Energy relaxation of a superconducting charge qubit via Andreev processes

R. M. Lutchyn and L. I. Glazman
W.I. Fine Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455, USA
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We study fundamental limitations on the energy relaxation rate of a superconducting charge qubit with a large-gap Cooper-pair box, $\Delta_b > \Delta_r$. At a sufficiently large mismatch between the gap energies in the box Δ_b and in the reservoir Δ_r, "quasiparticle poisoning" becomes ineffective even in the presence of nonequilibrium quasiparticles in the reservoir. The qubit relaxation still may occur due to higher-order (Andreev) processes. In this paper, we evaluate the qubit energy relaxation rate T_1^{-1} due to Andreev processes.

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I. INTRODUCTION

A large number of recent experimental studies[1–9] indicate the presence of quasiparticles in superconducting single-charge devices at low temperatures. The operation of these devices, of which the best known is the Cooper-pair box qubit, requires $2e$-periodic dependence of the charge of the box on its gate voltage, and thus an introduction of an unpaired electron (quasiparticle) in the Cooper-pair box (CPB) is a significant problem. The superconducting charge qubit operates at the degeneracy point for Cooper pairs, $N_g=1$, with N_g being the dimensionless gate voltage. For equal gap energies in the Cooper-pair box and reservoir, $\Delta_b=\Delta_r$, the states of the qubit at $N_g=1$ are unstable with respect to quasiparticle tunneling to the box. The quasiparticle changes the charge state of CPB from even to odd and lowers the charging energy. This phenomenon, commonly referred to as "quasiparticle poisoning," is well known from the studies of the charge parity effect in superconductors, see, for example, Ref. 10 and references therein. Quasiparticle poisoning can degrade the performance of the charge qubit in two ways. First, it causes the operating point of the qubit to shift stochastically on the time scale comparable with the measurement time.[6] Second, it contributes to the decoherence.[11] One of the approaches to improve the performance of charge qubits is to use superconducting gap engineering. In most single-charge superconducting devices, quasiparticle poisoning can be suppressed even in the presence of nonequilibrium quasiparticles in the reservoir by engineering a large mismatch between Δ_b and Δ_r. Gap energies in superconductors can be modified by oxygen doping,[2] applying a magnetic field,[4,5] and adjusting layer thickness.[7,8] In this paper, we study the fundamental limitations on the energy relaxation time in a charge qubit with a large gap in the box, $\Delta_b > \Delta_r$.

For equal gap energies in the box and reservoir, $\Delta_b=\Delta_r$, the energy relaxation rate due to quasiparticle poisoning[11] is

$$\frac{1}{T_1} \propto \frac{g_T n_{\rm qp}}{\hbar \nu_F}\sqrt{\frac{T}{E_J}}, \tag{1}$$

with $n_{\rm qp}$, g_T, and ν_F being the density of quasiparticles in the reservoir, dimensionless conductance of the junction, and density of states at the Fermi level, respectively. The relaxation rate $1/T_1$ in Eq. (1) was derived under the assumption that an unpaired electron tunnels from the reservoir to the box to minimize the energy of the system. Indeed, for $\Delta_b=\Delta_r$, the odd-charge state of the CPB has lower energy at $N_g=1$ due to the Coulomb blockade effect. By properly engineering superconducting gap energies (i.e., inducing large gap mismatch, $\Delta_b > \Delta_r$), one can substantially reduce the quasiparticle tunneling rate to the Cooper-pair box. Suppose initially that the qubit is in the excited state with energy $E_{|+\rangle}$, and the quasiparticle is in the reservoir with energy E_p. Upon quasiparticle tunneling to the box, the minimum energy of the final state is $E_f^{\min}=\Delta_b+E_{N+1}$ with E_{N+1} being the energy of the CPB in the odd-charge state. Therefore, the threshold energy for a quasiparticle to tunnel to the box is $E_p^{\min}=\Delta_b+E_{N+1}-E_{|+\rangle}$, see also Fig. 1. If $E_p^{\min}-\Delta_r \gtrsim E_J \gg T$, only exponentially small fraction of quasiparticles are able to tunnel into the island. (Note that the energy difference between excited and ground states of a charge qubit is E_J, while the energy of the qubit in the excited state is $E_{|+\rangle}=E_c+E_J/2$. Here, E_c, E_J, and T are the charging energy of the CPB, the Josephson energy associated with the tunnel junction, and the temperature, respectively.) Thus, the contribution to the qubit relaxation rate T_1^{-1} from the processes involving real quasiparticle tunneling to the island becomes

$$\frac{1}{T_1} \propto \frac{g_T n_{\rm qp}}{\hbar \nu_F}\exp\left(-\frac{\Delta_b-\Delta_r-E_c-E_J/2}{T}\right) \tag{2}$$

and is much smaller than the one of Eq. (1). [To obtain Eq. (2), we used the fact that $E_{N+1}=0$ at $N_g=1$.] However, there

