Calculation of critical states of superconducting multilayers based on numerical solution of the Ginzburg-Landau equations for superconducting plates

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The numerical method is used to analyze the critical state of superconducting multilayers. The method is based on self-consistent solutions of the Ginzburg-Landau system of nonlinear equations, which describe the behavior of a superconducting plate carrying transport current in a magnetic field, provided that there are no vortices inside the plate. The field-dependent critical currents computed for plates are used to determine the critical current as a function of the applied magnetic field strength, local magnetic field, and current distributions for multilayers in parallel magnetic fields. The mutual influence of the superconducting layers is assumed to be realized only via a magnetic field. The method makes it possible to account for the peak effect in multilayered superconductors. Our results give an alternative approach to explain different scaling laws that describe flux pinning in the two most common commercial superconductors, NbTi and Nb_3Sn. A simple method is proposed for analyzing the critical states of multilayers in magnetic fields of arbitrary strength, based on elementary transformations of the critical current-density distribution over individual layers in a zero applied magnetic field.

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I. INTRODUCTION

Most studies on the critical states of superconductors rely on the theory of the interaction between the vortex system and crystalline defects.[1] Analysis of this problem is complicated by the variety of quantum properties of the superconductor vortex lattice as an elastic medium described by nonlinear electrodynamics. For this reason, simplifying assumptions, such as the London approximation for the vortex system or model distributions of the magnetic field in superconductors, are invoked.[1] This frequently leads to poor agreement between theoretical calculations and experimental results.[2] Even in the simplest case of ordered defects, as in a multilayer placed in parallel magnetic fields, the calculation of critical current density is a difficult task. The most interesting results in this area were obtained in Refs. 3 and 4. In Ref. 3, the Ginzburg-Landau equations were solved to find the field-dependent critical current density in a multilayer for a magnetic field strength close to the upper critical field. In Ref. 4, the critical current was found in the London approximation by representing the vortex lattice as a set of linear chains and analyzing their interaction with the layers making up the multilayer. The scope of both studies was substantially limited by assuming that the vortex lattice matches the multilayer structure in the limit of weak order-parameter modulation. For superconductors of this kind, the last condition results in low critical current density, whereas a more interesting case for practical applications are superconductors with strong pinning centers, i.e., superconducting multilayers characterized by large amplitudes of order-parameter modulation.

We propose here an alternative method for analyzing the critical state of a superconducting multilayer based on the exact solution of the Ginzburg-Landau equations for a thin film.[5] Analyses of the behavior of bulk superconductors in magnetic fields based on the Ginzburg-Landau theory[6] have been presented in numerous studies.[7,8] Recent publications have been focused on mesoscopic superconductors of various geometries.[9–11] In this paper, we consider a vortex-free state, in which critical current density is equivalent to depairing current density, as a basis for analyzing the critical state of superconducting multilayers carrying transport current perpendicular to a magnetic field applied parallel to their surfaces. We consider a multilayer consisting of superconducting layers in the vortex-free Meissner state and assume that their mutual influence is mainly due to their interaction with the magnetic field. This approach makes it possible to develop a rigorous analysis of the properties of these superconducting structures. Note that the description of a superconducting plate in a parallel magnetic field based on the model of a vortex-free state has a limited scope. It was shown in Ref. 12 that vortices begin to penetrate into a film when the field strength reaches

$$H_s(D) \sim \phi_0/D^2,$$

where ϕ_0 is the magnetic flux quantum and D is the film thickness. This field is substantially stronger than the lower critical field H_{c1} characteristic of conventional bulk type II superconductors. Furthermore, in the limit of $D<\lambda$ (the magnetic field penetration depth), the highest "superheated" field in the unstable Meissner phase corresponds to an even stronger applied field strength:[13]

$$H_s \approx \phi_o/2\pi\xi D,$$

where ξ is the coherence length. This field strength restricts the applicability of our approach in the limit $\kappa \gg 1$, where κ is the Ginzburg-Landau parameter. Thus, the present approach is valid in a sufficiently wide range of magnetic field strengths and film thicknesses. For thin films $(D \leqslant \xi)$, the vortex-free approach is valid up to the upper critical magnetic field. In this paper, we consider type II superconductors with $\kappa > 1$, which are of primary practical interest.

This area of research is very important, because the Ginzburg-Landau equations are of fundamental importance and their exact solutions can be used to deal with various problems in superconductivity, including validation of these equations themselves as applied to high-temperature superconductors. The results of this study lead to a better understanding of the processes taking place in real superconducting structures. Moreover, to increase the critical current, artificial pinning structures are often used. For example, the critical current density of 4.25×10^9 A/m^2 was achieved at 5 T by forming a lamellar type artificial pinning structure composed of Nb and Nb-50 wt %Ti thin layers.[14]

II. STATEMENT OF THE PROBLEM

Let us consider a stack of long and wide superconducting plates of thickness D in a magnetic field H applied parallel to the layers. Each plate carries transport current perpendicular to the applied field. The transport current I_t is defined as the current density multiplied by the plate thickness, i.e., the current per unit plate width. The calculation of critical current for this structure is divided into two steps. First, a self-consistent solution to the Ginzburg-Landau equations is used to find the dependence of the critical current I_c on the applied magnetic field strength H for an individual plate, which is assumed to be in the vortex-free state. Second, the critical current is determined for a multilayer by finding an optimal distribution of transport current over individual plates.

