Theory of macroscopic quantum tunneling in high-T_c c-axis Josephson junctions

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We study macroscopic quantum tunneling (MQT) in c-axis twist Josephson junctions made of high-T_c superconductors in order to clarify the influence of the anisotropic order parameter symmetry (OPS) on MQT. The dependence of the MQT rate on the twist angle γ about the c axis is calculated by using the functional integral and the bounce method. Due to the d-wave OPS, the γ dependence of standard deviation of the switching current distribution and the crossover temperature from thermal activation to MQT are found to be given by $\cos 2\gamma$ and $\sqrt{\cos 2\gamma}$, respectively. We also show that a dissipative effect resulting from the nodal quasiparticle excitation on MQT is negligibly small, which is consistent with recent MQT experiments using $Bi_2Sr_2CaCu_2O_{8+\delta}$ intrinsic junctions. These results indicate that MQT in c-axis twist junctions becomes a useful experimental tool for testing the OPS of high-T_c materials at low temperature, and suggest high potential of such junctions for qubit applications.

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I. INTRODUCTION

The phenomenon of macroscopic quantum tunneling (MQT) has attracted much attention of experimentalists and theorists for many years.[1–3] Among several works on MQT, Josephson junctions have been intensively studied. In the current biased Josephson junctions, the states of nonzero supercurrent can move to lower-lying minima of the potential through the potential barrier by MQT. Ivanchenko and Zi'lberman showed the possibility of observing MQT in such systems.[4] As was predicted by Caldeira and Leggett, MQT is suppressed by the dissipation effect.[5,6] Later, MQT and the dissipative effects on MQT were experimentally observed in s-wave Josephson junctions.[7–10]

Recently, the MQT theories for s-wave Josephson junctions[5,6,11–14] have been extended to d-wave systems.[15–21] It was claimed that the influence of the quasiparticle excitation on MQT is negligible despite the existence of the line nodes which result from the d-wave order parameter symmetry (OPS).[22,23] Therefore, in c-axis Josephson junctions, e.g., intrinsic junctions[24] or cross-whisker junctions,[25–27] the crossover temperature T^* from thermal activation (TA) to MQT was predicted to be quite high. This can be ascribed not only to the weak dissipative nature[15,17,18] but also the large Josephson plasma frequency ω_p of d-wave junctions, i.e., $T^* \propto \omega_p$.[1,2] On the other hand, in the case of in-plane d-wave junctions which are parallel to the CuO_2 plane, e.g., grain boundary junctions[28,29] or ramp-edge junctions,[30] it was found that the zero energy Andreev bound states (ZESs)[31–34] give the Ohmic dissipative effect on MQT.[16,18,19] Later, the above MQT theories have been extended to explore macroscopic quantum coherence[35–37] and propose a d-wave phase qubit.[38]

The TA-related phenomena in d-wave junctions were already observed experimentally by many groups.[39–48] Until recently, however, no experiments of MQT have been reported. Observations of MQT in d-wave junctions were performed by Bauch *et al.*[49] and Inomata *et al.*,[50] using YBaCuO biepitaxial grain boundary junctions,[29,51–54] and high-quality $Bi_2Sr_2CaCu_2O_{8+\delta}$ (Bi2212) intrinsic junctions, respectively. Subsequently, several groups have observed MQT[55–59] and microwave assisted MQT[55,60–62] in such systems. They reported that T^* of c-axis (Bi2212 intrinsic) junctions is high (0.5–1 K)[50,55–57] compared with the high-quality s-wave junction in which T^* is at most 0.3 K.[63] This result is consistent with theoretical predictions.[15,17,18]

In the previous MQT studies for d-wave junctions, the advantage of a large gap value of d-wave superconductors has been mainly emphasized. On the other hand, in d-wave junctions, there appears a degree of freedom, i.e., a directional dependence of the anisotropic order parameter. This directional dependence produces many intriguing phenomena.[22,23,33,34] Additionally, angular dependence of the Josephson current in d-wave junctions explicitly reflects the d-wave OPS.[26,64,65] Therefore, how MQT depends on the relative angle of the lobe directions between two order parameters is an interesting problem.

In the present paper, we study the dependence of the MQT rate (the inverse lifetime of the metastable state) on the twist angle γ for the c-axis twist Josephson junction (see Fig. 1) by use of the functional integral and the bounce method. We also investigate favorable conditions for the realization of MQT at high crossover temperature T^*. Recently, it was found that intrinsic junction stacks exhibit a gigantic enhancement of the MQT rate.[55,66–70] In the present paper, however, we treat a single junction in order to clarify the effect of the anisotropic OPS which is peculiar to high-T_c superconductors. Note that such single intrinsic junctions made of Bi2212 single crystals have recently been fabricated by use of the Ar-ion etching method.[71,72] We will also present

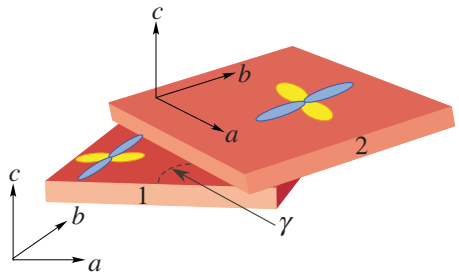

FIG. 1. (Color online) Schematic picture of the c-axis twist Josephson junction. γ is the twist angle around the c axis.

supplementary explanations and results of the previous papers[15,17,18] in which the quasiparticle dissipation effect on MQT is mainly discussed.

The organization of the present paper is as follows. In Sec. II, we derive the effective action for d-wave junctions and formulate the theory for the calculation of T^*. In Sec. III, we present calculated results of T^*. The advantages of c-axis twist Josephson junctions for qubit applications are briefly discussed in Sec. IV. In Sec. V, a summary of the results in the present paper is given.

II. FORMULATIONS

A. Effective action for d-wave junctions

Let us start from a microscopic model of the d-wave superconductor/insulator/d-wave superconductor Josephson junction, described by the grand canonical Hamiltonian,

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_T + \mathcal{H}_Q, \tag{1}$$

where $\mathcal{H}_1$ and $\mathcal{H}_2$ are Hamiltonians for the d-wave superconductors 1 and 2 as follows:

$$\mathcal{H}_{1(2)} = \sum_{\sigma} \int d\boldsymbol{r} \psi^{\dagger}_{1(2)\sigma}(\boldsymbol{r}) \left(-\frac{\hbar^2 \nabla^2}{2m} - \mu \right) \psi_{1(2)\sigma}(\boldsymbol{r}) - \frac{1}{2} \sum_{\sigma,\sigma'} \int d\boldsymbol{r} d\boldsymbol{r}' \psi^{\dagger}_{1(2)\sigma}(\boldsymbol{r}) \psi^{\dagger}_{1(2)\sigma'}(\boldsymbol{r}') \times g_{1(2)}(\boldsymbol{r}-\boldsymbol{r}') \psi_{1(2)\sigma'}(\boldsymbol{r}') \psi_{1(2)\sigma}(\boldsymbol{r}), \tag{2}$$

where m is the electron mass, μ is the chemical potential, and $\psi(\psi^{\dagger})$ is the fermion field operator. In order to obtain the anisotropic order parameter, the anisotropic attractive interaction $g(\boldsymbol{r}-\boldsymbol{r}')$ has to be taken into account. The third term in Hamiltonian (1), i.e.,

$$\mathcal{H}_T = \sum_{\sigma} \int d\boldsymbol{r} d\boldsymbol{r}' [t(\boldsymbol{r},\boldsymbol{r}') \psi^{\dagger}_{1\sigma}(\boldsymbol{r}) \psi_{2\sigma}(\boldsymbol{r}') + \text{h.c.}] \tag{3}$$

describes the tunneling of electrons between the two sides of the junctions, and

$$\mathcal{H}_Q = \frac{(Q_1 - Q_2)^2}{8C} \tag{4}$$

is the charging Hamiltonian where C is the capacitance of the junction and $Q_{1(2)}$ is the operator for the charge on the superconductor 1 (2), which can be written as

$$Q_{1(2)} = e \sum_{\sigma} \int d\boldsymbol{r} \psi^{\dagger}_{1(2)\sigma}(\boldsymbol{r}) \psi_{1(2)\sigma}(\boldsymbol{r}). \tag{5}$$

The procedure to derive the effective action is the same as s-wave junctions.[11,13,14,73] By using the functional integral method,[74,75] the ground partition function $\mathcal{Z}$ for the system can be written as

$$\mathcal{Z} = \int \mathcal{D}\bar{\psi}_1 \mathcal{D}\psi_1 \mathcal{D}\bar{\psi}_2 \mathcal{D}\psi_2 \exp\left[-\frac{1}{\hbar} \int_0^{\hbar\beta} d\tau \mathcal{L}(\tau) \right], \tag{6}$$

where $\beta = 1/k_B T$, $\psi(\bar{\psi})$ is the Grassmann field which corresponds to the fermionic field operator $\psi(\psi^{\dagger})$, and the Lagrangian $\mathcal{L}$ is given by

$$\mathcal{L}(\tau) = \sum_{\sigma} \sum_{i=1,2} \int d\boldsymbol{r} \bar{\psi}_{i\sigma}(\boldsymbol{r},\tau) \partial_{\tau} \psi_{i\sigma}(\boldsymbol{r},\tau) + \mathcal{H}(\tau). \tag{7}$$

