Magnetic-field-induced Coulomb blockade in small disordered delta-doped heterostructures

V. Tripathi[1] and M. P. Kennett[2]
[1]*Department of Theoretical Physics, Tata Institute of Fundamental Research, Homi Bhabha Road, Mumbai 400005, India*
[2]*Physics Department, Simon Fraser University, 8888 University Drive, Burnaby, British Columbia, Canada V5A 1S6*
(Received 29 May 2007; revised manuscript received 9 August 2007; published 19 September 2007)

At low densities, electrons confined to two dimensions in a delta-doped heterostructure can arrange themselves into self-consistent droplets due to disorder and screening effects. We use this observation to show that, at low temperatures, there should be resistance oscillations in low density two dimensional electron gases as a function of the gate voltage, which are greatly enhanced in a magnetic field. These oscillations are intrinsic to small samples and give way to variable range hopping resistivity at low temperatures in larger samples. We discuss recent experiments where similar physical effects have been interpreted within a Wigner crystal or charge density wave picture.

DOI: 10.1103/PhysRevB.76.115321 PACS number(s): 73.20.−r, 05.60.−k, 73.23.Hk, 75.47.−m

I. INTRODUCTION

The interplay between disorder and interactions in spatially inhomogeneous electronic states is an important ingredient in the physics of many strongly correlated electronic materials. Two dimensional electron gases (2DEGs) provide an ideal laboratory to gain insight into the effects of disorder and interactions as device properties such as disorder or carrier concentration can be tuned in the growth process or by external means such as a gate. Recent experiments[1] on small, disordered, delta-doped devices suggest a new set of unusual resistance oscillations in a perpendicular magnetic field. The resistance as a function of gate voltage is featureless at zero magnetic field, but at nonzero fields develops peaks that are evenly spaced in gate voltage and whose magnitudes grow with increasing magnetic field while the positions of the peaks are relatively unaffected. Similar effects have been previously observed in low density 2DEGs[2–4] and arrays of quantum dots.[5]

In earlier work on the same devices,[1–3] there was the remarkable observation that the electron tunneling distance (extracted from the magnetoresistance) is directly proportional to the average electron separation r_{ee} in the 2DEG. This was used to argue in favor of a charge density wave (CDW) or Wigner crystal (WC) picture.[6] The value of r_s in these devices is around 5, which is much less than the prediction of $r_s=37$ for Wigner crystallization in two dimensions in a clean system, but close to the prediction of $r_s \simeq 7.5$ in disordered systems.[7]

A number of theoretical studies have shown that the charge distribution in disordered delta-doped heterostructures at $r_s > 1$ but too small for a WC is likely to be neither Fermi liquid nor WC, but a droplet[8–11] or "emulsion" phase.[12] The simplest picture for the droplet phase is one where nonlinear screening by electrons is unable to dominate the disorder-induced potential barriers between regions of localized electrons.[8–10] A dropletlike phase may also arise at comparatively higher electron densities in disordered quantum Hall insulators[4,11] due to the interplay of the localization effects of disorder and screening, where the screening is now sensitive to whether or not the electrons in different parts of the system belong to a partially or completely filled Landau level. A third, and somewhat different, picture proposed for low disorder is one of an emulsion or stripelike phase with crystalline regions in an electron liquid background.[12]

Nanoscale electronic inhomogeneity has also been observed in diverse strongly correlated electron systems.[13] Some of the above mechanisms might be responsible for phase separation in many of these systems. In particular, electronic inhomogeneity in the superconductor BSCCO has been attributed to localization effects of the disorder in oxygen doping.[14]

In this paper, we use the first of the above mentioned droplet pictures to argue that the experiments in Refs. 1–3 manifest Coulomb blockade effects greatly enhanced by a magnetic field. We ignore quantum Hall physics in our treatment of the droplet phase taking note of the fact that compared to quantum Hall insulators believed to have a droplet phase, these experiments were performed on devices with strong disorder and low electron density, and no quantum Hall effect was seen at high fields. We make specific predictions about the conditions under which this magnetic-field-induced Coulomb blockade will occur, and find that our model is very successful in explaining resistance versus temperature data in Ref. 1. In particular, such a Coulomb blockade should be a signature of an electron droplet phase.