We write the Ginzburg-Landau equations[5] in a Cartesian coordinate system (x,y,z) with the y and z axes parallel to the plate surface and the z axis parallel to the magnetic field, assuming that transport current flows along the y axis. The vector potential $\mathbf{A}$ has only one component, $\mathbf{A}=e_y A(x)$. These equations may be written in the dimensionless form:

$$\frac{d^2U}{dx_\lambda{}^2} - \psi^2 U = 0, \tag{1}$$

$$\frac{d^2\psi}{dx_\lambda{}^2} + \kappa^2(\psi - \psi^3) - U^2\psi = 0, \tag{2}$$

where ψ is the superconducting order parameter. We introduce dimensionless quantities U, $b(x_\lambda)$, and $j(x_\lambda)$ instead of the dimensional potential A, magnetic induction B, and current density j_s:

$$A = \frac{\phi_0}{2\pi\lambda}U, \quad B = \frac{\phi_0}{2\pi\lambda^2}b, \quad b = \frac{dU}{dx_\lambda}, \quad j(x_\lambda) = j_s\left(\frac{c\phi_0}{8\pi^2\lambda^3}\right)^{-1}$$

$$= -\psi^2 U, \quad x_\lambda = \frac{x}{\lambda}, \tag{3}$$

where $\mathbf{B}=\mathrm{curl}\,\mathbf{A}$, j_s is the current density, and c is the speed of light in a vacuum. Since the transport current I_t carried by the plate generates the magnetic field

$$H_I = \frac{2\pi}{c}I_t, \tag{4}$$

the total field strengths at the plate surfaces are $H\pm H_I$. Accordingly, the following boundary conditions correspond to Eq. (1):

$$b\big|_{x_\lambda=0} = h - h_I, \quad b\big|_{x_\lambda=d} = h + h_I, \tag{5}$$

where $h=H/H_\lambda$, $h_I=H_I/H_\lambda$, $d=D/\lambda$, $H_\lambda=\phi_0/2\pi\lambda^2$.

Equation (2) is subject to standard boundary conditions on the plate surfaces:[1]

$$\left.\frac{d\psi}{dx_\lambda}\right|_{x_\lambda=0} = 0, \quad \left.\frac{d\psi}{dx_\lambda}\right|_{x_\lambda=d} = 0. \tag{6}$$

We remind the reader that the field penetration depth λ and the coherence length depend on the temperature. Thus, the expressions above depend implicitly on temperature and, formally, are valid for arbitrary values of T. (Though the Ginzburg-Landau equations themselves are applicable only in the limit $T\to T_c$.)

To find a self-consistent solution to Eqs. (1) and (2), we use the following iterative procedure. Introducing a trial function $\psi(x_\lambda)$, we solve Eq. (1) for $U(x_\lambda)$. We substitute the resulting $U(x_\lambda)$ into Eq. (2) and use boundary conditions (6) to find a new $\psi(x_\lambda)$. Then, we solve Eq. (1) and repeat the procedure until both $\psi(x_\lambda)$ and $U(x_\lambda)$ become invariant and can therefore be adopted as a self-consistent solution to the system. It is obvious that the solution obtained by this method is stable with respect to small perturbations (see Ref. 15 for details). The critical current I_{ci} carried by the ith plate is set equal to the value of I_t corresponding to $\psi(x_\lambda)\equiv 0$. Thus, we find the critical current per unit width of a superconducting plate as a function of the applied magnetic field strength h. A more detailed description of this method was presented in Ref. 17, where we found the temperature dependence of depairing critical current density for several values of applied magnetic field strength and plate thickness. Moreover, it was shown in Ref. 17 that the expression for the Ginzburg-Landau critical current corresponding to the zero applied magnetic field is valid for films of thickness comparable to coherence length and magnetic field penetration depth. Its value can be estimated by using the Ginzburg-Landau theory with a constant order parameter:[16]

$$I_c = \frac{1}{3\sqrt{6}}\frac{c}{\pi}H_{cm}\frac{D}{\lambda(T)}, \tag{7}$$

where H_{cm} is the thermodynamic critical field. This expression is obtained for $D\ll\lambda,\xi$.

Proceeding to the second step, we seek the critical current for the multilayer.[17] We assume that adjacent superconducting layers are separated by relatively thick insulating layers, i.e., the Josephson coupling between the layers is negligible. To allow for electrical coupling between the superconducting layers, we assume that they are connected by superconducting links at $y=\pm\infty$. We seek such a distribution of transport current over the layers so that the transition to the normal state occurs in all layers simultaneously. If h_i is the magnetic field corresponding to the ith layer, then the current per unit width of the film in the critical state equals the critical current $I_c(h_i)$, which is determined by the numerical solution of the Ginzburg-Landau equations obtained in the first step. Under this condition, each layer in the structure carries a corresponding critical current. The current flowing through the ith plate generates the magnetic field given by Eq. (4), which is independent of the distance from the plate and is in opposite directions on opposite sides of the plate. According to the field superposition principle, we must add up the contributions of all the layers to find the magnetic field that acts on the ith superconducting layer:

$$h_i = h + \sum_{j=1}^{i-1} h_{tj} - \sum_{j=i+1}^{N} h_{tj}, \tag{8}$$

where h_{tj} is the dimensionless magnetic field generated by the transport current carried by the jth layer. The magnetic field distribution over the layers that corresponds to their simultaneous transition to the normal state is found by successive approximation. First, we set some initial conditions. For example, we assume that the magnetic field that acts on each layer is equal to the applied field, and the corresponding critical current per unit width of the film is $I_c(h)$. Then, we combine relations (4) and (8) to find the magnetic field for the ith layer. Using the previously calculated function $I_c(h)$, we determine the critical currents for the layers in the respective magnetic fields h_i and substitute them into Eqs. (4) and (8) to find new values of h_i. The iterative process is terminated when the change in the critical currents from cycle to cycle becomes negligible. Note that this method can also be applied to analyze the critical states of multilayers consisting of different layers.

