Mesoscopic Aharonov-Bohm loops in a time-dependent potential: Quasistationary electronic states and quantum transitions

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Motivated by the interest to testable, exactly solvable models of quantum behavior in the time-dependent potentials, which may be important in studying (and in proof) basic quantum mechanical laws, as well as in considering the possibility of using the electronic components suggested to hypothetical "quantum computers," we develop a method of solving the Schrödinger equation in a certain class of time-periodic and space-dependent one-dimensional potentials. In particular, it is shown that the quasistationary electronic state in the one-dimensional cyclic mesoscopic metallic ring in a rotating-potential field displays periodic variation of quasienergy in the function of magnetic flux threading the ring (the Aharonov-Bohm effect) and oscillation, superposed on the monotonous dependence, in the function of angular velocity of rotating potential (the effect similar to Rabi and/or Bloch oscillation). At large speed of rotation, quasienergy decreases rather than increases with the increase of angular velocity. The dependence of quasienergy on flux in space periodic potential displays standard hc/e periodicity as well as the periodicity with a larger period, Nhc/e, where N is the number of sites in the loop, corresponding to one flux quantum per lattice site. This is an interference effect similar to one observed in the fractional quantum Hall effect but, unlike in the latter, not requiring the concept of "fractional electron charge," e/N. The physical significance of the quasienergy states is clarified by studying the quantum transitions between the states as well as by investigating the energy flow between the ring and the external source of the potential.

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I. INTRODUCTION

The impressive 80-year progress in quantum mechanics, owing to the epistemological role which the Schrödinger wave function Ψ plays in the explanation, prediction, and description of various phenomena and properties of atoms, complex molecules, mesoscopic systems, and solids, resulted in the idea of the possibility of "quantum computation,"[1] i.e., the usage of reversible time-dependent unitary transformation of Ψ resulting in the (0,1) to (1,0) (and vice versa) probability changes in certain "qubits" (quantum bits) mimicking the bits of classical computers, but processing in the "quantum parallelism" regime thus believed to make the computation exponentially faster than that of the polynomially fast classical computers.

The natural suggestions for electronic qubits are mesoscopic quantum phenomena of which the Coulomb-blockade charge oscillations in small metallic grains[2–5] and persistent currents[6–9] in small metallic rings related to the Aharonov-Bohm effect[10] are the best known. Recent discoveries of thin metallic wires (carbon nanotubes)[11,12] finalize this arsenal of mesoscopic and nanoscopic devices, making possible the realization of the quantum networks for superfast computers, as well as for other fast (multiprocessor) computational architectures such as the synergetic[13] or neural[14,15] computers.

In the application to quantum computation, it is often suggested that the system of coupled qubits operates under the condition of a special space arrangement of qubits, within certain time steps during which the external potentials are applied to qubits. The properly designed unitary evolution of qubits allows solving certain mathematical problems untractable by classical computers, e.g., the discrete Fourier transform in the known Shor factorization algorithm.[16] Such processes can run properly if we assume that the wave function plays the ontological role,[17,18] i.e., represents the exact amplitudes of probability changes, with phases of the qubit components changing coherently.

At this level of quantum integration, in addition to the severe criticism related to daunting technological problems, fundamental problems arise related to the well known fact that the quantum Schrödinger equation for Ψ has never been derived on a microscopic basis, and was not thoroughly tested in the case of stand-alone electronic systems (rather than a macroscopic ensemble of systems), especially regarding the possibility of a coherent bit flip in a finite time interval.

The aim of this paper is to study the time-dependent behavior of persistent currents in ballistic (i.e., free of impurities or imperfections) Aharonov-Bohm loops. The simplest realization of a regular time oscillation of persistent current can be achieved by introducing a periodic potential rotating around the ring in the near vicinity of the latter, thus forming moving barriers (at potential $V>0$) or potential traps (at $V<0$) for electrons. The other case is the rotating electric field perpendicular to the magnetic field, as well as the rotating periodic lattice in the form of Kronig-Penney barriers. (Similar time-dependent problems have been considered earlier).[19,20]

The mechanical rotation of a potential source is a sketch of the time-dependent situation. In fact, fast rotating potentials can also be generated by the motion of solitons (magnetic, superconducting, etc.) near and along the path of the Aharonov-Bohm loop, as well as acoustic or laser-focused rotating photonic fields, possibly with the near-zone focusing of the latter to reach the domain of submicron resolution along the loop.