In previous work,[10] we derived expressions for the physical parameters of electron droplets in the nonlinear screening regime that is relevant to the experiments of interest here, and applied this picture to explain the experimentally observed density dependence of the tunneling distance without invoking a CDW picture.

We propose here that the resistance oscillations arise from the decrease of interdroplet tunneling due to shrinking of the localization length in strong magnetic fields. The decrease in interdroplet conductance is sufficient at larger magnetic fields to lead to a visible Coulomb blockade effect in samples where this is not clearly resolved at zero magnetic field. Considered together with our earlier explanation[10] for the density dependence of the tunneling length, we believe that our simple picture provides a complete description of the experiments in Ref. 1 without invoking ordered electronic states. The experiments we study fall in a parameter regime where WC ordering is not expected theoretically, hence more convincing evidence for WC ordering in these experiments is needed than has been offered to date. Our work does not preclude the possibility of WC ordering in heterostructures with low density and high mobility that have higher values of r_s.

The remainder of the paper is organized as follows. In Sec. II, we briefly describe the model and experimental parameters. Section III recapitulates the magnetic field and density dependences of the interdroplet tunneling conductance obtained in earlier work. The main analysis of the paper, namely, magnetic-field-induced Coulomb blockade, is presented in Sec. IV, and in Sec. V we give a discussion of the results.

II. MODEL AND EXPERIMENTAL PARAMETERS

Unless otherwise specified, we assume the following device parameters: the δ-doping density is $n_d=10^{12}$ cm^{-2}, the 2D electron density is $n_e\sim 10^{11}$ cm^{-2}, $\lambda=50$ nm is the distance of the δ layer from the 2DEG, and $d=300$ nm is the distance of the metallic gate electrode from the 2DEG.

The important parameters describing the spatial extent of the electron droplets are[10] $R_c=n_d^{1/2}/(\pi^{1/2}n_e)$, the length scale on which potential fluctuations are screened by electrons and $R_p=\sqrt{a_B(\lambda+z_0)}\sim 30$ nm, the mean droplet radius, where $z_0\sim 50$ nm is the extent of the wave function in GaAs perpendicular to the surface. This is to be contrasted with earlier predictions that $R_p\sim\lambda$.[8] Increasing the electron density has little effect on the droplet size. Extra electrons are accommodated by increasing the density of droplets.[8,10] The separation between droplet centers, $l_{ip}=2(n_d^{1/2}R_p/\pi^{1/2}n_e)^{1/2}=2\sqrt{R_cR_p}$, decreases with increasing n_e, and $l_{ip}\simeq 85$ nm. The droplets merge at high enough n_e when $R_p>l_{ip}$.

The important energy scales for the droplets are $E_{\text{barrier}}\simeq 58$ K, the difference between the binding energy E_B, and the highest occupied energy level $\Delta=\hbar^2\sqrt{\pi n_d}/2mR_p\simeq 36$ K. The typical number of electrons in a droplet is $N_e=\sqrt{\pi n_d}R_p\simeq 6$, which implies a mean level spacing in the droplets of $\delta\sim\Delta/N_e=6$ K. The distance r between the surfaces of two neighboring droplets is $r=l_{ip}-2R_p\simeq 20$ nm. The localization length for interdroplet tunneling can be obtained from the size of the barrier, $\xi=\hbar/\sqrt{2mE_{\text{barrier}}}\simeq 10$ nm, which is of the order of the Bohr radius in GaAs but should be regarded as a coincidence. For our chosen parameters, r does not exceed ξ by a large amount.

