma/dT$ .<sup>16,17</sup>

Significantly less is known about the role of the interaction correction in disordered 2D systems when the  $k_F l$  value tends to unity, i.e., at crossover from weak to strong localization. Experimentally, this effect was studied in the simplest single-valley electron 2D system GaAs/In<sub>1-x</sub>Ga<sub>x</sub>As/GaAs with small  $g$  factor.<sup>21</sup> It was shown that the net value of the interaction correction decreases rapidly with the  $\sigma_0$  decrease at  $\sigma_0 \lesssim (12-15)G_0$  ( $k_F l \lesssim 4-5$ ). Such a behavior can result from the compensation of the contributions of the singlet and triplet channels as well as from suppression of both contributions with decreasing  $\sigma_0$ . It is impossible to separate these two effects in the systems with the small value of the  $g$  factor. The situation changes drastically when dealing with a system with a large enough  $g$  factor. In this case the magnetic field can be used as a tool allowing us to control the ratio between the two different contributions because it strongly suppresses the triplet channel and leaves the singlet channel unchanged. As shown below, the hole 2D gas in strained GaAs/In<sub>x</sub>Ga<sub>1-x</sub>As/GaAs structures is a suitable object to study the renormalization of the interaction quantum correction with the conductivity decrease.

In this paper we report the results of experimental study of the evolution of the interaction correction to the conductivity in a  $p$ -type 2D system with decreasing Drude conductivity within the range from  $\approx 30G_0$  to  $\approx 3G_0$  when the ballistic contribution of the  $h-h$  interaction is small (at a high value of the Drude conductivity,  $\sigma_0 > 30G_0$ , these structures were studied in our previous paper, Ref. 22). First, we will outline the procedures used for extracting the diffusion part of the interaction correction and the value of the Fermi liquid parameter  $F_0^\sigma = -\gamma_2/(1 + \gamma_2)$  from the dependences of  $\rho_{xx}$  and  $\rho_{xy}$  on the temperature and magnetic field. Then, we will discuss the change of  $F_0^\sigma$  with decreasing Drude conductivity. Finally, we will show that the reduction of the interaction correction with the decreasing Drude conductivity results from both the compensation of the singlet and triplet channels and from the arising of a prefactor  $\alpha_i < 1$  in the conventional expression for the interaction correction.<sup>2</sup>