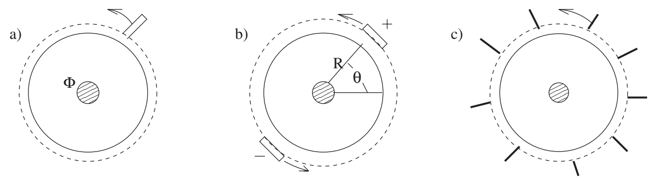

FIG. 1. Sketch of rotating potentials: (a) isolated barrier, (b) rotating capacitor, and (c) rotating Kronig-Penney potential.

The above mentioned problems fall in the range of potentials of generic form

$$V(\theta,t) = \sum_j V_j \delta[\theta - (\omega t + \alpha_j)_{2\pi}], \tag{1}$$

where θ is an angle defining position of point at the loop and $(X)_{2\pi}$ means $X \bmod 2\pi$, i.e., X with a proper number of integer periods 2π subtracted such that X falls within the interval $0 \leqslant \theta < 2\pi$. Equation (1) corresponds to the rotation of potential along the ring with the angular velocity ω.

The solution of the Schrödinger equation with such a potential can be presented, according to the Floquet theorem, in the form $\Psi = e^{-i\lambda_1\theta} e^{-i\lambda_2 t} f(\theta,t)$, where f is periodic in θ and in t. In the physical language, λ_1 and λ_2 are the Bloch quasimomenta. The space quasimomentum λ_1 is also called "crystal momentum" (see, e.g., Kittel's book.[21]). In a closed (circular) periodic structure, the crystal momentum should be put equal to zero or be a multiple of 2π (in the extended zone representation) because of the single valuedness of the wave function at $\theta=0$ and $\theta=2\pi$. The "time quasimomentum" has been nominated in Ref. 22 as the quasienergy.

The wave function of the electron in the time-periodic potential in a ring is therefore presented in the form of Zeldovich's quasistationary state[22] with quasienergy ε,

$$\Psi = \phi e^{-i\varepsilon t} = \sum_{n=-\infty}^{\infty} \phi_n e^{in(\theta-\omega t)} e^{-i\varepsilon t}, \tag{2}$$

where ϕ is a periodic function of time with a period $T = 2\pi/\omega$.

II. EXACTLY SOLVABLE TIME-DEPENDENT MODELS

We consider three simple models of rotating potentials satisfying Eq. (1), namely:

(a) a ballistic one-dimensional loop with a single rotating impurity atom or aggregate of atoms (quantum dot, $1d$ electronic island or trap) which satisfies Eq. (1) with $j=1$ and $V<0$, or a rotating barrier which is obtained at $V>0$ [Fig. 1(a)];

(b) an Aharonov-Bohm loop in crossed electric and magnetic fields, with the magnetic field perpendicular to loop plane and the electric field in the plane of the loop [Fig. 1(b)] (The effect of the magnetic field can be produced by the vector-potential field created by a thin, infinitely long solenoid localized inside the loop. We will simulate the homogeneous electric field by the electric potentials $\pm V$ localized at opposite sides of the loop.);

(c) a periodic $1d$ lattice of electronic dots ($V<0$) or antidotes ($V>0$) assuming that the ring circumference L matches the integer number N of lattice periods $a=L/N$ [Fig. 1(c)].

A. Rotating electronic island or barrier

Assume that the δ-functional electric potential is rotating around the one-dimensional conducting loop of radius R [Fig. 1(a)]. The Schrödinger equation for the loop

$$-K\left(\frac{\partial}{\partial\theta} - i\nu\right)^2 \Psi + V\delta(\theta - (\omega\tau)_{2\pi})\Psi = i\partial\Psi/\partial t,$$

$$0 \leqslant \theta < 2\pi, \tag{3}$$

where ω is the angular velocity of rotation, $K=\pi h/\mu L^2$ is the kinetic energy (in units of Planck constant $\hbar=h/2\pi$), and μ is the electron mass. V is the height of the potential barrier for electron when V is positive, and the depth of the electronic trap is at $V<0$. The quantity ν is the ratio of the magnetic flux within the loop (produced either by a uniform magnetic field B such that $\Phi=\pi R^2 B$ or by a thin, infinitely long solenoid inserted into the loop and carrying magnetic flux Φ) to the single-electron flux quantum $\Phi_0=hc/e$.

