Resonant transmission via magnetically bound states in periodic quantum structures

Yuan Ping Chen,[1] Yue E. Xie,[1] and Xiao Hong Yan[2]
[1]*Institute of Modern Physics and Department of Physics, Xiangtan University, Xiangtan 411105, China*
[2]*College of Science, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China*
(Received 22 April 2007; revised manuscript received 2 August 2007; published 28 September 2007)

We study the magnetotransport properties of two typical periodic structures. It is found that the magnetic field has a dramatic impact on the resonant splittings via bound states in the structures. For an open periodic structure, the original $(n-2)$-fold resonant splitting rule at the beginning of the first conductance step is changed to $(n-1)$-fold splitting rule by magnetic modulation. The coupling effect of the edge states is very important to form the magnetically bound states corresponding to the $n-1$ resonant splitting peaks. For an electric superlattice consisting of n barriers, the magnetic modulation turns the $n-1$ low quasibound states into true bound states; thus, the lower-energy resonant splitting in conductance disappears. Instead, $n-1$ resonant peaks via magnetically bound states appear at the beginning of the first conductance step. The high states are mainly confined in the potential barriers rather than in the wells, and their wave functions in different regions are coupled together by the edge states. One can find the "fingerprint" of the edge states from the probability densities of the high states.

DOI: 10.1103/PhysRevB.76.115439 PACS number(s): 73.21.−b, 75.47.−m

I. INTRODUCTION

Transmission resonance via the bound state is a typical quantum phenomenon in periodic quantum structures.[1–6] It is well known that there exists $(n-1)$-fold resonant splitting in the conductance for an electromagnetic superlattice, which consists of n identical potential or magnetic barriers.[1–3] Each of the resonant splitting peaks is induced by a quasibound state with amplitude confined within the quantum wells. A similar resonant splitting rule is also found in the open periodic structure such as a periodic multiwaveguide.[4–6] As the periodic multiwaveguide consists of n constrictions, $n-1$ resonant splitting peaks appear in the lower-energy region of the conductance profile. The quasibound states corresponding to the peaks are mainly confined in the stubs or wide regions.[5,6] By analogy with the electromagnetic superlattice, the stub in an open structure is usually regarded as an attractive well while the constriction is a repulsive barrier. However, recent research to the high quasibound states in the open periodic structure indicates that the effect of a stub on these states is a repulsive barrier rather than a well.[6] This is explained by the effective mass picture. At the high energies, the negative effective mass reverses the sign of the potential, which leads to each stub becoming repulsive rather than attractive. In this case, an open periodic structure consisting of n constrictions ($n-1$ stubs) is equivalent to an electromagnetic superlattice with $n-1$ potential barriers. So, there are $(n-2)$-fold resonant splitting peaks at the beginning of the first conductance step. The high quasibound states corresponding to the higher-energy peaks are mainly localized in the constrictions. However, most of the former studies are focused on the transmission resonance via quantum bound states in the case of no magnetic modulation. As a magnetic field is applied on a periodic structure, the edge states[7–10] along the boundaries of the structure will extremely affect the quantum bound states in the structure. Thus, some new magnetically bound states[11–13] will exist in the structure. Accordingly, some interesting transmission resonance via the new states will appear in conductance.

The purpose of this paper is to study the effect of magnetic field on the transmission resonance and quantum states in periodic structures. The models we studied are two typical periodic structures, as shown in Fig. 1. One is a multiwaveguide with geometrical constrictions. The other is an electric superlattice with potential barriers or wells. We discuss the influence of the magnetic modulation on both low and high quasibound states in the two structures. It is found that the energies and the lifetimes of the low quasibound states increase with the strength of the magnetic field. The low states in the electric superlattice are even changed to true bound states by strong modulation, which directly results in the disappearance of the lower-energy resonant peaks in conductance. The magnetic modulation also has a notable effect on the higher-energy electron states in the two structures. Through the coupling effect of the edge states, $n-1$ high quasibound states will exist in the multiwaveguide structure including n constrictions. Accordingly, $n-1$ resonant peaks via the quasibound states appear at the beginning of the first

