

Scattering approach to current and noise in interacting mesoscopic systems

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We propose an extension of the Landauer-Buttiker scattering theory to include effects of interaction in the active region of a mesoscopic conductor structure. The current expression obtained coincides with those derived by different methods. A general expression for the noise is also established. These expressions are then discussed in the case of strongly sequential tunneling through a double-barrier resonant tunneling structure.

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I. INTRODUCTION

The study of electronic transport through a mesoscopic system was pioneered in the full quantum mechanical framework by Landauer for a single conduction channel,¹ and then developed by Buttiker for multichannel and multiprobe systems using the scattering matrix approach.^{2,3} The theory, now known as the Landauer-Buttiker scattering theory (or scattering theory, for short), has become one of the standard ways to treat the problem of transport through nanoscale electronic devices, which consist of at least two terminals connected to a domain called scattering or active region. The heart of this theory is the assumption that the scattering region only separates the incident wave into transmitted and reflected parts. Consequently, the structure is characterized by a unique set of transmission $\{T_n\}$ (reflection $\{R_n\}$) coefficients which satisfies the basic probability conservation law, $T_n + R_n = 1$. The transport characteristics of the structure are then expressed in terms of these coefficients (see Ref. 4 for an overview). The scattering theory has successfully described the steady-state transport through mesoscopic structures in the ballistic regime. However, in most electronic semiconductor devices, scattering of carriers on impurities or/and lattice vibrations are usually important, and may even govern the transport process. To solve scattering problems, different approaches such as the nonequilibrium Green's function⁵⁻¹⁰ or/and two-particle Green's function^{11,12} have been developed. Unfortunately, such approaches are often complicated and computationally expensive that it is not easy to gain a quantitative description of interaction effects.

In contrast, the approach based on the scattering matrix is very simple in computation and transparent in physics. Recently, it has been efficiently developed to include the effect of incoherent transport on current-voltage characteristics in nanodevices.¹³ To capture such effects, the scattering method has been extended by considering each scattering center as a fictitious probe. The probe is then characterized by a phenomenological parameter interpreted as the local chemical potential, which determines a quasiequilibrium state of the probe. The conservation of current at each fictitious probe, which is used to determine the local chemical potential, implies that when an electron escapes into the probe, another electron has to be reinjected immediately therefrom to the conductor in such a way that there is no phase correlation between two events.¹⁴ Actually, the scattering region, which

includes a lot of scattering centers, can play a role such as a special intermediate source or/and sink of particles. Similar to the real sources and/or sinks (the contacts), the current in the intermediate region is conserved globally rather than locally. Moreover, there should be a phase correlation between the incoming and outgoing waves. Therefore, a suitable description is required to feature such an intermediate region. In this work, we develop the scattering method in such a way that it is able to distinguish appropriately the fluxes of coherent and incoherent particles passing through the scattering region. The approach is then applied to discuss the current-voltage (I - V) and current noise (S - V) characteristics in double-barrier resonant tunneling structures (DBRTS). It results in a current formula coinciding with that available in literature, and a different general formula for the current-noise power. From these formulas, we can recover all available conclusions of the current and noise from the limit of purely coherent to purely sequential tunneling regime.

The paper is organized as follows: Section II presents our key point for the development of the scattering theory. A current formula is then derived. In Sec. III, we derive a general formula for the shot noise power. We discuss in Sec. IV some important aspects resulting from general expressions obtained in the previous sections. Finally, a brief conclusion is given in Sec. V.

II. AVERAGE CURRENT FORMULA

The study is started by considering a structure characterized by a sole lattice temperature and a chemical potential, which can be determined by the applied bias for each real contact. The current, in principle, is the expectation value of the current operator, which is generally written in the form

$$\hat{I}(z, t) = \frac{\hbar e}{2mi} \sum_{\sigma} \int d\mathbf{r}_{\perp} \left[\Psi_{\sigma}^{\dagger}(\mathbf{r}, t) \frac{\partial \Psi_{\sigma}(\mathbf{r}, t)}{\partial \mathbf{r}} - \frac{\partial \Psi_{\sigma}^{\dagger}(\mathbf{r}, t)}{\partial \mathbf{r}} \Psi_{\sigma}(\mathbf{r}, t) \right], \quad (1)$$

where $\Psi_{\sigma}(\mathbf{r}, t)$ is the field operator with the spin index σ . Here, we neglect this index for simplicity and replace the sum by the factor $\eta_s = 2$ for the spin degeneracy. The transport is assumed to be along one direction so that we can expand the field operators in terms of transverse modes,