Equation (3) takes the form

$$[K(n-\nu)^2 - n\omega - \varepsilon]\phi_n + \frac{V}{2\pi}\sum_{n'=-\infty}^{\infty} \phi_{n'} = 0, \tag{4}$$

which solves for

$$\phi_n = -\frac{V}{2\pi}\frac{X}{K(n-\nu)^2 - n\omega - \varepsilon}, \tag{5}$$

where

$$X = \sum_{n=-\infty}^{\infty} \phi_n. \tag{6}$$

Summing by n and assuming $X \neq 0$, we receive the equation

$$1 + \frac{V}{2\pi K}\sum_{n=-\infty}^{\infty}\frac{1}{(n-\alpha)^2 - \Lambda^2} = 0, \tag{7}$$

in which

$$\alpha = \nu + \omega/2K \quad \text{and} \quad \Lambda^2 = \varepsilon + \nu\omega + \omega^2/4K. \tag{8}$$

By making use of the identity[23]

$$\lim_{N\to\infty}\sum_{n=-N}^{N}\frac{1}{n+x} = \pi\cot\pi x \tag{9}$$

and expanding the summand in Eq. (7) to simple fractions, we transform this equation to a simpler form,

$$\cos 2\pi\Lambda + \frac{V}{2K}\frac{\sin 2\pi\Lambda}{\Lambda} = \cos 2\pi\left(\nu + \frac{\omega}{2K}\right). \tag{10}$$

Solved for Λ, this equation determines the quasienergy

$$\varepsilon = \Lambda^2 - \nu\omega - \omega^2/4K. \tag{11}$$

Equation (10) coincides with the Kronig-Penney relation for the energy states in a one-dimensional periodic rectangular

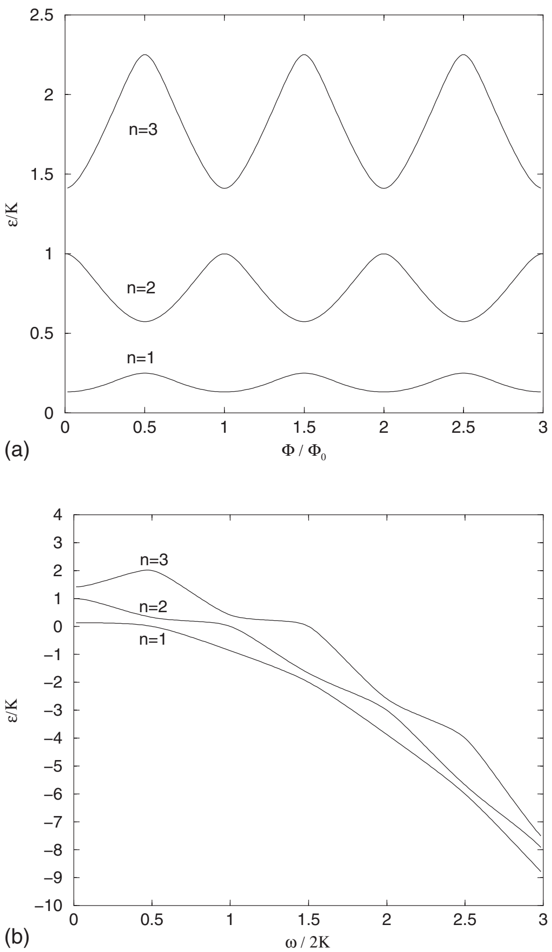

FIG. 2. Lowest quasienergy bands $n=1,2,3$ in a loop with one isolated barrier (a) as a function of magnetic flux at $\omega=0$ and (b) as a function of frequency at $\nu=0$. Coupling parameter $\upsilon=5$, where $\upsilon=\pi V/K$.