III. MAGNETIC FIELD AND DENSITY DEPENDENCE OF INTERDROPLET TUNNELING

Having summarized the physical properties of the electron droplets, we briefly review their implications for magnetotransport. Unlike a dirty semiconductor where the electrons are localized at pointlike impurity sites,[15] l_{ip} is comparable with the droplet diameter. This scenario was studied in Ref. 16 where it was shown that the resistance between two droplets behaves as

$$\frac{\mathcal{R}(B)}{\mathcal{R}(0)}=e^{(B/B_0)^2}\frac{1}{\cosh^2(B/B_1)}, \tag{1}$$

with[10] $B_0\sim\phi_0/(\pi y_0 l_{ip})$ and $B_1\sim 2\phi_0/(\pi l_{ip}^2)$. We estimate that the spread of the wave function under the barrier in the direction perpendicular to the tunneling is $y_0\sim\sqrt{\xi r}$. B_1 is the field below which interference effects are significant. The magnetoresistance data in Ref. 1 can be explained with Eq. (1) without invoking a CDW or WC scenario. In particular, $B_0^{-2}\simeq Ar_{ee}^3$, which was the primary motivation for suggesting a CDW. The good agreement with experiment is strong evidence for the existence of electron droplets. Equation (1) expresses the inverse of the barrier transparency between droplets, and at large fields, the transparency decreases exponentially with increasing B. This implies that droplets become isolated from each other with increasing field, and it is then natural to expect that Coulomb blockade effects will strengthen with magnetic field, similar to those recently observed in a lattice of quantum dots.[5]

IV. MAGNETIC-FIELD-INDUCED COULOMB BLOCKADE

We now consider the properties of the magnetic-field-induced Coulomb blockade. The charging energy of a single droplet, E_c, differs from the bare value $E_c^0=e^2/(8\pi\epsilon_0\kappa R_p)$ (that of a single metallic sphere) if there is a gate voltage V_g that couples to the droplet through the gate capacitance C_g to give a gate charge $q_g=C_gV_g$, which takes values in the interval $[0,1/2]$. For nonzero q_g, $E_c(q_g)=E_c^0(1-2q_g)$, and hence the charging energy takes values between 0 and $E_c^0\simeq 20$ K (for $R_p\simeq 30$ nm).

In the devices of interest, there are many neighboring droplets which have a depolarizing effect, renormalizing the bare charging energy. There is charge screening for distances greater than R_c, and if $R_c\lesssim 7R_p$ (easily realized in experiment, since $R_p\sim 0.6R_c$), one can assume that the droplet array only has nearest neighbor interactions. For a hexagonal array, we estimate the effective charging energy as $E_c^{\text{eff}}\simeq 0.22E_c^0\simeq 4.4$ K, and so expect strong renormalization of the droplet charging energy.

To calculate the excitation energy of an N_e-electron droplet, we need to consider the level separation δ as well as the effective charging energy $(1-2q_g)E_c^{\text{eff}}$. Due to the small size of the droplet, the Fermi energy E_F can lie between levels ε_{N_e+1} and ε_{N_e}, corresponding to N_e+1 and N_e electrons in the droplet, respectively. The energy to add an electron to the droplet is thus

$$E_{\text{exc}}^{N_e+1,N_e}=(1-2q_g)E_c^{\text{eff}}+\min[\varepsilon_{N_e+1}-E_F,\varepsilon_{N_e+1}-\varepsilon_{N_e}]. \tag{2}$$

The gate voltage tunes both q_g and E_F, so that the first term oscillates between 0 and E_c^{eff} and the second between 0 and δ. Equation (2) is for an isolated droplet, and tunneling into the droplet will reduce E_{exc}.

In a magnetic field, the energies ε_{N_e} are modified due to both Zeeman splitting and orbital effects. For fields of the order of a tesla, as is the case in Ref. 1, the Zeeman splitting $g\mu_BB\ll\delta$ can be ignored. Orbital excitations will generically be affected by a magnetic field, and when the cyclotron energy $\hbar\omega_c=\hbar eB/m\sim\delta$, one can use the Darwin-Fock theory[17] to determine the single-particle energy levels. In our case, for $B=1$ T, $\hbar\omega_c/2\simeq 10$ K is of the order of $\delta\simeq 6$ K. However, at our level of analysis, such an improvement in accuracy does not strongly affect our results. This is also borne out by experiment, where the positions of the Coulomb blockade peaks and/or troughs in a resistance versus gate voltage plot do not shift with field in the magnetic field range considered.[4]