The magnetic field distribution over a multilayer consisting of similar layers in a zero applied magnetic field can be found by a simpler method. Suppose that the number of layers is odd. First, consider a three-layer structure. By virtue of its symmetry, it is obvious that the central layer is in a zero magnetic field and the corresponding critical current $I_c(0)$ is determined by the numerical solution of the Ginzburg-Landau equations obtained in the first step. The magnetic field H_{I3} acting on each outer layer is generated by the other two:

$$H_{I3} = \frac{2\pi}{c}[I_c(0) + I_{c3}],$$

and the corresponding critical current I_{c3} is

$$I_{c3} = I_c\left(\frac{2\pi}{c}[I_c(0) + I_{c3}]\right). \tag{9}$$

This quantity can be found by fitting. It is obvious that there exists a unique value of $I_{c3}(h=0)$ satisfying Eq. (9) if the initial $I_c(h)$ is a monotonically decreasing function. In a five-layer structure, the three central layers exhibit similar behavior in the critical state, because the magnetic fields generated by the outer layers compensate each other, and the corresponding critical current

$$I_{c5} = I_c\left(\frac{2\pi}{c}[I_c(0) + 2I_{c3} + I_{c5}]\right) \tag{10}$$

can also be found by fitting. Adding two outer layers and calculating the corresponding critical current by the method described above, we can find the critical current for a multilayer consisting of any number of layers. The critical current in the added outer layers can be expressed as

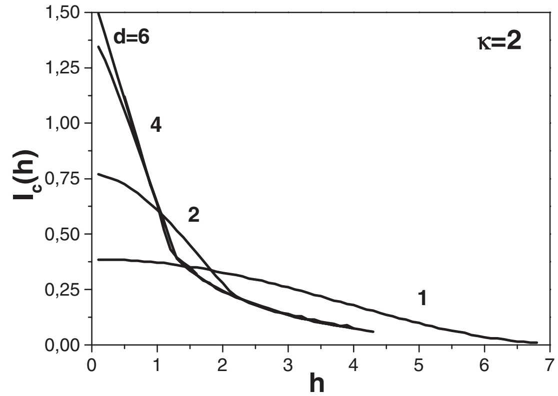

FIG. 1. Critical current vs applied magnetic field $h=H/H_\lambda$ for $\kappa=2$ and several values of dimensionless plate thickness d (indicated at the curves).

$$I_{cN} = I_c\left(\frac{2\pi}{c}\left[I_c(0) + I_{cN} + 2\sum_{n=1}^{L-1} I_{c2n+1}\right]\right), \tag{11a}$$

where $L=(N-1)/2$, and

$$I_{cN} = I_c\left(\frac{2\pi}{c}\left[I_{cN} + 2\sum_{n=1}^{L-1} I_{c2n}\right]\right), \tag{11b}$$

where $L=N/2$, for an even or odd number of layers, with $N>2$ and $N>3$, respectively. The critical current of the layers I_{cN} decreases as the number of layers increases; conversely, the magnetic field acting on the external layers increases. The present analysis shows that a unique distribution of I_{ci} over the layers in a zero magnetic field exists in the case of a monotonically decreasing $I_c(h)$. Thus, if the field-dependent critical current $I_c(h)$ for a single layer is known (e.g., from experiment), the distribution of transport current over the layers in a zero applied magnetic field can readily be found for the critical state of a multilayer consisting of an arbitrary number of layers.

An important consequence of the method is that the maximum value of the critical current of a multilayer $I^m_{\mathrm{cmax}}(0)$ is limited by the upper critical magnetic field of the layers. It is evident that, if the magnetic field acting on the external layers reaches the upper critical magnetic field, these layers turn into the normal nonsuperconducting state; therefore, as the number of additional layers increases, the critical current remains unchanged. This condition defines $I^m_{\mathrm{cmax}}(0)$:

$$I^m_{c\,\max}(0) = \sum_{i=1}^{N} I_{ci} = \frac{c}{2\pi} h_{c2}.$$

III. CALCULATION RESULTS

Figure 1 shows $I_c(h)$ calculated for $\kappa=2$ and several values of the superconducting plate thickness d in the first step. The curves demonstrate that critical current decreases with layer thickness for small h, whereas $I_c(h)$ is higher for thin plates as compared to relatively thick ones at a moderate field strength. Moreover, thin superconducting plates can carry relatively low transport currents without dissipation in much stronger fields, as compared to thick ones. Note also that, for layers of thickness $d=6$, the curves of $I_c(h)$ are indistinguishable, i.e., critical current becomes independent of plate thickness. It should be noted that, for thick plates, the upper limit of the applied magnetic field at which the superconductivity takes place is close to $h \approx h_{c2}=4$. A further increase in the applied magnetic field leads to complete suppression of superconductivity even at zero transport current in the vortex-free limit. Our calculations show that, for thick plates, the critical current disappears sharply, rather than gradually, at $h \approx h_{c2}$. This is accompanied by a step of the order parameter ψ_0 at the boundary from a finite value to zero. The functions $I_c(h)$ obtained here can be interpreted as the upper limits for the critical currents carried by superconducting plates with different thicknesses D in magnetic fields of different strength h.

Figure 2(a) illustrates the behavior of the average critical current calculated by the method described above as a function of the magnetic field,

$$\langle I_c \rangle = \frac{1}{N}\sum_{I=1}^{N} I_{ci}, \tag{12}$$

where N is the number of layers in a multilayer (indicated at each curve), for $\kappa=10$ and $d=0.3$, i.e., $D=3\xi$. When the number of layers is relatively small (several dozen for layered structures of the type analyzed here), the average critical current is close to the critical current of a single layer. The magnetic field generated by the layers increases as their number increases, and their increasing role manifests itself by a decrease in critical current. This effect weakens as the magnetic field increases, and the average critical current approaches the value of $I_c(h)$ for a single layer. Conversely, $\langle I_c(h) \rangle$ for the multilayers approaches the linear law $\langle I_c(h) \rangle \sim (h_{c2}-h)$ with an increase in the number of layers up to a few thousand.