$\{\chi_{\alpha,n}(\mathbf{r}_\perp)\}$, and of incident and outgoing waves,

$$\Psi_\alpha(\mathbf{r},t) = \int dE e^{-iEt/\hbar} \sum_n \frac{\chi_{\alpha,n}(\mathbf{r}_\perp)}{\sqrt{2\pi\hbar v_{\alpha,n}(E)}} [a_{\alpha,n}(t) e^{ik_{\alpha,n}z} + b_{\alpha,n}(t) e^{-ik_{\alpha,n}z}], \quad (2)$$

$$\Psi_\alpha^\dagger(\mathbf{r},t) = \int dE e^{iEt/\hbar} \sum_n \frac{\chi_{\alpha,n}^*(\mathbf{r}_\perp)}{\sqrt{2\pi\hbar v_{\alpha,n}(E)}} [a_{\alpha,n}^\dagger(t) e^{-ik_{\alpha,n}z} + b_{\alpha,n}^\dagger(t) e^{ik_{\alpha,n}z}], \quad (3)$$

where the index α is added to refer to the terminal the current is measured in; $a_{\alpha,n}^\dagger(a_{\alpha,n})$, and $b_{\alpha,n}^\dagger(b_{\alpha,n})$ are operators creating (annihilating) an electron in incoming and outgoing states of the mode n , respectively, $k_{\alpha,n} = \sqrt{2m(E - E_{\alpha,n})/\hbar^2}$, and $v_{\alpha,n}$ is the velocity. Accordingly, the current operator (1) can be rewritten in the form⁴

$$\hat{I}_\alpha(z,t) = \frac{e}{\pi\hbar} \sum_n \int dE dE' e^{i(E-E')t/\hbar} [a_{\alpha,n}^\dagger(E) a_{\alpha,n}(E') - b_{\alpha,n}^\dagger(E) b_{\alpha,n}(E')]. \quad (4)$$

Note that Eq. (4) was obtained with the same assumptions as in the original standard scattering theory. It is now our key point of the analysis. Indeed, in the standard scattering theory, one can set a linear relationship between outgoing wave and incoming wave amplitudes because of the phase coherence between them. However, in the presence of scattering processes such as the electron-phonon coupling, the phase of electrons is (randomly) changed when the particles jump into the scattering region. When they jump out, they have their own phase, which is setup by the interacting processes they have experienced. Accordingly, the scattering region itself should be considered as a quasi-independent intermediate source (sink) providing (absorbing) electrons to (from) the collection (emission) terminal. So in order to distinguish the coherent and incoherent particle fluxes through the structure, we introduce a couple of operators d_n and d_n^\dagger

which characterize the processes of annihilating and creating a particle in an interacting state in the scattering region. We thus propose the relations

$$b_{\alpha n} = \sum_l \left[\sum_\gamma s_{\alpha\gamma,nl} a_{\gamma l} + \lambda_{\alpha,nl} d_l \right], \quad (5)$$

$$b_{\beta m}^\dagger = \sum_k \left[\sum_\delta s_{\beta\delta,mk}^* a_{\delta k}^\dagger + \lambda_{\beta,mk}^* d_k^\dagger \right], \quad (6)$$

where $[s_{\alpha\gamma,nl}]$ still refers to the scattering matrix and $[\lambda_{\alpha,nl}]$ now refers to the dephasing matrix. The relationship among elements of these matrices is setup by the requirement that the operators (5) and (6) have to satisfy the anticommutation algebra, which results in

$$\sum_k \left[\sum_\gamma s_{\alpha\gamma,nk}(E) s_{\beta\gamma,mk}^*(E') + \lambda_{\alpha,nk}(E) \lambda_{\beta,mk}^*(E') \right] = \delta_{\alpha\beta} \delta_{nm} \delta(E - E'). \quad (7)$$

