Magnetoelastic ground state and waves in ferromagnet-nonmagnetic dielectric multilayer structures

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The periodic magnetoelastic ground state that spontaneously arises due to the magnetoelastic and thermoelastic interactions between layers of a multilayer structure is studied. Properties of this state, such as the equilibrium static elastic strains of the layers of the multilayer structure, the orientation of the static part of the magnetization of these layers depending on the value and orientation of the magnetic field and the temperature, are calculated. It is shown that the magnetoelastic ground state gives rise to three new terms in the energy, which describe different types of the effective magnetic anisotropy. Effects of the initial periodic inhomogeneities of the magnetic and elastic parameters as well as effects of the static periodic elastic stresses resulting from the spontaneous ground state on the spectrum of magnetoelastic waves and on the ferromagnetic resonance frequency of the multilayer structure are investigated.

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I. INTRODUCTION

In recent years, multilayer structures (MSs) with the periodic variation of some material parameter along the axis of this structure perpendicular to the plane of the layers are intensively investigated. Interaction of waves propagating along the MS axis with the periodic structure gives rise to the band structure of the wave spectrum analogous to that in crystals. Therefore, these MSs are often referred to as one-dimensional crystals—photonic, phononic, or magnonic crystals—depending on the physical nature of the periodically inhomogeneous parameter and, respectively, physical nature of waves interacting with that parameter.

In the last years, the magnonic crystals, in which the coupling between the spin and elastic waves is taken into account—the magnon-phononic crystals—have attracted the attention. The idea that spin and elastic waves in a ferromagnet should be considered in the framework of the united magnetoelastic (ME) continuum was suggested in the basic papers,[1–3] in which the ME wave theory was developed and ME resonance phenomena that arose in the region of the crossing of unperturbed spin and elastic wave dispersion curves were studied. Stimulated by these papers, the intensive theoretical and experimental investigations of the effects of interactions of spin and elastic waves in ferro-, ferri-, and antiferromagnets began, which are presented in numerous original papers as well as in reviews and books.[4–9] The main obstacle for the wide use of the ME resonance in applied devices is the smallness of the ME coupling parameter of microwave magnetic materials. One possible way to overcome this obstacle was suggested for the surface ME waves. The essence of this way is to consider such surface waves near the orientational phase transition point: the orientation of the magnetization vector becomes unstable and the transverse magnetic susceptibility increases rapidly in this region that results in increasing the effective ME coupling. By now, this effect has been extensively investigated both theoretically and experimentally (see the review in Ref. 10 and references therein). Originally, these waves were considered at the boundary of the surfaces of bilayer or three-layer materials. Due to development of the MS technology in recent years, the theory of surface ME waves was developed for multilayer materials[11–13] (see also the review in Ref. 14). In the present paper, we are not concern with this attractive effect because it arises for the surface ME waves propagating perpendicular to the MS axis along the surface contact of magnetic and nonmagnetic layers, that is, in the situation when the MS does not exhibit properties of the one-dimensional magnon-phononic crystal.

There is another way to gain the effective ME coupling for the body waves propagating along the MS axis different from that for the surface waves. It is based on the resonance interaction of these waves with the periodic structure of the MS. The development of the theory of the resonance interaction of spin and elastic waves with each other as well as with the periodic structure of the material is required for the purposeful preparation and investigation of these MSs. The theory of propagation of the body ME waves perpendicular to MS layers only begins to develop.[15–17] The theory of the ME wave spectrum for the case of alternating ferromagnetic and nonmagnetic layers was developed in Ref. 15. The ME wave spectrum for the general case of alternating two ferromagnetic layers with different magnetic and elastic properties was considered in Refs. 16 and 17. The propagation of ME waves against a background of the only initially created periodic inhomogeneous structure of the magnetic and elastic parameters was considered in all these papers. Here, we show that the spontaneous periodically inhomogeneous ME ground state arises, which results from the ME and thermoelastic interactions between the layers in the MS. We determine the properties of this ground state and study effects of both the initial periodic inhomogeneities of the magnetic and elastic parameters of the MS and the static periodic elastic stresses, which are due to the spontaneous ME ground state, on the ME wave spectrum.