Treating electron droplets as effectively quantum dots, we now turn to consider the resistance that arises due to tunneling between droplets as a function of temperature. The tunneling rate Γ between two droplets or between a droplet and leads is proportional to the corresponding transmission probability (and hence both droplet spacing and magnetic field) and the density of states on either side of the barrier. For sequential tunneling at temperatures T such that $\hbar\Gamma \ll k_B T \ll \delta, E_{\rm exc}$, the Coulomb blockade gives rise to the droplet conductance[18,19]

$$G_{CB} = \frac{G_0}{\cosh^2\left(\frac{E_{\rm exc}}{2k_BT}\right)}, \quad G_0 = \frac{g_s e^2}{4hk_BT}\frac{\Gamma_l\Gamma_r}{\Gamma_l+\Gamma_r}, \tag{3}$$

where G_0 is the peak value of the conductance of a droplet connected through (forward) scattering rates $\Gamma_{l,r}$ with its left and right neighbors and g_s is the spin degeneracy. At temperatures lower than $\hbar\Gamma/k_B$, G_0 saturates to $g_s e^2/h$. In Eq. (3), magnetic field controls Γ, whereas density (or E_F) controls $E_{\rm exc}$. There are also parallel conductance channels, such as resonant cotunneling[20,21] which is 1 order higher in the (small) tunneling probability and significant only at very low temperatures. For a single droplet (say the ith), the transmission probability $\mathcal{T}_{\rm cotunn}$ associated with cotunneling is (when $k_BT \ll \hbar\Gamma$)

$$\mathcal{T}^{(i)}_{\rm cotunn} = \frac{\Gamma^{(i)}_l\Gamma^{(i)}_r}{2(\Gamma^{(i)}_l+\Gamma^{(i)}_r)}\frac{\Gamma^{(i)}}{(E^{(i)}_{\rm exc}/\hbar)^2+(\Gamma^{(i)}/2)^2}, \tag{4}$$

where the total decay width $\Gamma^{(i)}$ includes inelastic as well as the elastic contributions, $\Gamma^{(i)}_{l,r}$. As inelastic processes are usually present, $\Gamma^{(i)} > \Gamma^{(i)}_l + \Gamma^{(i)}_r$.

For a string of $\mathcal{N}$ droplets, the total cotunneling transmission $\mathcal{T}$ is the product of the cotunneling transmissions, $\mathcal{T} = \Pi_{i=1}^{\mathcal{N}} \mathcal{T}^{(i)}_{\rm cotunn}$, of individual droplets in the string, and hence the cotunneling conductance is $G_{\rm cotunn} = g_s(e^2/h)\mathcal{T}$. This contribution is thus small unless there are uniformly spaced identical droplets, in which case there can be a contribution at resonance.[21]

The relative importance of cotunneling and activated conduction changes as the droplet separation, $l_{ip}-2r$, decreases toward ξ, implying that $\hbar\Gamma$ approaches $E_{\rm exc}$. This allows cotunneling to contribute to conductance at low temperatures. Assuming both contributions acting in parallel, we estimate the droplet resistance as

$$\mathcal{R} \simeq \frac{\cosh^2\left(\frac{E_{\rm exc}}{2k_BT}\right)}{G_0 + G_{\rm cotunn}\cosh^2\left(\frac{E_{\rm exc}}{2k_BT}\right)}. \tag{5}$$

Equation (5) is not valid in the high or very low temperature limits since it implies $\mathcal{R}\to 0$ as $1/T \to 0$ and does not include the fact that G_0 saturates as $T\to 0$. We use Eq. (5) to fit

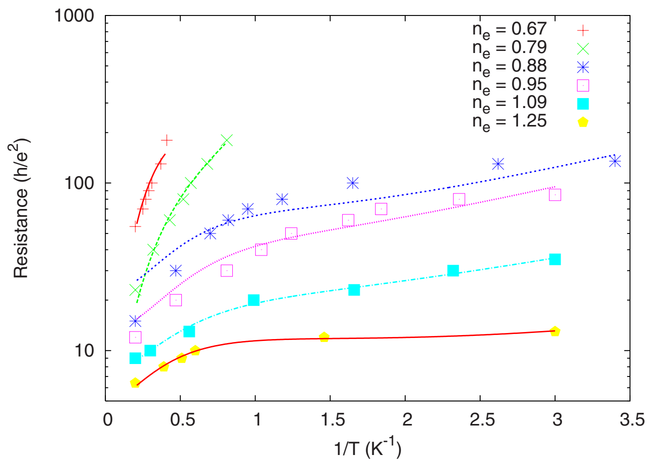

FIG. 1. (Color online) Temperature dependence of resistance data from Ref. 1 fitted to Eq. (5) with an additional series resistance. The fits yield an excitation energy $E_{\rm exc} \sim 1$ K $\sim \delta_{\rm sample}$. The electron densities n_e are shown in units of 10^{10} cm^{-2}.