Obviously, if operators d_m^\dagger and d_m are missing in Eq. (7) ($\lambda_\alpha=0$), the scattering matrix becomes unitary and the usual probability conservation, $T_n + R_n = 1$, is maintained. Under the current consideration, such unitary property is broken, but Eq. (7) still leads to a more general conservation law as will be seen later. We now denote

$$\begin{aligned} A_{\delta\gamma}^{kl}(\alpha, E, E') &= \delta_{\alpha\delta} \delta_{\alpha\gamma} \delta_{kl} - \sum_n s_{\alpha\delta,nk}^*(E) s_{\alpha\gamma,nl}(E'), \\ B_{\delta}^{kl}(\alpha, E, E') &= \sum_n s_{\alpha\delta,nk}^*(E) \lambda_{\alpha,nl}(E'), \\ \bar{B}_{\delta}^{kl}(\alpha, E, E') &= \sum_n \lambda_{\alpha,nk}^*(E) s_{\alpha\delta,nl}(E'), \\ C^{kl}(\alpha, E, E') &= \sum_n \lambda_{\alpha,kn}^*(E) \lambda_{\alpha,nl}(E'), \end{aligned} \quad (8)$$

the current operator is then rewritten in the form

$$\begin{aligned} \hat{I}_\alpha(z,t) &= \frac{e}{\pi\hbar} \sum_{\delta\gamma} \sum_{kl} \int dE dE' e^{i(E-E')t/\hbar} A_{\delta\gamma}^{kl}(\alpha, E, E') a_{\delta k}^\dagger(E) a_{\gamma l}(E') - \frac{e}{\pi\hbar} \sum_{\delta} \sum_{kl} \int dE dE' e^{i(E-E')t/\hbar} B_{\delta}^{kl}(\alpha, E, E') a_{\delta k}^\dagger(E) d_l(E') \\ &\quad - \frac{e}{\pi\hbar} \sum_{\delta} \sum_{kl} \int dE dE' e^{i(E-E')t/\hbar} \bar{B}_{\delta}^{kl}(\alpha, E, E') d_k^\dagger(E) a_{\delta l}(E') - \frac{e}{\pi\hbar} \sum_{kl} \int dE dE' e^{i(E-E')t/\hbar} C^{kl}(\alpha, E, E') d_k^\dagger(E) d_l(E'). \end{aligned} \quad (9)$$

This equation obviously expresses all possible physical processes of particles such as the transmission from lead to lead and from lead to intermediate region to lead, and the relaxation itself. To get the expectation value of this operator, it should be noticed that the average is now taken in quantum interacting states due to the presence of incoherent agents such as phonons. However, we simply use

$$\langle a_{\alpha,n}^\dagger(E) a_{\beta,m}(E') \rangle = \delta_{\alpha\beta} \delta_{mn} \delta(E - E') f_\alpha(E), \quad (10)$$

$$\langle a_{\alpha,n}^\dagger(E) d_m(E') \rangle = 0, \quad (11)$$

$$\langle d_n^\dagger(E) d_m(E') \rangle = \delta_{nm} \delta(E - E') f_w(E), \quad (12)$$

where $f_\alpha(E)$ is, as usual, the Fermi-Dirac function of electrons in the α contact, and f_w is referred to as the occupation probability in interacting states in the device active region. Note that the appearance of $\delta_{mn}\delta(E-E')$ in Eq. (12) does not mean only elastic processes being included, but it means f_w as the occupation probability of charges in excited states. The average of the second and third terms can be set to 0 because they are equal in magnitude but with opposite sign, and therefore, exactly cancel each other. After some simple algebra taking into account Eq. (7), the current in the emitter is written as

$$I_e(t) = \frac{2e}{h} \sum_n \int dE \{ [1 - R_{e,n}(E)] [f_e(E) - f_w(E)] - T_n(E) [f_c(E) - f_w(E)] \}. \quad (13)$$