Let us define a quantity analogous to the bulk pinning force: $P_v(h)=\langle I_c \rangle h$. Note that the individual layers considered here are in a vortex-free state, i.e., they are inhomogeneous only at their boundaries. This model provides a good approximation of the real state of superconductor-insulator-superconductor junctions because the formation of the strongest pinning centers is due to vortex-boundary interaction. Figure 2(b) shows the bulk pinning force as a function of magnetic field strength calculated by this method. The most important result demonstrated here is the deviation from the scaling law, which was found experimentally in the work.[18] In accordance with this law, the temperature and field dependences of the pinning force follow scaling laws of the form

$$P_v \propto H_{c2}^n (h/h_{c2})^x (1-h/h_{c2})^y .$$

Our calculations demonstrate that as the role played by the layers increases with their number, the slope $\partial P_v(h)/\partial h$ in the weak-field limit decreases, and the maximum of the curve of $P_v(h)$ moves toward higher field strengths, while its shape tends to that described by

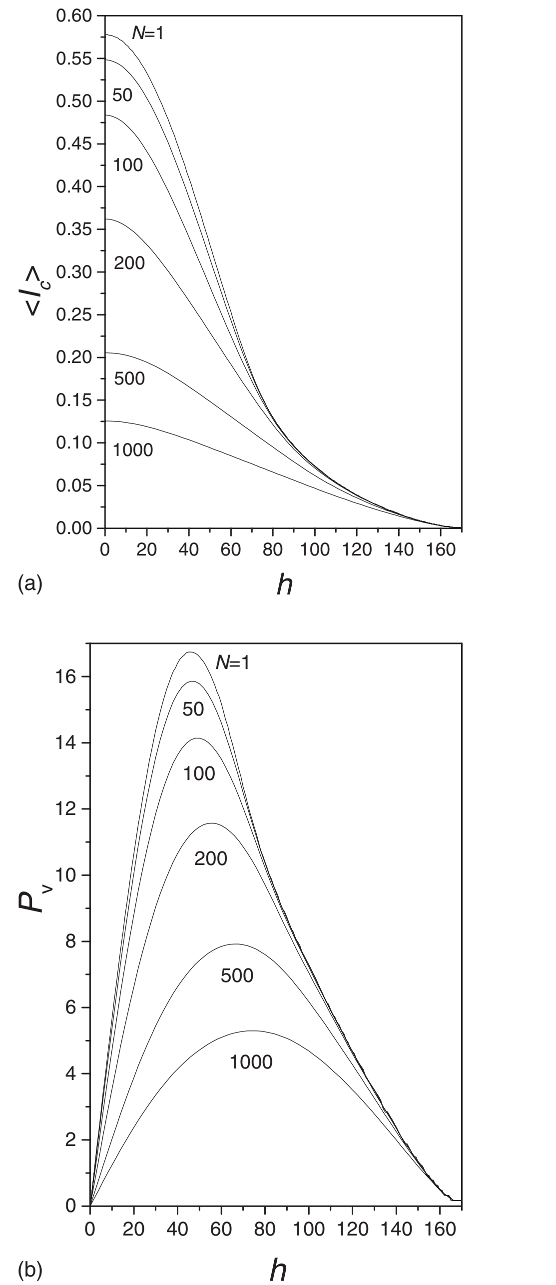

FIG. 2. Dependence of critical current density (a) and bulk pinning force (b) on applied magnetic field for several structures with differing numbers of layers (indicated at the curves) for $D=3\xi$ and $\kappa=10$.

$$p_v(h) \sim h(h_{c2}-h). \tag{13}$$

Figures 3 and 4 show distributions of transport current and individual magnetic field layers calculated by the present method for analyzing the critical states of superconducting multilayers for different N. Figures 3(a) and 3(b) show the distributions for $N=500$, while for comparison Figs. 4(a) and 4(b) show the distributions for $N=50$. Here, $\langle b \rangle$ denotes the magnetic induction averaged over the thickness of a layer. In both cases, the critical current density varies from layer to layer. At zero magnetic field strength, its distribution reaches a maximum in the central layers. In accordance with Eqs.

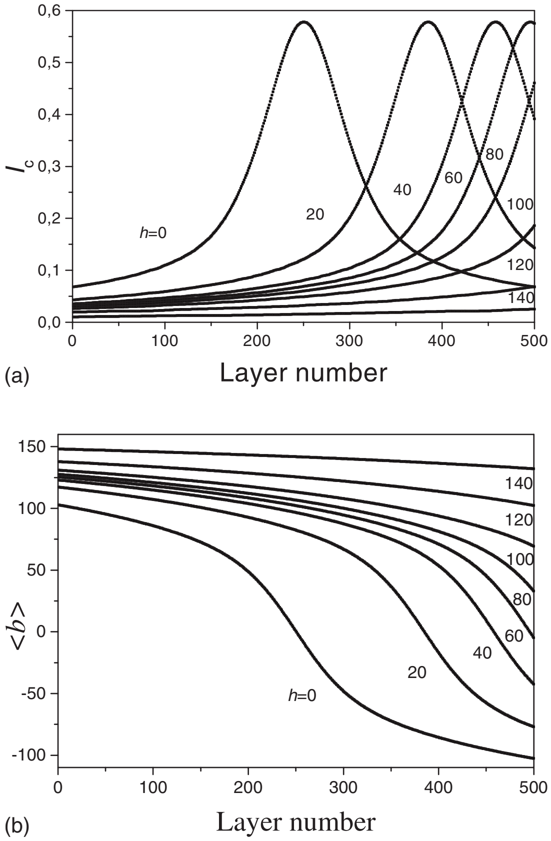

FIG. 3. Dependence of critical current (a) and local magnetic induction (b) on the location of a layer in a structure consisting of 500 for $D=3\xi$ and $\kappa=10$. The numbers at the curves are applied magnetic field strengths.