resistance versus T data from Ref. 1, and in so doing additionally assume a series resistance R_0, so that the total resistance $R = R_0 + \mathcal{R}$. We also assume a phenomenological form for $G_{\rm cotunn} = G_{\rm cotunn}(T=0)(1+cT+aT^2)$, with a and c nonnegative fitting parameters. Physically, the quadratic-T dependence comes from inelastic cotunneling and the linear-T dependence arises from the linear suppression of the tunneling density of states due to the Anderson orthogonality catastrophe when the droplet is coupled asymmetrically to the leads,[22] which is the generic situation. We find excellent agreement with experiment at almost all densities, as is evident in Fig. 1, and we extract $E_{\rm exc} \simeq 1$ K at most values of n_e. We note that extrapolation of the resistance in the Coulomb blockade regime to $B=0$ leads to a resistance that is always greater than $h/2e^2$, implying that there is, effectively, never more than one conductance channel open for transport, in agreement with the success of Eq. (5) in fitting the data.

As V_g is varied, the droplet energy levels cross E_F, and since $E_{\rm exc}$ also depends on V_g [see Eq. (2)], the excitation energy for a droplet will vary between 0 and $E_c^{\rm eff}+\delta$. If there are many droplets, which we believe is the case here, we assume that the excitation energies are uniformly distributed in the interval $[0, E_c^{\rm eff}+\delta]$. In Fig. 2, we plot resistance as a function of n_e for illustrative purposes. We used Eqs. (3)–(5), along with the schematic form $E_{\rm exc} = \frac{E_0}{2}\left[1+f\cos\left(\frac{2\pi(n_e-n_0)}{\Delta n}\right)\right]$, where $0 \lesssim f \lesssim 1$ (we choose $f=0.6$ here), $E_0 = 1$ K, $T=1$ K, $\Delta n = 0.6\times 10^{10}$ cm^{-2}, and $n_0 = 0.5\times 10^{10}$ cm^{-2}. We assume that $\Gamma_l = \Gamma_r = \Gamma$ and the n_e and B dependence of Γ is determined by Eq. (1). The behavior seen in Fig. 2 can be deduced very easily by looking at limits of Eq. (5). In the limit that $G_0 \ll G_{\rm cotunn}$, $\mathcal{R} \sim \frac{1}{G_{\rm cotunn}}$, whereas when $G_0 \gg G_{\rm cotunn}$, $\mathcal{R} \sim \frac{1}{G_0}\cosh^2\left(\frac{E_{\rm exc}}{2k_BT}\right)$. We expect that both $G_{\rm cotunn}$ and G_0 will decrease with n_e, but G_0 will decrease faster since $\Gamma^{(i)}$ includes inelastic as well as elastic contributions, which are likely to be less density dependent. This would imply a crossover from oscillations to increasing resistance with decreasing density, exactly as shown in Fig. 2.