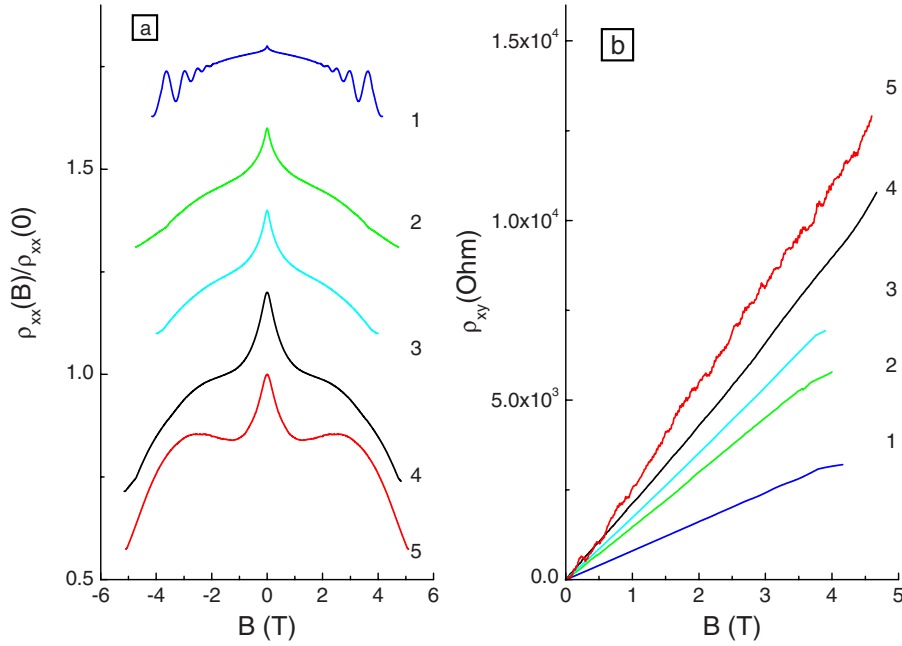


FIG. 1. (Color online) The magnetic field dependences of  $\rho_{xx}$  (a) and  $\rho_{xy}$  (b) at  $T=1.4$  K for different gate voltages, which are characterized by the following values of  $p$ ,  $\sigma_0$ , and  $\sigma(T=1.4$  K):  $8 \times 10^{11}$   $\text{cm}^{-2}$ ,  $59.6G_0$ , and  $56.9G_0$  (curves 1);  $4.5 \times 10^{11}$   $\text{cm}^{-2}$ ,  $9.9G_0$ , and  $6.8G_0$  (curves 2);  $3.9 \times 10^{11}$   $\text{cm}^{-2}$ ,  $8.1G_0$ , and  $4.37G_0$  (curves 3);  $3 \times 10^{11}$   $\text{cm}^{-2}$ ,  $3.9G_0$ , and  $0.36G_0$  (curves 4);  $2.6 \times 10^{11}$   $\text{cm}^{-2}$ ,  $3.5G_0$ , and  $0.027G_0$  (curves 5). Structure 3856. For clarity, the curves in the panel (a) are separated in vertical direction by the value of 0.2.

## II. EXPERIMENT

We measured the temperature and magnetic field dependences of  $\rho_{xx}$  and  $\rho_{xy}$  in the heterostructures GaAs/ $\text{In}_x\text{Ga}_{1-x}\text{As}$ /GaAs grown by metal-organic vapor phase epitaxy on semi-insulating GaAs substrate. The lattice mismatch between  $\text{In}_x\text{Ga}_{1-x}\text{As}$  and GaAs results in biaxial compression of the quantum well. Two structures, 3856 and 3857, of nominally identical design were studied. They consist of a 250-nm-thick undoped GaAs buffer layer, carbon  $\delta$  layer, a 7-nm spacer of undoped GaAs, a 10-nm  $\text{In}_{0.2}\text{Ga}_{0.8}\text{As}$  well, a 7-nm spacer of undoped GaAs, a carbon  $\delta$  layer, and a 200-nm cap layer of undoped GaAs. The samples were mesa etched into standard Hall bars and then an Al gate electrode was deposited by thermal evaporation onto the cap layer through a mask. Varying the gate voltage  $V_g$  we were able to change the hole density  $p$  and mobility  $\mu$  within the following ranges:  $p=(2.5 \cdots 8.0) \times 10^{11}$   $\text{cm}^{-2}$ ,  $\mu=(1000 \cdots 5700)$   $\text{cm}^2/\text{V s}$ .

The magnetic field dependences of  $\rho_{xx}$  and  $\rho_{xy}$  at  $T=1.4$  K at different gate voltages for one of the samples investigated are presented in Fig. 1. It is clearly seen that despite the very large difference in conductivity values at  $B=0$ , the magnetoresistance (MR) curves  $\rho_{xx}(B)$  are very similar: the sharp negative MR at low magnetic field, which results from suppression of the interference contribution to the conductivity, is followed by the paraboliclike MR caused by the interaction correction.<sup>23</sup>

Since our goal is to study the interaction correction let us briefly explain the method allowing us to extract it from the experimental data. Under our experimental conditions the parameter  $T\tau$  is small enough ( $T\tau < 0.1$ ) and therefore the

main contribution comes from the diffusion part of the interaction correction. The unique property of the diffusion part is that it contributes to  $\sigma_{xx}$  but not to  $\sigma_{xy}$ . This fact opens a possibility to extract this correction reliably even when the correction value is small. The most straightforward way is to find such contribution to  $\sigma_{xx}$  which is absent in  $\sigma_{xy}$ . We extract these contributions by making use of the structure of the components of the conductivity tensor  $\sigma_{xx}$  and  $\sigma_{xy}$ . As shown in Ref. 24 the weak localization correction and the ballistic part of the interaction corrections are reduced to renormalization of the transport relaxation time and can be accounted for through the temperature and magnetic field dependence of the mobility. Thus  $\sigma_{xx}$  and  $\sigma_{xy}$  can be written as