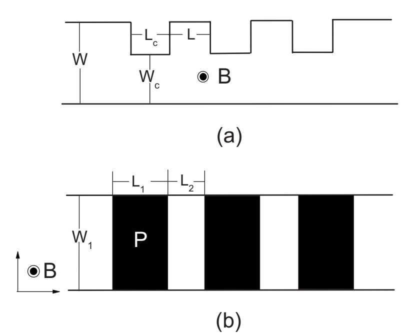

FIG. 1. (a) Schematic view of a periodic multiwaveguide, where a finite superlattice is connected to two leads with width W. The basic cell consists of a stub, with length L and width W, connected to a constriction of length L_c and width W_c. (b) Schematic view of a quasi-one-dimensional electric superlattice with width W_1, where potential wells with length L_2 are embedded in the potential barriers with length L_1 and height P.

conductance step. As to the high quasibound states in the electric superlattice under a magnetic field, electrons in these states are mainly localized in the potential barriers rather than in the wells. The critical effect of the edge states on the forming of the magnetically bound states is clearly shown in the probability density plots of the high quasibound states.

II. MODEL AND METHOD

Let us consider two periodic structures shown in Fig. 1. Figure 1(a) is an open periodic multiwaveguide in which a finite superlattice is connected to two leads with width W. The finite superlattice consists of stubs with size $L\times W$ and constrictions with size $L_c\times W_c$. Figure 1(b) is a quasi-one-dimensional electric superlattice with width W_1, in which the potential wells with length L_2 are embedded in the potential barriers with length L_1. The conductance of the two quantum structures can be calculated by the lattice Green's function (LGF) method.[14–16] In terms of the LGF scheme, a system is divided into a set of effective square lattices with lattice constant a. One uses the following tight-binding Hamiltonian to describe the effective discretized system:[14]

$$H=\sum_{i,j}(\varepsilon_{i,j}+P_{i,j})|i,j\rangle\langle i,j|+\sum_{i,j}V_1(|i,j\rangle\langle i,j+1|+\text{H.c.})+\sum_{i,j}V_2(|i+1,j\rangle\langle i,j|+\text{H.c.}), \quad (1)$$

where $\varepsilon_{i,j}$ and $P_{i,j}$ represent the site energy and the additional potential at the (i,j) site, respectively, while V_1 and V_2 respectively represent the transverse and longitudinal hopping energies between nearest neighboring sites. Generally, $\varepsilon_{i,j}=-4V$ and $V_1=V_2=V=-\hbar^2/2m^*a^2$. As a magnetic field, B is perpendicularly applied on the structure, $V_2=V\exp(ieBa^2j/\hbar)$.[14] By using the recursive Green's function technique,[15] the Green's function of the system can be found and then the conductance G of the system can be obtained.[14,16]

In addition, to calculate the eigenenergy E of a structure and the corresponding wave function Ψ, one can write the Hamiltonian of the system as

$$H=H_0+\Sigma, \quad (2)$$

where H_0 is the Hamiltonian of the structure without leads and Σ is the total self-energies of the two leads. Solving the eigenequation $H\Psi=E\Psi$, one can obtain the eigenenergy E and wave function Ψ. In general, the eigenvalue is a complex whose imaginary part is associated with the lifetime of the eigenstate. We will measure all lengths in units of lattice constant a, while the electron energy E, potential height P, and magnetic field $H=\hbar\omega_c(\omega_c=eB/m^*)$ are all in units of $-V$.