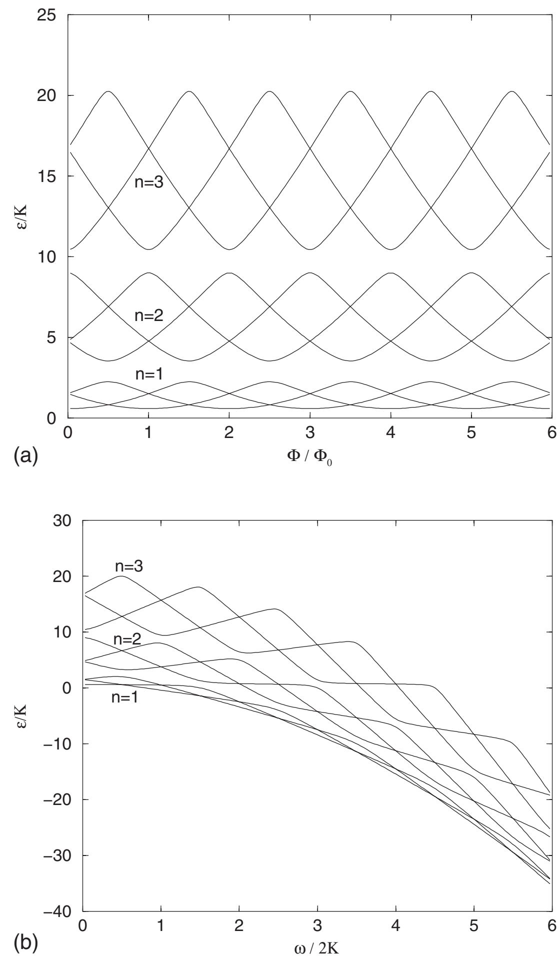

FIG. 3. Groups of lowest quasienergy states $k=0,1,2$ corresponding to band numbers $n=1,2,3$, respectively, in a three-site loop ($N=3$) (a) as a function of magnetic flux at $\omega=0$ and (b) as a function of frequency at $\nu=0$ (b). Coupling parameter $\upsilon=5$, where $\upsilon=\pi V/2K$.

potential.[21] The solution to Eq. (10) is multivalued, $\Lambda=\Lambda^{(m)}$, with $m=1,2,\ldots$ numerating the quasienergy bands in the time-periodic potential.

The magnetic flux and angular velocity of rotation enter into Eq. (10) in a symmetric way. However, their contributions to quasienergy [Eq. (11)] are not equivalent. It is interesting to note that the quasienergy *decreases* with the increase of ω at large angular velocity, which can be interpreted as an effect of blocking the standard classical ω^2 (positive) variation with the angular velocity due to spatial quantization (Λ remains of the order 1 an any ω), and then the quantum (negative) ω dependence shows up (see Figs. 2 and 3 below).

At the values of ν and ω such that $\nu+\omega/2K=0$, by recollecting the value of parameters ν and ω, we receive

$$B=-\frac{2\mu c}{e}\omega. \tag{12}$$

The relation $dB/d\omega=-2\mu c/e$ in the mesoscopic normal-conducting ring is equivalent to that in the macroscopic superconductor (the "London moment"[24]). London's interpretation was based on a suggestion (which was later supported and elucidated microscopically by Bardeen *et al.*[25]) that the wave function of the superconductor is rigid with respect to the adiabatic change of the magnetic field, resulting in the decompensation of diamagnetic and paramagnetic components of the currents and, therefore, in the appearance of the nonzero superconducting persistent current at $\mathbf{B}\neq 0$. A similar property of "rigidity" of the wave function of electron in the mesoscopic nonsuperconducting metallic ring is the mechanism of mesoscopic persistent current in the Aharonov-Bohm loop.[26,27] The current changes periodically with magnetic flux with a period hc/e in mesoscopic ring and with the period $hc/2e$ in the superconductor. The reason for the latter is the pairing of electrons, and a kind of Bose condensation, of Cooper pairs in superconductor whereas the mesoscopic persistent current is a single-electron topological effect related to the spatial quantization of electronic states in the ring. The rigidity of the wave function in the normal-metallic ring means the following.

