

## Landau levels analysis by using symmetry properties of mesoscopic Hall bars

B. Jouault,<sup>1,2</sup> O. Couturaud,<sup>1,2</sup> S. Bonifacie,<sup>1,2</sup> D. Maily,<sup>3</sup> and C. Chaubet<sup>1,2</sup>

<sup>1</sup>Université Montpellier 2, Groupe d'Etude des Semiconducteurs, cc074, pl. E. Bataillon, 34095 Montpellier cedex 5, France

<sup>2</sup>CNRS, UMR 5650, cc074, pl. E. Bataillon, 34095 Montpellier cedex 5, France

<sup>3</sup>CNRS-LPN, Route de Nozay, 91460 Marcoussis, France

(Received 5 June 2007; published 9 October 2007)

We use the resistance fluctuations (RFs) appearing in the integer quantum Hall regime to scan the density of states of a very thin Hall bar. By applying a dc voltage on a top gate, we analyze the correlation properties of the various resistances as a function of the magnetic field and the carrier density. In the gate voltage-magnetic field plane, these RFs follow lines with slopes quantized in unit of filling factor and the slope of these RFs depends on their correlation properties.

DOI: [10.1103/PhysRevB.76.161302](https://doi.org/10.1103/PhysRevB.76.161302)

PACS number(s): 73.23.-b, 73.43.-f

The electron-electron interaction was thought to play a marginal role in the integer quantum Hall effect (IQHE) which appears in two-dimensional electron systems (2DES). The IQHE is well explained by single-particle models.<sup>1</sup> But recent experiments performed on mesoscopic transistors have questioned the validity of this picture. In these small samples, resistances fluctuations (RFs) are observed in transition regions between Hall plateaus.<sup>2,3</sup> When studied in a function of the magnetic field  $B$  and the carrier density  $n_s$ , these RFs show an unexpected behavior: they all move parallel to lines of filling factor  $\nu$  integer. Also, very similar patterns have been observed in local measurements of the electronic compressibility  $\kappa(B, n_s)$  by using single electron transistors on top of a 2DES.<sup>4</sup> In this case, variations of the compressibility follow stripes parallel to the line's  $\nu$  integer. These observations seem incompatible with a single particle description which *a priori* should force all lines to pass at  $(0,0)$ , so the fluctuations should follow lines of constant  $\nu$ . Indeed, recent numerical simulations<sup>5,6</sup> show how nonlinear screening effects enforce these fluctuations to move parallel, as observed experimentally.

In this paper, we report a detailed investigation of these RFs in low mobility samples. We take advantage of the geometry of the six-probes mesoscopic Hall bars we use. Indeed, it was shown recently that correlation properties of the RFs measured with different combinations of contacts directly reflect the symmetry of the conductance matrix and the occupancy of the last populated Landau level (LPLL).<sup>8,9</sup> If the LPLL is almost filled [Fig. 1(b)], Jain-Kivelson (JK) scattering processes between opposite edge states of the central part of the Hall bar via electron antidots correlate the longitudinal resistances measured on both sides of the device, while the Hall resistance remains quantized.<sup>7</sup> In this case, the conductance matrix is mainly not symmetric, reflecting the chirality of these processes. On the contrary, when the LPLL is almost empty [Fig. 1(c)], transport can only occur through tunneling from one dot to another, which implies that the conductance matrix of the device is symmetric and that the Hall resistances are correlated with the longitudinal resistances.<sup>10</sup> Using  $n_s$  and  $B$  as parameters, we discriminate the RFs induced by both kinds of tunneling processes by their symmetry properties. These two regimes correspond to the low energy tail and to the high energy tail of the density

of states (DOS)  $\rho_B(\epsilon)$  of the LPLL. We also find that RFs, as observed in previous experiments, follow quantized slopes in the  $(B-n_s)$  map.

