Frequency-temperature crossover in the conductivity of disordered Luttinger liquids

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The temperature (T) and frequency (ω) dependent conductivity of weakly disordered Luttinger liquids is calculated in a systematic way both by perturbation theory and from a finite temperature renormalization group (RG) treatment to leading order in the disorder strength. Whereas perturbation theory results in ω/T scaling of the conductivity, such scaling is violated in the RG treatment. We also determine the nonlinear field dependence of the conductivity, whose power law scaling is different from that of temperature and frequency dependence.

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I. INTRODUCTION

Interacting one-dimensional (1D) electron systems display a large variety of unusual and interesting phenomena, since not only interactions but also external potentials (periodic or random) and thermal fluctuations have pronounced effects on the behavior of these systems.[1,2] The hallmark of interaction effects in 1D systems is the power law single particle density of states observable in the backscattering from a single impurity in a Luttinger liquid (LL).[3] While the LL spectral function has been observed experimentally,[4] the rich behavior of collective scattering by random impurities has eluded experimental observation so far.

In this paper, we focus on the effect of many weak (Gaussian) impurities in a 1D disordered system. For noninteracting electrons, this problem can be solved exactly.[5] Interaction effects are mostly treated perturbatively and described by a dephasing length, which cuts off interference corrections. While this regime of weak interactions has been studied thoroughly,[6–8] less is known about the opposite regime of strong interactions. For attractive interactions, an unpinning transition as a function of the interaction strength was found.[9–11] In addition, the power law exponent describing the energy dependence of impurity scattering was predicted to flow as a function of energy.[10] Finite temperature effects were partially incorporated by truncating the renormalization group (RG) flow at the de Broglie wavelength of plasmon excitations.[10] However, for a complete study of the thermal to quantum crossover, quantum and thermal fluctuations must be treated on an equal footing.[12]

We calculate the frequency, temperature, and electric field dependence of the conductivity for spinless 1D fermions to leading order in the disorder strength but for arbitrary short range interactions. We go beyond previous approaches[9–11,13] in several respects. From a technical point of view, we (i) present a systematic approach to calculate the conductivity from a bosonic self-energy in the spirit of Refs. 14 and 15, both in perturbation theory and from a finite temperature renormalization group.[16,17] This approach can be generalized to higher orders in the impurity strength to obtain weak localization corrections. We (ii) explain that due to a symmetry property of the self-energy, an imaginary time RG[10,16] has a ballistic density propagator although the retarded density propagator is diffusive. From a physics point of view, we (iii) present results for the frequency-temperature crossover including the renormalization of the interaction strength, and show that simple ω/T scaling is violated in the RG solution. In addition, we (iv) calculate the nonlinear field dependence of the conductivity, which is characterized by a nonstandard power law exponent. Our predictions for weak disorder are in reach of present experimental technology, for instance, in carbon nanotubes[18–20] and polydiacetylen,[21] where the influence of strong disorder has already been observed. Especially promising experimental systems are quantum Hall line junctions[23] and bent two-dimensional electron systems in a strong magnetic field,[22] in which both interaction and disorder strength can be tuned. We expect our results to be applicable to the metallic state.[22]

II. MODEL

We consider particles moving in a random potential $U(x)=\Sigma_i U_i \delta(x-x_i)$ due to point scatterers with random positions x_i and density n_{imp}. The random potential strength U_i has moments $\overline{U_i}=0$ and $\overline{U_i U_j}=U_{\mathrm{imp}}^2 \delta_{ij}$. Throughout the paper, we use units with $\hbar=1$, $k_B=1$. For noninteracting electrons with Fermi velocity v, one finds a mean free path $1/\ell_0 = 2n_{\mathrm{imp}}U_{\mathrm{imp}}^2/v^2$ by using standard perturbation theory. We will use ℓ_0 as an abbreviation for the disorder strength also in the interacting case. In the replica formalism, a system of spinless interacting 1D electrons is described by the bosonic action

$$\mathcal{S}^{(n)} = \sum_{\alpha,\beta} \int_0^L dx \int_0^{1/T} d\tau \left\{ \frac{v}{2\pi K} \left[(\partial_x \varphi_\alpha)^2 + \frac{1}{v^2} (\partial_\tau \varphi_\alpha)^2 \right] \delta_{\alpha\beta} - \frac{1}{\ell_0} \frac{v^2 \Lambda^2}{8\pi^2} \int_0^{1/T} d\tau' \cos p[\varphi_\alpha(x,\tau) - \varphi_\beta(x,\tau')] \right\}. \quad (1)$$