We use the Hubbard-Stratonovich transformation[76,77]

$$e^{-1/\hbar \int_0^{\hbar\beta} d\tau \int d\boldsymbol{r} d\boldsymbol{r}' \bar{\psi}_{\uparrow}(\boldsymbol{r}',\tau) \bar{\psi}_{\downarrow}(\boldsymbol{r},\tau) g(\boldsymbol{r}-\boldsymbol{r}') \psi_{\downarrow}(\boldsymbol{r},\tau) \psi_{\uparrow}(\boldsymbol{r}',\tau)} = \int \mathcal{D}\bar{\Phi}(\boldsymbol{r},\boldsymbol{r}',\tau) \mathcal{D}\Phi(\boldsymbol{r},\boldsymbol{r}',\tau) \exp\left\{ \frac{1}{\hbar} \int_0^{\hbar\beta} d\tau \int d\boldsymbol{r} d\boldsymbol{r}' \times \left[-\frac{|\Phi(\boldsymbol{r},\boldsymbol{r}',\tau)|^2}{g(\boldsymbol{r}-\boldsymbol{r}')} + \bar{\Phi}(\boldsymbol{r},\boldsymbol{r}',\tau) \psi_{\downarrow}(\boldsymbol{r},\tau) \psi_{\uparrow}(\boldsymbol{r}',\tau) + \bar{\psi}_{\uparrow}(\boldsymbol{r},\tau) \bar{\psi}_{\downarrow}(\boldsymbol{r}',\tau) \Phi(\boldsymbol{r},\boldsymbol{r}',\tau) \right] \right\}, \tag{8}$$

in order to remove the term ψ^4 in the Hamiltonian $\mathcal{H}(\tau)$. This introduces a complex order parameter field $\Phi(\boldsymbol{r},\boldsymbol{r}';\tau)$. The resulting action is only quadratic in the Grassmann field, so that the functional integral over this number can readily be performed explicitly.

Next, we perform the variable transformation (or gauge transformation)[11,13,14]

$$\Phi(\boldsymbol{r}_a,\boldsymbol{r}_b,\tau) = \Delta(\mathbf{R},\mathbf{r},\tau) \exp[i\phi(\mathbf{R},\mathbf{r},\tau)], \tag{9}$$

$$\bar{\Phi}(\boldsymbol{r}_a,\boldsymbol{r}_b,\tau) = \Delta(\mathbf{R},\mathbf{r},\tau) \exp[-i\phi(\mathbf{R},\mathbf{r},\tau)]. \tag{10}$$

Here, Δ is a real field, and $\boldsymbol{R} = (\boldsymbol{r}_a + \boldsymbol{r}_b)/2$ and $\mathbf{r} = \boldsymbol{r}_a - \boldsymbol{r}_b$ are center of mass and relative coordinates, respectively. We assume a slow variation of the order parameter in space and time, and hence $\Delta(\mathbf{R},\mathbf{r},\tau) \cong \Delta(\mathbf{r})$ and $\phi(\mathbf{R},\mathbf{r},\tau) \cong \phi(\mathbf{R},\tau)$ are satisfied. We also introduce auxiliary voltage field V which couples to the charge operator. Then, the partition function $\mathcal{Z}$ becomes

$$\mathcal{Z} = \int \mathcal{D}\Delta_1 \mathcal{D}\Delta_2 \mathcal{D}\phi_1 \mathcal{D}\phi_2 \mathcal{D}V \times \exp\left\{ -\int_0^{\hbar\beta} \frac{d\tau}{\hbar} \int d\boldsymbol{r}_a d\boldsymbol{r}_b \times \left[\frac{|\Phi_1|^2}{g_1(\boldsymbol{r}_a - \boldsymbol{r}_b)} + (1 \leftrightarrow 2) + \frac{CV^2}{2} \right] + \mathrm{Tr} \ln G^{-1} \right\}. \tag{11}$$

Here, G is a 4×4 matrix Green's function

$$G^{-1}(\boldsymbol{r}_a, \tau_a, \boldsymbol{r}_b, \tau_b) = \begin{pmatrix} \hat{G}_1^{-1}(\boldsymbol{r}_a, \tau_a, \boldsymbol{r}_b, \tau_b) & -\hat{t}(\boldsymbol{r}_a, \boldsymbol{r}_b)\delta(\tau_a - \tau_b) \\ -\hat{t}^\dagger(\boldsymbol{r}_a, \boldsymbol{r}_b)\delta(\tau_a - \tau_b) & \hat{G}_2^{-1}(\boldsymbol{r}_a, \tau_a, \boldsymbol{r}_b, \tau_b) \end{pmatrix}, \tag{12}$$

where

$$\hat{t}(\boldsymbol{r}_a, \boldsymbol{r}_b) = \begin{pmatrix} t(\boldsymbol{r}_a, \boldsymbol{r}_b) e^{i(\phi_1(\mathbf{R},\tau) - \phi_2(\mathbf{R},\tau))/2} & 0 \\ 0 & -t^*(\boldsymbol{r}_a, \boldsymbol{r}_b) e^{-i(\phi_1(\mathbf{R},\tau) - \phi_2(\mathbf{R},\tau))/2} \end{pmatrix}, \tag{13}$$

$$\hat{G}_{1(2)}^{-1} = \left\{ -\hbar \frac{\partial}{\partial \tau} \hat{\tau}_0 + i\hbar (v_{1(2)} \cdot \nabla) \hat{\tau}_0 + \left[\frac{\hbar^2 \nabla^2}{2m} + \mu - \frac{m}{2} v_{1(2)}^2 + i\left(\frac{\hbar}{2} \frac{\partial \phi_1}{\partial \tau} - eV_{1(2)} \right) \right] \hat{\tau}_3 - \hat{\Delta}_{1(2)} \right\} \delta(\boldsymbol{r}_a - \boldsymbol{r}_b) \delta(\tau_a - \tau_b). \tag{14}$$

In the last equation, we introduced the Pauli matrix $\hat{\tau}_i$, the identity matrix $\hat{\tau}_0$, the pair potential

$$\hat{\Delta}_{1(2)} = \begin{pmatrix} 0 & \Delta_{1(2)}(\mathbf{r}) e^{-i\phi_{1(2)}(\mathbf{R},\tau)} \\ \Delta_{1(2)}(\mathbf{r}) e^{i\phi_{1(2)}(\mathbf{R},\tau)} & 0 \end{pmatrix},$$

the superfluid velocity $v_{1(2)} = -(\hbar/2m)\nabla\phi_{1(2)}$, and $V_{1(2)} = +(-)V/2$. In the following, we assume that the phase varies slowly in space, namely, $v_{1(2)} \approx 0$. The functional integrals over the modulus of the order parameter field Δ and the voltage field V are performed by the saddle-point method, which lead to the gap equation and the Josephson equation, respectively.[11,13,14] We also assume that the tunneling matrix element t is finite only in the vicinity of the insulating barrier. Then, the partition function $\mathcal{Z}$ is reduced to a single functional integral over the phase difference $\phi = \phi_1 - \phi_2$,

$$\mathcal{Z} = \int \mathcal{D}\phi \exp\left[-\frac{1}{\hbar} \int_0^{\hbar\beta} \frac{C}{2} \left(\frac{\hbar}{2e} \frac{\partial \phi}{\partial \tau} \right)^2 d\tau + \mathrm{Tr} \ln G^{-1} \right]. \tag{15}$$

Next, we expand $\ln G^{-1}$ up to the second order in the tunneling matrix element t, i.e.,

$$\mathrm{Tr} \ln G^{-1} \approx \mathrm{Tr} \ln \underline{\hat{g}}^{-1} - \frac{1}{2} \mathrm{Tr}(\underline{\hat{g}}\underline{\hat{t}}\underline{\hat{g}}\underline{\hat{t}}), \tag{16}$$

with

$$\underline{\hat{g}}^{-1} = \begin{pmatrix} \hat{G}_1^{-1}(\boldsymbol{r}_a, \tau_a, \boldsymbol{r}_b, \tau_b) & 0 \\ 0 & \hat{G}_2^{-1}(\boldsymbol{r}_a, \tau_a, \boldsymbol{r}_b, \tau_b) \end{pmatrix}, \tag{17}$$

$$\underline{\hat{t}} = \begin{pmatrix} 0 & \hat{t}(\boldsymbol{r}_a, \boldsymbol{r}_b)\delta(\tau_a - \tau_b) \\ \hat{t}^\dagger(\boldsymbol{r}_a, \boldsymbol{r}_b)\delta(\tau_a - \tau_b) & 0 \end{pmatrix}. \tag{18}$$

After some calculations, we obtain

$$\mathcal{Z} = \int \mathcal{D}\phi(\tau) \exp\left(-\frac{\mathcal{S}_{\mathrm{eff}}[\phi]}{\hbar} \right), \tag{19}$$

where the effective action $\mathcal{S}_{\mathrm{eff}}$ is given by[15,82,83]

$$\mathcal{S}_{\mathrm{eff}}[\phi] = \int_0^{\hbar\beta} d\tau \frac{C}{2} \left[\frac{\hbar}{2e} \frac{\partial \phi(\tau)}{\partial \tau} \right]^2 - \int_0^{\hbar\beta} \int_0^{\hbar\beta} d\tau d\tau' \times \left[\alpha(\tau - \tau') \cos \frac{\phi(\tau) - \phi(\tau')}{2} - \beta(\tau - \tau') \cos \frac{\phi(\tau) + \phi(\tau')}{2} \right]. \tag{20}$$