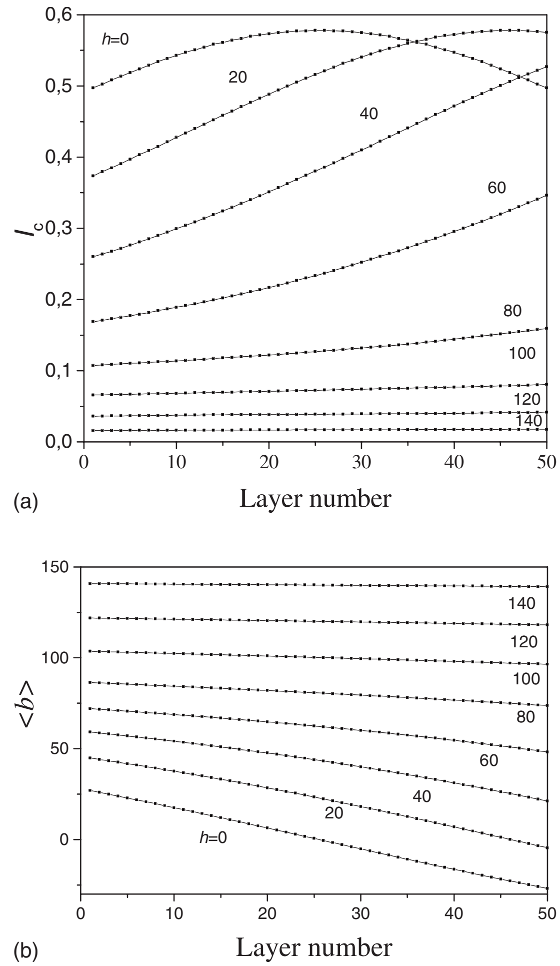

FIG. 4. The same as in Fig. 3, but for 50 layers.

(11a) and (11b), the critical current of the layers I_{cN} decreases as the number of layers increases; conversely, the magnetic field acting on the external layers increases as the number of layers increases. When a magnetic field is applied, the maximum shifts toward one of the multilayer boundaries.

The role played by the layers increases as d increases and as κ decreases. Figure 5(a) shows the $\langle I_c(h)\rangle$ curve for a multilayer with $\kappa=3$ and $d=1$, i.e., $D=3\xi$. Note that the steplike disappearance of the critical current in a single layer at $h\approx h_{c2}$ results in a series of smaller current steps of the critical current of the multilayers at strong magnetic fields. In accordance with Eq. (12), the value of these steps is inversely proportional to N. Evidently, $\langle I_c(h)\rangle$ tends to a linear dependence as N increases: $\langle I_c(h)\rangle \sim (h_{c2}-h)$ at strong magnetic fields. In comparison with Fig. 2(a), a decrease in κ results in an increasing influence of each layer, and when the number of layers becomes several dozen, the magnetic field contribution of the layers becomes comparable to the applied magnetic field, and $\langle I_c(h)\rangle$ is significantly smaller than $I_c(h)$ for one layer when the difference $(h_{c2}-h)$ is comparable to the upper critical magnetic field h_{c2}. As a result, $P_v(h)$ is transformed into relation (13) in this case [Fig. 4(b)]. However, the transport current and magnetic field distributions [Figs. 6(a) and 6(b)] over the layers are similar to $I_c(N)$ and $b(N)$ shown in Figs. 3 and 4.

Note that the transport current distribution predicted for the critical state is similar to its distribution in a zero field, but is also shifted toward a boundary. This similarity has the following simple explanation. It is obvious that the applied magnetic field h is equivalent to the superposition of the magnetic fields generated by l virtual layers, if two conditions are satisfied. First, the total strength of the magnetic fields generated by these layers in each real layer must be equal to the corresponding applied field strength, $\Sigma_{j=1}^{l} h_{tj}=h$, where h_{tj} is the magnetic field generated by the jth virtual layer. Second, the transport-current distribution across the multilayer obtained by combining N real layers with l virtual ones in the critical state must be similar to the symmetric critical current-density distribution across a multilayer consisting of $N+l$ real layers at zero magnetic field strength. Unlike the number of real layers, the number of virtual layers increases as h increases. The shift in the distribution with an increase in the magnetic field looks as if some real layers become virtual, but new layers "enter" from another boundary at the same time. The value of the shift is determined by the selection of the number of virtual layers. However, the central maximum of the critical current in the multilayer obtained by adding l virtual layers will not exactly match the shifted critical-current maximum in a real multilayer if the shift is not a multiple of the half-period of the layered struc-

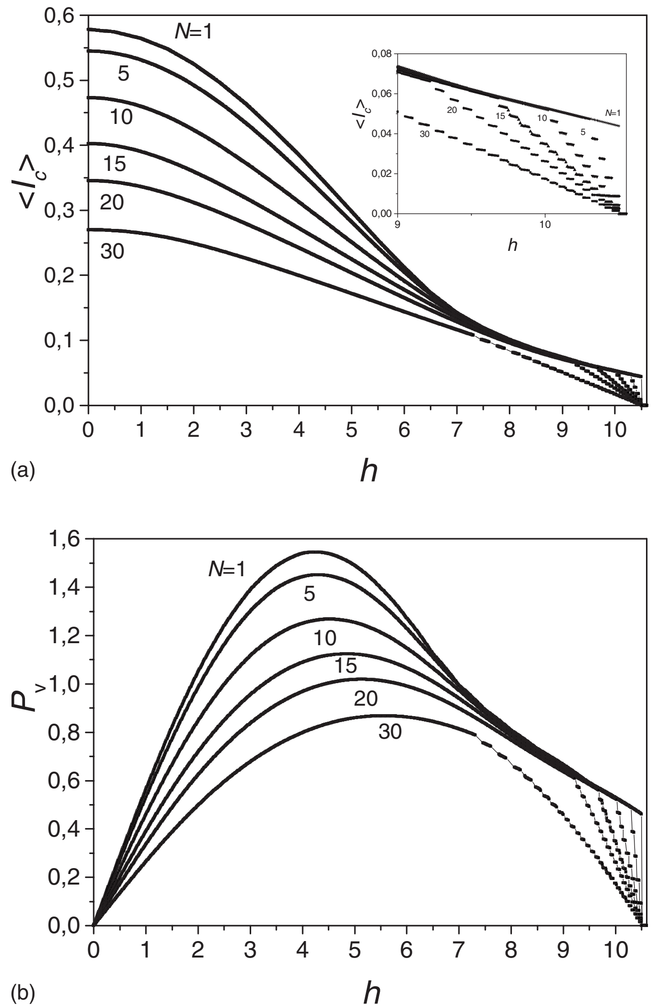

FIG. 5. The same as in Fig. 2, but for a structure with $D=3\xi$ and $\kappa=3$. The inset in Fig. 5(a) gives $\langle I_c\rangle(h)$ in the extended scale.