We studied in detail several Hall bars as sketched in Fig. 1(a). They were of width  $1.5 \mu\text{m}$  and of length  $L=15 \mu\text{m}$ , etched from the same GaAs/In<sub>0.15</sub>Ga<sub>0.85</sub>As/AlGaAs wafer. Ti/Au Schottky gates were deposited on top of them. They were cooled in a dilution fridge with a base temperature of 30 mK. Measurements were done using standard lock-in techniques with excitation currents of 0.1–1 nA at a frequency of 7 Hz. The 2DES in the InGaAs quantum well has a carrier density  $n_s=9 \times 10^{15} \text{m}^{-2}$  and a mobility of  $3 \text{m}^2/\text{Vs}$  at  $T=4.2 \text{K}$  or below, for an applied gate voltage  $V_g=0 \text{V}$ . Applying a negative gate voltage reduces both  $n_s$  and the electrical width  $W$  of the Hall bar. Indeed,  $W$  can be estimated by  $W=LR_{\square}/R_L$ , where  $R_L=R_{14,65}$  ( $=R_{14,23}$ ) is the longitudinal resistance at  $B=0 \text{T}$ ,<sup>11</sup> and  $R_{\square}(V_g)$  is the resistance per square obtained by Van der Pauw measurements.<sup>12</sup> The evolution of  $W(V_g)$  is shown in Fig. 2(a) and is in good agreement with three-dimensional calculations of the coupled Fermi and Poisson equations. These calculations also show that the lateral potential in the Hall bar is a square well at  $V_g=0 \text{V}$  but becomes parabolic at  $V_g \approx -1 \text{V}$ .

Previous studies demonstrated the existence of this parabolic potential in disordered quantum wires.<sup>13</sup> In high mag-

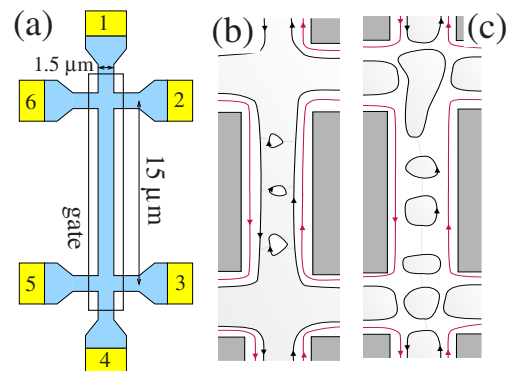


FIG. 1. (Color online) (a) Sketch of the  $15 \mu\text{m}$  long and  $1.5 \mu\text{m}$  wide Hall bars used in the experiments.  $B$  is oriented so as edge states turn anticlockwise. (b), (c) The two regimes observed in the tails of the  $\nu=1-\nu=2$  SdH transition.

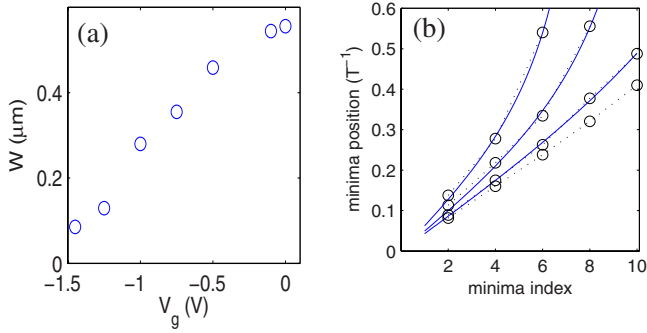


FIG. 2. (Color online) (a) Electrical width  $W$  as a function of the gate voltage  $V_g$  at  $T=4.2$  K. (b) Open dots: inverse of the SdH minima position measured at 4.2 K as a function of the LL index, for different gate voltages. From bottom to top:  $V_g = -0.5$  V,  $-0.65$  V,  $-1$  V,  $-1.25$  V. Blue/dark gray solid lines: best fits according to Ref. 13.