The second term in Eq. (20) describes the dissipation due to the quasiparticle tunneling. The third term describes the tunneling of Cooper pairs (the Josephson tunneling). The only difference of the effective action between d-wave and s-wave junctions is the dependence of Δ on the relative coordinate $\boldsymbol{r}$, and is included in the two kernels. The dissipation kernel $\alpha(\tau)$ and the Josephson kernel $\beta(\tau)$ are given in terms of the diagonal and off-diagonal components of the Matsubara Green functions in Nambu space, denoted by $\mathcal{G}$ and $\mathcal{F}$,

$$\alpha(\tau) = -\frac{2}{\hbar} \sum_{\boldsymbol{k},\boldsymbol{k}'} |t(\boldsymbol{k},\boldsymbol{k}')|^2 \mathcal{G}_1(\boldsymbol{k},\tau) \mathcal{G}_2(\boldsymbol{k}',-\tau), \tag{21}$$

$$\beta(\tau) = -\frac{2}{\hbar} \sum_{\boldsymbol{k},\boldsymbol{k}'} |t(\boldsymbol{k},\boldsymbol{k}')|^2 \mathcal{F}_1(\boldsymbol{k},\tau) \mathcal{F}_2^\dagger(\boldsymbol{k}',-\tau). \tag{22}$$

The Green functions are given by

$$\mathcal{G}(\boldsymbol{k},\omega_n) = -\frac{\hbar(i\hbar\omega_n + \xi_{\boldsymbol{k}})}{(\hbar\omega_n)^2 + \xi_{\boldsymbol{k}}^2 + \Delta(\boldsymbol{k})^2}, \tag{23}$$

$$\mathcal{F}(\boldsymbol{k},\omega_n) = \frac{\hbar\Delta(\boldsymbol{k})}{(\hbar\omega_n)^2 + \xi_{\boldsymbol{k}}^2 + \Delta(\boldsymbol{k})^2}, \tag{24}$$

where $\xi_{\boldsymbol{k}}=\hbar^2\boldsymbol{k}^2/2m-\mu$ and $\hbar\omega_n=(2n+1)\pi/\beta$ is the fermionic Matsubara frequency (n is an integer). Information about the anisotropy of the order parameter is included in $\Delta(\boldsymbol{k})$. In the case of the cuprate high-T_c superconductors (the $d_{x^2-y^2}$ OPS), $\Delta(\boldsymbol{k})$ is given by $\Delta(\boldsymbol{k})=\Delta_0\cos 2\theta$. Here, θ is an angle between $\boldsymbol{k}$ and a axis.

B. Effective action for current-biased *c*-axis twist junctions

Let us turn to the calculation of the effective action for the c-axis twist Josephson junctions (see Fig. 1). We define γ as the twist angle about the c axis ($0 \leqslant \gamma < \pi/4$).[25] Such junctions can be fabricated by using the single crystal whisker of Bi2212.[26,27,78,79] Takano *et al.* measured the γ dependence of the c-axis Josephson critical current and showed clear evidence of the $d_{x^2-y^2}$ OPS of Bi2212.[26,78,80]

In what follows, we assume that the tunneling between the two superconductors is described in terms of the coherent tunneling, i.e.,

$$|t(\boldsymbol{k},\boldsymbol{k}')|^2 = |t|^2\delta_{\boldsymbol{k}_\parallel,\boldsymbol{k}'_\parallel}, \tag{25}$$

where $\boldsymbol{k}_\parallel$ is the momentum parallel to the ab plane. For simplicity, we also assume that each superconductor consists of a single CuO_2 layer, $\Delta_1(\boldsymbol{k})=\Delta_0\cos 2\theta$, and $\Delta_2(\boldsymbol{k})=\Delta_0\cos 2(\theta+\gamma)$ (see Fig. 1). Moreover, we consider the low temperature limit ($k_BT \ll \Delta_0$). In the case where the Josephson junction is biased by an externally applied current $I_{\rm ext}$, we have to add an additional potential contribution linear in ϕ.[11,13] At this level of approximation, the effective action $\mathcal{S}_{\rm eff}$ of the current biased c-axis twist Josephson junction has the form

$$\mathcal{S}_{\rm eff}[\phi] = \int_0^{\hbar\beta} d\tau\left[\frac{M}{2}\left(\frac{\partial\phi(\tau)}{\partial\tau}\right)^2 + U(\phi)\right] + \mathcal{S}_{\rm diss}[\phi], \tag{26}$$

$$\mathcal{S}_{\rm diss}[\phi] = -\int_0^{\hbar\beta}\int_0^{\hbar\beta} d\tau d\tau' \alpha(\tau-\tau')\cos\frac{\phi(\tau)-\phi(\tau')}{2}, \tag{27}$$

where $M=C(\hbar/2e)^2$ is the mass and $U(\phi)$ is the tilted washboard potential

$$U(\phi) = -E_J[\cos\phi(\tau) + \eta\phi(\tau)]. \tag{28}$$

In this equation, η is given by $\eta=I_{\rm ext}/I_C$, and $E_J=(\hbar/2e)I_C$ is the Josephson coupling energy, where

$$I_C = -\frac{2e}{\hbar}\int_0^{\hbar\beta} d\tau\beta(\tau) \tag{29}$$

is the Josephson critical current. In the derivation of the Josephson term, we adopted the local approximation.[1,81] Details of this approximation and its justification are given in Appendix A.

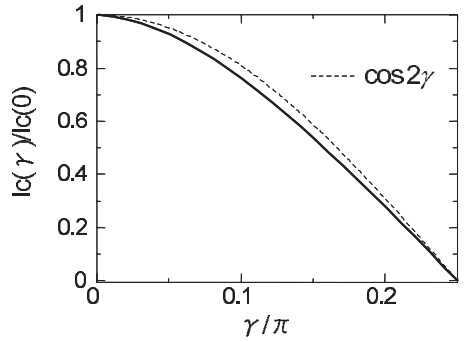

FIG. 2. The twist angle γ dependence of the critical current I_C. Broken line is proportional to $\cos 2\gamma$.

By substituting Eq. (22) into Eq. (29), we can obtain the γ dependence of I_C, i.e.,

$$I_C(\gamma) = \frac{2e}{\hbar}|t|^2N_0^2\Delta_0\int_0^{2\pi}\frac{d\theta}{2\pi}\cos 2\theta\cos 2(\theta+\gamma) \times\int_0^{\infty} dy\frac{1}{\sqrt{y^2+\cos^2 2\theta}}\frac{1}{\sqrt{y^2+\cos^2 2(\theta+\gamma)}}, \tag{30}$$

where N_0 is the density of states at the Fermi energy. Especially at $\gamma=0$, we get a well-known result[33]

$$I_C(0) = \frac{2e}{\hbar}|t|^2N_0^2\Delta_0. \tag{31}$$

Figure 2 shows the twist angle γ dependence of the critical current I_C. It is approximately proportional to $\cos 2\gamma$. Note that for $\gamma > \pi/4$, the sign of the current changes. Thus, in this case, the π junction is formed. These behaviors are attributed to the d-wave OPS of high-T_c superconductors.

Next, we will calculate the dissipation kernel $\alpha(\tau)$ for the c-axis twist junctions. In the case of the c-axis junction with $\gamma=0$ (e.g., single intrinsic junctions),[71,72] the node-to-node quasiparticle tunneling is always possible. Then, the explicit and asymptotic forms of $\alpha(\tau)$ at low temperature are obtained as

$$\alpha(\tau) = \frac{|t|^2N_0^2\Delta_0^2}{2\pi^2\hbar}\int_0^{2\pi} d\theta\cos^2 2\theta K_1\left(\frac{|\tau|}{\hbar}\Delta_0|\cos 2\theta|\right)^2 \tag{32}$$

$$\approx \frac{3\hbar^2}{16\pi\Delta_0}\frac{R_Q}{R}\frac{1}{|\tau|^3} \quad \text{for } \frac{\Delta_0|\tau|}{\hbar} \gg 1, \tag{33}$$

where K_1 is the modified Bessel function, $R_Q=h/4e^2$ is the resistance quantum, and

$$\frac{1}{R} = \frac{2e^2}{\hbar}|t|^2N_0^2 \tag{34}$$

is the inverse of the normal state resistance R. Equation (33) gives the super-Ohmic dissipation which agrees with the pre-

vious results[15,82,83] based on the coherent tunneling approximation [Eq. (25)]. The present model shows a stronger dissipation effect than the incoherent tunneling model [$\alpha(\tau) \sim 1/\tau^4$] (Ref. 81) and a more realistic tunneling model based on the first-principle band calculation[84] [$\alpha(\tau) \sim 1/|\tau|^5$].[85] Despite this fact, the quasiparticle dissipation effect on MQT in the present coherent tunneling model is still quite weak as will be shown in Sec. III. See Appendix B for the derivation of these asymptotic forms of the kernel $\alpha(\tau)$.