II. MODEL AND EQUATIONS OF EQUILIBRIUM AND MOTION

Let us consider the infinite MS consisting of alternating ME ferromagnetic and nonmagnetic elastic layers with the

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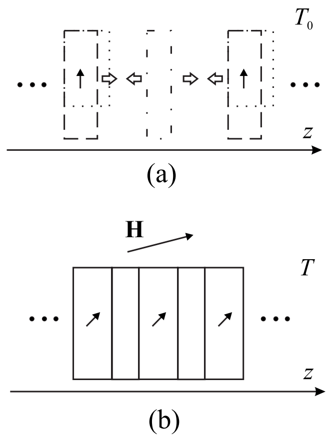

FIG. 1. Scheme of the (a) preparation and (b) investigation of the MS.

thicknesses l_1 and l_2, respectively ($l=l_1+l_2$ is the MS period). We assume that the z axis is perpendicular to the planes of the layers. We write the energy densities of the ferromagnetic ($\mathcal{H}_1$) and nonmagnetic ($\mathcal{H}_2$) layers in the form

$$\mathcal{H}_1 = \alpha(\partial\mathbf{M}/\partial\mathbf{x})^2/2 - \mathbf{MH} - \mathbf{MH}_m/2 + \lambda_1 U_{ii}^2/2 + \mu_1 U_{ij}^2 + \gamma M_i M_j U_{ij} - K_1\kappa_1\Delta T U_{ii},$$

$$\mathcal{H}_2 = \lambda_2 V_{ii}^2/2 + \mu_2 V_{ij}^2 - K_2\kappa_2\Delta T V_{ii}, \quad (1)$$

where $\mathbf{M}$ is the magnetization vector, α is the exchange constant, $\mathbf{H}_m$ is the demagnetizing field, $\mathbf{H}$ is the external constant magnetic field, $U_{ij}=(\partial U_i/\partial x_j+\partial U_j/\partial x_i)/2$ and $V_{ij}=(\partial V_i/\partial x_j+\partial V_j/\partial x_i)/2$ are the strain tensors, $\mathbf{U}$ and $\mathbf{V}$ are the elastic displacement vectors of the ferromagnetic and nonmagnetic layers, respectively, λ_n and μ_n are Lamé's constants, $K_n=\lambda_n+2\mu_n/3$ are the coefficients of the bulk contraction, κ_n are the coefficients of the linear thermal expansion of the ferromagnetic ($n=1$) and nonmagnetic ($n=2$) layers, and γ is the constant of the ME coupling. Here and in the following, the summation over repeated subscripts is assumed. We assume that the MS is studied at the temperature $T=T_0+\Delta T$ and in the magnetic field $\mathbf{H}$ different from the temperature T_0 and the field $\mathbf{H}_0$ at the MS preparation. Namely, this situation usually corresponds to the conditions of experimental investigations or applications of the MS.

It is evident that MS properties (the ME ground state and spectrum) depend on the method of the MS preparation. We assume that this method corresponds to the situation of the absence of elastic stresses resulting from the interaction between the MS layers at the temperature $T=T_0$ and the homogeneous orientation of the magnetization vectors $\mathbf{M}$ in the plane of the magnetic layers. The principal features of the MS preparation are shown in Fig. 1(a) in more detail, where the layers are conventionally shown before their joining into the MS. Dotted lines represent the form of the ferromagnetic layers without the magnetization (or without the ME coupling). We assume that each ferromagnetic layer has passed through the state with the spontaneous magnetization and spontaneous magnetostriction strains before the joining of the layers. These spontaneous magnetostriction strains correspond to the orientation $\mathbf{M}=\mathbf{M}_{00}$ of the magnetization vector (black arrows) in the layer plane along some direction the same for all ferromagnetic layers. This situation is shown in Fig. 1(a) by dashed lines. Such spontaneously strained ferromagnetic layers are joined to nonmagnetic layers, shown in Fig. 1(a) by dotted-dashed lines, creating a continuous MS without elastic stresses between the layers. To take into account spontaneous strains of the ferromagnetic layers, the displacements $\mathbf{U}$ in the energy density $\mathcal{H}_1$ are reckoned from the state corresponding to the absence of the spontaneous strains (dotted lines) and the displacements $\mathbf{V}$ in the energy density $\mathcal{H}_2$ are reckoned from the elastic state corresponding to the moment of joining of the layers (dotted-dashed line). The investigation or application of the MS is carried out in the external magnetic field $\mathbf{H}$ at the temperature T different from the preparation field and temperature T_0 [Fig. 1(b)]. Solid lines here correspond to the equilibrium strains of the MS layers in the field $\mathbf{H}$ at the temperature T, whereas arrows show the equilibrium orientation $\mathbf{M}_0$ of the magnetization vector of the ferromagnetic layers in these field and temperature.