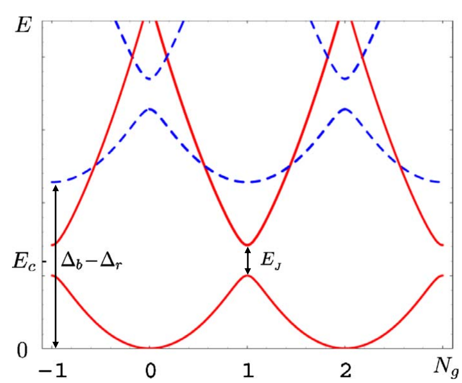

FIG. 1. (Color online) The spectrum of the Cooper-pair box as a function of the dimensionless gate voltage for a large-gap mismatch, $\Delta_b > \Delta_r$. The solid and dashed lines correspond to even- and odd-charge states of the box, respectively.

is also a mechanism of energy relaxation originating from the higher-order tunneling processes (Andreev reflection). The contribution of these processes to the qubit relaxation is activationless and can be much larger than the one of Eq. (2). In the rest of the paper, we study qubit energy relaxation due to Andreev processes in detail.

II. THEORETICAL MODEL

Dynamics of the Cooper-pair box coupled to the superconducting reservoir through the tunnel junction is described by the Hamiltonian

$$H = H_C + H^b_{\mathrm{BCS}} + H^r_{\mathrm{BCS}} + H_T. \tag{3}$$

Here, H^b_{BCS} and H^r_{BCS} are BCS Hamiltonians for the box and reservoir; $H_C = E_c(Q/e - N_g)^2$ with N_g and Q being the dimensionless gate voltage and the charge of the CPB, respectively. We consider the following energy scale hierarchy: $\Delta_b > \Delta_r$, $E_c > E_J \gg T$. In order to distinguish between Cooper pair and quasiparticle tunneling, we present Hamiltonian (3) in the form[12]

$$H = H_0 + V \quad \text{and } V = H_T - H_J. \tag{4}$$

Here, $\mathrm{H}_0 = H_C + H^b_{\mathrm{BCS}} + H^r_{\mathrm{BCS}} + H_J$, and H_J is the Hamiltonian describing the Josephson tunneling,

$$H_J = |N\rangle\langle N|H_T \frac{1}{E - H_0} H_T|N+2\rangle\langle N+2| + \mathrm{H.c.}$$

The matrix element $\langle N|H_T\frac{1}{E-H_0}H_T|N+2\rangle$ is proportional to the Josephson energy E_J. The perturbation Hamiltonian V defined in Eq. (4) is suitable for the calculation of the quasiparticle tunneling rate. The tunneling Hamiltonian for the homogeneous insulating barrier is

$$H_T = \sum_\sigma \int d\mathbf{x}d\mathbf{x}'[T(\mathbf{x},\mathbf{x}')\Psi^\dagger_\sigma(\mathbf{x})\Psi_\sigma(\mathbf{x}') + \mathrm{H.c.}], \tag{5}$$

where $\mathbf{x}$ and $\mathbf{x}'$ denote the coordinates in the CPB and reservoir, respectively, and $T(\mathbf{x},\mathbf{x}')$, in the limit of a barrier with low transparency, is defined as

$$T(\mathbf{x},\mathbf{x}') = \frac{1}{4\pi^2}\sqrt{\frac{\mathcal{T}}{\nu_F^2}}\delta^2(\mathbf{r}-\mathbf{r}')\delta(z)\delta(z')\frac{\partial}{\partial z}\frac{\partial}{\partial z'}. \tag{6}$$