In the case of nonzero γ, we get

$$\alpha(\tau) = \frac{|t|^2 N_0^2 \Delta_0^2}{2\pi^2 \hbar} \int_0^{2\pi} d\theta |\cos 2\theta \cos 2(\theta+\gamma)| \times K_1\left(\frac{|\tau|}{\hbar}\Delta_0 |\cos 2\theta|\right) K_1\left(\frac{|\tau|}{\hbar}\Delta_0 |\cos 2(\theta+\gamma)|\right). \tag{35}$$

The asymptotic behavior of $\alpha(\tau)$ for nonzero γ is numerically estimated as an exponential function, i.e.,

$$\alpha(\tau) \sim \exp\left[-\sin(2\gamma)\frac{\Delta_0|\tau|}{\hbar}\right], \tag{36}$$

for $\Delta_0|\tau|/\hbar \gg 1$. This exponential behavior is ascribed to the suppression of the node-to-node quasiparticle tunneling by the finite value of γ,[15,82] and is similar to that of the conventional s-wave junctions with the constant order parameter Δ: $\alpha(\tau) \sim \exp(-2\Delta|\tau|/\hbar)$.[11,13] For nonzero γ, if the phase $\phi(\tau)$ varies slowly with time on the scale given by $\hbar/\Delta_0$, we can expand $\phi(\tau)-\phi(\tau')$ in Eq. (27) with respect to $\tau-\tau'$, which results in

$$\mathcal{S}_{\rm diss}[\phi] \approx \frac{\delta M(\gamma)}{2} \int_0^{\hbar\beta} d\tau \left[\frac{\partial\phi(\tau)}{\partial\tau}\right]^2, \tag{37}$$

where

$$\delta M(\gamma) = \frac{1}{2}\int_0^\infty d\tau \tau^2 \alpha(\tau) = \frac{\hbar^2|t|^2 N_0^2}{\Delta_0} \int_0^{2\pi} \frac{d\theta}{2\pi} \frac{[\cos^2 2\theta + \cos^2 2(\theta+\gamma)]E(k) - 2|\cos 2\theta \cos 2(\theta+\gamma)|K(k)}{[|\cos 2\theta| + |\cos 2(\theta+\gamma)|][|\cos 2\theta| - |\cos 2(\theta+\gamma)|]^2} \tag{38}$$

for $\gamma \neq 0$ with $k=[|\cos 2\theta| - |\cos 2(\theta+\gamma)|]/[|\cos 2\theta| + |\cos 2(\theta+\gamma)|]$. Here, $E(k)$ and $K(k)$ are complete elliptic integrals of the second and first kinds, respectively. Hence, under the above condition, the dissipation action $\mathcal{S}_{\rm diss}$ acts as a kinetic term so that the effect of the quasiparticle dissipation results in an increase of the mass, i.e., $M \to M + \delta M$ (the mass renormalization). This result indicates that the influence of the quasiparticle dissipation for nonzero γ is quite weaker than that for $\gamma=0$, and hence it also gives negligible contribution to MQT as will be shown in the next section. It should be remarked that the increase of the mass δM is inversely proportional to Δ_0.

C. Macroscopic quantum tunneling rate

We will calculate the γ dependence of the MQT rate at low temperatures. Note that, in the previous studies of MQT in d-wave c-axis junctions,[15,17,18] only the limiting cases of γ, i.e., $\gamma=0$ and $\pi/8$, have been considered. The MQT rate is defined by the formula[1,2,86]

$$\Gamma = \lim_{\beta\to\infty} \frac{2}{\beta} \mathrm{Im} \ln \mathcal{Z}. \tag{39}$$

In order to determine Γ, we employ the bounce approximation.[1,2,86] When the barrier is low enough for the MQT to occur but still so high that the bounce approximation can be valid, Γ is given by

$$\Gamma \approx A \exp\left(-\frac{S_B}{\hbar}\right), \tag{40}$$

where $S_B = \mathcal{S}_{\rm eff}[\phi_B]$ is the bounce exponent, which is the value of the action $\mathcal{S}_{\rm eff}$ evaluated along the bounce trajectory $\phi_B(\tau)$. Assuming that $I_{\rm ext}$ is close to I_C, we can approximate the washboard potential $U(\phi)$ as a quadratic-plus-cubic one. Then, we obtain, for $\gamma=0$,[15,17,18]

$$\Gamma(0,\eta) \approx \Gamma_0(0,\eta) \exp\left[-B(0,\eta) - c_0 \frac{\hbar I_C(0)}{\Delta_0^2} \times \sqrt{\frac{\hbar}{2e}\frac{I_C(0)}{C}}(1-\eta^2)^{5/4}\right], \tag{41}$$

and for $\gamma \neq 0$

$$\Gamma(\gamma,\eta) \approx \Gamma_0(\gamma,\eta) \exp[-B(\gamma,\eta)]. \tag{42}$$

In Eqs. (41) and (42), $\eta = I_{\rm ext}/I_C(\gamma)$, $c_0 = (27\pi/8)\int_0^\infty dx [x^4/\sinh^2(\pi x)] \ln(1+x^{-2}) \approx 0.14$,

$$B(\gamma,\eta) = \frac{6}{5e}\sqrt{\frac{\hbar}{2e} I_C(\gamma) C} \frac{\delta M(\gamma)}{M} (1-\eta^2)^{5/4}, \tag{43}$$

and $\Gamma_0(\gamma,\eta)$ is the decay rate without the quasiparticle dissipation, i.e.,

$$\Gamma_0(\gamma,\eta) = 12\omega_p(\gamma,\eta)\sqrt{\frac{3U_0(\gamma,\eta)}{2\pi\hbar\omega_p(\gamma,\eta)}}\exp\left[-\frac{36U_0(\gamma,\eta)}{5\hbar\omega_p(\gamma,\eta)}\right], \tag{44}$$

where barrier height U_0 and the Josephson plasma frequency ω_p are given by

$$U_0(\gamma,\eta) = \frac{\hbar I_C(\gamma)}{3e}(1-\eta^2)^{3/2}, \tag{45}$$

$$\omega_p(\gamma,\eta) = \sqrt{\frac{\hbar I_C(\gamma)}{2eM}}(1-\eta^2)^{1/4}. \tag{46}$$

For $\gamma=0$, $\delta M(0)$ is the mass increment due to the high-frequency components ($\omega > \omega_p$) of the dissipation kernel [Eq. (32)] and is given by

$$\delta M(0) = \frac{\hbar^2 N_0^2 |t|^2}{\pi^2 \Delta_0}\int_0^1 dx \frac{x^2}{\sqrt{1-x^2}}\int_0^{\Delta_0/\hbar\omega_p(0,\eta)} dss^2 K_1(sx)^2. \tag{47}$$

The second term in the exponent of Eq. (41) results from the low-frequency components ($\omega < \omega_p$) of the dissipation kernel $\alpha(\tau)$ [Eq. (32)].

D. Switching current distribution and crossover temperature

The switching current distribution $P(\eta)$ is related to the MQT rate $\Gamma(\gamma,\eta)$ as[7,87]

$$P(\eta) = \frac{1}{v}\Gamma(\gamma,\eta)\exp\left[-\frac{1}{v}\int_0^\eta \Gamma(\gamma,\eta')d\eta'\right], \tag{48}$$

where $v \equiv |d\eta/dt|$ is the sweep rate of the external bias current. The mean value $\langle\eta\rangle$ and the square mean value $\langle\eta^2\rangle$ of the switching current are, respectively, expressed by

$$\langle\eta\rangle \equiv \int_0^1 d\eta' P(\eta')\eta', \tag{49}$$

$$\langle\eta^2\rangle \equiv \int_0^1 d\eta' P(\eta')\eta'^2. \tag{50}$$

Then, the standard deviation σ_n of the switching current distribution $P(\eta)$ is defined by

$$\sigma_n = I_C\sqrt{\langle\eta^2\rangle - \langle\eta\rangle^2}. \tag{51}$$

In actual MQT experiments, the temperature T dependence of σ_n is measured. At high temperature, the TA decay dominates the escape process. Then, the escape rate Γ is given by the Kramers formula[1,2,88]

$$\Gamma(\gamma,\eta) = \frac{\omega_p(\gamma,\eta)}{2\pi}\exp\left[-\frac{U_0(\gamma,\eta)}{k_B T}\right]. \tag{52}$$

Below T^*, the escape process is dominated by MQT. The crossover temperature T^* is determined from an intersection point of σ_n between T-independent MQT and T-dependent TA process. In the low-dissipation (underdamping) cases, T^* is approximately given by[89–91]