netic fields, when Landau levels (LLs) are formed, the lateral potential induces a high energy tail in the DOS of each LL because of the relatively important weight of the states close to the edges. This asymmetry can be detected by deviations from linearity of the Shubnikov-de Haas (SdH) minima positions in  $B^{-1}$ , when plotted versus their indices.<sup>14</sup> We indeed observe a deviation of the SdH minima at  $T=4.2$  K in our samples, which is reported in Fig. 2(b). This deviation is not so clearly observed at lower  $T$ , because universal fluctuations of conductance appear at  $B \leq 2$  T and blur the SdH oscillations. On the other hand, at  $V_g \approx 0$  V, the Hall plateaus of the Hall resistances with values  $h/ne^2$ ,  $n=1, 2$ , etc. are shifted to lower  $B$  and do not cross the line of the classical resistance  $R_{class} = B/n_s e$ , possibly because the 2DES develop a low energy tail due to the proximity of ionized impurities.<sup>15</sup>

We now discuss how the LL DoS can be monitored at very low  $T$  by analyzing the correlations of the resistances components. In Fig. 3(c), we plot a grayscale map of  $R_L = R_{14,65}$  as a function of  $B$  and  $V_g$ . White regions correspond to a high resistance, black regions to a low resistance. In addition to this grayscale map, Figs. 3(a), 3(b), and 3(d) give cross sections of  $R_L(B, V_g)$  at  $B=13.5$  T,  $V_g = -1.4$  V, and  $V_g = -0.7$  V, respectively. These three figures also plot different cross sections of the Hall resistance  $R_H = R_{14,53}$ , allowing a direct comparison between the two regimes: (i)  $R_H + R_L$  quantized versus (ii)  $R_H$  quantized and  $R_L$  nonzero. In Fig. 3(a), the plot of the sum  $R_L + R_H$  is also indicated, showing that the first regime, at  $B=13.5$  T, occurs when  $V_g < -1$  V for the  $\nu=1-2$  transition.

The second regime is well visible for  $V_g > -1$  V in the same figure. This second regime is also well evidenced in Fig. 3(b) at  $B=4-5$  T for the  $\nu=2-4$  transition and at  $B=9-11$  T for the  $\nu=1-2$  transition: the SdH peaks increase their amplitude and extend to their low- $B$  side, up to the point where they cover a large part of the Hall plateau. A better insight is obtained by calculating the normalized cross-correlation function:<sup>16</sup>  $g(V, B) \propto \int_{\Delta} R_L(V') R_H(V') dV'$ , where  $\bar{R}(V) = R(V) - \int_{\Delta} R(V') dV' / \Delta$ ,  $\Delta$  is a sliding interval of width 9 mV centered on  $V$ . The exact size of this interval has little influence on the results. The green shaded regions in Fig. 3(c) correspond to a strong correlation ( $g > 0.7$ ) between