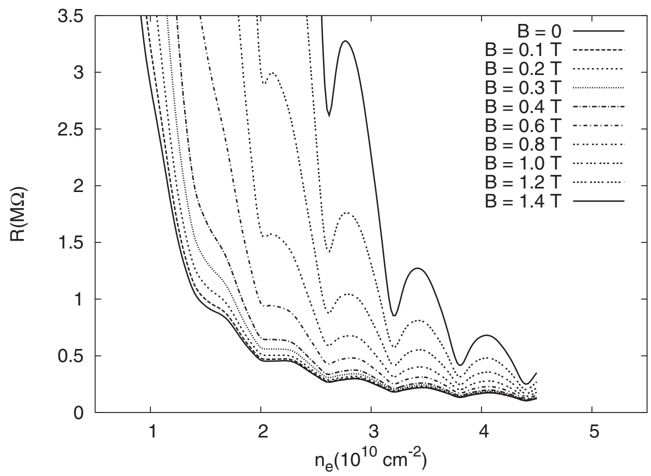

FIG. 2. Plot showing magnetic-field-induced Coulomb blockade calculated using Eq. (5). For small values of the magnetic field, the interdroplet tunneling is strong enough to wash out Coulomb blockade effects. Large magnetic fields reduce the interdroplet tunneling by shrinking the localization length, leading to an enhanced visibility of Coulomb blockade oscillations.

The temperature dependence of thermally activated transport depends on the size of the sample. For a sample large enough to contain many droplets, there will be some with arbitrarily low activation energy, in which case transport will proceed via variable range hopping (VRH),[23–25] and we do not expect to see Coulomb blockade oscillations.[26] The crossover length for Arrhenius to Mott VRH behavior may be estimated as follows. For Mott VRH in two dimensions, the characteristic hopping distance D_{Mott} is given by $D_{\mathrm{Mott}} = (\xi/\pi\nu k_B T)^{1/3}$, where ν is the density of available localized states per unit area:

$$\nu \approx \frac{1}{\pi(l_{ip}/2)^2(\delta + E_c^{\mathrm{eff}})}. \tag{6}$$

This gives $D_{\mathrm{Mott}} \simeq 60T^{-1/3}$ nm K$^{-1/3}$, and at $T=50$ mK, $D_{\mathrm{Mott}} \simeq 160$ nm. In Ref. 1, an Arrhenius law in temperature is observed, implying that the active device area should be less than D_{Mott}^2. The total number of droplets N_p in an area πD_{Mott}^2 is

$$N_p = \frac{4D_{\mathrm{Mott}}^2}{l_{ip}^2} = \left(\frac{2\xi(\delta + E_c^{\mathrm{eff}})}{l_{ip}k_B T}\right)^{2/3}, \tag{7}$$

which is about 15 for the typical parameters we assume. Some low density samples do show VRH behavior,[27] and hence we assume that the effective size of the sample is of the order of D_{Mott}, so the mean level separation in the sample is approximately $\delta_{\mathrm{sample}} \approx \frac{(E_c^{\mathrm{eff}}+\delta)}{N_p}$, which is about 0.75 K. The separation of the magnetoresistance peak from a trough corresponds to an energy scale of $\simeq 1$ K, in good agreement with both the activation energy deduced from experiment and with δ_{sample}.

As V_g is varied, excited states in different droplets successively come into resonance; these resonances are associated with the minima of resistance. In between the minima, if the sample is small, no droplet in the system is in resonance with E_F and the resistance will show a maximum. The observability of resistance oscillations therefore crucially depends on the samples being small. This seems to be borne out in experiment.[1]

At large electron densities, the oscillations will be less visible for two reasons. First, as n_e increases, l_{ip} decreases, as does E_c^{eff}, which reduces δ_{sample}. This reduces the contrast between resonant and Coulomb blockaded states. Second, the interdroplet tunneling distance, $D \propto 1/\sqrt{n_e}$, decreases[10] and hence Γ increases. Ultimately, when the localization condition, $D/\xi \gtrsim 1$, cannot be satisfied, the system becomes well conducting and no Coulomb blockade oscillations are possible.

Increasing magnetic field reduces Γ, with relatively little effect on excitation energies which improves the visibility of the Coulomb blockade. In experiment, the resistance oscillations tend to appear above a threshold field, which we estimate to be where the magnetoresistance switches from negative to positive, in the vicinity of $B_1 \propto n_e$, where B_1 is defined below Eq. (1).

V. DISCUSSION

In some respects, the issues discussed here are similar to observations in *one dimensional* wires of variations in the conductance periodic in n_e.[28] These were initially described in terms of a CDW, whereas later investigations appear to have convincingly demonstrated that the oscillations are due to Coulomb blockade effects.[29,30] It was found that the Coulomb blockade effects strengthened as a magnetic field was applied, consistent with the picture proposed here.[30] However, the Coulomb blockade did not rely on magnetic field for visibility.