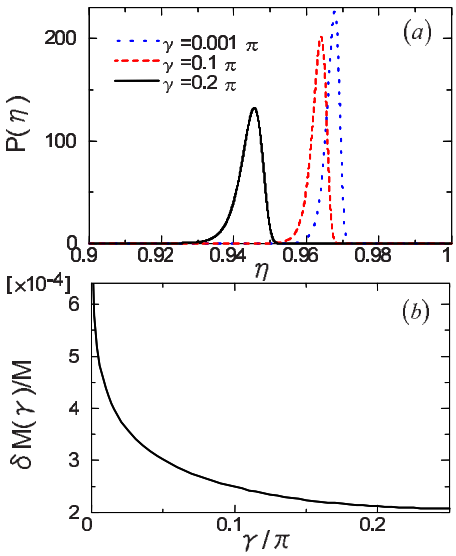

FIG. 3. (Color online) The twist angle γ dependences of (a) the switching current distribution $P(\eta)$ and (b) the mass increment δM due to the quasiparticle dissipation.

$$T^*(\gamma) = \frac{\hbar\omega_p(\gamma,\eta=\langle\eta\rangle)}{2\pi k_B}. \tag{53}$$

III. RESULTS

In this section, we will numerically calculate the MQT rate Γ and the crossover temperature T^* for the c-axis twist junctions. In the following we choose $v=42.4$ mA/s per $I_C(0)$, $I_C(0)=48.54$ μA, $C=76.26$ fF, and $\Delta_0=30$ meV unless explicitly mentioned. These parameters were used in a MQT experiment of Bi2212 intrinsic junctions.[50]

Figures 3(a) and 3(b) exhibit the γ dependences of the switching current distribution $P(\eta)$ and the mass increment δM due to the quasiparticle dissipation, respectively. As γ increases, $P(\eta)$ becomes smeared, and the peak position shifts to the smaller values of η. This is because I_C decreases with increasing γ, which makes the switching process more likely to occur even for small η. As shown in Fig. 3(b), the ratio $\delta M/M$ is of the order of $10^{-4} \ll 1$ for all γ. Therefore, the mass renormalization effect in twist junctions is quite weak, especially for large γ. This γ dependence can be explained as follows. As γ increases, the node-to-node quasiparticle tunneling becomes more suppressed. Therefore, the effect of the quasiparticle dissipation, namely, the mass increment δM, is decreasing with increasing γ.

In Fig. 4, the standard deviation σ_n of the switching current distribution $P(\eta)$ is depicted for various γ. The crossover temperature T^* decreases with increasing γ. In the TA regime, the switching current is proportional to $T^{2/3}$.[87,88] In Fig. 5, we show γ dependences of (a) σ_n and (b) T^*. They decrease monotonically with the increase of γ. Their γ dependences are approximately given by $\cos 2\gamma$ and $\sqrt{\cos 2\gamma}$, respectively. The γ dependence of T^* can be explained as follows. Due to the d-wave OPS, $I_C(\gamma)$ is nearly proportional

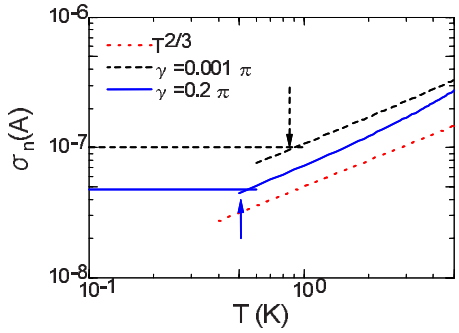

FIG. 4. (Color online) The standard deviation σ_n of the switching current distribution $P(\eta)$ for various γ. The dotted line is proportional to $T^{2/3}$. The arrows point to the crossover temperature T^* calculated from the approximate formula (53).

to $\cos 2\gamma$ as shown in Fig. 2. Thus, we obtain

$$T^* \propto \omega_p \propto \sqrt{I_C(\gamma)} \propto \sqrt{\cos 2\gamma} \tag{54}$$

if we neglect the γ dependence of $\langle \eta \rangle$. Hence, T^* is almost proportional to $\sqrt{\cos 2\gamma}$.

For applications to, e.g., d-wave phase qubit,[38] higher T^* is desirable for high temperature qubit operation. Thus, we will study the dependences of T^* on other parameters (v, I_C, and C) by changing these parameters, and clarify the condition for realizing high T^* in the case of $\gamma=0$.

Figure 6(a) shows the sweep rate v dependence of T^*. Note that v has to be much smaller than the plasma frequency ω_p in order to apply our theory. The crossover temperature T^* is a decreasing function of v. However, it quite weakly depends on v. More sensitive parameters are the critical current $I_C(0)$ and capacitance C. Figure 6(b) displays the $I_C(0)$ dependence of T^*, where $I_C(0) \propto \Delta_0$ [see Eq. (31)].

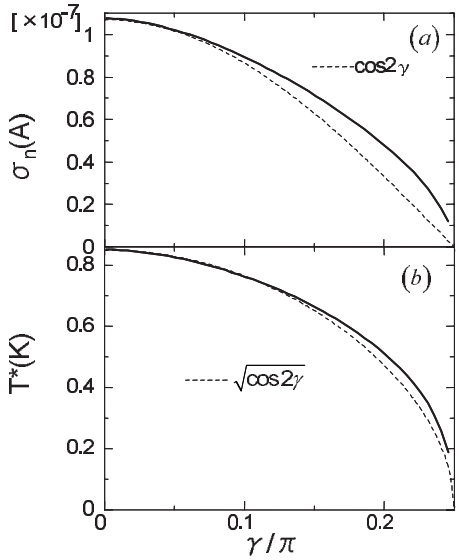

FIG. 5. The twist angle γ dependences of (a) the standard deviation σ_n of the switching current distribution $P(\eta)$ and (b) the crossover temperature T^*. The broken lines are proportional to $\cos 2\gamma$ and $\sqrt{\cos 2\gamma}$ in (a) and (b), respectively.

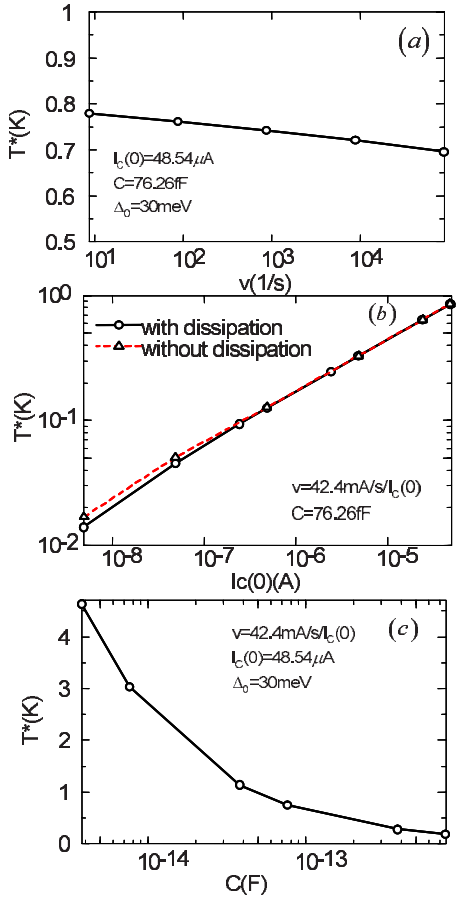

FIG. 6. (Color online) The dependences of the crossover temperature T^* on (a) the sweep rate v, (b) the critical current $I_C(0)$ which is proportional to Δ_0 [we set $I_C(0)=48.54$ μA at $\Delta_0=30$ meV], and (c) the capacitance C for c-axis junction with $\gamma=0$.

As I_C increases, T^* increases. On the other hand, T^* decreases with increasing C, as shown in Fig. 6(c). The obtained parameter dependences of T^* are consistent with the simple formula, $T^* \propto \sqrt{I_C/C}$. In Fig. 6(b), we also show the result without the quasiparticle dissipation effect. Since the ratio $\delta M/M$ is inversely proportional to Δ_0, the influence of the quasiparticle dissipation becomes stronger for smaller $I_C(0)$. However, the effect of dissipation is still weak even for $I_C(0) \sim 1$ nA, which is consistent with the recent experiment with a low J_c Bi2212 surface intrinsic Josephson junction.[59]

Note that, for large I_C, the Josephson penetration depth λ_J of the junction becomes small. In the case of $I_C(0)=10^{-4}$ A, λ_J is typically of the order of 1 μm. In order to apply our MQT theory, the size of the junction should be smaller than λ_J. It should be also noted that T^* can be approximately expressed by

$$T^* \propto \sqrt{\frac{\Delta_0}{\varepsilon_r \rho}}, \tag{55}$$

where ε_r and ρ are the relative permittivity and the normal-state resistivity of the junctions, respectively. Therefore, T^* is

almost independent of the size of the junction. Thus, in order to obtain high T^*, large magnitude of the gap Δ_0 and small magnitudes of the relative permittivity ε_r and the resistivity ρ are desirable.

Finally, let us comment on the effect of the quasiparticle dissipation on MQT. As was mentioned above, this effect is quite weak even at $\gamma=0$, in which the node-to-node quasiparticle tunneling is possible. In fact, we have confirmed that even when the dissipation term is neglected, the obtained results are almost the same. Note that, in order to see the quasiparticle dissipation effect on MQT more clearly, we have to choose at least 3 orders smaller magnitude of the gap Δ_0 as seen from Eq. (38) and Fig. 3(b).

IV. APPLICATION TO QUBITS

In this section, we briefly discuss the advantage of *c*-axis twist junctions for the qubit application.