Let us consider the equations of motion in each type of the layer. They are the Landau-Lifshitz equation for the magnetization vector $\mathbf{M}$ and the equation of motion of the elasticity theory for the displacement vector $\mathbf{U}$ in the ferromagnetic layer,

$$\dot{\mathbf{M}} = -g[\mathbf{M}\times\mathbf{H}^{(e)}], \quad \rho_1\ddot{U}_i = \partial\sigma_{ij}^{(U)}/\partial x_j, \quad (2)$$

where g is the gyromagnetic ratio, and the equation of motion of the elasticity theory for the displacement vector $\mathbf{V}$ in the nonmagnetic layer,

$$\rho_2\ddot{V}_i = \partial\sigma_{ij}^{(V)}/\partial x_j, \quad (3)$$

where ρ_1 and ρ_2 are the densities of the ferromagnetic and nonmagnetic layers, respectively. Expressions for the effective magnetic field $\mathbf{H}^{(e)}$, stress tensor $\sigma_{ij}^{(U)}$ of the ferromagnetic layer, and stress tensor $\sigma_{ij}^{(V)}$ of the nonmagnetic layer have the form

$$\mathbf{H}^{(e)} = -\frac{\partial\mathcal{H}_1}{\partial\mathbf{M}} + \frac{\partial}{\partial\mathbf{x}}\frac{\partial\mathcal{H}_1}{\partial(\partial\mathbf{M}/\partial\mathbf{x})} = \alpha\Delta\mathbf{M} + \mathbf{H} + \mathbf{H}_m - 2\gamma(M_iU_{ix}\mathbf{i} + M_iU_{iy}\mathbf{j} + M_iU_{iz}\mathbf{l}),$$

$$\sigma_{ij}^{(U)} = \partial\mathcal{H}_1/\partial U_{ij} = \lambda_1 U_{kk}\delta_{ij} + 2\mu_1 U_{ij} + \gamma M_i M_j,$$

$$\sigma_{ij}^{(V)} = \partial\mathcal{H}_2/\partial V_{ij} = \lambda_2 V_{kk}\delta_{ij} + 2\mu_2 V_{ij}. \quad (4)$$

Here, $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{l}$ are the unit vectors of the x, y, and z axes; δ_{ij} is Kronecker's delta symbol. We represent $\mathbf{M}$, $\mathbf{U}$, and $\mathbf{V}$ in the form

$$\mathbf{M} = \mathbf{M}_0 + \mathbf{m}(t), \quad \mathbf{U} = \mathbf{u}^{(0)} + \mathbf{u}(t), \quad \mathbf{V} = \mathbf{v}^{(0)} + \mathbf{v}(t), \quad (5)$$

where $\mathbf{M}_0$, $\mathbf{u}^{(0)}$, and $\mathbf{v}^{(0)}$ are the static equilibrium orientation of the magnetization vector and the static equilibrium elastic displacements of the ferromagnetic and nonmagnetic layers, respectively, in the applied external magnetic field $\mathbf{H}$; $\mathbf{m}(t)$, $\mathbf{u}(t)$, and $\mathbf{v}(t)$ are the dynamic components of the same values. By using the set of equations (5), we represent $\mathbf{H}^{(e)}$, $\sigma_{ij}^{(U)}$, and $\sigma_{ij}^{(V)}$ as the sum of the static and dynamic parts:

$$\mathbf{H}^{(e)} = \mathbf{H}_0^{(e)}(\mathbf{M}_0,\mathbf{u}^{(0)}) + \mathbf{h}^{(e)}(\mathbf{M}_0,\mathbf{u}^{(0)},\mathbf{m},\mathbf{u}),$$

$$\sigma_{ij}^{(U)} = \sigma_{ij}^{(0u)}(\mathbf{M}_0,\mathbf{u}^{(0)}) + \sigma_{ij}^{(u)}(\mathbf{M}_0,\mathbf{u}^{(0)},\mathbf{m},\mathbf{u}),$$

$$\sigma_{ij}^{(V)} = \sigma_{ij}^{(0v)}(\mathbf{v}^{(0)}) + \sigma_{ij}^{(v)}(\mathbf{v}^{(0)},\mathbf{v}). \tag{6}$$