Here, we have used the eigenvalue representation of the matrices $[s_{ee}^*(E)s_{ee}(E)]$ and $[s_{ec}^*(E)s_{ec}(E)]$, which correspond to the reflection ($R_{e,n}$) and transmission (T_n) probabilities of a transmission event through the whole structure in the conduction channel n . Note that we generally distinguish the reflection coefficients at each contact but, of course, a unique set of transmission coefficients $\{T_n\}$ governs the coherent transport through the whole structure.¹⁵

A similar formula is also easily written down for the current measured at the collector. The conservation of current allows symmetrizing the current formula by evaluating the occupation factor f_w in the scattering region,

$$f_w = \frac{\Delta_e f_e + \Delta_c f_c}{\Delta_e + \Delta_c}, \quad (14)$$

where $\Delta_{e(c),n} = 1 - (R_{e(c),n} + T_n) \geq 0$. Substituting this result into Eq. (13) yields

$$I_e(t) = \frac{2e}{h} \sum_n \int dE T_n^{tot}(E) [f_e(E) - f_c(E)], \quad (15)$$

where $T_n^{tot} = T_n + \frac{\Delta_{e,n}\Delta_{c,n}}{\Delta_{e,n} + \Delta_{c,n}}$ is regarded as the effective or total transmission coefficient. This result is really interesting because of its simple and well-known form of the Landauer formula. As expected, the total current is a sum of two contributions: the coherent (associated with T_n) and sequential (associated with the second term in T^{tot}) parts. Equation (15) leads to the conductance

$$G = \frac{2e^2}{h} \sum_n \left(T_n + \frac{\Delta_{e,n}\Delta_{c,n}}{\Delta_{e,n} + \Delta_{c,n}} \right), \quad (16)$$

which is also coincident with that derived using other methods, such as the voltage-probe method¹⁶ and the NEGF method.⁸

III. CURRENT NOISE POWER FORMULA

We now go further to establish a formula for the current noise when both direct and indirect tunneling processes are taken into account. As usual, we introduce the operator $\delta\hat{I}_\alpha(t) = \hat{I}_\alpha(t) - \langle \hat{I}_\alpha(t) \rangle$ and define the correlation function

$S_{\alpha\beta}(t, t')$ between the currents in contact α and contact β as

$$S_{\alpha\beta}(t, t') = \langle \delta\hat{I}_\alpha(t) \delta\hat{I}_\beta(t') + \delta\hat{I}_\beta(t') \delta\hat{I}_\alpha(t) \rangle. \quad (17)$$

The current noise power $S_{\alpha\beta}(\omega)$ is then obtained as the Fourier transform of the time correlation function by noticing that $S_{\alpha\beta}(t, t')$, in fact, depends on $t - t'$.

Using the formula of current operator, Eq. (9), in calculating the noise power leads to evaluating the expectation value of the products of two pairs of creation and annihilation operators in the interacting states. The products of operators of the same kind can be evaluated using the same algebra as that in the noninteracting case, i.e.,

$$\begin{aligned} \langle a_{\alpha,k}^\dagger(E_1) a_{\beta,l}(E_2) a_{\gamma,m}^\dagger(E_3) a_{\delta,n}(E_4) \rangle \\ \simeq \delta_{\alpha\delta} \delta_{\beta\gamma} \delta_{kn} \delta_{lm} \delta(E_1 - E_4) \delta(E_2 - E_3) f_\alpha(E_1) \\ \times [1 - f_\beta(E_2)] + \delta_{\alpha\beta} \delta_{\gamma\delta} \delta_{kl} \delta_{mn} \delta(E_1 - E_2) \\ \times \delta(E_3 - E_4) f_\alpha(E_1) f_\gamma(E_3) \end{aligned} \quad (18)$$

and

$$\begin{aligned} \langle d_k^\dagger(E_1) d_l(E_2) d_m^\dagger(E_3) d_n(E_4) \rangle \\ \simeq \delta_{kn} \delta_{lm} \delta(E_1 - E_4) \delta(E_2 - E_3) f_w(E_1) [1 - f_w(E_2)] \\ + \delta_{kl} \delta_{mn} \delta(E_1 - E_2) \delta(E_3 - E_4) f_w(E_1) f_w(E_3), \end{aligned} \quad (19)$$