$$\sigma_{xx}(B, T) = \frac{ep\mu(B, T)}{1 + \mu^2(B, T)B^2} + \delta\sigma_{xx}^{hh}(B, T), \quad (1)$$

$$\sigma_{xy}(B, T) = \frac{ep\mu^2(B, T)B}{1 + \mu^2(B, T)B^2}, \quad (2)$$

where  $\delta\sigma_{xx}^{hh}(B, T)$  is the diffusion part of the interaction correction. If the Zeeman splitting is very small as compared with the temperature,  $\delta\sigma_{xx}^{hh}$  is magnetic field independent. It has the form<sup>2,6-8</sup>

$$\frac{\delta\sigma_{xx}^{hh}(T)}{G_0} = \alpha_i \left[ 1 + 3 \left( 1 - \frac{\ln(1 + F_0^\sigma)}{F_0^\sigma} \right) \right] \ln T\tau, \quad (3)$$

where the first term in square brackets is the exchange or the Fock contribution while the second one is the Hartree contribution (the triplet channel). For the following, we enter

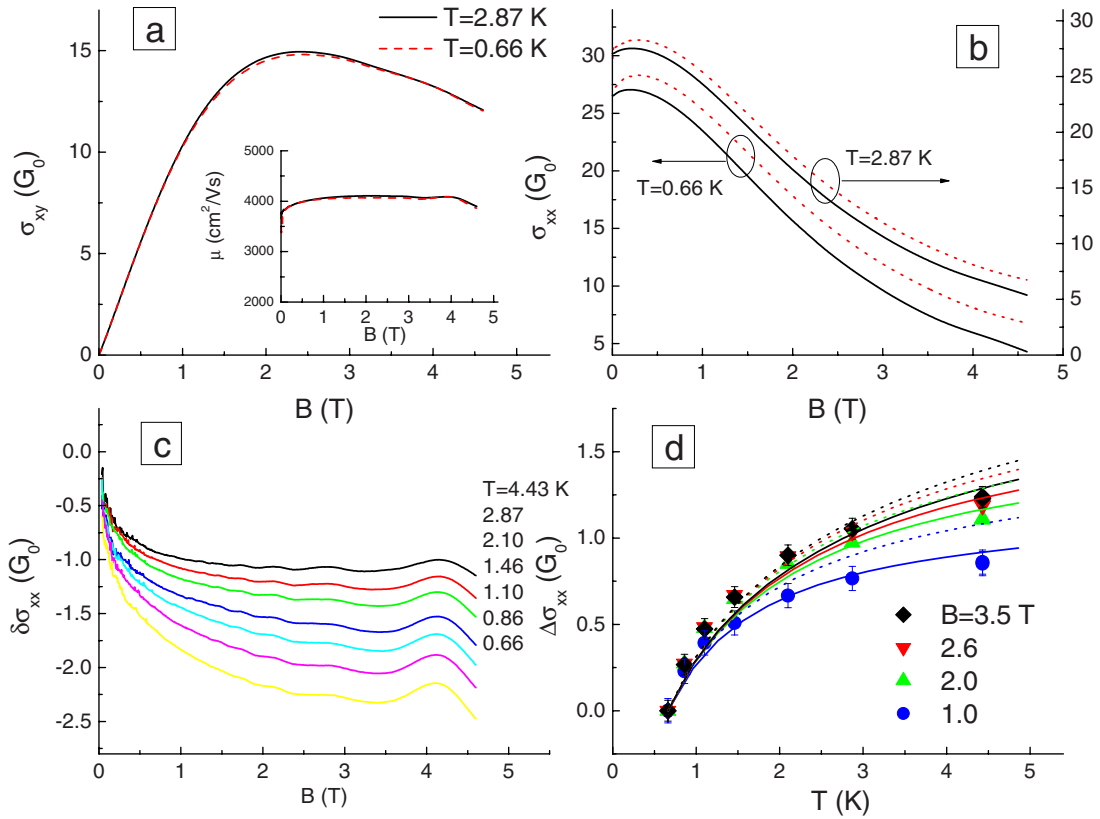


FIG. 2. (Color online) (a) The experimental magnetic field dependences of  $\sigma_{xy}$  for two temperatures. The inset shows the magnetic field dependence of the mobility calculated from  $\sigma_{xy}(B)$  for two temperatures with  $p=5.4 \times 10^{11} \text{ cm}^{-2}$ . (b) The  $\sigma_{xx}$  vs  $B$  dependences for two temperatures. The solid curves are the data, the dotted curves are the first term of Eq. (1) calculated as described in the text. (c) The magnetic field