As was shown in the previous section, the quasiparticle dissipation on MQT is negligibly small. This result strongly indicates the high potentiality of *c*-axis twist junctions for *d*-wave qubit.[38,92–101]

Moreover, in the case of the twist junction with $\gamma > \pi/4$, the π junction is formed (see Sec. II B). A superconducting ring with the π junction exhibits a spontaneous current without an external magnetic field and the corresponding magnetic flux is half a flux quantum $\Phi_0=\hbar/2e$ in the ground state.[102] Thus, the high-T_c superconductor ring[103,104] including the *c*-axis twist junction with $\gamma > \pi/4$ becomes a quiet qubit[92,95,105,106] that can be efficiently decoupled from the fluctuation of the external field.

In order to realize the *d*-wave quantum computer with large number of qubits, we have to tune the circuit parameter (ω_p) of each qubit independently. In actual *d*-wave junctions, however, precise control of ω_p, e.g., by changing the thickness of the insulating barrier or by the oxygen doping, is very difficult. Our results in Sec. III clearly show that we can artificially and precisely control the qubit parameter $\omega_p \propto T^*$ only by varying γ of *c*-axis twist junctions. Therefore, high controllability of qubit parameters is another advantage of *c*-axis twist junctions for realizing scalable quantum computers.

V. SUMMARY

In this paper, we have investigated the influence of the *d*-wave OPS on MQT in *c*-axis twist Josephson junctions by use of the functional integral method and the bounce approximation. Due to the *d*-wave OPS, the twist angle γ dependences of standard deviation σ_n of the switching current distribution and the crossover temperature T^* are approximately given by $\cos 2\gamma$ and $\sqrt{\cos 2\gamma}$, respectively. Therefore, MQT in *c*-axis twist junction becomes a useful experimental tool for testing OPS of high-T_c superconductors at low temperature regimes. Moreover, the influence of the quasiparticle dissipation is found to be very weak. This result indicates the high potential of *c*-axis twist junctions for qubit applications.

Throughout this paper, we have considered MQT in the single junctions where the quasiparticle dissipation effect is found to be negligibly small. On the other hand, in multiple stack intrinsic junctions, the long-range capacitive coupling between junctions[107,108] gives rise to the $O(N^2)$ enhancement of the MQT rate,[55,66–70] where N is the number of the stack. Therefore, the effect of the long-range capacitive coupling on the quasiparticle dissipation in multiple stack junctions will be an interesting subject of future studies.

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APPENDIX A: NONLOCALITY OF THE β TERM

The effective action in Eq. (20) includes two terms, which are nonlocal in the imaginary time. In this paper, we have studied dissipation effects caused by the second term (the α term) of Eq. (20), while the third term (the β term) has been treated by the local approximation: The Josephson coupling is calculated by assuming that the local component at $\tau \sim \tau'$ is dominant in the double integral of the β term. In this approximation, we can separate the integral by changing variables as $T=(\tau+\tau')/2$ and $t=\tau-\tau'$. Then, the Josephson energy is defined as

$$E_J \equiv -\int_0^{\hbar\beta} dt\beta(t). \tag{A1}$$

As a result, the Josephson term in the effective action takes the usual form

$$-E_J\int_0^{\hbar\beta} dT \cos\phi(T). \tag{A2}$$

This approximation is justified in the *s*-wave superconducting junction where the β term has an exponential form $[\beta(\tau)\sim e^{-2\Delta_0|\tau|/\hbar}]$. In the high-$T_c$ junctions, however, the β term shows a power-law decay, and the local approximation

is not justified in general. For example, it has been discussed in Ref. 35 that the Ohmic power-law decay of the β term may affect the phase transition of the Josephson junctions made of d-wave superconductors. In this appendix, we study the nonlocal effect of the β term, and show that the local approximation is actually justified in the calculation of the MQT rate.

Let us start with the effective action within the local approximation, which is given as

$$S = \frac{\hbar^2}{4E_C}\int_0^{\hbar\beta}\left[\frac{\partial\phi(\tau)}{\partial\tau}\right]^2 d\tau - E_J\int_0^{\hbar\beta} d\tau(\cos\phi(\tau) + \eta\phi(\tau)). \tag{A3}$$

Here, we neglected the dissipative term. After changing the phase variable as $\psi=\phi-\pi/2+\sqrt{2(1-\eta)}$, the action is written as

$$S = \frac{\hbar^2}{4E_C}\int_0^{\beta\hbar} d\tau\left(\frac{d\psi}{d\tau}\right)^2 + E_J\int_0^{\beta\hbar} d\tau\left[-\frac{1}{6}\psi^3 + \frac{1}{2}\sqrt{2(1-\eta)}\psi^2\right], \tag{A4}$$

where $E_c=(2e)^2/2C$. Here, the potential term is approximated by a cubic polynomial. We further rescale the variables as $\tilde{\tau}=\omega_0\tau$ and $\psi=\sqrt{2(1-\eta)}\tilde{\psi}$, where $\hbar\omega_0=\sqrt{2E_CE_J}(2(1-\eta))^{1/4}$. Then, the action is obtained as

$$\frac{S}{\hbar} = \sqrt{\frac{E_J}{2E_C}}(2(1-\eta))^{5/4}\bar{S}_{\rm local}, \tag{A5}$$

$$\bar{S}_{\rm local} = \int d\tilde{\tau}\left[\frac{1}{2}\left(\frac{d\tilde{\psi}}{d\tilde{\tau}}\right)^2 + \frac{1}{2}\tilde{\psi}^2 - \frac{1}{6}\tilde{\psi}^3\right]. \tag{A6}$$

The same variable change can be done for the general action (20) by using the expansion around the metastable state. The result is given by

$$\bar{S}_{\rm nonlocal} = \int d\tilde{\tau}\frac{1}{2}\left(\frac{d\psi}{d\tilde{\tau}}\right)^2 + \int d\tilde{\tau}\int d\tilde{\tau}'\,\tilde{\beta}(\tilde{\tau}-\tilde{\tau}') \times\left(-\frac{1}{6}\bar{\psi}^3 + \frac{1}{2}\bar{\psi}^2\right), \tag{A7}$$

where $\tilde{\beta}(\tilde{\tau})=\beta(\tilde{\tau})/E_J$ and $\bar{\psi}=(\tilde{\psi}(\tau)+\tilde{\psi}(\tau'))/2$. The normalized action $\bar{S}$ only depends on the properties of the β term, and is independent of other parameters such as the current, Josephson energy, and charging energy. The value $\bar{S}$ for the bounce solution determines the prefactor in the exponent of the tunneling rate, and is given by 4.8 in the local approximation.

In order to deal with the nonlocal β term, we need the short-time cutoff τ_0. This cutoff is of the order of $\hbar/\Delta_0$ in the d-wave junction considered in this paper, and is much shorter than the plasma frequency ω_0 in actual junctions. In the following discussion, we focus on the limit $\omega_0\tau_0\to 0$. For the purpose of this appendix, it is sufficient to consider the following simplified kernel ($s>0$) (see also Appendix B)

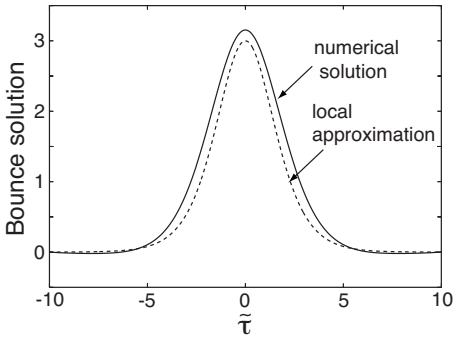

FIG. 7. Numerically obtained bounce solution for the action with the Ohmic nonlocal β term for $\omega_0\tau_0=1$ is shown by the solid line. The usual bounce solution for the action with local approximation, which is given by $\psi_0(\tilde{\tau})=3/\cosh^2(\tilde{\tau}/2)$, is also shown by the broken line.

$$\tilde{\beta}(\tilde{\tau}) = \frac{A_s(\omega_0\tau_0)^s}{(\omega_0\tau_0)^{s+1}+|\tilde{\tau}|^{s+1}}, \tag{A8}$$

$$A_s = \frac{s+1}{2\pi}\sin\left(\frac{\pi}{s+1}\right), \tag{A9}$$

which decays in the power-law form $1/|\tau|^{s+1}$ in the long-time limit ($\tau\gg\tau_0$), by noting the sum rule $\int d\tilde{\tau}\tilde{\beta}(\tilde{\tau})=1$.