where f_w is defined by Eq. (14). However, it is inappropriate to treat similarly the products of two pairs of different operators since they manifest the correlation of charges transferring through different regions of the structure, the many-particle effects. Generally, we set

$$\begin{aligned} \langle a_{\alpha,k}^\dagger(E_1) d_l(E_2) d_m^\dagger(E_3) a_{\delta,n}(E_4) \rangle \\ = \delta_{\alpha\delta} \delta_{kn} \delta_{lm} \delta(E_1 - E_4) \delta(E_2 - E_3) \Xi_\alpha(E_1, E_2) \end{aligned} \quad (20)$$

and

$$\begin{aligned} \langle d_m^\dagger(E_1) a_{\alpha,l}(E_2) a_{\delta,m}^\dagger(E_3) d_n(E_4) \rangle \\ = \delta_{\alpha\delta} \delta_{kn} \delta_{lm} \delta(E_1 - E_4) \delta(E_2 - E_3) \tilde{\Xi}_\alpha(E_1, E_2), \end{aligned} \quad (21)$$

where $\Lambda_\alpha = \Xi_\alpha + \tilde{\Xi}_\alpha$ implies the correlation of an occupied state in contact α with an empty one in the scattering region, and vice versa [see Eq. (28)]. In the result, we obtain a general formula for the noise power, which may be expressed in four terms

$$S_{\alpha\beta}(\omega) = S_{\alpha\beta}^1 + S_{\alpha\beta}^2 + S_{\alpha\beta}^3 + S_{\alpha\beta}^4, \quad (22)$$

which are defined as

$$\begin{aligned} S_{\alpha\beta}^1 = \frac{e^2}{\pi\hbar} \sum_{\gamma\delta} \int dE [A_{\gamma\delta}(\alpha, E, E + \hbar\omega) A_{\delta\gamma}(\beta, E \\ + \hbar\omega, E)]' \{ f_\gamma(E) [1 - f_\delta(E + \hbar\omega)] + f_\delta(E + \hbar\omega) \\ \times [1 - f_\gamma(E)] \}, \end{aligned} \quad (23)$$

$$\begin{aligned} S_{\alpha\beta}^2 = \frac{e^2}{\pi\hbar} \sum_\gamma \int dE [B_\gamma(\alpha, E, E + \hbar\omega) \bar{B}_\gamma(\beta, E \\ + \hbar\omega, E)]' \{ \Xi_\gamma(E, E + \hbar\omega) + \tilde{\Xi}_\gamma(E + \hbar\omega, E) \}, \end{aligned} \quad (24)$$

$$S_{\alpha\beta}^3 = \frac{e^2}{\pi\hbar} \sum_{\gamma} \int dE [\bar{B}_{\gamma}(\alpha, E, E + \hbar\omega) B_{\gamma}(\beta, E + \hbar\omega, E)]' \times \{\tilde{\Xi}_{\gamma}(E + \hbar\omega, E) + \Xi_{\gamma}(E, E + \hbar\omega)\}, \quad (25)$$

$$S_{\alpha\beta}^4 = \frac{e^2}{\pi\hbar} \int dE [C(\alpha, E, E + \hbar\omega) C(\beta, E + \hbar\omega, E)]' \{f_w(E) \times [1 - f_w(E + \hbar\omega) + f_w(E + \hbar\omega)[1 - f_w(E)]]\}. \quad (26)$$

Here, we use the notation $[\dots]'$ for the trace of matrix in brackets.