Here, Λ denotes the cutoff in momentum space and the interaction strength is parametrized by K, with $K<1$ for repulsive and $K>1$ for attractive interactions. The smooth part of the density is described by $\frac{1}{\pi}\partial_x \varphi(x)$, giving rise to a forward scattering term. Although it strongly influences static corre-

lation functions,[17] it has no effect on dynamic ones and is hence neglected in this analysis. For charge density waves, we have $p=1$, and $p=2$ for spinless Luttinger liquids. In the following, we let $p=2$, and the formulas for $p=1$ can be obtained by replacing $K\to K/4$. The optical conductivity can be calculated from the retarded boson Green function as

$$\sigma(q,\omega) = e_0^2\kappa \frac{-i\omega}{q^2 - \frac{\omega^2}{v^2} + \pi^2\kappa\Sigma^R(q,\omega)}. \tag{2}$$

Here, e_0 is the electron charge, and the compressibility $\kappa = \frac{K}{\pi v}$ is used as a generalized density of states for interacting systems. All disorder effects are contained in the retarded self-energy Σ^R.

III. PERTURBATION THEORY

We use the cosine term in Eq. (1) as a vertex and keep in each term of its power series two field operators as external operators and contract all other operators with respect to the quadratic part of Eq. (1). This amounts to replacing the cosine by its best quadratic approximation. The perturbative self-energy is found to be

$$\Sigma^{\rm per}_{\alpha\beta}(\tau) = -\frac{v^2\Lambda^2}{\pi^2\ell_0}\delta_{\alpha\beta}\left[e^{-2G_0(\tau)} - \delta(\tau)\int_0^{1/T} d\tilde{\tau} e^{-2G_0(\tilde{\tau})}\right]. \tag{3}$$

For the bare local bosonic Matsubara Green function we use the expression (see the Appendix)

$$G_0(\tau) = K\ln\left(1 + \left|\frac{\sin(\pi T\tau)}{\pi T/v\Lambda}\right|\right). \tag{4}$$

Proper regularization of the Green function is especially important in the RG approach, where the cutoff flows to zero and the Green function needs to be evaluated for energies larger than the cutoff. We calculate the retarded self-energy via the analytic continuation

$$\Sigma^R(t) = -2\Theta(t)\mathrm{Im}\,\Sigma(\tau\to it), \tag{5}$$

with $\Theta(t)$ denoting the Heavyside step function. After Fourier transforming, one obtains a scaling form of the perturbative self-energy,

$$\pi^2\kappa\Sigma^{R,\rm per}_{\alpha,\beta}(\omega) = -\delta_{\alpha\beta}\frac{1}{\xi^2}\left(\frac{\omega}{\omega_p}\right)^{2K-1} F\left(\frac{\omega}{2\pi T}\right), \tag{6}$$

where $\xi=\Lambda^{-1}(l_0\Lambda/K^2)^{1/(3-2K)}$ is the renormalized mean free path and $\omega_p=v/\xi$ the renormalized scattering rate. The scaling function F is given by

$$F(x) = -4x^{1-2K}\,\Gamma(-2K) \times\left[\frac{|\Gamma(1-K-ix)|^{-2}}{\cosh(\pi x) - i\cot(\pi K)\sinh(\pi x)} - \Gamma^{-2}(1-K)\right]. \tag{7}$$

The perturbative expression [Eq. (6)] is valid for temperatures much larger than the mean level spacing $1/(\xi\kappa)$ on the scale of the renormalized mean free path. However, for weak interactions, $K\approx 1$, localization corrections can only be neglected for temperatures $T\gg\frac{1}{|1-K|^3}\frac{1}{\xi\kappa}$.[7,8] At finite frequencies, $\omega\gg\omega_p$, the perturbative expressions are valid for all temperatures.[24] In the derivation of Eq. (7), the limit $T,\omega \ll v\Lambda$ was taken and the regularization in Eq. (4) was neglected. The imaginary part of this scaling function is a good approximation for all values of K, its real part only for $K\lesssim 0.5$; however, for $K\approx 1$ and larger, the real part of Σ can be neglected compared to the ω^2 term in Eq. (2). Interestingly, the part $i\cot(\pi K)\sinh(\pi x)$ giving rise to the imaginary part of the self-energy vanishes identically for all $x=in$ with integer n, i.e., it does not appear in the Matsubara self-energy. The scaling function has the limiting forms $\mathrm{Im}[F(x)]\sim 1$ for $x\to\infty$ and $\mathrm{Im}[F(x)]\sim x^{2-2K}$ for $x\to 0$. Using the asymptotic behavior of $F(x)$, we find for the real part of the conductivity