III. RESULTS AND DISCUSSION

In Fig. 2, the magnetoconductance for the multiwaveguide, as shown in Fig. 1(a) with two constrictions, are calculated. Without magnetic modulation ($H=0$), there is a sharp resonant peak at the low-energy region of the conductance in Fig. 2(a). The peak is caused by a low quasibound state whose probability density is displayed in the inset of Fig. 2(a). The electrons in the state are confined in the center of the T junction. As a magnetic field is applied on the structure [Figs. 2(b) and 2(c)], the lower-energy peak shifts to high energy and becomes sharper with the increase of the magnetic modulation. This indicates that the energy and lifetime of the low quasibound state increase with the strength of magnetic field. However, the low state will not be changed to a true bound state (with infinite lifetime) by strong modulation due to the open channel of the structure. This can also be illustrated by the left inset of Fig. 2(c), which shows the probability density of the low state corresponding to the first peak in Fig. 2(c). One can find that the magnetic modulation just slightly shrinks the wave function of the low state. While in the higher-energy regions of Figs. 2(b) and 2(c), a resonant peak induced by magnetic field appears at the beginning of the first conductance step. The peak corresponds to a high quasibound state in the structure. In the right inset of Fig. 2(c), the probability density of the high state is depicted. Why does the high quasibound state only exist in the open structure under a magnetic modulation? It is originated from the quantum states of higher-energy electrons. According to the effective mass picture, the effect of a stub to the electrons with energies at the beginning of the first conductance step is a repulsive barrier rather than a well.[6] So, the quantum states of these electrons are mainly localized in the constrictions. In the case of no magnetic field, a high quasibound state cannot exist in the multiwaveguide including two constrictions due to the fact that the electron states in the constrictions directly couple with the continuum states in the leads. In the case of magnetic modulation, however, edge states will form along the edge of the T junction. The electron states in the two constrictions will communicate with each other by the edge states; as a result, a quasibound state, such as that in the right

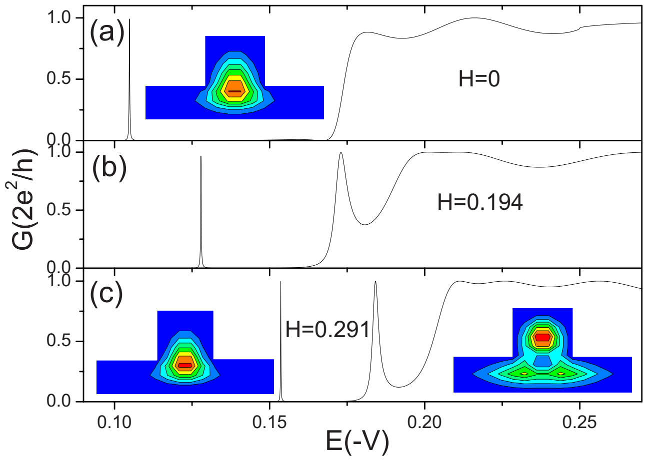

FIG. 2. (Color online) Conductance versus electron energy for a periodic multiwaveguide including two constrictions under different magnetic fields H. (a) $H=0$, (b) $H=0.194$, and (c) $H=0.291$. Results are for the case $W=16$, $L=8$, $W_c=6$, and $L_c=10$. Inset of (a): probability density of the quasibound state corresponding to the first peak in (a). Left and right insets of (c): probability densities of the quasibound states corresponding to the first and second peaks in (c), respectively.

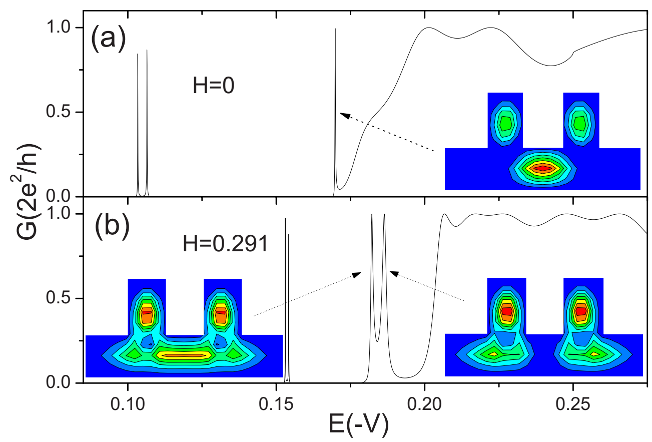

FIG. 3. (Color online) Conductance versus electron energy for a periodic multiwaveguide including three constrictions under different magnetic fields H. (a) $H=0$ and (b) $H=0.291$. Other parameters are the same as in Fig. 2. Inset of (a): probability density of the quasibound state corresponding to the third peak in (a). Left and right insets of (b): probability densities of the quasibound states corresponding to the third and fourth peaks in (b), respectively.