Here, $\mathcal{T}$ is the transmission coefficient of the barrier, and $\mathbf{r}$ and z are the coordinates in the plane of the tunnel junction and perpendicular to it, respectively. Hamiltonian (5) along with the above definition of $T(\mathbf{x},\mathbf{x}')$ properly takes into account the fact that in the tunnel-Hamiltonian approximation, the wave functions turn to zero at the surface of the junction.[13,14] In terms of the transmission coefficient $\mathcal{T}$, the dimensionless conductance of the tunnel junction g_T can be defined as $g_T = \mathcal{T}S_J k_F^2/4\pi = \frac{1}{3}\mathcal{T}N_{\mathrm{ch}}$, where S_J is the area of the junction and N_{ch} is the number of transverse channels in the junction.

The energy relaxation rate of the qubit due to higher-order processes is given by

$$\Gamma_A = \frac{2\pi}{\hbar}\sum_{p,p'} 2|A_{p'p}|^2\delta(E_{p'} - E_p - E_J)f_F(E_p)[1 - f_F(E_{p'})]. \tag{7}$$

Here, $f_F(E_p)$ is the Fermi distribution function with $E_p = \sqrt{\varepsilon_p^2 + \Delta_r^2}$ being the energy of a quasiparticle in the reservoir. The amplitude $A_{p'p}$ is given by the second order perturbation theory in V,

$$A_{p'p} = \langle -, E_{p'\uparrow}|V\frac{1}{E_i - H_0}V| +, E_{p\uparrow}\rangle. \tag{8}$$

At $E_c \gg E_J$ and $N_g = 1$, the eigenstates of the qubit are given by the symmetric and antisymmetric superpositions of two charge states, i.e., $|-\rangle = \frac{|N\rangle + |N+2\rangle}{\sqrt{2}}$ and $|+\rangle = \frac{|N\rangle - |N+2\rangle}{\sqrt{2}}$ with the corresponding eigenvalues $E_{|\pm\rangle} = E_c \pm E_J/2$. In the initial moment of time, the qubit is prepared in the excited state and the quasiparticle is in the reservoir, i.e., $|+, E_{p\uparrow}\rangle \equiv |+\rangle \otimes |E_{p\uparrow}\rangle$. The energy of the initial state is $E_i = E_p + E_{|+\rangle}$. The denominator in the amplitude [Eq. (8)] corresponds to the formation of the virtual intermediate state when the quasiparticle has tunneled to the island from the reservoir. Since a quasiparticle is a superposition of a quasielectron and a quasihole, the contributions to $A_{p'p}$ come from two interfering paths,

$$A_{p'p} = \frac{1}{2}\langle N+2, E_{p'\uparrow}|V\frac{1}{E_i - H_0}V|N, E_{p\uparrow}\rangle - \frac{1}{2}\langle N, E_{p'\uparrow}|V\frac{1}{E_i - H_0}V|N+2, E_{p\uparrow}\rangle. \tag{9}$$

To calculate the amplitude $A_{p'p}$, we use the particle-conserving Bogoliubov transformation,[15–17]

$$\gamma^\dagger_{n\sigma} = \int d\mathbf{x}[U_n(\mathbf{x})\Psi^\dagger_\sigma(\mathbf{x}) - \sigma V_n(\mathbf{x})\Psi_{-\sigma}(\mathbf{x})R^\dagger],$$

$$\gamma_{n\sigma} = \int d\mathbf{x}[U_n(\mathbf{x})\Psi_\sigma(\mathbf{x}) - \sigma V_n(\mathbf{x})\Psi^\dagger_{-\sigma}(\mathbf{x})R]. \tag{10}$$

The operators $R^\dagger$ and R transform a given state in an N-particle system into the corresponding state in the $(N+2)$- and $(N-2)$-particle systems, respectively, leaving the quasiparticle distribution unchanged, i.e., $R^\dagger|N\rangle = |N+2\rangle$. Thus, quasiparticle operators $\gamma^\dagger_{n\sigma}$ and $\gamma_{n\sigma}$ defined in Eq. (10) do conserve particle number.[18] The transformation coefficients $U_n(\mathbf{x})$ and $V_n(\mathbf{x})$ are given by the solution of the Bogoliubov–de Gennes equation. For spatially homogeneous superconducting gap Δ, the functions $U_n(\mathbf{x})$ and $V_n(\mathbf{x})$ can be written as $U_n(\mathbf{x}) = u_n\phi_n(\mathbf{x})$ and $V_n(\mathbf{x}) = v_n\phi_n(\mathbf{x})$. The coherence factors u_n and v_n are given by

$$u_n^2 = \frac{1}{2}\left(1 + \frac{\varepsilon_n}{E_n}\right) \quad \text{and } v_n^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_n}{E_n}\right).$$