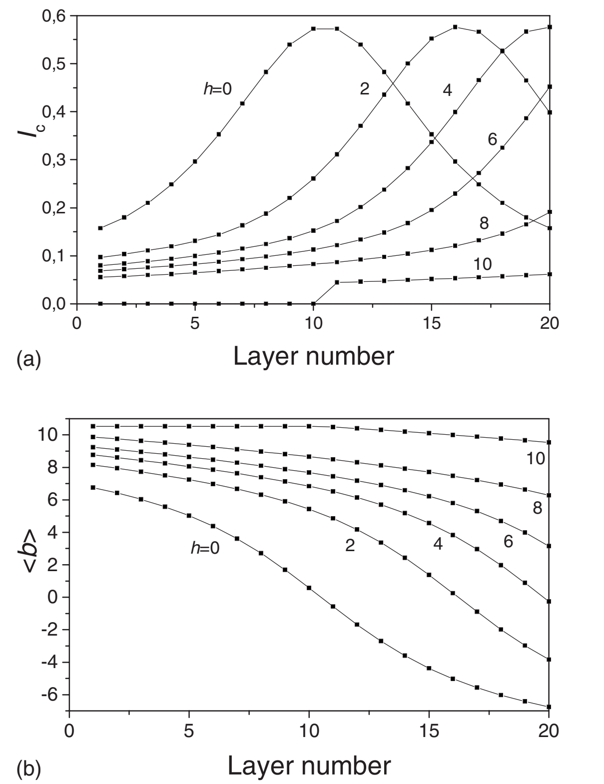

FIG. 6. The same as in Fig. 3, but for a layer structure consisting of 20 layers for $D=3\xi$ and $\kappa=3$.

ture. The error due to this mismatch can be reduced by using the following algorithm: the maximum of the critical-current distribution over individual layers is shifted with a step equal to the multilayer half-period, and then the average critical-current densities and applied magnetic field strengths corresponding to the resulting distributions are calculated. Based on this method for calculating I_{ci}, we conclude that, since the distribution of I_{ci} in a zero magnetic field is unique, the distribution of I_{ci} in a nonzero magnetic field is also unique. Therefore, a known $I_c(h)$ for a single superconducting layer can be used to calculate the average critical current as a function of magnetic field strength and to determine the distribution of transport current over a layered structure in the critical state.

By using this method, we can account for the transformation of the dependence $\langle I_c\rangle(h)$ into a linear dependence and $P_v(h)$ into Eq. (13) as the number of layers increases. Evidently, the critical current of a layer structure is equal to the sum of the critical currents of each layer: $I_c^m=\Sigma_{i=1}^N I_{ci}$. When we change h, the shift of the current distribution over layers $I_c(N)$ results in a change in the critical current (ΔI_c^m): $\Delta I_c^m=\Delta I_{c1}^m-\Delta I_{c2}^m$, where ΔI_{c1}^m is the critical current of the new real layers that enter and ΔI_{c2}^m is the critical current of new virtual layers. Since the applied magnetic field is defined by the critical current of the virtual layers, $\Delta h\sim\Delta I_{c2}^m$. As N increases, ΔI_{c1}^m tends to zero: $\Delta I_{c1}^m\rightarrow 0$. As a result, $\Delta I_c^m\approx -\Delta I_{c2}^m\sim -\Delta h$. This explains the transformation of $\langle I_c\rangle(h)$ into a linear dependence and $P_v(h)$ into Eq. (13). Our results provide an alternative approach for explaining different scaling laws that describe flux pinning in the two most common commercial superconductors, NbTi and Nb_3Sn.[19] For NbTi, the scaling law for the pinning is very close to Eq. (13). Conversely, for Nb_3Sn, the scaling law is close to $h^{0.5}(1-h)^2$. The first law is of greater interest for practical applications because the critical current is larger for a strong magnetic field $h\rightarrow h_{c2}$ than in the second case. Usually, this difference is explained by the difference between the microstructures of both materials.[19] In our case, the second scaling law is close to $P_v(h)$ of the multilayer structure when the magnetic field of the transport current is significantly smaller than the upper critical magnetic field. Conversely, the first law (13) results from the influence of the magnetic field of the transport current. Moreover, the second law can be transformed into the first law simply by increasing the number of layers. Our explanation is realistic, since the upper critical magnetic field of NbTi is significantly lower than H_{c2} of Nb_3Sn.

Note that the average critical current is a monotonically decreasing function of magnetic field strength in the model considered here for multilayers consisting of any number of identical layers. The resulting transport-current and magnetic