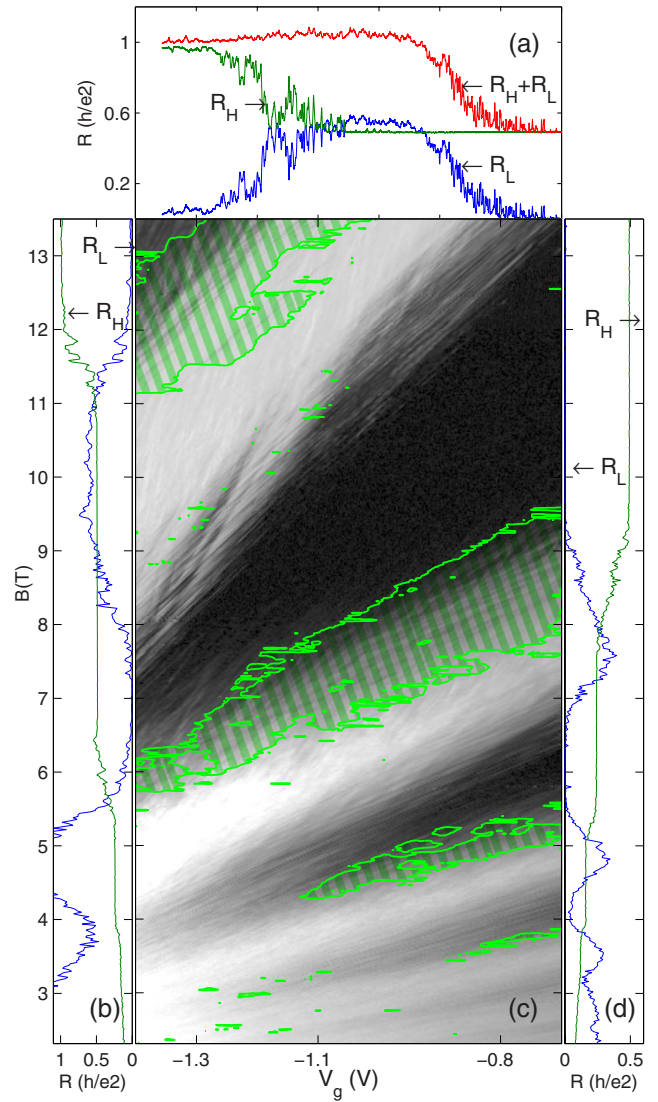


FIG. 3. (Color online) (a)  $R_L(V_g)$  (blue/dark gray),  $R_H(V_g)$  (green/light gray), and  $R_H(V_g) + R_L(V_g)$  (red/light gray) are reported for  $B=13.5$  T. (b)  $R_L(B)$  (blue/dark gray) and  $R_H(B)$  (green/light gray) at  $V_g = -1.4$  V. (d)  $R_L(B)$  (blue/dark gray) and  $R_H(B)$  (green/light gray) at  $V_g = -0.7$  V. (c) Full  $V_g$ - $B$  gray map of  $R_L$ . The (green/light gray) shaded and contoured regions correspond to a calculated correlation  $g > 0.7$  between  $R_H$  and  $R_L$ . The correlated regions may also be seen by the eye in (b) and (d).  $T=120$  mK.

$R_H$  and  $R_L$ . The correlation can also be checked by the eye in Figs. 3(b) and 3(c). For  $V_g \geq -0.8$  V, most of the  $R_L$  SdH peaks are correlated with  $R_H$ , but as  $V_g$  decreases, the correlated parts shrink. Then, the lower the  $V_g$ , the greater the weight for the states in the high energy tail. This is in agreement with the formation of a one-dimensional (1D) wire at low  $V_g$  and with our preliminary observations reported above. The increasing importance of these states is clearly seen for at least three SdH transitions in Fig. 3(c). We conclude that this analysis provides information on the symmetry of the LL DOS.

We can now have a closer look at the shift of the RFs with  $B$  and  $V_g$ . Figure 4 shows grayscale maps of  $R_L(B, V_g)$  and  $R_H(B, V_g)$  around the transition between the  $\nu=1$  and  $\nu=2$

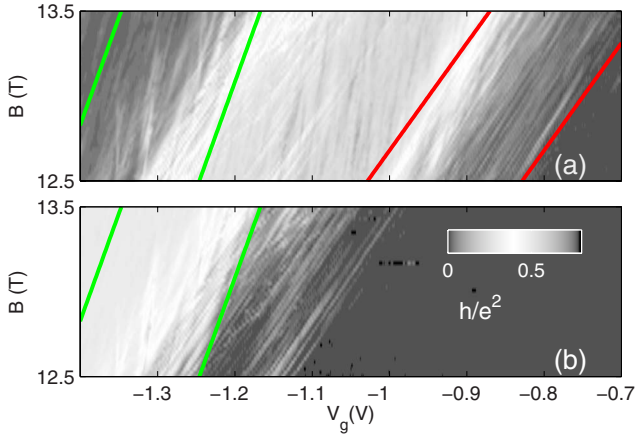


FIG. 4. (Color online)  $V_g$ - $B$  map of  $R_L$  (top) and  $R_H$  (bottom). The two thick red/gray lines have a slope corresponding to  $\nu=2$  and sandwich fluctuations appearing both in  $R_{14,23}$  and  $R_{14,65}$  that are attributed to JK scattering between opposite edge states. The two thick green/light gray lines correspond to a  $\nu=1$  slope and sandwich fluctuations appearing both in  $R_L$  and  $R_H$ .  $T=120$  mK.