The wave function of electron in a one-dimensional ring is const $e^{in\theta}$. It remains unchanged, i.e., rigid, at the adiabatic change of magnetic flux within a certain range of magnetic flux value, $(n-1/2)\Phi_0<\Phi<(n+1/2)\Phi_0$ because the energy $\varepsilon=\frac{1}{2}(n-\Phi/\Phi_0)^2$ is separated by a finite gap from the nearest higher energy states $\varepsilon=\frac{1}{2}(n\pm 1-\Phi/\Phi_0)^2$. Current $J=-\partial\varepsilon/\partial\Phi=J_0(n-\Phi/\Phi_0)$ remains nonzero in the ground state, provided the magnetic flux is on. This can be interpreted as an effect of "carrying along" of the electronic flow in the field of vector-potential flow **A**. Formally, the origin of persistent current is due to two simultaneous mechanisms: (1) the violation of time-reversal symmetry in the magnetic field and (2) the existence of energy gap in the electron spectrum providing the rigidity of the wave function as discussed above. These two circumstances have been recognized as early as in 1970. However, it took time to mentally accept this "unusual" aspect of the Aharonov-Bohm effect, and the microelectronic technology was not ready until the mid-1980s to manipulate the submicron size conductors.[28] Note that it was recognized at a later time that persistent current in nanoscopic (almost atomic) structures can arise as a spontaneous current without the, or in presence of infinitely small, magnetic field,[9,29,30] provided the ring is stable against a structural Jahn-Teller transformation by adhesing the ring atoms on the tight-binded solid nonmetallic surface (or, almost equivalently, developing the $J(\Phi)$ dependencies with the abnormally large $dJ/d\Phi$ slope at $\Phi=0$, with almost a fump of $J(\Phi)$ near the zero magnetic field).

In the case of the standard persistent current ($J=0$ at $\Phi=0$), the current changes under rotation periodically with the magnetic flux (the Aharonov-Bohm effect[10]) and with the angular velocity of rotation (the effect analogous to the Rabi oscillation[31–33]).

B. Rotating electric field

The rotating electric field [Fig. 1(b)], which will be presented with a potential $V(\theta)=V\cos(\theta-\omega t)$ (the Mathieu problem at $\omega=0$), will be modeled here by two opposite-sign δ potentials of magnitude V at the opposite sides of the ring,

$$V(\theta,t)=V\delta[\theta-(\omega t)_{2\pi}]-V\delta[\theta-(\omega t+\pi)_{2\pi}],\quad 0\leqslant\theta<2\pi. \tag{13}$$

The Schrödinger equation is solved with the Fourier expansion [Eq. (2)], resulting in an equation for the Fourier coefficients,

$$[K(n-\nu)^2-n\omega-\varepsilon]\phi_n+\frac{V}{2\pi}\sum_{n'=-\infty}^{\infty}[1-(-1)^{n+n'}]\phi_{n'}=0. \tag{14}$$

By introducing the notations

$$X=\sum_{n=-\infty}^{\infty}\phi_n,\quad Y=\sum_{n=-\infty}^{\infty}(-1)^n\phi_n \tag{15}$$

and summing both sides of Eq. (14) (after being solved for ϕ_n) by n, we receive a set of coupled equations for X, Y,

$$\left(1+\frac{V}{2\pi}S_1\right)X-\frac{V}{2\pi}S_2Y=0,\quad \frac{V}{2\pi}S_2X+\left(1-\frac{V}{2\pi}S_1\right)Y=0. \tag{16}$$

In these formulas,

$$S_1-S_2=\sum_{m=-\infty}^{\infty}\frac{1}{K(2m+1-\nu)^2-\varepsilon-(2m+1)\omega-\varepsilon} \tag{17}$$

and

$$S_1+S_2=\sum_{m=-\infty}^{\infty}\frac{1}{K(2m-\nu)^2-\varepsilon-2m\omega-\varepsilon}. \tag{18}$$

By using the formula for such sums derived in Sec. II A, we receive the identity

$$\cos^2 2\pi\Omega-(V/4K\Omega)^2\sin^2 2\pi\Omega=\cos^2\pi(\nu+\omega/2K), \tag{19}$$

which should be solved for Ω to determine the quasienergy

$$\varepsilon=4\Omega^2-\nu\omega-\omega^2/4K. \tag{20}$$

Figure 2(a) shows an example of flux dependence of quasienergy for three lowest quasienergy bands for the value of positive potential V such that the "coupling parameter" $\nu=\pi V/K=5$. The quasienergy oscillates as function of magnetic flux with the Aharonov-Bohm period $\Delta\Phi=hc/e$ corresponding to a single-electron charge e. Similarly, Fig. 2(b) shows the ω dependence of three lowest quasienergy bands on the angular velocity of rotation. Classically, we expect that the energy should increase with ω, but in the quantum regime it decreases monotonically and oscillates as a function of ω with a period $\Delta\omega=2K$.