Here, $E_n = \sqrt{\varepsilon_n^2 + \Delta^2}$; ε_n and $\phi_n(\mathbf{x})$ are exact eigenvalues and eigenfunctions of the single-particle Hamiltonian, which may include a random potential $\mathcal{V}(\mathbf{x})$, e.g., due to impurities. The single-particle energies ε_n and wave functions $\phi_n(\mathbf{x})$ are defined by the following Shrödinger equation:

$$\left[-\frac{\hbar^2}{2m}\vec{\nabla}^2+\mathcal{V}(\mathbf{x})\right]\phi_n(\mathbf{x})=\varepsilon_n\phi_n(\mathbf{x}).$$

In the presence of time-reversal symmetry, u_n, v_n, and $\phi_n(\mathbf{x})$ can be taken to be real. Then, with the help of Eq. (10), we obtain the amplitude of the process $A_{p'p}$,

$$A_{p'p}=\frac{1}{2}\int d\mathbf{x}_1 d\mathbf{x}_1' d\mathbf{x}_2 d\mathbf{x}_2' T(\mathbf{x}_1,\mathbf{x}_1')T(\mathbf{x}_2,\mathbf{x}_2')[U_{p'}(\mathbf{x}_1')V_p(\mathbf{x}_2') - U_p(\mathbf{x}_1')V_{p'}(\mathbf{x}_2')]\sum_k \frac{U_k(\mathbf{x}_1)V_k(\mathbf{x}_2)}{E_p+\delta E_+-E_k}, \quad (11)$$

where $\delta E_+ \equiv E_{|+\rangle}-E_{N+1}=E_c+E_J/2$. The minus sign in the parentheses here reflects the destructive interference between quasielectron and quasihole contributions, see also Eq. (9).

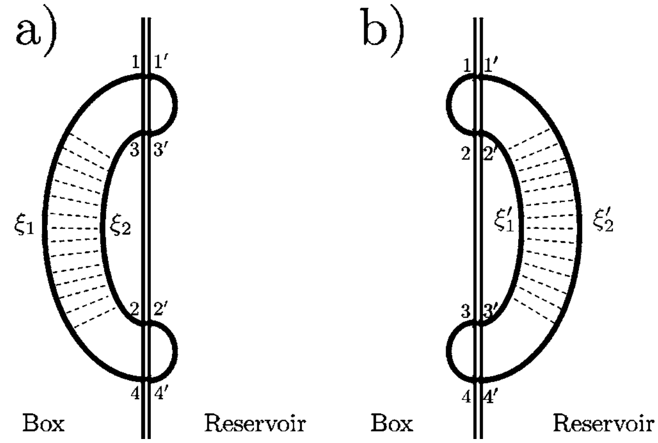

FIG. 2. The diagrams corresponding to the interference of electron trajectories in the (a) box and (b) reservoir. The contribution of the diagrams with interference in both electrodes (not shown) is much smaller than the one of the above diagrams (Ref. 19).

III. DISORDER AVERAGING

It is well known that Andreev conductance is sensitive to disorder, see, for example, Refs. 19 and 20. Similarly, the rate Γ_A is affected by electron backscattering to the tunnel junction, see Fig. 2. If a quasiparticle bounces off the walls of the box or impurities many times, it is reasonable to expect the chaotization of its motion. Thus, one is prompted to consider ensemble-averaged quantities rather than their particular realization. Using Eqs. (7) and (11), we obtain