plateaus, where the LPLL is partially filled. Obviously, the RFs move along lines of slopes corresponding either to  $\nu=1$  or  $\nu=2$ , as yet observed in Refs. 2 and 3. In all samples studied, this fine structure can be classified in accordance with the two regimes discussed above:

- (1) On the low  $\nu$  tail of the transition, correlated RFs are observed between  $R_H$  and  $R_L$ . These RFs follow a slope  $\nu=1$  and are sandwiched between the two green lines in Fig. 4.
- (2) On the high  $\nu$  tail of the transition, RFs appear on both sides of the Hall bar in  $R_{14,65}(R_L)$  and  $R_{14,23}(R'_L)$  while both  $R_{14,53}(R_H)$  and  $R_{14,62}(R'_H)$  remain quantized. These RFs follow a slope corresponding to  $\nu=2$  and, for the sake of clarity, are sandwiched between the two red lines in Fig. 4.

The lines of Fig. 4 are guides for the eye corresponding to  $\nu=n_s B e/h$ , where  $n_s=V_g C/e$ ;  $C$  is determined by the characteristics of the heterostructure and does not depend on the magnetic field.<sup>17</sup>  $n_s(V_g)$  is extracted from the classical Hall resistance taken at low fields between 0 and 2 T. The slope of the thick solid gray (red) lines corresponds to  $\nu=2$  while the slope of the light gray (green) lines corresponds to  $\nu=1$ . The correspondence between the slopes of the RFs and their correlation properties has not been reported in the literature yet, although this is an expected result, in agreement with the previous observations of Refs. 2 and 4, who concluded that dots and antidots give rise to fluctuations of different slopes.

We now point out that, because of the one-to-one relation between the number of states and the number of flux quanta, two different models: (i) Coulomb blockade between compressible regions, and (ii) single electron tunneling through localized states of the disordered potential,<sup>18</sup> lead to the same experimental consequences—that the RFs slopes are quantized. The former model was used in Refs. 2 and 4; we give a simple derivation of the latter below, by modeling a sample containing  $i$  completely filled LLs, while the partially filled LL contains  $n$  dots of size  $S$  per unit area. The filling factor is given by  $\nu=i+nS$ . Besides, the density  $n_s$  varies as  $dn_s=\nu/\Phi_0 dB+\rho_B d\epsilon_F$ , where  $\Phi_0=h/e$ ,  $\epsilon_F$  is the Fermi energy, and the energy reference is the LL center. When following

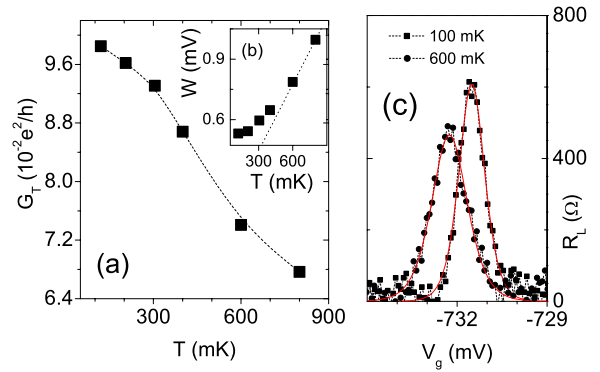


FIG. 5. (Color online)  $T$  dependence of the first peak observed on the high- $\nu$  side. (a) Peak height  $G_T$  as a function of  $T$ ; (b) Peak width  $W$  as a function of  $T$ ; solid squares correspond to data, dotted lines are guides for the eye. (c)  $R_L(V_g)$  measurements showing the peak structure for  $T=100$  and 600 mK. Best fits are indicated by red (gray) lines. Peak at 600 mK has been shifted for clarity.

one RF in the  $(B, V_g)$  plane,  $\epsilon_F$  is continuously pinned on the energy  $E$  of the localized state through which the tunnel processes occur and we get  $dE=d\epsilon_F$ . The energy variation  $dE$  of the localized state induced by a magnetic field variation  $dB$  can also be determined by flux conservation:  $\rho_B dE \approx d\Phi/\Phi_0$ , where  $d\Phi=-nSdB$ . Combining these equations, the RF slope is quantized:  $dn_s/dB=i/\Phi_0$ . A similar conclusion can be drawn for antidots.