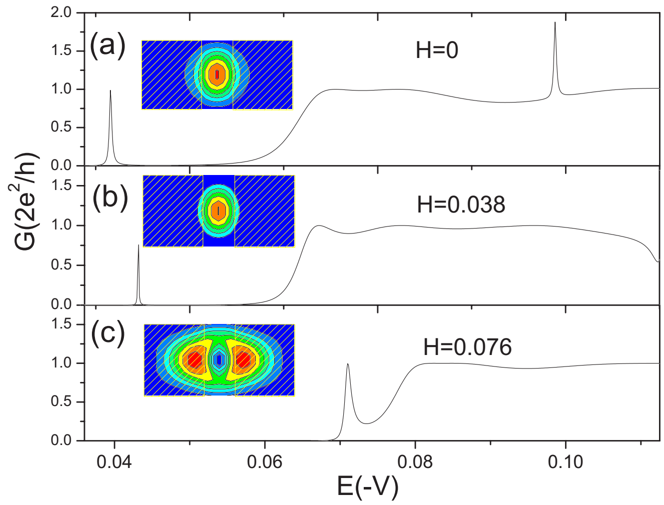

FIG. 4. (Color online) Conductance versus electron energy for an electric superlattice which includes two potential barriers under different magnetic fields H. (a) $H=0$, (b) $H=0.038$, and (c) $H=0.076$. Results are for the case $W_1=20$, $L_1=20$, $L_2=10$, and $P=0.033$. Insets in (a)–(c): probability densities of the quasibound states corresponding to the first resonant peaks in (a)–(c), respectively. The shadow regions represent potential barriers.

inset of Fig. 2(c), exists in the structure. One can find that the state is confined with big probabilities around the T junction rather than in the center of the T junction. While the wave function in different constrictions is connected together by the coupling effect of the edge states. The result demonstrates that the edge states are very important to form the high quasibound state in the periodic structure.

In Fig. 3, we show the conductance as a function of electron energy for a multiwaveguide including three constrictions. Due to the structure consisting of two T junctions, two resonant peaks via low quasibound states appear in the lower-energy region of conductance as $H=0$ [see Fig. 3(a)]. Meanwhile, a high quasibound state, as that shown in the inset of Fig. 3(a), can also exist in the structure because the structure to the higher-energy electrons is equivalent to a double-barrier structure.[6] Accordingly, a higher-energy resonant peak appears in Fig. 3(a). However, in Fig. 3(b), there are two resonant peaks at the beginning of the first conductance step. It indicates that under magnetic modulation, there are two high quasibound states existing in the structure. In the left and right insets of Fig. 3(b), the probability densities of the two states are depicted. The states are results of symmetric and antisymmetric superpositions of two adjacent quasibound states, as shown in the right inset of Fig. 2(c). One can expect that as a magnetic field is applied on the multiwaveguide consisting of n constrictions, $n-1$ high quasibound states will exist in the structure and $(n-1)$-fold resonant splitting peaks will appear at the beginning of the first conductance step. In addition, the difference between the common state and the magnetic state can be distinctly distinguished by comparing the inset of Fig. 3(a) and the insets of Fig. 3(b). The wave function of the common state is independently localized in the three constrictions due to lack of coupling, while the wave function of the magnetic state is connected together by the coupling of the edge states.

We next consider the transmission resonance via magnetically bound states in an electric superlattice, as shown in Fig. 1(b). Figure 4 shows the calculated conductance for the superlattice including two barriers under different magnetic fields. As $H=0$, a resonant peak is exhibited below the first threshold energy [see Fig. 4(a)]. The peak corresponds to a low quasibound state whose probability density is depicted in the inset of Fig. 4(a). As expected, the electrons in the state are mainly confined in the potential well. Under magnetic modulation, the resonant peak shifts quickly to high energy and becomes sharper, as shown in Fig. 4(b), while in the inset we show the probability density of the quasibound state corresponding to the peak. One can find that the quasibound state is tightly squeezed into the potential well by the magnetic modulation. Gradually, the low quasibound state will change to a true bound state with the increase of the magnetic field. Thus, as H is increased to 0.076 [Fig. 4(c)], no resonant peak appears in the lower-energy region of the conductance profile. Instead, a higher-energy resonant peak is shown at the beginning of the first conductance step. This peak is caused by a high quasibound state whose probability density is shown in the inset of Fig. 4(c). It is found that the high state is mainly localized in the two barriers rather than in the middle well. This indicates that the effect of a real potential well on the state acts as a repulsive barrier, which can also be explained by the simple effective mass picture.[6] As in the case of the multiwaveguide, a high quasibound state cannot exist in the electric superlattice including two potential barriers without magnetic modulation. However, under strong magnetic modulation, edge states will form along the boundaries of the structure. Through the coupling of these edge states, the electron states localized in the two barriers can communicate with each other, and then a magnetically bound state as shown in the inset of Fig. 4(c) forms. From the probability density of the state, one can obviously find the "fingerprint" of the edge states around the potential well, while the wave function of the magnetic state is coupled together by the edge states.