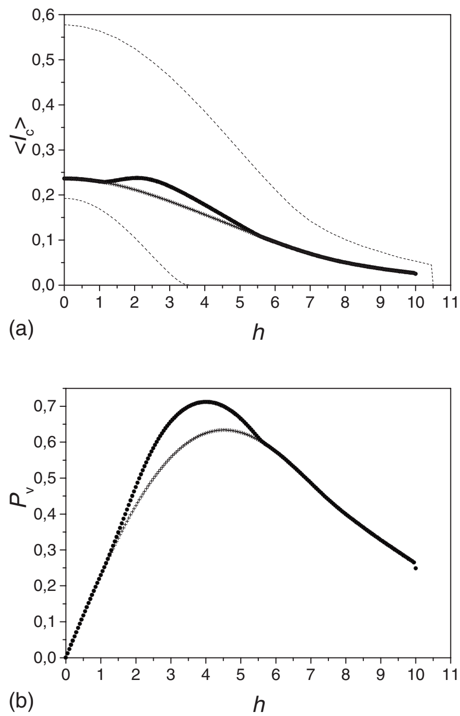

FIG. 7. The same as in Fig. 2, but for a multilayer structure consisting of two types of superconducting layers having different Ginzburg-Landau parameters: for the first 10 layers $\kappa=3$, and for the last 10 layers $\kappa=1$. The thickness of each layer equals λ. Circles show the dependences for the forward direction of the current and crosses show the dependences for the backward direction of the current. For comparison, the upper and lower dashed lines in Fig. 7(a) show the $I_c(h)$ dependences for an individual layer with $\kappa=3$ and 1, respectively.

field distributions over layers are different from those predicted by using the Bean[20] and Anderson-Kim[21] models. In the Bean model, the critical current density is constant across the sample, and the magnetic field varies as a linear function. In the Anderson-Kim model, the magnetic field is characterized by a parabolic distribution. At moderate field strengths ($1 \ll h \ll h_{c2}$), the magnetic field distributions shown in Figs. 3(b), 4(b), and 6(b) agree with those predicted by the Anderson-Kim model, whereas the distributions obtained for strong fields ($h \leqslant h_{c2}$) tend to exhibit linear behavior, as in the Bean model. Since the magnetic fields generated by the low critical currents corresponding to strong applied fields are weak, the transport current is uniformly distributed over layers in the critical state.

Interesting results are obtained for inhomogeneous multilayers consisting of different superconducting layers. Figure 7(a) shows the behavior of the average current calculated by using the iterative process as a function of the magnetic field. In this case, the multilayer consists of two types of superconducting layers having different Ginzburg-Landau parameters: for the first, ten layers $\kappa=3$, and for the second, ten layers $\kappa=1$. The thickness of each layer equals λ. Unlike multilayers consisting of identical layers, the behavior of $\langle I_c \rangle(h)$ depends on the direction of the transport current, i.e., they are different for the forward (I_c^+) and backward (I_c^-) directions of the current. Moreover, one of these magnetic field dependences demonstrates the presence of a peak. This is the peak effect, which is well known in superconductivity and was first observed by Raffy *et al.*[22] The interpretation of this effect was given by Ami and Maki[3] in terms of commensurability between the multilayer period and the vortex lattice spacing. Our calculations show that this effect can be explained by taking into account the heterogeneity of superconductors. It should be noted that the magnetic field dependences of the individual layers, which form the multilayer structure, are monotonically decreasing functions, as usual (see Fig. 7). Figure 7(b) demonstrates the magnetic field dependences $P_v(h)$ for the forward and backward directions of the transport current. These dependences are different, as are $\langle I_c \rangle(h)$ in these cases.

The peak effect is explained by superposition of the applied magnetic field and the field induced by the current flowing along the layers. The transport current induces the magnetic field along the applied field on a boundary of the multilayer structure and in the opposite direction on the other boundary. If the current changes its direction, the field also changes its direction. Thus, on the first boundary, the field of the transport current is directed along the external magnetic field and they are added. On the opposite boundary, the field of the current is directed oppositely to the applied magnetic field and decreases the total magnetic field. For homogeneous structures, the change in the induced magnetic field has no effect on the $\langle I_c \rangle(h)$ dependence because the structure is symmetric—all the layers are identical. For inhomogeneous structures, this results in different $\langle I_c \rangle(h)$ dependences. Figures 8(a) and 8(b) demonstrate transport current distributions over individual layers for these two cases to illustrate the conclusion. At low applied magnetic fields, the magnetic field induced by superconducting layers with $\kappa=3$ exceeds the upper critical magnetic field of layers with $\kappa=1$ (h_{c2}^1) in both directions of the transport current. Moreover, in the second case, when the magnetic field of the transport current is directed along the applied field, the sum of the external field and the field of transport current exceeds h_{c2}^1 in any magnetic field. Conversely, in the first case, there is a range of applied fields ($1 < h < 5.5$) where their sum is smaller than h_{c2}^1. In this range, layers with $\kappa=1$ also carry superconducting current in the critical state. As a result, this forms a maximum on the $\langle I_c \rangle(h)$ dependence in the first case. It is evident that the ratio I_c^+/I_c^- depends on the magnetic field. The asymmetry for critical currents of opposite polarities was first detected by Kadin *et al.*[23] In the opinion of the author of that study, the asymmetry is related to the asymmetric profile of the samples across the layers (niobium deposited on silicon layers, as it was in the experimental work, may be different from silicon on niobium layers). Unlike our work, their approach does not explain the magnetic field dependence of the ratio I_c^+/I_c^-, which was detected in the experiment.

Even a deviation in the physical properties of only one of the layers can lead to noticeable results. Figures 9(a) and 9(b) show the behavior of the average current and of P_v for the forward and backward directions of the transport current as a function of the magnetic field, which were calculated by using an iterative process. In this case, the multilayer also

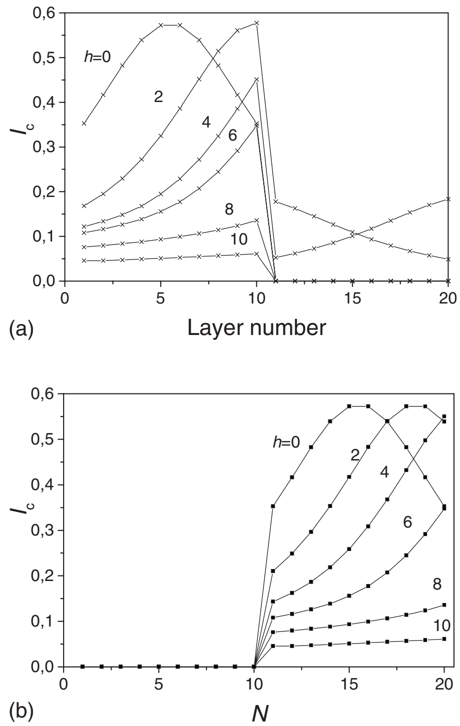

FIG. 8. Dependence of critical current on the location of a layer in a structure consisting of two types of superconducting layers having different Ginzburg-Landau parameters: for the first 10 layers $\kappa=3$, and for the last 10 layers $\kappa=1$ for the forward (a) and for the backward (b) directions of the current. The thickness of each layer equals λ. Numbers at the curves are applied magnetic field strengths.