In the following, we claim that the disorder is not efficiently screened by electron-electron interactions in our samples. The first argument comes from a fine analysis of the RFs evolution in the  $(B, V_g)$  map. Recently, Sohrmann *et al.*<sup>6</sup> calculated the inverse electronic compressibility of a 2DES and showed that even in a noninteracting picture, most of the fluctuations follow lines of slope  $\nu$  integer, while some lines have a varying slope interpreted as a fingerprint of scattering resonances on the disordered potential. We indeed observe such a behavior, e.g., in Fig. 4(b),  $V_g \approx -1.2$  V,  $B \approx 12.6$  T. Also, the fluctuations we observe are not rigorously parallel but grow slightly closer as  $B$  increases, as expected if they reflect the LL degeneracy.

The second argument comes from the  $T$  dependence of the RFs. Figure 5 shows the  $T$  dependence of the first peak observed in the SdH low- $B$  tail, at  $V_g=-0.72$  V and  $B=13.5$  T. This peak follows a slope corresponding to  $\nu=2$ , and we associate it with JK transitions through one (maybe several) antidot. Its amplitude decreases when  $T$  decreases, as expected when tunneling through only one electron state is possible.<sup>7</sup> This occurs when  $k_B T$  is large compared to the intrinsic linewidth  $\Gamma$  of the levels but small compared to the level spacing  $\Delta E$ , and implies that flat compressible regions are not formed.<sup>20</sup> We checked (Fig. 5) that the usual equation<sup>19</sup> that describes this regime:

$$G = G_T \cosh^{-2} \left( \frac{V_g - V_0}{W} \right), \quad (1)$$

where  $V_0$  is the peak position,  $W=2k_B T/\alpha$ ,  $\alpha=|d\epsilon/dV_g|$  is the leverage factor and  $G_T \propto 1/T$ , gives a good fit of the peak lineshape, using the relation  $G \approx 4R_L(e^2/h)^2$ . Equation (1)



reasonably fits all RFs observed for  $V_g > -0.8$  V at  $B = 13.5$  T. In the high- $B$  tail of the SdH peak, fluctuations overlap each other, which hinders any definitive interpretation.

Between the two tails of the transitions, when  $R_L$  reaches its maximum value, the situation is blurred by other phenomena. We usually observe additional RFs following a slope  $\nu = 1$  in the longitudinal resistance [see Fig. 4(a),  $V_g \approx -1$  V,  $B \approx 13$  T]. They are attributed to a parasitic coupling between two adjacent LLs, because (i) these RFs are correlated neither with the other longitudinal resistance, nor with the Hall resistances; (ii) they are also observed in lower transitions, in particular in the  $\nu=2-\nu=4$  transition of Fig. 3; (iii) they are visible in samples in which the SdH peak maximum exceeds  $h/2e^2$ . These RFs are usually associated with others of slope  $\nu=2$ , appearing in the longitudinal resistances and correlated with the Hall resistances [see Fig. 4(a) and 4(b),  $V_g \approx -1.1$  V,  $B \approx 13$  T]. We ascribe these RFs to additional

scatterings appearing in the lateral probes themselves.