As in the standard scattering theory, although $S_{\alpha\beta}$ is written at arbitrary frequency, Eqs. (22)–(26) are only applicable to the case of low frequency (for detailed discussions, see Ref. 4). In this work, we are interested only in the zero-frequency noise for the two-terminal systems so that we can explicitly achieve a general expression, which reads

$$S_{ee}(0) = \frac{4e^2}{h} \sum_n \int dE \{T_n [f_e(1 - f_e) + f_c(1 - f_c)] + T_n(1 - T_n) \times (f_e - f_c)^2\} + \frac{4e^2}{h} \sum_n \int dE \{\Delta_{e,n}^2 [f_e(1 - f_e) + f_w(1 - f_w)] + \Delta_{e,n}(1 - \Delta_{e,n})\Lambda_e\} + \frac{4e^2}{h} \sum_n \int dE \Delta_{e,n} T_n \times [(f_e - f_c)(1 - 2f_e) - (\Lambda_e - \Lambda_c)]. \quad (27)$$

It is clear that each term in Eq. (27) can be associated with an explicit and simple physical meaning. Indeed, the first term, which coincides with the main results in Refs. 2 and 3, and the second one are the thermal and partition noise of the coherent and incoherent tunneling currents, respectively, while the third term is interpreted as the correlation between the two current components. Apart from this correlation, i.e., the correlation between the direct and indirect tunneling events, Eq. (27) also depends on $\Lambda_{e(c)}$, the correlation among the charges transferred between the scattering region and the contacts, which is governed by the full statistics of the former region. On average, $\Lambda_{e(c)}$ can be well approximated by

$$\Lambda_{e(c)} = \Lambda_{e(c)}^0 = f_{e(c)}[1 - f_w] + f_w[1 - f_{e(c)}], \quad (28)$$

and our result, Eq. (27), then reduces exactly to Eq. (18) in Ref. 17, which was derived using the Keldysh Green functions.

IV. DISCUSSION

In this section, we shall discuss our main results, Eqs. (15) and (27), derived in the previous sections. It is certainly not necessary to do so for the current formula (15) since it absolutely coincides with that obtained by different approaches.^{16,17} So we shall discuss only the noise expression. Although Eq. (27) has a simple form and has a clear physical meaning for each term, it is difficult to go quantitatively in more detail because of some undetermined phenom-

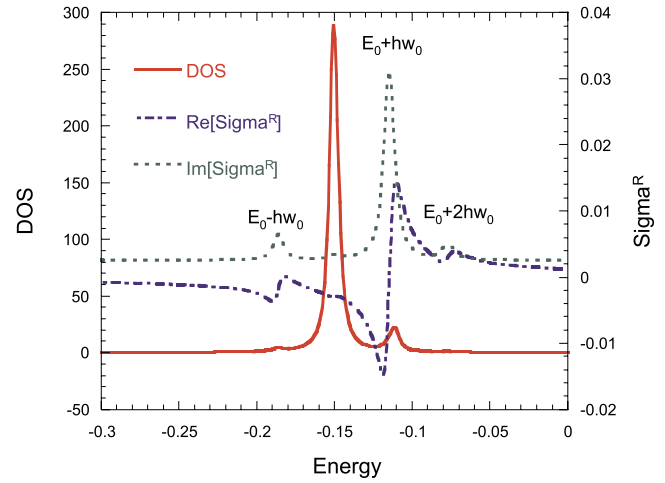


FIG. 1. (Color online) Density of states (red solid line) and retarded self-energy (real part, moss-green dashed line; imagine part, green dot-dashed line) describing the microscopic picture of the inelastic electron-phonon interaction.

enological parameters such as Δ_e, Δ_c , etc. However, in this section, we shall qualitatively discuss some important aspects of this formula. The discussion is for the typical double-barrier resonant tunneling structure. After that, we illustrate this discussion using concrete results obtained in the framework of a microscopic description.

First of all, it is instructive to see a bit the microscopic picture of the electron interaction, e.g., the electron-phonon coupling, in the scattering region. The fact is that an inelastic interaction process may result in significant peaks corresponding to excited states in the density of states (DOS) picture ($E_0 + n\hbar\omega_0$, where $\hbar\omega_0$ is the phonon energy, n is an integer number, and E_0 is a bound state in the quantum well). The transport of electrons from the emitter to the collector can then be described in the following way. Even if the level E_0 is not available for the tunneling (lower than the emitter conduction band bottom), electrons can jump into the well due to the presence of available excited states in the permitted energy interval. They then either relax to other states in the well before escaping to the collector or coherently tunnel out from these excited states. Apparently, the height of excited peaks in the DOS determines the coherent tunneling probabilities $\{T_n\}$, i.e., the higher the excited peaks, the greater T_n (see Fig. 1). The total current then, as pointed out in the previous sections, can be seen as comprising two components, sequential and coherent.