We did not discuss how nonlinear screening is affected by the presence of a magnetic field. The screening of 2D electrons in a disordered potential in a magnetic field was discussed in Refs. 31 and 32, focusing on the regime where disorder is not too strong. We note that experiment appears to provide some of the solution. Measurements of localized states in the quantum Hall regime[4] that support a dotlike picture at low densities find the local electronic compressibility to be essentially independent of magnetic field. We also ignored possible correlations of donor charges in the dopant layer—including these worsened the agreement with experiment.[10] However, donor correlations are likely to be relevant in some cases.[33]

In summary, we use the picture of electron droplets to establish that for low density 2DEGs in disordered delta-doped heterostructures, there can be a magnetic-field-induced Coulomb blockade. We provide evidence for this picture by using a model for resistance as a function of temperature based on the idea of electron droplets acting like quantum dots to successfully fit experimental data. The ideas we present here may have wider applicability in inhomogeneous strongly correlated electron systems.

ACKNOWLEDGMENTS

We thank D. Khmelnitskii, N. Cooper, A. Ghosh, and M. Baenninger for useful discussions and A. Ghosh and M. Baenninger for sharing unpublished data. V.T. thanks TIFR and DST for financial support [Sanction No. 100/IFD/154/2007-08], and M.P.K. acknowledges support from NSERC and an SFU President's Research Grant. We also acknowledge the support of Trinity College, Cambridge where this work began.

[1] M. Baenninger, A. Ghosh, M. Pepper, H. E. Beere, I. Farrer, P. Atkinson, and D. A. Ritchie, Phys. Rev. B **72**, 241311(R) (2005).

[2] A. Ghosh, M. Pepper, H. E. Beere, and D. A. Ritchie, Phys. Rev. B **70**, 233309 (2004).

[3] A. Ghosh, C. J. B. Ford, M. Pepper, H. E. Beere, and D. A. Ritchie, Phys. Rev. Lett. **92**, 116601 (2004).

[4] S. Ilani, J. Martin, E. Teitelbaum, J. H. Smet, D. Mahalu, V. Umansky, and A. Yacoby, Nature (London) **427**, 328 (2004).

[5] A. Dorn, T. Ihn, K. Ensslin, W. Wegscheider, and M. Bichler, Phys. Rev. B **70**, 205306 (2004).

[6] M. Baenninger, A. Ghosh, M. Pepper, H. E. Beere, I. Farrer, P. Atkinson, and D. A. Ritchie, arXiv:0707.3543 (unpublished).

[7] B. Tanatar and D. M. Ceperley, Phys. Rev. B **39**, 5005 (1989); S. T. Chui and B. Tanatar, Phys. Rev. Lett. **74**, 458 (1995).

[8] V. A. Gergel' and R. A. Suris, Zh. Eksp. Teor. Fiz. **75**, 191 (1978) [Sov. Phys. JETP **48**, 95 (1978)].

[9] J. A. Nixon and J. H. Davies, Phys. Rev. B **41**, 7929 (1990); J. Shi and X. C. Xie, Phys. Rev. Lett. **88**, 086401 (2002); M. M. Fogler, Phys. Rev. B **69**, 121409(R) (2004); **69**, 245321 (2004); **70**, 129902(E) (2004); J. Wiebe, Chr. Meyer, J. Klijn, M. Morgenstern, and R. Wiesendanger, *ibid.* **68**, 041402 (2003).

[10] V. Tripathi and M. P. Kennett, Phys. Rev. B **74**, 195334 (2006).

[11] S. Ilani, A. Yacoby, D. Mahalu, and H. Shtrikman, Science **292**, 1354 (2001); J. Martin, S. Ilani, B. Verdene, J. Smet, V. Umansky, D. Mahalu, D. Schuh, G. Abstreiter, and A. Yacoby, *ibid.* **305**, 980 (2004).

[12] B. Spivak and S. A. Kivelson, Phys. Rev. B **70**, 155114 (2004); J. Phys. IV **131**, 255 (2005); R. Jamei, S. Kivelson, and B. Spivak, Phys. Rev. Lett. **94**, 056805 (2005).