To summarize, we have observed a direct correspondence between the slope of the RFs appearing on the tails of the Hall transition and the correlation properties of these RFs. This confirms other previous observations that tunneling processes through dots or antidots give rise to two different quantized slopes in the  $(B, V_g)$  map. Our experiment does not allow a precise determination of the relative influence of disorder and electron-electron interaction in the quantization of the RFs slopes. However, our results balance in favor of a single particle interpretation, and we conclude that a rough quantization of the RFs slopes in mesoscopic IQHE is not a clear signature of electron-electron interaction.

We wish to thank C. Zhou and M. Berciu for fruitful discussions. This work is supported by the Action Concertée Nanostructure NANO HALL of the French Ministry of Research.

<sup>1</sup>For a review, see R. E. Prange and S. M. Girvin, *The Quantum Hall Effect* (Springer-Verlag, New York, 1990); T. Chakraborty and P. Pietiläinen, *The Quantum Hall Effects* (Springer, Berlin, 1995).

<sup>2</sup>D. H. Cobden, C. H. W. Barnes, and C. J. B. Ford, *Phys. Rev. Lett.* **82**, 4695 (1999).

<sup>3</sup>T. Machida, S. Ishizuka, S. Komiyama, K. Muraki, and Y. Hirayama, *Phys. Rev. B* **63**, 045318 (2001).

<sup>4</sup>S. Ilani, J. Martin, E. Tetelbaum, J. H. Smet, D. Mahalu, V. Umanski, and A. Yacobi, *Nature (London)* **427**, 328 (2004).

<sup>5</sup>A. Struck and B. Kramer, *Phys. Rev. Lett.* **97**, 106801 (2006).

<sup>6</sup>C. Sohrmann and R. A. Römer, *Phys. Status Solidi C* **3**, 313 (2006); *New J. Phys.* **9**, 97-22 (2007).

<sup>7</sup>J. K. Jain and S. A. Kivelson, *Phys. Rev. Lett.* **60**, 1542 (1988).

<sup>8</sup>E. Peled, D. Shahar, Y. Chen, E. Diez, D. L. Sivco, and A. Y. Cho, *Phys. Rev. Lett.* **91**, 236802 (2003).

<sup>9</sup>C. Zhou and M. Berciu, *Phys. Rev. B* **72**, 085306 (2005).

<sup>10</sup>S. Datta, *Electronic Transport in Mesoscopic Systems* (Cambridge University Press, Cambridge, U.K., 1997).

<sup>11</sup>We use the standard notation  $R_{ij,kl}$  if one measures the voltage between leads  $k$  and  $l$  when a current goes from leads  $i$  to  $j$ .

<sup>12</sup>L. J. van der Pauw, *Philips Res. Rep.* **13**, 1 (1958).

<sup>13</sup>K.-F. Berggren, G. Roos, and H. van Houten, *Phys. Rev. B* **37**, 10118 (1988).

<sup>14</sup>R. G. Mani and K. v. Klitzing, *Phys. Rev. B* **46**, 9877 (1992).

<sup>15</sup>R. J. Haug, R. R. Gerhardt, K. v. Klitzing, and K. Ploog, *Phys. Rev. Lett.* **59**, 1349 (1987); R. J. Haug, K. v. Klitzing, and K. Ploog, *Phys. Rev. B* **35**, 5933 (1987).

<sup>16</sup>E. Peled, Y. Chen, E. Diez, D. C. Tsui, D. Shahar, D. L. Sivco, and A. Y. Cho, *Phys. Rev. B* **69**, 241305(R) (2004).

<sup>17</sup>V. Mosser, D. Weiss, K. v. Klitzing, K. Ploog, and G. Weimann, *Solid State Commun.* **58**, 5 (1986).

<sup>18</sup>C. Zhou and M. Berciu, *Nat. Phys.* (to be published).

<sup>19</sup>This is basically the derivative of the Fermi distribution function. See also A. A. M. Staring, H. van Houten, C. W. J. Beenakker, and C. T. Foxon, *Phys. Rev. B* **45**, 9222 (1992).

<sup>20</sup>I. Karakurt, V. J. Goldman, J. Liu, and A. Zaslavsky, *Phys. Rev. Lett.* **87**, 146801 (2001).