Substituting Eqs. (5) and (6) into Eqs. (2) and (3) and separating the static and dynamic parts, we obtain the set of static equations for the calculation of the MS ground state,

$$[\mathbf{M}_0 \times \mathbf{H}_0^{(e)}] = 0,$$

$$\partial\sigma_{ij}^{(0u)}/\partial x_j = 0,$$

$$\partial\sigma_{ij}^{(0v)}/\partial x_j = 0, \tag{7}$$

and the set of equations of motion,

$$\dot{\mathbf{m}} = -g([\mathbf{m}\times\mathbf{H}_0^{(e)}] + [\mathbf{M}_0\times\mathbf{h}^{(e)}] + [\mathbf{m}\times\mathbf{h}^{(e)}]),$$

$$\rho_1\ddot{u}_i = \partial\sigma_{ij}^{(u)}/\partial x_j,$$

$$\rho_2\ddot{v}_i = \partial\sigma_{ij}^{(v)}/\partial x_j. \tag{8}$$

Both of the obtained sets of equations should be supplemented with appropriate boundary conditions for the magnetization and elastic displacements of the adjacent layers. We take the magnetic boundary conditions in the form

$$\partial\mathbf{M}/\partial\mathbf{n}|_{z_0} = 0, \tag{9}$$

that corresponds to the absence of any pinning of the magnetization vector $\mathbf{M}$ at the boundary z_0 between the ferromagnetic and nonmagnetic layers, and $\mathbf{n}$ is the normal to the surface of the ferromagnetic layer. The elastic boundary conditions have the form

$$(\mathbf{U} - \mathbf{u}^{(00)})|_{z_0} = \mathbf{V}|_{z_0},$$

$$n_j\sigma_{ij}^{(U)}|_{z_0} = n_j\sigma_{ij}^{(V)}|_{z_0}. \tag{10}$$

The former condition corresponds to the continuity of elastic displacements at the boundary of the adjacent layers, whereas the latter condition corresponds to the equality to zero of the sum of the forces at the same boundary. We take into account in the first equation of the set of equations (10) that the nonmagnetic layers are joined to the ferromagnetic layers, which have the spontaneous magnetostriction displacements $\mathbf{u}^{(00)}$ corresponding to the orientation $\mathbf{M}_{00}$ of the magnetization vector (the general expression for the spontaneous magnetostriction strains $u_{ij}^{(00)}$ will be given below). Substituting Eqs. (5) into Eqs. (9) and (10) and taking into account that $\mathbf{n}$ is parallel to the z axis and $\mathbf{M}_0$ is homogeneous in the volume of the ferromagnetic layer, we obtain boundary conditions for the static components in the form

$$(\mathbf{u}^{(0)} - \mathbf{u}^{(00)})|_{z_0} = \mathbf{v}^{(0)}|_{z_0},$$

$$\sigma_{iz}^{(0u)}|_{z_0} = \sigma_{iz}^{(0v)}|_{z_0}, \tag{11}$$

and boundary conditions for the dynamic components in the form

$$\partial\mathbf{m}/\partial z|_{z_0} = 0,$$

$$\mathbf{u}|_{z_0} = \mathbf{v}|_{z_0},$$

$$\sigma_{iz}^{(u)}|_{z_0} = \sigma_{iz}^{(v)}|_{z_0}. \tag{12}$$

First, we will derive the MS ground state, that is, we will solve the equilibrium equations, Eqs. (7), taking into account the boundary conditions, Eqs. (11). Then, we will derive the spectrum of plane ME waves propagating along the z axis against a background of the obtained ground state using the equations of motion, Eqs. (8), and the boundary conditions, Eqs. (12).