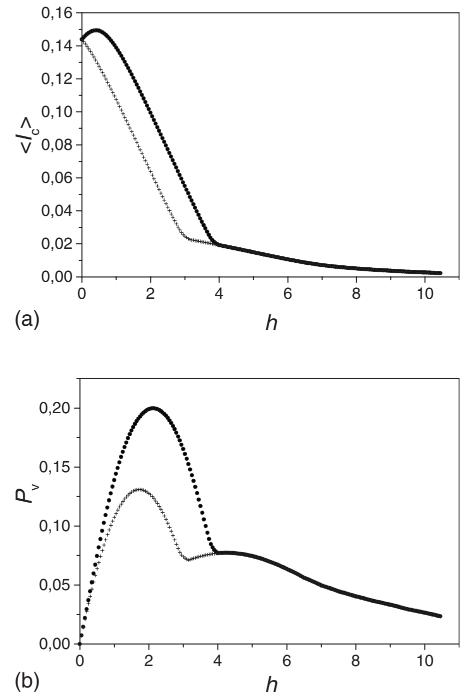

FIG. 9. The same as in Fig. 7, but the Ginzburg-Landau parameters for the first layer $\kappa=3$, and for the last 19 layers $\kappa=1$. The thickness of each layer equals λ.

consists of superconducting layers of two types, and the total number of layers is 20. The thickness of each layer equals λ. However, the Ginzburg-Landau parameter for the first layer $\kappa=3$, and for the last 19 layers $\kappa=1$. We can see that the behavior of $\langle I_c \rangle(h)$ and $P_v(h)$ also remarkably depends on the direction of the transport current. Moreover, the $P_v(h)$ dependences differ drastically from these dependences for homogeneous multilayers. The main difference is the presence of the two maxima on the curves. For a multilayer consisting of identical layers, there is only one peak on these dependences. Similar curves with two maxima were detected in experimental works,[24,25] which investigated the pinning force in multilayers. Usually, the existence of the additional maximum is explained by the matching of the vortex lattice with the layered structure. In our case, this results from the current distribution features in the inhomogeneous multilayers.

IV. CONCLUSIONS

The main results of the present study can be summarized as follows.

The nonlinear Ginzburg-Landau equations are solved numerically to calculate critical current as a function of magnetic field strength and both current and magnetic field distributions over layers for superconducting multilayers in parallel applied magnetic fields. The method enables one to calculate these dependences not only for homogeneous but also for inhomogeneous multilayers. The problem is solved for vortex-free layers whose mutual influence is entirely due to their interaction with a magnetic field. A simple method is proposed for calculating and analyzing the critical states of layered structures in magnetic fields of arbitrary strength, based on elementary transformations of the critical current-density distribution over individual layers in a zero applied magnetic field. Our results give a new approach for explaining different scaling laws that describe flux pinning in the two most common commercial superconductors, NbTi and Nb_3Sn. The method makes it possible to account for the peak effect and the asymmetry for the critical currents of opposite polarities in multilayered superconductors.

It should also be noted that the results presented here will not change significantly if the Josephson coupling between layers separated by a dielectric is taken into account, because the Josephson current density is lower than the depairing current density by several orders of magnitude, and its effect on the order-parameter modulation amplitude and the transport-current distribution is weak. However, if superconducting layers are separated by normal-metal layers, the proximity effect induces superconductivity in the normal conductors and suppresses the order parameter in the superconductors. In the thin normal-metal layer limit, this leads to weak superconducting order-parameter modulation in the layered structure. In this case, the Ginzburg-Landau equations can be solved for a structure placed in a parallel magnetic field whose strength is close to the upper critical field.[3] Note that weak modulation of the order parameter corresponds to weak interaction between Abrikosov vortices and the lattice induced by inhomogeneities, i.e., to low critical current density.

The vortex-free Meissner state of the layers assumed in this study is an important restriction. One can say that in our case the critical current is determined by the vortex formation conditions in the superconducting plates, and the pinning centers are the boundaries of the plates. The dependence $\langle I_c \rangle (h)$ obtained here can be interpreted as an upper limit for the critical current carried by a multilayer in a parallel magnetic field. Moreover, it is well known[26] that a perfect lock-in of the Josephson vortices parallel to the layers is possible in the superconducting multilayers even in a magnetic field directed at a small angle to the layers. Thus, the vortex-free state in the superconducting layers can exist not only in a magnetic field applied perfectly parallel to the layers. Of course, above some critical angle, the perpendicular field component will induce pancake vortices in the layers, i.e., the critical current density in the inclined field will be determined by the pinning of these vortices.

The results of this study are significant for applied superconductivity because artificial pinning structures are often used to increase the critical current.[14] However, the vortices do exist in superconductors used for practical application, and their interaction with pinning centers is one of the main factors that determines the critical current density. Nevertheless, the rapid development of technology gives one the hope that it will soon be possible to create more advanced microstructures, specifically for improving both critical current and the upper critical magnetic field. With this aim, it would be interesting to prepare a superconductor consisting of a set of superconducting microrods separated by an insulator. Thus, the vortex-free limit can be used and the main results of our paper are correct for this structure, which is very interesting for practical applications. However, the exact results can be obtained by solving the two-dimensional Ginzburg-Landau equations for microrods placed in an external magnetic field.

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