$$\mathrm{Re}\,\sigma \approx \sigma_0\begin{cases} \dfrac{\left(\dfrac{2\pi T}{\omega_p}\right)^{2-2K}\dfrac{K\Gamma(2K)}{\Gamma^2(K)}}{1+\left(\dfrac{\omega}{\omega_p}\right)^2\left(\dfrac{2\pi T}{\omega_p}\right)^{4K-4}\dfrac{K^2\Gamma^2(2K)}{\Gamma^4(K)}}, & \dfrac{\omega}{T}\ll 1 \\ \left(\dfrac{\omega}{\omega_p}\right)^{-4+2K}\dfrac{1}{K\Gamma(2K)}, & \dfrac{\omega}{T}\gg 1. \end{cases} \tag{8}$$

Here, $\sigma_0=e_0^2\kappa\xi v$. For $K=1$, the Drude conductivity $\sigma = \sigma_0/(1+\omega^2v^2/\ell_0^2)$ is recovered. For $K>3/2$, perturbation theory is valid for all frequencies, and the conductivity is imaginary with a $1/\omega$ divergence characteristic for a superconductor.

IV. RENORMALIZATION

Since the derivation of the flow equations is well documented in the literature, we will here quote only the result.[16,17] At finite temperature, the flow equations are given by

$$\frac{dK}{dl} = -8u^2KB\left(K,\frac{K}{2t}\right)\coth\frac{K}{2t}, \tag{9a}$$

$$\frac{du^2}{dl} = \left[3 - 2K\coth\frac{K}{2t}\right]u^2, \tag{9b}$$

$$B(K,y) = \int_0^y \frac{\tau^2 d\tau}{\left[1+\left(\dfrac{2y}{\pi}\sin\dfrac{\pi\tau}{2y}\right)^2\right]^K}\frac{\cosh(y-\tau)}{\cosh y}. \tag{9c}$$

Here, the dimensionless temperature t obeys the flow equation $\frac{dt}{dl}=t$ with initial value $t_0\equiv t(l=0)=TK/v\Lambda$, and the dimensionless disorder strength u^2 has the initial value $u_0^2 = \frac{1}{\ell_0\Lambda}\frac{K^2}{2\pi}$. Calculating the self-energy by integrating over a momentum shell $\Lambda\exp(-\delta l)<|k|<\Lambda$, we obtain

$$\delta\Sigma_{\alpha\beta}(\tau) = 2K \frac{\cosh\left[v\Lambda\left(\tau - \frac{1}{2T}\right)\right]}{\sinh\left(\frac{v\Lambda}{2T}\right)} \delta l\, \Sigma^{\mathrm{per}}_{\alpha\beta}(\tau), \qquad (10)$$

for $0<\tau<1/T$. Due to the symmetry property $\delta\Sigma_{\alpha\beta}(\tau) = \delta\Sigma_{\alpha\beta}(1/T-\tau)$ in this interval, the low frequency expansion of the Fourier transform $\delta\Sigma_{\alpha\beta}(\omega_n)$ starts with a term $O(\omega_n^2)$, which renormalizes the corresponding term in the action and gives rise to the flow of K in the RG. The analytical continuation $i\omega_n \to \omega + i\eta$ would not give rise to a diffusive term in the boson propagator. Instead, one has to perform the analytic continuation in the time domain. Although the diffusive part of Σ^R does not feed back in the RG, it determines the conductivity in an essential way. The incremental change of the retarded self-energy is

$$\delta\Sigma^R(\omega,T,l) = \frac{\Lambda^3 v^2 u^2 e^{-3l} 2\delta l}{\pi K \sinh(v\Lambda/2T)} \int_0^\infty d\tilde{t}\, \frac{e^{i\omega\tilde{t}} - 1}{\left[1 + \frac{\sinh^2(\pi T\tilde{t})}{(\pi T/v\Lambda)^2}\right]^K} \times \mathrm{Im}\left\{ \exp\left(\frac{2K}{i} \arctan\left[\frac{\sinh(\pi T\tilde{t})}{\pi T/v\Lambda} \right] \right) \times \cosh\left(-\frac{v\Lambda}{2T} + i\tilde{t}v\Lambda \right) \right\}. \qquad (11)$$