$$\langle\Gamma_A\rangle=\frac{\pi}{\hbar}\Bigg\langle\sum_{p,p'}\int\prod_{i=1..4}d\mathbf{x}_i d\mathbf{x}_i' T(\mathbf{x}_1,\mathbf{x}_1')T(\mathbf{x}_2,\mathbf{x}_2')T(\mathbf{x}_3,\mathbf{x}_3')T(\mathbf{x}_4,\mathbf{x}_4')[u_{p'}v_p\phi_{p'}(\mathbf{x}_1')\phi_p(\mathbf{x}_2')-u_pv_{p'}\phi_p(\mathbf{x}_1')\phi_{p'}(\mathbf{x}_2')]$$
$$\times[u_{p'}v_p\phi_{p'}(\mathbf{x}_3')\phi_p(\mathbf{x}_4')-u_pv_{p'}\phi_p(\mathbf{x}_3')\phi_{p'}(\mathbf{x}_4')]\sum_k\frac{u_kv_k\phi_k(\mathbf{x}_1)\phi_k(\mathbf{x}_2)}{E_p+\delta E_+-E_k}\sum_{k'}\frac{u_{k'}v_{k'}\phi_{k'}(\mathbf{x}_3)\phi_{k'}(\mathbf{x}_4)}{E_p+\delta E_+-E_{k'}}$$
$$\times\delta(E_{p'}-E_p-E_J)f_F(E_p)[1-f_F(E_{p'})]\Bigg\rangle. \quad (12)$$

Here, the brackets $\langle\cdots\rangle$ denote averaging independently over different realizations of the random potential in the box and reservoir. In order to average over the disorder in the CPB, one has to calculate the following correlation function:

$$I\equiv\left\langle\sum_{k,k'}\frac{u_kv_k\phi_k(\mathbf{x}_1)\phi_k(\mathbf{x}_2)}{E_p+\delta E_+-E_k}\frac{u_{k'}v_{k'}\phi_{k'}(\mathbf{x}_3)\phi_{k'}(\mathbf{x}_4)}{E_p+\delta E_+-E_{k'}}\right\rangle = \int\frac{\Delta_b^2 d\xi_1 d\xi_2}{4E(\xi_1)E(\xi_2)}\frac{\langle K_{\xi_1}(\mathbf{x}_1,\mathbf{x}_2)K_{\xi_2}(\mathbf{x}_3,\mathbf{x}_4)\rangle}{[E_p+\delta E_+-E(\xi_1)][E_p+\delta E_+-E(\xi_2)]}, \quad (13)$$

where $K_\xi(\mathbf{x}_1,\mathbf{x}_2)=\sum_k\phi_k(\mathbf{x}_1)\phi_k(\mathbf{x}_2)\delta(\epsilon_k-\xi)$ and $E(\xi)=\sqrt{\xi^2+\Delta_b^2}$. The correlation function $\langle K_{\xi_1}(\mathbf{x}_1,\mathbf{x}_2)K_{\xi_2}(\mathbf{x}_3,\mathbf{x}_4)\rangle$ consists of reducible and irreducible parts,

$$\langle K_{\xi_1}(\mathbf{x}_1,\mathbf{x}_2)K_{\xi_2}(\mathbf{x}_3,\mathbf{x}_4)\rangle=\langle K_{\xi_1}(\mathbf{x}_1,\mathbf{x}_2)\rangle\langle K_{\xi_2}(\mathbf{x}_3,\mathbf{x}_4)\rangle + \langle K_{\xi_1}(\mathbf{x}_1,\mathbf{x}_2)K_{\xi_2}(\mathbf{x}_3,\mathbf{x}_4)\rangle_{\mathrm{ir}}. \quad (14)$$

The reducible part can easily be calculated by relating $\langle K_\xi(\mathbf{x}_1,\mathbf{x}_2)\rangle$ to the ensemble-averaged Green's function: $\langle K_\xi(\mathbf{x}_1,\mathbf{x}_2)\rangle\equiv-\frac{1}{\pi}\,\mathrm{Im}\langle G_\xi^R(\mathbf{x}_1,\mathbf{x}_2)\rangle=\nu_F f_{12}$. (Upon averaging over disorder, one can neglect the energy dependence of the density of states here, i.e., $\langle\nu_F(\xi)\rangle=\nu_F$. The function f_{12} is given by $f_{12}=\langle e^{i\mathbf{k}(\mathbf{x}_1-\mathbf{x}_2)}\rangle_{\mathrm{FS}}$ with $\langle\cdots\rangle_{\mathrm{FS}}$ being the average over electron momentum on the Fermi surface. For three-dimensional system, the function f_{12} is equal to $f_{12}=\frac{\sin(k_F|\mathbf{x}_1-\mathbf{x}_2|)}{k_F|\mathbf{x}_1-\mathbf{x}_2|}$.) The irreducible part $\langle K_{\xi_1}(\mathbf{x}_1,\mathbf{x}_2)K_{\xi_2}(\mathbf{x}_3,\mathbf{x}_4)\rangle_{\mathrm{ir}}$ can be expressed in terms of the classical diffusion propagators—diffusons and Cooperons, see, for example,