First, we calculate the bounce solution of the action including nonlocal β term for the Ohmic case ($s=1$) numerically. We discretize the integral in the action (A7) as

$$\bar{S} \sim \sum_i \frac{1}{2}\left(\frac{\psi_{i+1}-\psi_i}{\Delta\tau}\right)^2\Delta\tau + \sum_{i,j}\beta_{i,j}\left[\frac{1}{2}\left(\frac{\psi_i+\psi_j}{2}\right)^2 - \frac{1}{6}\left(\frac{\psi_i+\psi_j}{2}\right)^3\right]\Delta\tau, \tag{A10}$$

where $\Delta\tau$ is a time slice and $\beta_{i,j}=\beta(\tau_i-\tau_j)\Delta\tau$. The stationary condition $\delta S=0$ is expressed by $\partial S/\partial\psi_i=0$, and the bounce solution is obtained by the equation

$$0 = -\frac{\psi_{i+1}+\psi_{i-1}-2\psi_i}{(\Delta\tau)^2} + \sum_j\beta_{i,j}\left[\left(\frac{\psi_i+\psi_j}{2}\right) - \frac{1}{2}\left(\frac{\psi_i+\psi_j}{2}\right)^2\right]. \tag{A11}$$

Here, we apply the Newton method to solve these nonlinear equations. For the actual calculation, we introduce a long-time cutoff $\tau_{\rm max}$, and discretize the range $[-\tau_{\rm max},\tau_{\rm max}]$ with $2N$ time slices. We consider the open boundary condition ($\psi_{-N}=\psi_N=0$), and choose $N=400$ and $\tau_{\rm max}=10$ ($\Delta\tau=0.025$).

We show the bounce solution for $\omega_0\tau_0=1$ by the solid line in Fig. 7. The calculated bounce solution does not deviate so much from the usual bounce solution $[\psi_0(\tilde{\tau})=3/\cosh^2(\tilde{\tau}/2)]$ for the local approximation, which is drawn by the broken line in this figure. This result indicates that the nonlocality of the β term is not crucial for the shape of the bounce solution.

We show the action of the bounce solution, $\bar{S}_{\rm nonlocal}$, as a function of the short-time cutoff τ_0 by the solid line in Fig. 8.

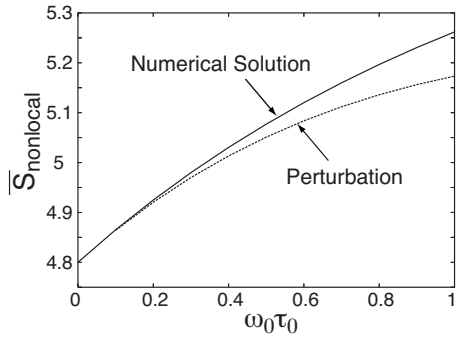

FIG. 8. The normalized bounce action $\bar{S}$ as a function of the cutoff parameter τ_0 obtained by numerical calculation (solid line) and perturbative method (dotted line). In the limit $\omega_0\tau_0 \to 0$, the bounce action becomes 4.8, which corresponds to the local approximation.

We observe that the value of the action does not change so much from 4.8, which corresponds to the local approximation. Moreover, in the limit $\omega_0\tau_0 \to 0$, the calculated action approaches 4.8. Similar results are obtained also for the general exponent s.

We can also discuss analytically the irrelevance of the nonlocality of the β term by the perturbative method. We introduce a parameter λ, which expresses the nonlocality of the β term, and write the action as

$$\bar{S}(\lambda) = \bar{S}_{\rm local} + \lambda \Delta \bar{S}, \tag{A12}$$

$$\Delta \bar{S} = \int d\tilde{\tau} d\tilde{\tau}' \tilde{\beta}(\tilde{\tau} - \tilde{\tau}') \left(\frac{1}{2}\tilde{\psi}^2 - \frac{1}{6}\tilde{\psi}^3 \right) - \int d\tilde{\tau} \left(\frac{1}{2}\tilde{\psi}^2 - \frac{1}{6}\tilde{\psi}^3 \right). \tag{A13}$$

The local approximation corresponds to $\lambda = 0$, while the action including the full nonlocal β term is obtained for $\lambda = 1$. Here, we develop the perturbation theory with respect to λ. Up to the first order of λ, the action is evaluated as

$$\bar{S} \simeq \bar{S}_{\rm local}[\psi_0(\tau)] + \lambda \Delta \bar{S}[\psi_0(\tau)], \tag{A14}$$

where $\psi_0(\tilde{\tau}) = 3/\cosh^2(\tilde{\tau}/2)$ is the nonperturbed bounce solution and $\bar{S}_{\rm local}[\psi_0(\tau)] = 4.8$. We show this approximate action with $\lambda = 1$ for the Ohmic damping by the dotted line in Fig. 8. As seen in this figure, the perturbation method gives a reliable result especially in the limit $\omega_0\tau_0 \to 0$.

In the following, we show results by the perturbation method, which indicates the irrelevance of the nonlocality of the β term for a general exponent s by focusing on the limit $\omega_0\tau_0 \to 0$. We can continue calculation from Eq. (A14) as follows:

$$\Delta \bar{S}[\psi_0(\tau)] = \frac{1}{16} \int d\tilde{\tau} \int d\tilde{\tau}' \tilde{\beta}(\tilde{\tau} - \tilde{\tau}')(\psi' - \psi)^2(\psi' + \psi - 2), \tag{A15}$$

where $\psi = \psi_0(\tau)$ and $\psi' = \psi_0(\tau')$. We can calculate the leading correction with respect to small $\omega_0\tau_0$. For $0 < s < 2$, we obtain

$$\Delta \bar{S}[\psi_0(\tau)] = \frac{A_s}{16} (\omega_0\tau_0)^s \gamma(s), \tag{A16}$$

where $\gamma(s)$ is a numerical factor, which only depends on s as follows:

$$\gamma(s) = \int d\tilde{\tau} d\tilde{\tau}' \frac{(\psi' - \psi)^2}{(\tilde{\tau} - \tilde{\tau}')^{s+1}} (\psi' + \psi - 2). \tag{A17}$$

For Ohmic damping ($s = 1$), we obtain $\Delta S \simeq 0.084(\omega_0\tau_0)$. For $s > 2$, we have

$$\Delta \bar{S}[\psi_0(\tau)] = \frac{A_s(\omega_0\tau_0)^2}{3} \Gamma\left(\frac{s-2}{s+1} \right) \Gamma\left(\frac{s+4}{s+1} \right). \tag{A18}$$

We note that, for any s, the correction by the nonlocality of the β term disappears in the limit of $\omega_0\tau_0 \to 0$. Therefore, we conclude that the nonlocal effect can be neglected at least in the calculation of the MQT rate.

Finally, we comment on nonlocal effects of the β term in macroscopic quantum coherence. In this case, we have to treat the path with many instantons. The interaction between instantons governs the coherence of the superposition between distinct macroscopic states. In this calculation, there is a possibility that the nonlocal effects of the β term become important. A detailed discussion along this line is an interesting future problem.

APPENDIX B: DERIVATION OF THE ASYMPTOTIC FORMS OF $\alpha(\tau)$ AND $\beta(\tau)$

Here, we derive the asymptotic forms of $\alpha(\tau)$ and $\beta(\tau)$ for c-axis junctions at $\gamma = 0$.[3,15,81,85] We define the order parameters as $\Delta_1(\theta) = \Delta_2(\theta) = \Delta_0 \cos 2\theta \equiv \Delta(\theta)$. The dissipation kernel $\alpha(\tau)$ and the Josephson kernel $\beta(\tau)$ are defined as

$$\alpha(\tau) = -\frac{2}{\hbar} \sum_{k,k'} |t(k,k')|^2 \mathcal{G}_1(k,\tau) \mathcal{G}_2(k',-\tau), \tag{B1}$$

$$\beta(\tau) = -\frac{2}{\hbar} \sum_{k,k'} |t(k,k')|^2 \mathcal{F}_1(k,\tau) \mathcal{F}_2^{\dagger}(k',-\tau). \tag{B2}$$

Here, Matsubara Green's functions at the zero temperature are given by

$$\mathcal{G}_1(k,\tau) = -\frac{1}{2}\left(1 + \frac{\xi_k}{E_k} \right) e^{-(\tau/\hbar)E_k} \Theta(\tau) + \frac{1}{2}\left(1 - \frac{\xi_k}{E_k} \right) e^{(\tau/\hbar)E_k} \Theta(-\tau), \tag{B3}$$

$$\mathcal{G}_2(k',-\tau) = \frac{1}{2}\left(1 - \frac{\xi_{k'}}{E_{k'}} \right) e^{-(\tau/\hbar)E_{k'}} \Theta(\tau) - \frac{1}{2}\left(1 + \frac{\xi_{k'}}{E_{k'}} \right) e^{(\tau/\hbar)E_{k'}} \Theta(-\tau), \tag{B4}$$

$$\mathcal{F}_1(k,\tau) = \frac{\Delta_k}{2E_k} e^{-(|\tau|/\hbar)E_k}, \tag{B5}$$

TABLE I. Asymptotic forms of the dissipation kernel $\alpha(\tau)$ and the spectral density $J(\omega)$ for several types of the Josephson junction (JJ). Here, "Exp." denotes a function of the form $\exp(-a|\tau|)$ with a constant a. "Gap type" is defined by its Fourier transformation.