The variables K, v, and Λ are functions of the RG scale l as determined from the flow equations [Eqs. (9a) and (9b)]. Note that there is no renormalization of κ. The retarded self-energy to be used in the conductivity equation (2) is obtained by integrating over the flow parameter l from zero to infinity. Our calculation to $O(u^2)$ is valid if either a finite temperature stops the RG flow equations (9a) and (9b) or a finite frequency cuts off the integral over $\delta\Sigma^R(l)$, giving rise to the condition $T \gg \frac{1}{\xi\kappa}$ or $\omega \gg \frac{v}{\xi}$ discussed earlier. The frequency-temperature crossover of self-energy and conductivity is displayed in Fig. 1. One clearly sees that the power law exponent of the frequency dependence changes with energy. Besides the scale dependence of exponents, the conductivity is characterized by the limiting forms in Eq. (8). The temperature dependence of the dc conductivity is compared to the approximate result[10] in Fig. 2. We observe qualitative agreement between the two approaches. Due to the flow of the Luttinger parameter K, ω/T scaling of the self-energy is violated (see Fig. 3).

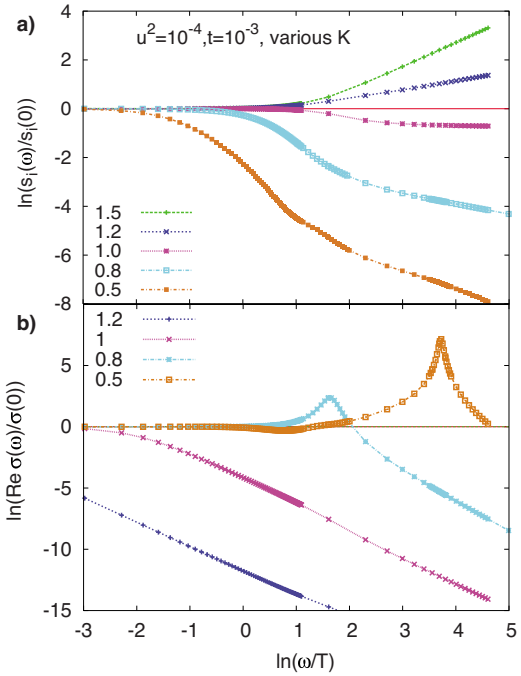

FIG. 1. (Color online) Plots for the self-energy and conductivity calculated using the finite temperature RG result [Eq. (11)]. (a) Imaginary part $s_i = \mathrm{Im}(\Sigma/\omega)$ of the self-energy for several values of K_0 (shown in the plots) at fixed $u_0^2 = 10^{-4}$ and $t_0 = 10^{-3}$. (b) Double logarithmic plot of the real part of the conductivity [Eq. (2)] for the same parameters, rescaled to 1 at $\omega = 0$.

V. NONLINEAR FIELD DEPENDENCE

In the regime of linear dc transport, the average excitation energy due to the external electric field is $e_0 E\ell(T)$. In the nonlinear regime, the electric field dependence $\ell(E,T)$ of the mean free path must be taken into account. The *frequency* and temperature dependence of ℓ can be directly calculated from the imaginary part of the self-energy via $\frac{1}{v\ell} = -\pi^2\kappa\, \mathrm{Im}[\Sigma^R(\omega,T)/\omega]$. In order to obtain the *electric field* and temperature dependence of ℓ, we recall that for scattering from a single impurity in a LL, the nonlinear conductance $G_{\mathrm{nl}}(V,T)$ and the ac conductance $G_{\mathrm{ac}}(\omega,T)$ are related by $G_{\mathrm{nl}}(V,T) = G_{\mathrm{ac}}(e_0 V,T)$ to leading order in the impurity strength. By analogy, the nonlinear field dependence of the mean free path can be obtained by replacing the frequency in the self-energy by the average electron energy $e_0 E\ell(E,T)$. In this way, $\ell(E,T)$ can be found as the solution of the equation