dependences of  $\delta\sigma_{xx}$  obtained as described in the text for different temperatures. (d) The temperature dependences of  $\Delta\sigma_{xx}$  at different magnetic fields. The symbols are the experimental results; curves are calculated dependences with  $F_0^\sigma = -0.4$  (solid curves) and with  $F_0^\sigma = -0.35$  (dotted curves). Structure 3857,  $V_g = 2.4 \text{ V}$ ,  $\sigma_0 \approx 30G_0$ .

here the prefactor  $\alpha_i$ . In Refs. 2 and 6–8, where the case of the high conductivity,  $\sigma_0 \gg G_0$ , was studied, it is equal to unity.

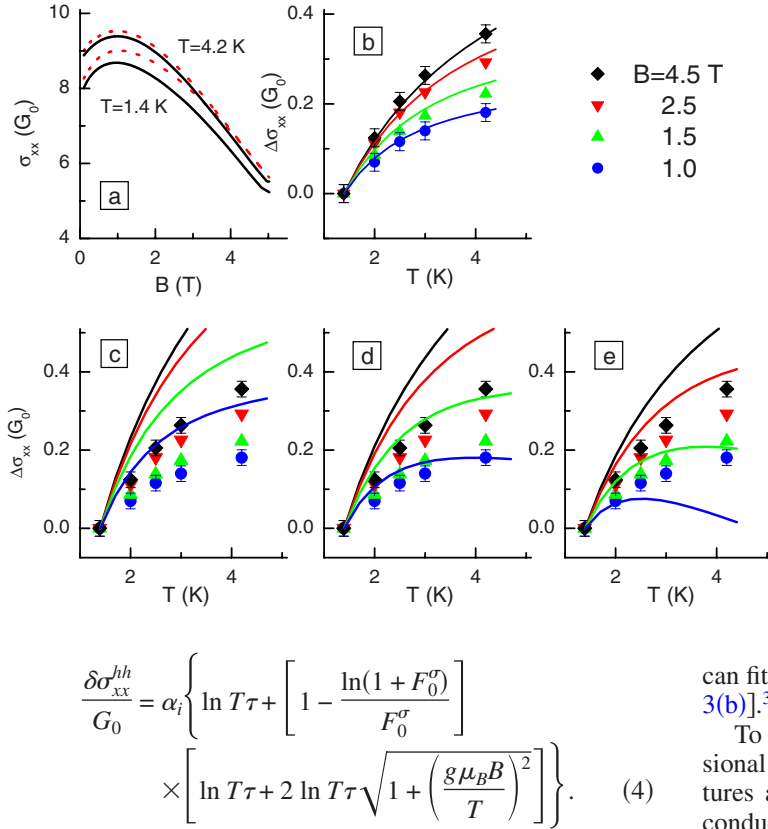
Thus knowing the hole density  $p$  we can find  $\mu(T, B)$  from experimental  $\sigma_{xy}$  vs  $B$  dependences [with the help of Eq. (2)] and then calculate the first term in Eq. (1). The difference between the experimental value of  $\sigma_{xx}$  and this term should give the diffusion part of the  $h-h$  correction to the conductivity. This method allows us to find  $\delta\sigma_{xx}^{hh}(B, T)$  for relatively low  $\sigma_0$ , when the interference contribution to MR is not negligible up to the high magnetic field.

In what follows we demonstrate how this method works considering the results obtained for one of the samples, fabricated on the basis of structure 3857.