C. Rotating periodic potential

The method of (θ,t) Fourier transform of the wave function allows for a solution of another δ-functional model, that of lattice of moving δ barriers [Fig. 1(c)]

$$V(\theta,t)=\sum_{l=0}^{N-1}V_l\delta\left[\theta-\left(\omega t+\frac{2\pi}{N}l\right)_{2\pi}\right],\quad 0\leqslant\theta<2\pi. \tag{21}$$

Here, N is the number of "atomic periods" of length $2\pi/N$ along the ring. Putting V_l=const=V, we receive

$$[K(n-\nu)^2-n\omega-\varepsilon]\phi_n+\frac{NV}{2\pi}\sum_{n'=-\infty}^{\infty}\phi_{n'}\delta_{(n'-n)_N,0}=0, \tag{22}$$

where n_N means $n \bmod N$. We introduce the composite indices $n=Np+k$, $n'=Np'+k'$, where p, p' are integers and $k,k'=1,2,\ldots,N-1$. The Kronecker symbol in Eq. (22) reduces to $\delta_{k,k'}$, which means that k is selected as an index specifying a particular type of solution. It follows from Eq. (22) that the dispersion relation for the eigenquasienergy is specified by the parameter k and by another parameter, n, determining the number of solutions to the equation

$$1+\frac{NV}{2\pi}\sum_{p=-\infty}^{\infty}\frac{1}{K(Np+k-\nu-\omega/2K)^2-Q^2}=0, \tag{23}$$

where Q is related to quasienergy ε,

$$\varepsilon=\varepsilon_{n,k}=N^2Q^2-\nu\omega-\omega^2/4K. \tag{24}$$

With the use of an identity (Sec. II A),

$$\sum_{n=-\infty}^{\infty}\frac{1}{(n+\alpha)^2-x^2}=\frac{\pi}{x}\frac{\sin 2\pi x}{\cos 2\pi x-\cos 2\pi\alpha}, \tag{25}$$

Eq. (23) takes form

$$\cos 2\pi Q+\frac{V}{2NK}\frac{\sin 2\pi Q}{Q}=\cos 2\pi\frac{k-\nu-\omega/2K}{N}. \tag{26}$$

Index $k=1,2,\ldots$. specifies the multivalued solutions to Eq. (26).

Combining all bands, quasienergy becomes periodic with the magnetic flux with two periods: $\Delta\nu=1$ and $\Delta\nu=N$ corresponding to single flux quanta $\Delta\phi=\phi_0$ and $\Delta\Phi=N\Phi_0$, respectively. The second periodicity corresponds to one flux quantum per one lattice site of the periodic potential. Single-quantum periodicity is clear from the possibility of replacing k to $k+1$ when ν changes to $\nu+1$. This is the standard Aharonov-Bohm effect. The $N\Phi_0$ periodicity is obtained when k is fixed. These types of energy versus flux dependence (together with the corresponding ε versus ω dependence) are illustrated in Figs. 3(a) and 3(b).

The anomalous periodicity $\Delta\Phi=\Phi_1=N\Phi_0=hc/q^*$, where Φ_0 is a single-electron flux quantum and $q^*=e/N$ is a fractional charge often discussed in the theory of quantum Hall effect.[34] What happens is that an electron shares N positions in the N unitary cells along the ring, thus making the charge e/N per one cell. Since there is no interaction between the charges and there is no way to separate the e/N charge from one cell, we do not see any reason to claim this situation the "fractional charge quantization," as is assumed in a similar (but clearly not identical) situation in fractional Hall effect in the $2d$ metallic layers.

III. QUANTUM TRANSITIONS BETWEEN THE QUASIENERGY STATES

A. Perturbation theory for quasistationary states

We suppose that the potential energy of the ring has a form $V(\theta,t)=V_0(\theta,t)+V_1(\theta,t)$, with small V_1 having the same period as $V_0(\theta,t)$, and solve perturbatively for V_1. For illustration, we consider the case of a single rotating-potential barrier (or the potential well) considered in Sec. II A.

We seek the solution to the Schrödinger equation $(H_0+V_1)\Psi=i\partial\Psi/\partial t$ in the form $\Psi=e^{-i\varepsilon t}\phi(\theta-\omega t)$ and find the correction to the wave function $\phi=\phi_0+\phi_1+\cdots$ and to the quasienergy $\varepsilon=\varepsilon_0+\varepsilon_1+\cdots$. In the case of a single barrier (Sec. II A), the zero and the first order of perturbation conditions read

$$[K(n-\nu)^2-n\omega-\varepsilon_0]\phi_n^0+\frac{\upsilon_0}{2\pi}\sum_{n'=-\infty}^{\infty}\phi_{n'}^0=0 \tag{27}$$

and

$$[K(n-\nu)^2-n\omega-\varepsilon_0]\phi_n^1+\frac{\upsilon_0}{2\pi}\sum_{n'=-\infty}^{\infty}\phi_{n'}^1=\varepsilon_1\phi_n^0-\sum_{m=-\infty}^{\infty}\upsilon_m\phi_{n-m}^0, \tag{28}$$

respectively, where υ_m is the mth Fourier harmonic of potential $V_1(\theta,t)$.