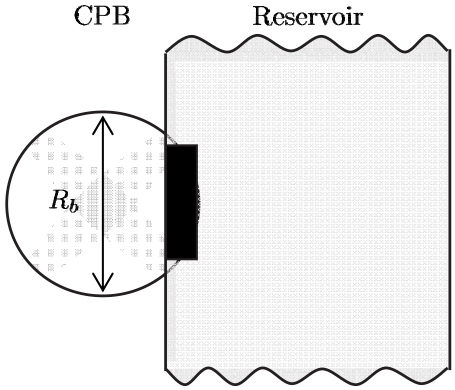

FIG. 3. The layout of the Cooper-pair box qubit considered in the text.

Ref. 21. In the absence of magnetic field, diffusons and Cooperons coincide, $\mathcal{P}_\omega(\mathbf{x}_1,\mathbf{x}_2)=\mathcal{P}^D_\omega(\mathbf{x}_1,\mathbf{x}_2)=\mathcal{P}^C_\omega(\mathbf{x}_1,\mathbf{x}_2)$, and the irreducible part of the correlation function [Eq. (14)] reads

$$\langle K_{\xi_1}(\mathbf{x}_1,\mathbf{x}_2)K_{\xi_2}(\mathbf{x}_3,\mathbf{x}_4)\rangle_{\rm ir}=\frac{\nu_F}{\pi}\,\mathrm{Re}[f_{14}f_{23}\mathcal{P}_{|\xi_2-\xi_1|}(\mathbf{x}_1,\mathbf{x}_3)+f_{13}f_{24}\mathcal{P}_{|\xi_2-\xi_1|}(\mathbf{x}_1,\mathbf{x}_4)]. \tag{15}$$

The spectral expansion of $\mathcal{P}_\omega(\mathbf{x}_1,\mathbf{x}_2)$ for the diffusive system is

$$\mathcal{P}_\omega(\mathbf{x}_1,\mathbf{x}_2)=\sum_n\frac{f_n^*(\mathbf{x}_1)f_n(\mathbf{x}_2)}{-i\omega+\gamma_n}. \tag{16}$$

Here, γ_n and $f_n(\mathbf{x})$ are the corresponding eigenvalues and eigenfunctions of the diffusion equation, $-D\vec{\nabla}^2f_n(\mathbf{x})=\gamma_nf_n(\mathbf{x})$, satisfying von Neumann boundary conditions in the box.

Equation (15) can be simplified in the case of large Thouless energy, i.e., $E_T\gg\Delta_b,\Delta_r,E_c,E_J$. (Here, $E_T=\hbar/\tau_D$ with $\tau_D\sim S_b/D$ being the time to diffuse through the box and S_b being the area of the island, see Fig. 3) This condition is fulfilled for a small aluminum island[22] with $S_b\ll 1\ \mu\mathrm{m}^2$ and mean free path $l\gtrsim 25$ nm,[23] when the time spent by the virtual quasiparticle in the box, $t\sim\hbar/(\Delta_b-\Delta_r-\delta E_+)$, is much longer than the classical diffusion time τ_D.[24] In this case, the irreducible part in Eq. (14) is given by the universal limit,

$$\langle K_{\xi_1}(\mathbf{x}_1,\mathbf{x}_2)K_{\xi_2}(\mathbf{x}_3,\mathbf{x}_4)\rangle_{\rm ir}=\frac{\nu_F}{V_b}\delta(\xi_1-\xi_2)(f_{14}f_{23}+f_{13}f_{24}). \tag{17}$$