Let us start with the case of high Drude conductivity,  $\sigma_0 \approx 30G_0$  (for the details of determination of  $\sigma_0$ , see Ref. 25). First for each temperature we have inverted the resistivity tensor whose components were measured experimentally and found the conductivity tensor components  $\sigma_{xy}$  and  $\sigma_{xx}$  [see Figs. 2(a) and 2(b)]. Then, using the obtained  $\sigma_{xy}$  vs  $B$  dependences we have found  $\mu(B)$  [shown in the inset in Fig. 2(a)] and calculated the experimental value of the first term in Eq. (1). Finally, subtracting the latter term from the experimental value of  $\sigma_{xx}$  we obtain  $\delta\sigma_{xx}$  [see Fig. 2(c)], which is identified with the diffusion part of the  $h-h$  correc-

tion  $\delta\sigma_{xx}^{hh}(B, T)$ . As seen from Fig. 2(b)  $\delta\sigma_{xx}$  is a small difference between two large quantities. That is why an accuracy in determination of  $\delta\sigma_{xx}^{hh}(B, T)$ , i.e., in the absolute value of the interaction correction, is sufficiently low. In particular, it is very sensitive to the value of hole density, which is experimentally known with some accuracy. However, the difference of the quantities  $\delta\sigma_{xx}$  taken at two temperatures for a given magnetic field (or taken at two magnetic fields for a given temperature) depends only slightly on the hole density and therefore is found with better accuracy.

In Fig. 2(d) we present the temperature dependences  $\Delta\sigma_{xx}(T, B) = \delta\sigma_{xx}(T, B) - \delta\sigma_{xx}(T_0, B)$ , where  $T_0$  is the lowest temperature, obtained for different magnetic fields. One can see that the higher the magnetic field, the stronger the change of  $\Delta\sigma_{xx}$  with the temperature. This dependence can be attributed to the Zeeman splitting which leads to suppression of the triplet channel and hence to the appearance of the magnetic field dependence of the interaction correction. Theoretically, the effect of Zeeman splitting has been considered in Refs. 8, 26, 27, and 6. However, the expressions derived there are too complicated and therefore inconvenient for the practical use. A much simpler expression, which well approximates these formulas, is<sup>22,28</sup>



In Fig. 2(d) we plot the curves calculated according to Eq. (4) with  $\alpha_i=1$ ,  $g=3$ ,<sup>29</sup> and different  $F_0^\sigma$  values. One can see that the curves calculated with  $F_0^\sigma=-0.4$  almost coincide with the experimental data.

Similar data treatment was carried out for the lower conductivity. In Fig. 3 we present the experimental and calculated magnetic field dependences of  $\sigma_{xx}$  [Fig. 3(a)] and  $\Delta\sigma_{xx}$  vs  $T$  dependences for different magnetic fields [Fig. 3(b)] for  $\sigma_0 \approx 11G_0$  ( $V_g=2.8$  V). As seen from Figs. 3(c)–3(e), it is impossible to describe the data by Eq. (4) with the prefactor  $\alpha_i=1$  for any  $F_0^\sigma$  values. This is not surprising because the theory predicts  $\alpha_i=1$  only for large  $\sigma_0$  value. However, one

FIG. 3. (Color online) (a) The magnetic field dependences of  $\sigma_{xx}$  for two temperatures. The solid curves are the data, the dotted curves are the first term of Eq. (1) calculated as described in text. (b)–(e) The temperature dependences of  $\Delta\sigma_{xx}$  at different magnetic fields. The symbols are the experimental results; the curves are theoretical dependences calculated with (b)  $\alpha_i=0.5$ ,  $F_0^\sigma=-0.43$ ; (c)  $\alpha_i=1$ ,  $F_0^\sigma=-0.45$ ; (d)  $\alpha_i=1$ ,  $F_0^\sigma=-0.5$ ; (e)  $\alpha_i=1$ ,  $F_0^\sigma=-0.55$ . Structure 3857,  $V_g=2.8$  V,  $p=4.4 \times 10^{11}$  cm<sup>-2</sup>,  $\sigma_0 \approx 11G_0$ .

can fit the data perfectly with  $\alpha_i=0.5$  and  $F_0^\sigma=-0.43$  [see Fig. 3(b)].<sup>30</sup>