Symmetry	Model of JJ	Kernel $\alpha(\tau)$	Spectral density $J(\omega)$	Refs.
s wave	JJ without shunt resistance	Exp.	Gap type	13
s wave	JJ with shunt resistance	$1/\tau^2$	ω	13
d wave	c-axis JJ ($\gamma \neq 0$) with coherent interlayer tunneling	Exp.	Gap type	15 and 82
d wave	c-axis JJ ($\gamma=0$) with coherent interlayer tunneling	$1/\tau^3$	ω^2	15, 82, and 83
d wave	c-axis JJ ($\gamma=0$) with incoherent interlayer tunneling	$1/\tau^4$	ω^3	81
d wave	Intrinsic JJ	$1/\tau^5$	ω^4	85
d wave	In-plane JJ without ZES	$1/\tau^3$	ω^2	16 and 97
d wave	In-plane JJ with ZES	$1/\tau^2$	ω	16, 19, and 98
d wave	In-plane s-wave/d-wave JJ	Exp.	Gap type	21

$$\mathcal{F}_2^{\dagger}(\boldsymbol{k}',-\tau)=\frac{\Delta_{\boldsymbol{k}'}}{2E_{\boldsymbol{k}'}}e^{-(|\tau|/\hbar)E_{\boldsymbol{k}'}}, \tag{B6}$$

where $E_{\boldsymbol{k}}=\sqrt{\xi_{\boldsymbol{k}}^2+\Delta_{\boldsymbol{k}}^2}$, $\Delta_{\boldsymbol{k}}=\Delta(\theta)$, and Θ is the step function.

Therefore, we have

$$\alpha(\tau)=\frac{1}{2\hbar}\sum_{\boldsymbol{k}}\sum_{\boldsymbol{k}'}|t_{\boldsymbol{k},\boldsymbol{k}'}|^2\left(1+\frac{\xi_{\boldsymbol{k}}}{E_{\boldsymbol{k}}}\right)\left(1-\frac{\xi_{\boldsymbol{k}'}}{E_{\boldsymbol{k}'}}\right)e^{-\tau/\hbar(E_{\boldsymbol{k}}+E_{\boldsymbol{k}'})}\Theta(\tau) \tag{B7}$$

$$+\frac{1}{2\hbar}\sum_{\boldsymbol{k}}\sum_{\boldsymbol{k}'}|t_{\boldsymbol{k},\boldsymbol{k}'}|^2\left(1-\frac{\xi_{\boldsymbol{k}}}{E_{\boldsymbol{k}}}\right)\left(1+\frac{\xi_{\boldsymbol{k}'}}{E_{\boldsymbol{k}'}}\right)e^{\tau/\hbar(E_{\boldsymbol{k}}+E_{\boldsymbol{k}'})}\Theta(-\tau), \tag{B8}$$

$$\beta(\tau)=-\frac{1}{2\hbar}\sum_{\boldsymbol{k}}\sum_{\boldsymbol{k}'}|t_{\boldsymbol{k},\boldsymbol{k}'}|^2\frac{\Delta_{\boldsymbol{k}}\Delta_{\boldsymbol{k}'}}{E_{\boldsymbol{k}}E_{\boldsymbol{k}'}}e^{-|\tau|/\hbar(E_{\boldsymbol{k}}+E_{\boldsymbol{k}'})}. \tag{B9}$$

First, we consider the coherent tunneling model where the momentum $\boldsymbol{k}$ dependence of the tunneling matrix element is given by $t_{\boldsymbol{k},\boldsymbol{k}'}=t\delta_{\boldsymbol{k}_\parallel,\boldsymbol{k}'_\parallel}$. This approximation is applicable to cross-whisker junctions with a clean insulating barrier. Then, the dissipation kernel $\alpha(\tau)$ and the Josephson kernel $\beta(\tau)$ can be calculated as

$$\alpha(\tau)=\frac{N_0^2|t|^2}{2\pi^2\hbar}\int_0^{2\pi}d\theta\left[\int_0^{\infty}d\xi e^{-|\tau|/\hbar\sqrt{\xi^2+\Delta(\theta)^2}}\right]^2, \tag{B10}$$

$$\beta(\tau)=-\frac{N_0^2|t|^2}{2\pi^2\hbar}\int_0^{2\pi}d\theta\Delta(\theta)^2\left[\int_0^{\infty}d\xi\frac{1}{E_{\boldsymbol{k}}}e^{-|\tau|/\hbar\sqrt{\xi^2+\Delta(\theta)^2}}\right]^2. \tag{B11}$$

By applying a formula for the modified Bessel functions K_1 and K_0, i.e.,

$$\int_0^{\infty}d\xi e^{-|\tau|/\hbar\sqrt{\xi^2+\Delta(\theta)^2}}=|\Delta(\theta)|K_1\left(\frac{|\tau||\Delta(\theta)|}{\hbar}\right), \tag{B12}$$

$$\int_0^{\infty}d\xi\frac{1}{E_{\boldsymbol{k}}}e^{-|\tau|/\hbar\sqrt{\xi^2+\Delta(\theta)^2}}=K_0\left(\frac{|\tau||\Delta(\theta)|}{\hbar}\right), \tag{B13}$$

we get Eqs. (32) and (33) as follows:[15,82,83]

$$\alpha(\tau)=\frac{N_0^2|t|^2\Delta_0^2}{2\pi^2\hbar}\int_0^{2\pi}d\theta\cos^2(2\theta)K_1\left(\frac{|\tau|\Delta_0}{\hbar}|\cos 2\theta|\right)^2$$
$$\approx\frac{3\hbar^2}{16\pi\Delta_0}\frac{R_Q}{R}\frac{1}{|\tau|^3}\quad\text{for }\tau\gg\frac{\hbar}{\Delta_0}, \tag{B14}$$

and the following expression of $\beta(\tau)$:

$$\beta(\tau)=-\frac{|t|^2N_0^2\Delta_0^2}{2\pi^2\hbar}\int_0^{2\pi}d\theta\cos^2(2\theta)K_0\left(\frac{|\tau|}{\hbar}\Delta_0|\cos 2\theta|\right)^2$$
$$\approx-\frac{\hbar^2R_Q}{16\pi\Delta_0R}\frac{1}{|\tau|^3}\quad\text{for }\tau\gg\frac{\hbar}{\Delta_0}. \tag{B15}$$

Next, we consider the incoherent tunneling model with a constant tunneling matrix: $t_{\boldsymbol{k},\boldsymbol{k}'}=t$, which is applicable to cross-whisker junctions with an imperfect dirty insulating barrier. After some calculations similar to the coherent tunneling model, we obtain[81]

$$\alpha(\tau)=\frac{|t|^2N_0^2\Delta_0^2}{2\pi^2\hbar}\left[\int_0^{2\pi}d\theta|\cos 2\theta|K_1\left(\frac{|\tau|}{\hbar}\Delta_0|\cos 2\theta|\right)\right]^2$$
$$\approx\frac{\hbar^3}{\pi^2\Delta_0^2}\frac{R_Q}{R_1}\frac{1}{\tau^4}\quad\text{for }\tau\gg\frac{\hbar}{\Delta_0}, \tag{B16}$$

where normal resistance R_1 is defined by

$$\frac{1}{R_1} = \frac{4\pi e^2}{\hbar}|t|^2 N_0^2. \tag{B17}$$

Note that the Josephson kernel $\beta(\tau)$ disappears due to the angular averaging in the incoherent tunneling model.

For intrinsic Josephson junctions (c-axis junction with $\gamma=0$), tunneling matrix element can be modeled as $t_{\boldsymbol{k},\boldsymbol{k}'}=t\cos^2(2\theta)\delta_{\boldsymbol{k}_\parallel,\boldsymbol{k}'_\parallel}$ from the first-principle band calculations,[84] where the node-to-node quasiparticle tunneling is inhibited. Within this model, we obtain[85]

$$\alpha(\tau) = \frac{N_0^2|t|^2\Delta_0^2}{2\pi^2\hbar}\int_0^{2\pi} d\theta \cos^4(2\theta) K_1\left(\frac{|\tau|\Delta_0}{\hbar}|\cos 2\theta|\right)^2$$

$$\approx \frac{45\hbar^4}{128\pi\Delta_0^3}\frac{R_Q}{R_2}\frac{1}{|\tau|^5} \quad \text{for } \tau \gg \frac{\hbar}{\Delta_0} \tag{B18}$$

and

$$\beta(\tau) = -\frac{N_0^2|t|^2\Delta_0^2}{2\pi^2\hbar}\int_0^{2\pi} d\theta \cos^4(2\theta) K_0\left(\frac{|\tau|\Delta_0}{\hbar}|\cos 2\theta|\right)^2$$

$$\approx -\frac{27\hbar^4 R_Q}{128\pi\Delta_0^3 R_2}\frac{1}{|\tau|^5} \quad \text{for } \tau \gg \frac{\hbar}{\Delta_0} \tag{B19}$$

with normal resistance R_2 which is defined as

$$\frac{1}{R_2} = \frac{e^2}{\hbar}|t|^2 N_0^2. \tag{B20}$$

Finally, let us summarize the results of the dissipation kernel $\alpha(\tau)$ in this appendix in Table I, including the results of s-wave[13] and in-plane d-wave junctions.[16,97,98] A remarkable feature of in-plane junctions is the emergence of ZES, which stem from the sign change of the order parameter.[31–34] In this table, we also show the spectral density $J(\omega)$ which is defined by the Fourier transformation of $\alpha(\tau)$[13,109]

$$\alpha(\tau) \propto \int_0^\infty \frac{d\omega}{2\pi} e^{-\omega|\tau|} J(\omega). \tag{B21}$$

This table indicates that a wide variety of quantum dissipations can be realized in d-wave junction only by changing the junction configuration and the barrier property.

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