Here, V_b is the volume of the box. Upon substituting Eqs. (14) and (17) into Eq. (13) and evaluating the integrals over energies ξ_1 and ξ_2, we obtain

$$I=4\nu_F^2f_{12}f_{34}L_1\left(\frac{E_p+\delta E_+}{\Delta_b}\right)+\nu_F^2\frac{\delta_b}{2\Delta_b}(f_{14}f_{23}+f_{13}f_{24})L_2\left(\frac{E_p+\delta E_+}{\Delta_b}\right), \tag{18}$$

where $\delta_b=1/\nu_FV_b$ is the mean level spacing in the box. The functions $L_1(y)$ and $L_2(y)$ are defined as

$$L_1(y)=\frac{1}{1-y^2}\arctan^2\left(\sqrt{\frac{1+y}{1-y}}\right),$$

$$L_2(y)=\int_1^\infty dx\frac{1}{\sqrt{x^2-1}}\frac{1}{x(x-y)^2}. \tag{19}$$

The expressions above are valid for $y<1$. The function $L_2(y)$ has the following asymptotes:

$$L_2(y)\approx\begin{cases}\pi/4+(4/3)y, & y\ll 1\\ \pi/2\sqrt{2}(1-y)^{3/2}, & 1-y\ll 1.\end{cases} \tag{20}$$

After substituting Eq. (18) into Eq. (12) and averaging over disorder in the reservoir, we obtain the following expression for $\langle\Gamma_A\rangle$:

$$\langle\Gamma_A\rangle=\frac{\pi\nu_F^2}{2\hbar}\int d\xi_1'd\xi_2'\delta(E(\xi_2')-E(\xi_1')-E_J)f_F[E(\xi_1')]\{1-f_F[E(\xi_2')]\}\int\prod_{i=1,\ldots,4}d\mathbf{x}_id\mathbf{x}_i'T(\mathbf{x}_1,\mathbf{x}_1')T(\mathbf{x}_2,\mathbf{x}_2')T(\mathbf{x}_3,\mathbf{x}_3')T(\mathbf{x}_4,\mathbf{x}_4')$$
$$\times\left\{4f_{12}f_{34}L_1\left[\frac{E(\xi_1')+\delta E_+}{\Delta_b}\right]+\frac{\delta_b}{2\Delta_b}(f_{14}f_{23}+f_{13}f_{24})L_2\left[\frac{E(\xi_1')+\delta E_+}{\Delta_b}\right]\right\}\left[1-\frac{\Delta_r^2}{E(\xi_1')E(\xi_2')}\right]\langle K_{\xi_1'}(\mathbf{x}_1',\mathbf{x}_3')K_{\xi_2'}(\mathbf{x}_2',\mathbf{x}_4')\rangle. \tag{21}$$

Here, $E(\xi')=\sqrt{\xi'^2+\Delta_r^2}$. The correlation function in the reservoir $\langle K_{\xi_1'}(\mathbf{x}_1',\mathbf{x}_3')K_{\xi_2'}(\mathbf{x}_2',\mathbf{x}_4')\rangle$ follows from Eqs. (14) and (15). Using Eq. (6) and evaluating the spatial integrals over the area of the junction as well as the integrals over energies ξ_1' and ξ_2', we finally obtain the answer for $\langle\Gamma_A\rangle$,

$$\langle\Gamma_A\rangle=\Gamma_1+\Gamma_2, \tag{22}$$

with Γ_1 and Γ_2 being defined as

$$\Gamma_1\approx\frac{2\pi}{\hbar}\frac{3C_1}{(4\pi^2)^2}\frac{g_T^2}{N_{\rm ch}}\sqrt{\frac{E_J}{2\Delta_r+E_J}}\frac{n_{\rm qp}}{\nu_F}L_1\left(\frac{\Delta_r+\delta E_+}{\Delta_b}\right) \tag{23}$$

and

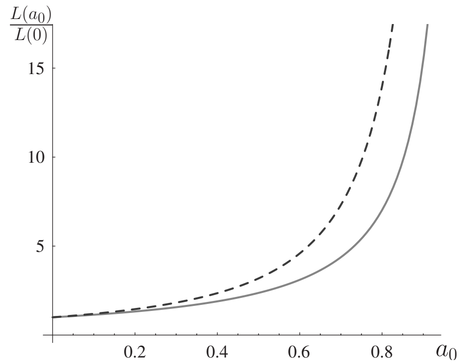

FIG. 4. The dependence of the functions $L_1(a_0)$ and $L_2(a_0)$ [normalized by $L_1(0)$ and $L_2(0)$, respectively] on the dimensionless parameter $a_0=(\Delta_r+\delta E_+)/\Delta_b$. The solid and dashed lines correspond to L_1 and L_2, respectively, and reflect the increase of the rates Γ_1 and Γ_2 with a_0. The expressions for $L_1(a_0)$ and $L_2(a_0)$ given by Eq. (19) are valid for $a_0 \ll 1-T/\Delta_b$.