To be sure that these changes in  $F_0^\sigma$  and  $\alpha_i$  are not occasional we carried out systematic studies of the both structures at successive decrease of the hole density and Drude conductivity. It was recognized that Eq. (4) with the two fitting parameters,  $\alpha_i$  and  $F_0^\sigma$ , describes well the experimental data down to  $\sigma_0 \approx (3.5 \pm 0.3)G_0$ . All the results for  $\alpha_i$  and  $F_0^\sigma$  are summarized in Fig. 4. The results of Ref. 22 for  $F_0^\sigma$ , obtained for  $\sigma_0 > 30G_0$ , are presented in Fig. 4(a) also. One can see that all data match well. The broader scatter of the data from Ref. 22 is due to the large ballistic contribution that complicated the determination of  $F_0^\sigma$ . Note that the  $\alpha$  vs  $\sigma_0$  data can be interpolated by the empirical formula

$$\alpha_i = 1 - \frac{4G_0}{\sigma_0}. \quad (5)$$

Let us first discuss the behavior of the Fermi liquid parameter  $F_0^\sigma$ . Its value as a function of the gas parameter  $r_s$  is

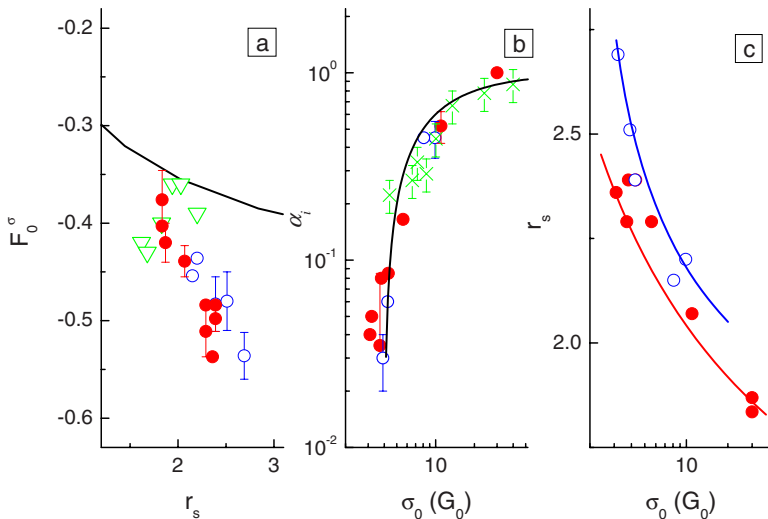


FIG. 4. (Color online) (a) The  $r_s$  dependences of  $F_0^\sigma$ . (b), (c) The  $\sigma_0$  dependence of the prefactor  $\alpha_i$  and the gas parameter  $r_s$ , respectively. Open and solid circles are the experimental data obtained in the present paper for structures 3856 and 3857, respectively. The triangles are data from Ref. 22. The crosses are the results of recalculation of the data obtained in Ref. 21. The curve in panel (a) is Eq. (6). The curve in panel (b) is the interpolating formula, Eq. (5). The curves in panel (c) are provided as a guide for the eye.

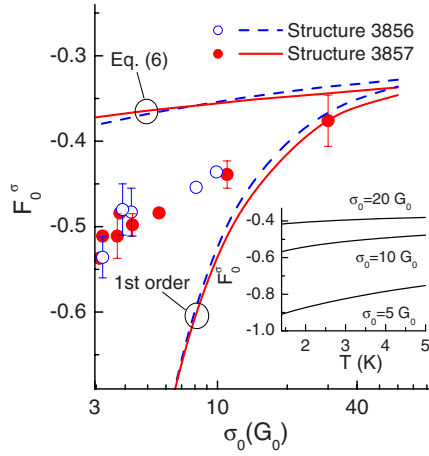


FIG. 5. (Color online) The  $\sigma_0$  dependence of the interaction parameter  $F_0^\sigma$ . The symbols are the experimental data. The curves are theoretical dependences calculated as described in text. The inset shows the behavior of  $F_0^\sigma$  within our temperature range obtained as solution of Eqs. (7) and (8) with different initial conditions  $\sigma(\xi=0)=\sigma_0$ .

plotted in Fig. 4(a). It is seen that as  $F_0^\sigma$  appreciably decreases with  $r_s$ , it becomes less than  $-0.454$  at  $r_s \approx 2$ . It is the value where the singlet and triplet channels compensate each other and the interaction correction in zeroth magnetic field changes the sign [see Eqs. (3) and (4)]. However, the change of the sign of the interaction correction does not result in the metalliclike behavior of the total conductivity. This is because the insulatinglike WL quantum correction dominates in our samples. Nevertheless, this fact manifests itself in our experiment. Since the triplet channel is suppressed with the  $B$  increase, the magnetic field inverts the sign of  $\delta\sigma_{xx}^{hh}$  again. So the magnetoresistance should be positive at low magnetic field and negative at high field. This fact graphically shows itself as the maximum in  $\rho_{xx}$  vs  $B$  dependence, which is evident at  $B \approx 2.8$  T for  $\sigma_0 \approx 3.5G_0$  [see Fig. 1(a)].