The zero order equation solves for the value of quasienergy ε_0. The solution to the first-order correction ϕ_n^1 gives

$$\phi_n^1=\frac{-\frac{\upsilon_0}{2\pi}X_0+\varepsilon_1\phi_n^0-\sum_{m=-\infty}^{\infty}\upsilon_m\phi_{n-m}^0}{K(n-\nu)^2-n\omega-\varepsilon_0}, \tag{29}$$

with $X_0=\Sigma_{n=-\infty}^{\infty}\phi_n^0$. Summing by n and taking into account the equation for ε_0, we receive the first-order correction to quasienergy,

$$\varepsilon_1=\sum_{m=-\infty}^{\infty}\upsilon_m Z_m/Z_0, \tag{30}$$

where

$$Z_m=\sum_{n=-\infty}^{\infty}\{[K(n-\nu)^2-n\omega-\varepsilon_0]\times[K(n-m-\nu)^2-(n-m)\omega-\varepsilon_0]\}^{-1}. \tag{31}$$

B. Quantum transitions

We assume that the Hamiltonian of the ring contains a small part of H_1 periodic with frequency Ω, which turns on at $t=0$

$$H_1=2\upsilon(\theta)\cos\Omega t \quad \text{at } t\geqslant 0, \tag{32}$$

and seek for the solution to the Schrödinger equation in the form

$$\Psi=\sum_k C_k(t)\phi_k^0(t)e^{-i\varepsilon_k^0 t}, \tag{33}$$

where index k enumerates the quasieigenstates of the unperturbed Hamiltonian. The solution to coefficients $C_k(t)$, perturbatively $C_k(t)=C_k^0+C_k^1(t)+\cdots$, gives in the first order

$$i\frac{\partial C_k^1}{\partial t}=\sum_m\sum_l\int_0^{2\pi}d\theta\phi_k^{0*}(\theta-\omega t)\upsilon_m\times e^{im\theta}\phi_l^0(\theta-\omega t)(e^{i\Omega t}+e^{-i\Omega t})e^{i(\varepsilon_k^0-\varepsilon_l^0)t}, \tag{34}$$

where υ_m are Fourier coefficients of $\upsilon(\theta)$. Integrating by θ gives the following for the amplitude $A_{k,l,m}$ of the transition between the k and l quasistationary states, involving m photons of frequency ω,

$$|A_{k,l,m}|^2 \simeq |\phi^{0*}_{k,n} v_m \phi^{0*}_{l,n+m}|^2 \frac{\sin^2(\varepsilon^0_k - \varepsilon^0_l - m\omega \pm \Omega)/2}{[(\varepsilon^0_k - \varepsilon^0_l - m\omega \pm \Omega)/2]^2}. \tag{35}$$

At large time, the probability of transition increases linearly with t and satisfies the "conservation" of energy with given m,

$$|A_{k,l,m}|^2 \propto 2\pi t \delta(\varepsilon^0_k - \varepsilon^0_l - m\omega \pm \Omega). \tag{36}$$

This relation differs from the standard Fermi "golden rule" by an extra energy $m\hbar\omega$. This condition means that the system in a rotating potential can radiate (or absorb) energy in multiple steps of rotation energy quantum, $\hbar\omega$. Unlike in the case of a stationary potential, there is no such thing as the "ground state" in which there would be no possibility of emission of radiation. Considering full energy conservation, it means that the rotating-potential device requires the work to be done by (or the work to be released from) this external device.

IV. DISCUSSIONS

Our investigation showed the existence of a solvable model of quantum mechanical behavior in systems with periodical space and time dependences, generated by external "classical" potentials. The advantage is that the model is exactly solvable and therefore suggests a possibility of experimental verification. The disadvantage is that the condition to satisfy the criteria necessary to distinguish between the classical and the quantum behavior is quite difficult to satisfy.