$$\Gamma_2 \approx \frac{2\pi}{\hbar}\frac{g_T^2}{8(4\pi^2)^2}\frac{\delta_b}{\Delta_b}\sqrt{\frac{E_J}{2\Delta_r+E_J}}\frac{n_{\rm qp}}{\nu_F}L_2\left(\frac{\Delta_r+\delta E_+}{\Delta_b}\right). \tag{24}$$

Here, C_1 is a numerical constant of the order of 1,

$$C_1=\frac{1}{\pi^3 k_F^2 S_J}\int_{k_F^2 S_J} d\mathbf{y}_1 d\mathbf{y}_2 d\mathbf{y}_3 d\mathbf{y}_4 P_{12}P_{13}P_{24}P_{34},$$

with $\mathbf{y}$ being a dimensionless coordinate in the plane of a tunnel junction and $P_{12}=\frac{\sin(|\mathbf{y}_1-\mathbf{y}_2|)-|\mathbf{y}_1-\mathbf{y}_2|\cos(|\mathbf{y}_1-\mathbf{y}_2|)}{|\mathbf{y}_1-\mathbf{y}_2|^3}$. The functions L_1 and L_2 are defined in Eq. (19), and their dependence on the ratio $(\Delta_r+\delta E_+)/\Delta_b$ is shown in Fig. 4. The rate Γ_1 describes the contribution from the reducible terms, see Eq. (14), and is similar to the ballistic case when electron scattering from the impurities or boundaries is negligible. The other term, Γ_2, reflects the enhancement of $\langle\Gamma_A\rangle$ in the diffusive limit due to the quantum interference of quasiparticle return trajectories[25] and originates from the irreducible contributions, see Fig. 2. In the case of $N_{\rm ch}\delta_b/\Delta_b \gg 1$, the contribution of this interference term becomes dominant, $\Gamma_2 \gg \Gamma_1$. The contribution of the interference in the reservoir to the rate Γ_2, see Fig. 2(b), is geometry dependent. For a typical charge qubit with the small junction connected to a large electrode, backscattering of electrons to the junction from the reservoir side gives much smaller contribution to Γ_2 than the similar one for the box side of the junction. In particular, for the layout of the qubit shown in Fig. 3, the contribution of the interference in the reservoir to Γ_2 is smaller than the one in the box by a factor $\frac{d_b}{d_r}\frac{\Delta_b}{E_T}\ln\left(\frac{\hbar D}{\Delta_r S_J}\right)\frac{L_1(a_0)}{L_2(a_0)} \ll 1$. [Here, $a_0=(\Delta_r+\delta E_+)/\Delta_b$, and $d_{b(r)}$ is the thickness of the superconducting film in the box (reservoir).] Therefore, we neglected the terms corresponding to the interference in the reservoir in Eq. (24).

IV. CONCLUSION

We have studied the fundamental limitations on the energy relaxation time in a charge qubit with a large-gap Cooper-pair box, $\Delta_b > \Delta_r$. For sufficiently large Δ_b, real quasiparticle transitions can be exponentially suppressed, and the dominant contribution to the charge qubit energy relaxation time T_1 comes from the higher-order (Andreev) processes, see Eq. (22). For realistic geometry of the charge qubits and the density of nonequilibrium quasiparticles in the reservoir $n_{\rm qp} \sim 10^{19}-10^{18}$ m^{-3},[11] we estimate the Andreev relaxation rate to be $\langle\Gamma_A\rangle \sim 10^{-1}-10^{-2}$ Hz. Thus, in the absence of other relaxation channels, the mismatch of gap energies leads to extremely long T_1 times. (For comparison, the quasiparticle-induced T_1 found in Ref. 11 for the charge qubit with equal gap energies was $T_1^{-1} \sim 10^5-10^3$ Hz.)

The charge qubit with a large gap in the box also permits to reduce quasiparticle-induced decoherence. Since real quasiparticle transitions into the island are suppressed, see Eq. (2), the dephasing time of the qubit is limited by the energy relaxation processes, i.e., $T_2 \approx 2/\langle\Gamma_A\rangle$.

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