Let us compare the experimental values of  $F_0^\sigma$  with theoretical calculations. According to Ref. 18 the following estimates for  $F_0^\sigma$  can be used when describing the interaction in the triplet channel:

$$F_0^\sigma \rightarrow -\frac{1}{2\pi} \frac{r_s}{\sqrt{2-r_s^2}} \ln \left( \frac{\sqrt{2} + \sqrt{2-r_s^2}}{\sqrt{2} - \sqrt{2-r_s^2}} \right), \quad r_s^2 < 2,$$

$$F_0^\sigma \rightarrow -\frac{1}{\pi} \frac{r_s}{\sqrt{r_s^2-2}} \arctan \sqrt{\frac{1}{2}r_s^2 - 1}, \quad r_s^2 > 2. \quad (6)$$

One can see in Fig. 4(a) that the experimental points strongly deviate downwards from the theoretical curve with increasing  $r_s$ . The possible reason of the deviation is the renormalization of the Fermi liquid constant  $F_0^\sigma$  with the decreasing Drude conductivity which strongly changes with  $r_s$ , [see Fig. 4(c)]. This is directly evident from Fig. 5, where both the experimental and theoretical [Eq. (6)]  $F_0^\sigma$  vs  $\sigma_0$  dependences are presented. When calculating the theoretical curves we have used the  $r_s$  vs  $\sigma_0$  dependences from Fig. 4(c). It is seen

that the lower the Drude conductivity the stronger the deviation.

Theoretically, the effect of renormalization of  $F_0^\sigma$  with the changing conductivity was studied in the framework of RG theory,<sup>3,6-8,16,17</sup> which took the interaction into account in the first order in  $1/\sigma$  exactly. According to this theory the temperature dependences of both  $\sigma$  and  $F_0^\sigma$  are the solutions of the system of differential equation

$$\frac{d\sigma}{d\xi} = - \left\{ 1 + 1 + 3 \left[ 1 - \frac{1 + \gamma_2}{\gamma_2} \ln(1 + \gamma_2) \right] \right\}, \quad (7)$$

$$\frac{d\gamma_2}{d\xi} = \frac{1}{\sigma} \frac{(1 + \gamma_2)^2}{2}, \quad (8)$$

where  $\xi = -\ln(T\tau)$ ,  $\gamma_2 = -F_0^\sigma/(1 + F_0^\sigma)$ , and  $\sigma$  is measured in units of  $G_0$ . The term  $1+1$  in braces is responsible for the weak localization and the interaction in a singlet channel which in the case of Coulomb interaction give equal contributions.

The above system of differential equations have been solved numerically with the following initial conditions. We suppose that the high-temperature conductivity is equal to the Drude conductivity:  $\sigma(\xi=0)=\sigma_0$ . This condition seems to be natural. It corresponds to that the diffusion part of interaction correction is equal to zero in accordance with Eq. (3) and the WL correction is much less than the Drude conductivity at  $T\tau=1$ . The second condition is  $\gamma_2(\xi=0)=-F_0^\sigma/(1 + F_0^\sigma)$  where  $F_0^\sigma$  is determined by Eq. (6).<sup>31</sup> Note that this system describes the conductivity as a function of the parameter  $T\tau$  (i.e., as a function of temperature). Experimentally, we are able to find the interaction contribution within only the relatively narrow temperature range,  $T=1.4-4.5$  K. Therefore it is more appropriate to compare the  $\sigma_0$  dependence of  $F_0^\sigma$  rather than the temperature one. The solutions obtained for several  $\sigma_0$  values, as the  $F_0^\sigma$  vs  $T$  dependence within the actual temperature range, are presented in the inset in Fig. 5. It is seen that this dependence is relatively weak. In order to compare these results with the experimental data for  $F_0^\sigma$  we have arithmetically averaged the calculated value of  $F_0^\sigma$  over this temperature interval. Namely, this averaged value is presented in Fig. 5 as a function of  $\sigma_0$ .