Now, look at the third term in Eq. (27), it expresses the correlation between the sequential and coherent components of current, and gives rise to a negative contribution,

$$S_3 = -\frac{8e^2}{h} \sum_n \int dE \Delta_{e,n} T_n (f_e - f_c)(f_e - f_w). \quad (29)$$

Here, we have used Eq. (28). The term becomes important if the coherent tunneling probability T_n is large. Accordingly, if the inelastic interaction process is strong enough, it may cause a reduction of shot noise. While the current increases

due to the opening of additional conduction channels, it is reasonable to expect a noise suppression. This point, comment (a), is contradictory to conclusions deduced from the voltage-probe approach,⁴ but it is commonly accepted (see Ref. 18, for instance) and will be illustrated later.

Next, we discuss the case of very small T_n , which corresponds to the tunneling induced by very low (insignificant) excited peaks of the DOS. In this limit, the first and last terms in Eq. (27) can be neglected and we keep only the second one,

$$S_{ee}(0) \approx \frac{4e^2}{h} \sum_n \int dE \{ \Delta_{e,n}^2 [f_e(1-f_e) + f_w(1-f_w)] + \Delta_{e,n}(1-\Delta_{e,n})\Lambda_e \}. \quad (30)$$

The quantities Δ_e and Δ_c are then directly interpreted as the probability that charges tunnel through the first and second barriers, respectively, from the probability conservation law, $\Delta_{e(c)} + R_{e(c)} \approx 1$. Consequently, they can be assumed as constants for simplicity. It should be kept in mind that in the interaction problems, effects of many particles generally have to be properly considered. Here, the information on these effects is contained in the corrections to f_w and to Λ_e and Λ_c . Equation (30) can now be estimated for zero temperature and high bias as

$$S_{ee}(0) \propto \frac{4e^2}{h} \frac{\Delta_e \Delta_c}{\Delta_e + \Delta_c} \left[\frac{\Delta_c^2}{\Delta_e + \Delta_c} + (1 - \Delta_e) \frac{\Delta_e + \Delta_c}{\Delta_c} \Lambda_e \right], \quad (31)$$

and at the same time,

$$I \propto \frac{2e}{h} \frac{\Delta_e \Delta_c}{\Delta_e + \Delta_c}. \quad (32)$$

The Fano factor, therefore, is equal to

$$F = \frac{\Delta_c^2}{\Delta_e + \Delta_c} + (1 - \Delta_e) \frac{\Delta_e + \Delta_c}{\Delta_c} \Lambda_e \\ = 1 - \frac{\Delta_e \Delta_c}{\Delta_e + \Delta_c} + (1 - \Delta_e) \frac{\Delta_e + \Delta_c}{\Delta_c} \delta\Lambda_e. \quad (33)$$

Here, we have used the expression $\Lambda_e = \Lambda_e^0 + \delta\Lambda_e = \Delta_c / (\Delta_e + \Delta_c) + \delta\Lambda_e$. Apparently, if the correction $\delta\Lambda_e$ is neglected, Eq. (33) results in a Fano factor smaller than 1 ($F < 1$). However, if the many-particle effects are appropriately taken into account, the following condition can be satisfied:

$$\delta\Lambda_e > \frac{\Delta_e}{1 - \Delta_e} \left(\frac{\Delta_c}{\Delta_e + \Delta_c} \right)^2. \quad (34)$$

Under this condition, the Fano factor becomes greater than 1, $F > 1$, i.e., the noise is enhanced. From Eqs. (33) and (34), we can additionally deduce the following two comments: (b) inelastic interaction processes may cause a weak enhancement of the Fano factor, and (c) it is easier to observe $F > 1$ in asymmetric structures, wherein $\Delta_e > \Delta_c$.