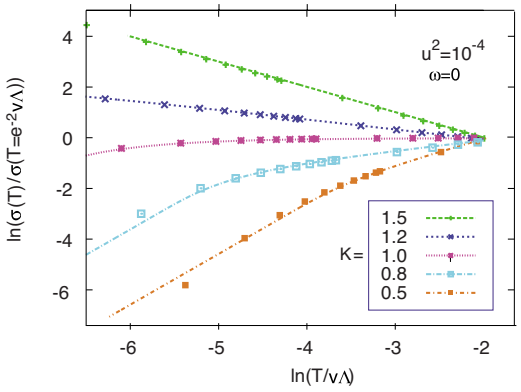

FIG. 2. (Color online) Temperature dependence of the dc conductivity in log-log representation for several values of K_0 and $u_0^2 = 10^{-4}$. Lines: Results obtained by truncation of the zero temperature RG equations (9a) and (9b) at the thermal de Broglie wavelength (cf. Ref. 10). Symbols: Conductivity calculated from finite temperature RG and the renormalized self-energy [Eq. (13)].

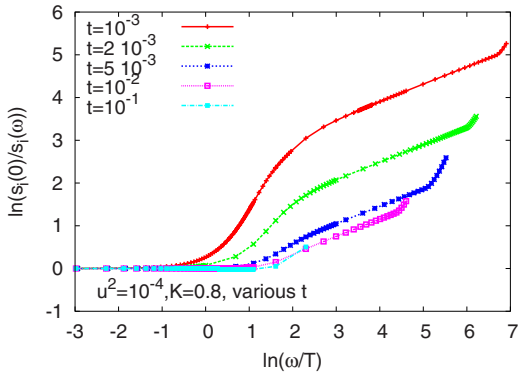

FIG. 3. (Color online) Double logarithmic plot of the imaginary part $s_i = \mathrm{Im}(\Sigma/\omega)$ of the self-energy for $K=0.8$ and different temperatures t_0. As K varies with frequency, curves for different temperatures do not collapse and ω/T scaling is violated.

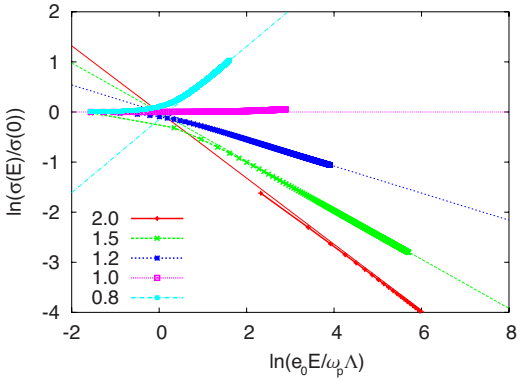

FIG. 4. (Color online) Field dependence of the nonlinear conductivity $\sigma_{\mathrm{nl}} \sim E^{(2-2K)/(2K-1)}$ for different values of K. The straight lines represent the power law.

$$\frac{1}{\ell(E,T)} = -\pi K \,\mathrm{Im}\left[\frac{\Sigma(e_0 E\ell(E,T),T)}{e_0 E\ell(E,T)}\right]. \tag{12}$$

The calculation presented here is a mean field approach, where the average mean free path and the average excitation energy of diffusive electrons are calculated self-consistently. We do not consider fluctuations in the energies of electrons and do not calculate the whole distribution function. This approximation is justified if the energy distribution of electrons is sharply peaked such that the average is much larger than fluctuations. As the power law dependence of the mean free path on energy is controlled by the *average* electron energy in a given spatial region and not by the energy of the scattered electron, electrons with a particularly large or small kinetic energy will equilibrate with the average equilibration rate. Hence, the energy distribution function will not have power law tails and our assumption of a sharply peaked distribution function is justified.