One can see that both the experimental and calculated (labeled as “1st order”) values of  $F_0^\sigma$  decrease with decreasing  $\sigma_0$ . However, the theory gives much faster decrease. Most probably such a discrepancy indicates that one should take into account the next terms in  $1/\sigma$  expansion in RG equations. To the best of our knowledge this has been done only for two particular cases inappropriate to our situation. The first case relates to multivalley ( $n_v \gg 1$ ) systems with  $\gamma_2 \ll 1$ .<sup>17</sup> The second one is single valley ( $n_v=1$ ) systems but with the large  $\gamma_2$  value.<sup>32</sup>

Thus realizing the crudity of the above estimations we, nevertheless, believe that the decrease of the experimental value of  $F_0^\sigma$  with the decreasing Drude conductivity results from the renormalization of the  $h-h$  interaction.

Strictly speaking, the RG equations (7) and (8) were derived in the absence of magnetic field, whereas the Zeeman splitting suppresses the triplet contributions to the right-hand

side of Eqs. (7) and (8).<sup>6,8,26,27</sup> Recently, it was shown that the effect of Zeeman splitting on conductivity can be used for extracting the dependence of  $F_0^\sigma$  on temperature.<sup>33–35</sup> In our analysis of experimental data, the effect of Zeeman splitting is taken into account only in the correction to the conductivity, Eq. (4), but not in the renormalization of the interaction parameter. In our case the temperature range in which the Zeeman splitting is strong,  $g\mu_B B > T$ , is a small fraction of the total interval which we use for averaging. Therefore we do not expect a significant difference in our results for  $F_0^\sigma$  due to taking into account the Zeeman splitting and consider that the comparison of our data with solutions of Eqs. (7) and (8) is almost correct.

Next we discuss the behavior of the prefactor  $\alpha_i$ . Figure 4(b) shows that  $\alpha_i$  decreases sharply when  $\sigma_0$  lowers. The behavior of the interaction correction with decreasing  $\sigma_0$  was studied experimentally for the  $n$ -type 2D structures in Ref. 21. The recalculated data from this paper presented in Fig. 4(b) by crosses demonstrate an analogous decrease also. The possible reason of such  $\alpha_i$  vs  $\sigma_0$  dependence is the interplay between the interference and the interaction which has not been taken into account in the RG theory.<sup>16,17</sup> As shown in Refs. 36 and 37 two additional terms in the expression for the conductivity arise if this interplay is allowed for [see Eq. 40 in Ref. 37]. One term depends on the magnetic field and leads to the appearance of the prefactor in WL magnetoresistance. The second one does not depend on the magnetic field,

and therefore it was away in Ref. 37. It is quite possible that namely this term leads to a decrease of  $\alpha_i$ , the prefactor in the interaction correction, with decreasing Drude conductivity. Another contribution to the prefactor  $\alpha_i$  is due to the second-loop interaction effect (which can be thought of as the next-order Altshuler-Aronov-type correction). This correction is known for the singlet channel in the unitary ensemble (strong magnetic field).<sup>38</sup> To the best of our knowledge, the impact of the interplay between the interaction and the interference upon the interaction correction to the conductivity as well as the second-loop contribution in the triplet channel for  $n_v=1$  is yet to be studied.

In summary, the behavior of the interaction contribution to the conductivity with decreasing Drude conductivity is determined both by the renormalization of the interaction constant  $F_0^\sigma$  and by the decrease of the prefactor  $\alpha_i$  in Eq. (4), and the latter is more pronounced.

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- <sup>30</sup>Strictly speaking, the value of the  $g$  factor in Eq. (4) is also modified by the interaction. We note that the hole density is relatively high under our experimental conditions,  $p \gtrsim 2.5 \times 10^{11} \text{ cm}^{-2}$ , and we believe that the role of the renormalization of the  $g$  factor due to the hole-hole interaction is negligible within our analysis.
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