In order to illustrate comments (a)–(c), we use a microscopic description for the process of inelastic electron-phonon interaction developed in Ref. 19 for the model of a

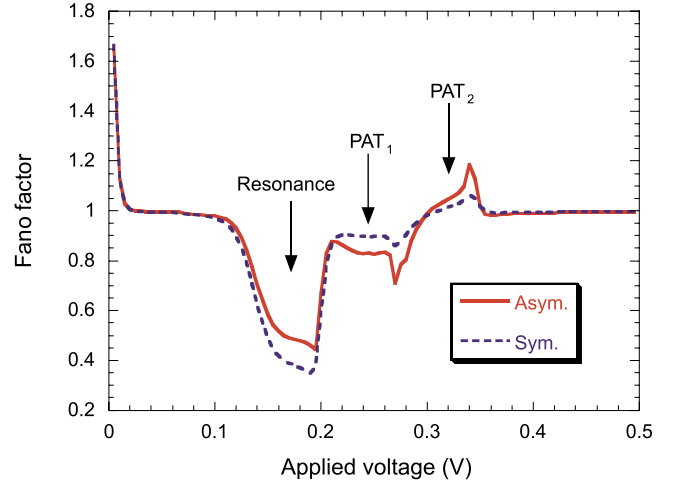


FIG. 2. (Color online) Fano factor curves extracted for the symmetric (green dashed line) and asymmetric (red solid line) barriers of the double-barrier resonant tunneling structures.

GaAs quantum well embedded between two GaAs/AlGaAs layers. Figure 1 plots the DOS and the retarded self-energy to show the formation of the excited states due to the interaction. The DOS exhibits a main resonant peak E_0 and a significant phonon-emission peak $E_0 + \hbar\omega_0$, while the self-energy exhibits three excited states, one due to phonon absorption $E_0 - \hbar\omega_0$ and two due to phonon emission (the one with $E_0 + \hbar\omega_0$ is strong and the other with $E_0 + 2\hbar\omega_0$ is weak), which are in accordance with the first and second phonon-assisted tunneling (PAT) regimes.

In Fig. 2, we plot two curves of the Fano factor which are extracted for the symmetric case, $\Delta_e = \Delta_c$, and the asymmetric one, $\Delta_e > \Delta_c$. Obviously, the figure shows a suppression of F in the first PAT regime and a weak enhancement in the second PAT regime. The figure also shows distinguishably that the noise in the PAT regime is more suppressed and more enhanced in the asymmetric case compared to the symmetric one, which is consistent with comment (c).

Thus, we have discussed some aspects of the general noise expression derived in Sec. 3. It was shown that the inelastic scattering process may induce either suppression or enhancement of the shot noise (at least in resonant tunneling structures). This conclusion, which was then additionally discussed in the framework of a microscopic description, is not in agreement with those deduced by other approaches in the literature. The reason is that in our consideration, the role of many-particle effects is strongly emphasized, while in other phenomenological works, it is not appropriately taken into account. It is important here to mention that the super-Poissonian noise of $F=1.6$, observed by Li *et al.*²⁰ in GaAs/Al_{0.4}Ga_{0.6}As-DBRTS, was ensured to come from the electron-phonon interaction process. This experimental data can be seen as a potential support to our discussion on the noise behavior.

V. CONCLUSION

In conclusion, we have developed the scattering theory in an appropriate manner such that it can describe the electron

transport through an intermediate region, wherein interacting agents can strongly destroy the phase of incident electrons or make them relax their energy. Our key point here is based on the idea of considering the scattering region as an intermediate source and/or sink of particles, which maintains the phase correlation among the collision processes. Therefore, we can distinguish the coherent and incoherent particle fluxes which contribute to the total current independently. A derived formula of the current [Eq. (15)] recovers available

results in the literature using different approaches. A different type of general formula for the current noise power [Eq. (27)] is also established, from which one can recover well-known results from the limits of purely coherent to purely sequential transport. A short discussion for the case of strongly incoherent tunneling in a double-barrier resonant tunneling structure is also presented and illustrated by a microscopic description.

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