The quantum interference in the model discussed shows an unexpected periodicity of physical properties (the quasienergy states and the transition probabilities between the states), particularly, the new periods of energy versus magnetic field and versus the angular velocity of rotation dependences. The latter effect resembles the Rabi oscillation[31–33] as well as the finite-band-width Bloch oscillation in metals and semiconductors,[35–37] but is not reducible to any one of those. The periodicity in the regular (θ,t) plane results in quasienergy oscillation with period, which can be written in terms of fractional charge of electron. We consider this as a mere coincidence with the widely discussed fractionalization of electronic charge in the (x,y) periodic space plane in the fractional quantum Hall effect.[34] We adhere to the point of view that quantum interference rather than the physical "falling apart" of electrons is a more adequate interpretation.

Considering the possibility of experimental realization of fast rotation, which was partly mentioned in the Introduction, we need to mention the numerous technological problems arising from the attempt to address the issue, by considering an example of rotating superconducting solitons (Josephson vortices) in long tunneling junctions[38,39] in the form of a periodic array ($1d$ periodic Abrikosov lattice) moving along the boundary between two superconductors in an annular[40–42] tunneling junction due to voltage V applied across the junction.

We suppose that the thin superconducting sheet is divided into two parts, S_1 and S_2, across a barrier shown by the dashed line in Fig. 4 and placed in a magnetic field larger than the Josephson critical field H_{c1} (termed H_s in Ref. 39) perpendicular to the sheet. The solitons (shown by closed loops representing the flow of supercurrent in solitons) move parallel to the barrier, making the periodic array rotate with the angular velocity $\omega = \Omega/N$, where N is the number of solitons in the junction and Ω is the Josephson frequency,

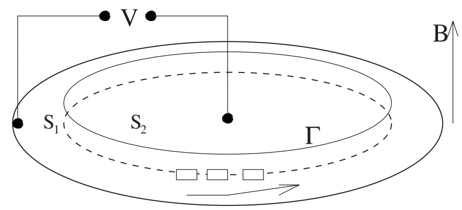

FIG. 4. Sketch of the suggested experiment with rotating superconducting solitons. dc voltage V is applied between the center and the periphery of the superconducting film of the Corbino disk geometry creating, in an external magnetic field perpendicular to the film, Josephson vortices (shown by closed loops) moving along the potential barrier separating superconducting regions S_1 and S_2 (shown by dashed line). Γ is the Aharonov-Bohm ring inspected by measuring the radiated/absorbed electromagnetic quanta due to transitions between the quasienergy states of the ring.

$$\Omega = 2eV/\hbar, \tag{37}$$

where V is a voltage applied between S_1 and S_2 across the tunneling barrier. The perturbation created by magnetic field focusing inside the solitons creates an effective periodic potential inside the Aharonov-Bohm loop Γ, which is assumed to be within the vicinity of the tunneling barrier. Quite large Ω is created in this way ($\Omega \approx 3\times 10^9$ rad/s for $V=1$ μV) such that the product $\omega\tau_\phi \gg 1$, where τ_ϕ is the dephasing time estimated in Ref. 43 as

$$\tau_\phi \simeq \frac{1}{\alpha^3}\frac{\hbar}{\epsilon_B}\left(\frac{L}{a_B}\right)^4, \tag{38}$$

where $\alpha = 1/137$ is the fine structure constant, and a_b and ϵ_B are the Bohr radius and the atomic Bohr energy, respectively.

The experiment should be done at temperature T such that $k_B T < \hbar v_F/L$, where v_F is the electron velocity. This condition is satisfied at $T \sim 10$ μK in the case of ring circumference L below 1 μm. Due to a large size of the Josephson vortex (say, $a \sim 10^3$ nm), all conditions are satisfied except the requirement of ballistic transport, $l > L$, where l is the elastic mean free path of electron, which is realistically expected to be of the order of the ring circumference. Therefore, the possibility of creating an appropriate environment for quantum experiments with rotating potentials in the mesoscopic system remains at present a thought experiment rather than a suggestion for actual experimentation, except in the case of extremely small (nanoscopic and molecular) systems.

However, we mention the possibility of mechanical rotation of the loop in exotic quantum systems such as super cooled highly rarefied gases (and, possibly, neutrons) investigated for Bose and Fermi condensation[44,45] and the acoustic or electromagnetic waves in metallic structures (in particular, in the wrapped to ring carbon nanotubes).

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