[13] S. H. Pan, J. P. O'Neal, R. L. Badzey, C. Chamon, H. Ding, J. R. Engelbrecht, Z. Wang, H. Eisaki, S. Uchida, A. K. Gupta, K.-W. Ng, E. W. Hudson, K. M. Lang, and J. C. Davis, Nature (London) **413**, 282 (2001); K. M. Lang, V. Madhavan, J. E. Hoffman, E. W. Hudson, H. Eisaki, S. Uchida, and J. C. Davis, *ibid.* **415**, 412 (2002); N. Mathur and P. B. Littlewood, Phys. Today **56** (1), 25 (2003); S. Nakatsuji, V. Dobrosavljevic, D. Tanaskovic, M. Minakata, H. Fukazawa, and Y. Maeno, Phys. Rev. Lett. **93**, 146401 (2004).

[14] K. McElroy, Jinho Lee, J. A. Slezak, D.-H. Lee, H. Eisaki, S. Uchida, and J. C. Davis, Science **309**, 1048 (2005).

[15] B. I. Shklovskii and A. L. Efros, Zh. Eksp. Teor. Fiz. **84**, 811 (1983) [Sov. Phys. JETP **57**, 470 (1983)]; B. I. Shklovskii, Fiz. Tekh. Poluprovodn. (S.-Peterburg) **17**, 2055 (1983) [Sov. Phys. Semicond. **17**, 1311 (1983)].

[16] M. E. Raikh and L. I. Glazman, Phys. Rev. Lett. **75**, 128 (1995).

[17] C. G. Darwin, Proc. Cambridge Philos. Soc. **27**, 86 (1930); V. Fock, Z. Phys. **47**, 446 (1928).

[18] L. I. Glazman and R. I. Shekhter, J. Phys.: Condens. Matter **1**, L5811 (1989).

[19] C. W. J. Beenakker, Phys. Rev. B **44**, 1646 (1991).

[20] B. Ricco and M. Ya. Azbel, Phys. Rev. B **29**, 1970 (1984); A. D. Stone and P. A. Lee, Phys. Rev. Lett. **54**, 1196 (1985).

[21] M. Büttiker, Phys. Rev. B **33**, 3020 (1986).

[22] A. Furusaki and K. A. Matveev, Phys. Rev. B **52**, 16676 (1995).

[23] N. F. Mott, J. Non-Cryst. Solids **1**, 1 (1968).

[24] V. Ambegoakar, B. I. Halperin, and J. S. Langer, Phys. Rev. B **4**, 2612 (1971).

[25] A. L. Efros and B. I. Shklovskii, J. Phys. C **8**, L49 (1975).

[26] A. B. Fowler, Proc. Natl. Acad. Sci. U.S.A. **84**, 4701 (1987).

[27] A. Ghosh, M. Pepper, D. A. Ritchie, E. H. Linfield, R. H. Harrell, H. E. Beere, and G. A. C. Jones, Phys. Status Solidi B **230**, 211 (2002).

[28] J. H. F. Scott-Thomas, S. B. Field, M. A. Kastner, H. I. Smith, and D. A. Antoniadis, Phys. Rev. Lett. **62**, 583 (1989); M. A. Kastner, S. B. Field, U. Meirav, J. H. F. Scott-Thomas, D. A. Antoniadis, and H. I. Smith, *ibid.* **63**, 1894 (1989); U. Meirav, M. A. Kastner, M. Heiblum, and S. J. Wind, Phys. Rev. B **40**, 5871(R) (1989).

[29] H. van Houten and C. W. J. Beenakker, Phys. Rev. Lett. **63**, 1893 (1989).

[30] A. A. M. Staring, H. van Houten, C. W. J. Beenakker, and C. T. Foxon, Phys. Rev. B **45**, 9222 (1992).

[31] A. L. Efros, Solid State Commun. **67**, 1019 (1988).

[32] A. L. Efros, F. G. Pikus, and V. G. Burnett, Phys. Rev. B **47**, 2233 (1993).

[33] C. Siegert, A. Ghosh, M. Pepper, I. Farrer, and D. A. Ritchie, Nat. Phys. **3**, 315 (2007).