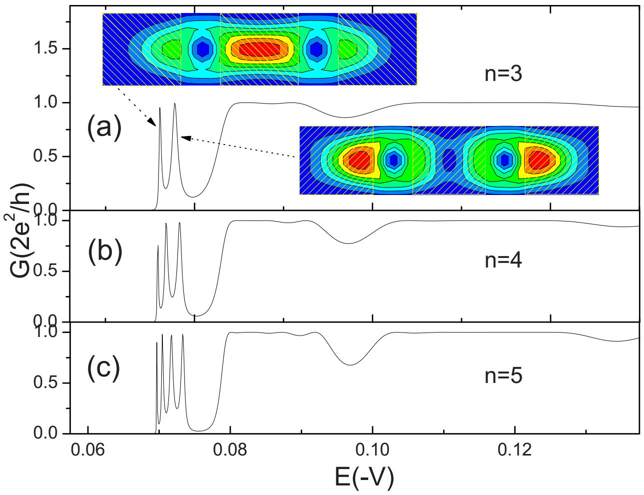

FIG. 5. (Color online) Conductance versus electron energy for an electric superlattice which includes n potential barriers under magnetic field $H=0.076$. (a) $n=3$, (b) $n=4$, and (c) $n=5$. Other parameters are the same as in Fig. 4. Upper and lower insets of (a): probability densities of the quasibound states corresponding to the first and second resonant peaks in (a), respectively. The shadow regions represent potential barriers.

In Fig. 5, the conductance for an electric superlattice including n barriers under magnetic modulation are calculated. There are $n-1$ resonant peaks at the beginning of the first conductance step. As mentioned above, these peaks are induced by the high quasibound states whose wave functions are mainly confined in the barriers. The upper and lower insets of Fig. 5(a), respectively, show the probability densities of the high states corresponding to the first and second peaks in Fig. 5(a). The two states are the results of symmetric and antisymmetric superpositions of the two adjacent quasibound states, as shown in the inset of Fig. 4(c). The fingerprint of the edge states and their coupling effect are found again from the probability densities of the two states.

IV. CONCLUSIONS

By using the LGF method, we study transmission resonance via quantum bound states in two typical periodic structures under a magnetic field. For the open periodic multiwaveguide consisting of n constrictions, the $n-1$ low-energy resonant peaks shift to high energy with the strength increase of the magnetic field, while the magnetic modulation turns the $(n-2)$-fold resonant splitting peaks at the beginning of the first conductance step into $(n-1)$-fold resonant splitting peaks. The high quasibound states corresponding to the higher-energy peaks are formed by the coupling effect of the edge states around the T junctions. For the electric superlattice consisting of n barriers, the $n-1$ lower-energy resonant peaks disappear under magnetic modulation because the low quasibound states in the structure change into true bound states. Instead, $n-1$ higher-energy resonant peaks via high quasibound states appear at the beginning of the first conductance step. The wave functions of the high states are mainly confined in the potential barriers rather than in the wells. One can clearly find the fingerprint of the edge states and their coupling effect from the probability density of the high state.

ACKNOWLEDGMENTS

This work was supported by the Scientific Research Fund of the Hunan Provincial Education Department (No. 05C103) and the Major Project of State Ministry of China (Grant No. 204099).

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