Using the perturbative expression in Eq. (6), the self-consistency condition in Eq. (12) simplifies to

$$\frac{\ell(E)}{\xi} = K\Gamma(2K)\left(\frac{e_0 E\ell(E)}{\omega_p}\right)^{2-2K}, \tag{13}$$

for temperatures $T \ll e_0 E\ell(E)$, and one obtains

$$\sigma_{\mathrm{nl}}(E) = \sigma_0[K\Gamma(2K)]^{1/(2K-1)}\left(\frac{e_0 E\xi}{\omega_p}\right)^{(2-2K)/(2K-1)}. \tag{14}$$

This zero temperature approximation is valid only for $K > \frac{1}{2}$, as for lower values of K a physical solution to Eq. (12) can be found only at finite temperature. In Fig. 4, $\sigma_{\mathrm{nl}}(E,T)$ as calculated from the perturbative self-energy [Eq. (6)] is shown.

For application to an experimental system of length L (see, e.g., Ref. 22), the conductance rather than the conductivity is needed. The four terminal conductance is directly obtained as σ/L, whereas for the calculation of the two terminal conductance, the effect of Fermi liquid contacts needs to be taken into account. In this case, one has to add the voltage drop due to the contact between reservoir and wire to the voltage drop along the wire, i.e., the contact resistance h/e_0^2 needs to be added to the wire resistance σ/L. In this way, one obtains

$$G = \left(\frac{h}{e_0^2} + \frac{L}{\sigma}\right)^{-1}. \tag{15}$$

On a more formal level, this relation can be obtained by generalizing the method[25] to the problem of a Luttinger liquid with many impurities.

Comparing our prediction [Eq. (14)] to the asymptotic voltage dependence $dI/dV \sim V^{-0.4}$ in Ref. 22, we obtain $K=4/3$ as compared to $K=1.2$ without using the self-consistency condition in Eq. (12). However, to determine the temperature and electric field dependence of K, experimental measurements over a wider temperature and voltage range are desirable.

VI. CONCLUSIONS

We have discussed frequency, temperature, and field dependence of charge transport in a disordered LL in a bosonized theory. The conductivity is obtained from the bosonic self-energy, which is integrated from the flow equations of a finite temperature RG. In contrast to single impurity physics in a LL, the power law exponent describing impurity scattering is scale dependent in the disordered case. The mean free path in nonlinear dc transport is self-consistently calculated by replacing frequency in the ac self-energy by the average electron energy acquired between scattering events.

It has recently come to our attention that the preprint in Ref. 26 contains a calculation of the dynamic conductivity similar to our perturbative result.

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APPENDIX: MATSUBARA GREEN FUNCTION

Here, we present a short derivation of the Matsubara Green function [Eq. (4)].

In imaginary time, the Green function is defined as

$$G_0(x,\tau) = \frac{2\pi T}{\Lambda L}\sum_{n,m}{}' \frac{1-e^{i(k_m x+\omega_n \tau)}}{\omega_n^2+k_m^2}, \tag{A1}$$

where $\omega_n = \frac{2\pi T}{K} n$ are dimensionless Matsubara frequencies (T in units of $\hbar v \Lambda$), $k_m = \frac{2\pi}{\Lambda L} m$ are momenta, and Σ'_m stands for the sum limited by $|k_m| \leqslant 1$.

The sum over n can be calculated exactly using

$$\sum_{n=-\infty}^{\infty} \frac{\cos(nx)}{n^2+a^2} = \frac{\pi}{|a|}\frac{\cosh((\pi-x)a)}{\sinh(\pi|a|)}, \quad 0 \leqslant x \leqslant 2\pi.$$

For large L, we replace the sum over m by a *soft cutoff* integral: $\Sigma'_m \to \int_{-\infty}^{\infty} \frac{dk e^{-|k|}}{2\pi/\Lambda L}$, which results in

$$G_0(x,\tau) = K\int_0^{\infty} \frac{dk\, e^{-k}}{k}\frac{1-e^{\tilde{\tau}k}+e^{-kK/T}(1-e^{\tilde{\tau}k})}{1-e^{-kK/T}},$$

$$\tilde{\tau} = \tau \bmod (K/T).$$

In the following, we drop the tilde for τ and get for real time ($\tau \to it$)

$$G_0(x,t) = 2K\int_0^{\infty} \frac{dk\, e^{-k}}{k} \sin^2(tk)\coth[kK/(2T)] + iK \arctan t,$$

which gives $G_0(x,t) = \ln(1+it)$ and $G_0(x,\tau) = \ln(1+\tau)$ for $T \to 0$. With the replacement $\tau \to |\frac{\sin(\pi\tau T/K)}{\pi T/K}|$ in the latter expression, we obtain the result given in Eq. (4) for finite temperature.

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