III. GROUND STATE

Let us consider the problem of the ME ground state of the MS. Explicit expressions of variables in Eqs. (7) have the form

$$\mathbf{H}_0^{(e)} = \mathbf{H} + \mathbf{H}_{0m} - 2\gamma(M_{0i}u_{ix}^{(0)}\mathbf{i} + M_{0i}u_{iy}^{(0)}\mathbf{j} + M_{0i}u_{iz}^{(0)}\mathbf{l}),$$

$$\sigma_{ij}^{(0u)} = \lambda_1 u_{kk}^{(0)}\delta_{ij} + 2\mu_1 u_{ij}^{(0)} + \gamma M_{0i}M_{0j},$$

$$\sigma_{ij}^{(0v)} = \lambda_2 v_{kk}^{(0)}\delta_{ij} + 2\mu_2 v_{ij}^{(0)}. \tag{13}$$

We start with solving the elastic equilibrium equations. The substitution of Eqs. (13) for $\sigma_{ij}^{(0u)}$ and $\sigma_{ij}^{(0v)}$ into Eqs. (7) results in the set of three equations

$$\lambda_1\partial u_{kk}^{(0)}/\partial x_i + 2\mu_1\partial u_{ij}^{(0)}/\partial x_j = 0, \quad i = x,y,z, \tag{14}$$

for the ferromagnetic layer and the analogous set with replacing λ_1 and μ_1 by λ_2 and μ_2 for the nonmagnetic layer. It is enough to consider only one of the obtained sets. The strains $u_{ij}^{(0)}$ do not depend on x and y because of the translational invariance in the xOy plane (but the displacements $\mathbf{u}^{(0)}$ do depend on x and y). Therefore, we obtain the set of equations

$$\partial u_{iz}^{(0)}/\partial z = 0, \quad i = x,y,$$

$$\lambda_1\partial u_{jj}^{(0)}/\partial z + 2\mu_1\partial u_{zz}^{(0)}/\partial z = 0. \tag{15}$$

As $u_{ij}^{(0)} = (\partial u_i^{(0)}/\partial x_j + \partial u_j^{(0)}/\partial x_i)$, these equations are the set of three equations in three components of the displacement vector $\mathbf{u}^{(0)}$. Instead of solving the set of equations in three components of the vector $\mathbf{u}^{(0)}$, we consider it as the set of equations in six components of the strain tensor $u_{ij}^{(0)}$. For that, this set should be supplemented with three more equations, which are the compatibility relationships[18]

$$\frac{\partial^2 u_{ik}^{(0)}}{\partial x_l \partial x_m} + \frac{\partial^2 u_{lm}^{(0)}}{\partial x_i \partial x_k} = \frac{\partial^2 u_{il}^{(0)}}{\partial x_k \partial x_m} + \frac{\partial^2 u_{km}^{(0)}}{\partial x_i \partial x_l}. \quad (16)$$

These compatibility relationships result from the fact that the six components $u_{ij}^{(0)}$ are not independent because they can be expressed in terms of three components of $\mathbf{u}^{(0)}$ [it is easy to show that the relationships described by Eqs. (16) are identities if they are expressed in terms of $\mathbf{u}^{(0)}$]. In our case, the only three relationships corresponding to second derivatives with respect to z remain from the six relationships described by Eqs. (16). Finally, we obtain the set of six equations for the six components of the strain tensor $u_{ij}^{(0)}$

$$\partial^2 u_{ij}^{(0)}/\partial z^2 = 0, \quad i,j = x,y,$$

$$\lambda_1 \partial u_{jj}^{(0)}/\partial z + 2\mu_1 \partial u_{zz}^{(0)}/\partial z = 0,$$

$$\partial u_{iz}^{(0)}/\partial z = 0, \quad i = x,y. \quad (17)$$

The components $u_{xx}^{(0)}$, $u_{yy}^{(0)}$, and $u_{xy}^{(0)}$ of the strain tensor are the relative elongations along the x and y axes and the shearing strains between the xOz and yOz planes, respectively. In view of symmetry of the problem, these components should be the symmetric functions in z relative to the medium plane of the layer. Because of this, it follows from the three first equations of the set of equations (17) that these components do not depend on z. Subject to it, the other three equations of the set of equations (17) give us that the strain components $u_{zz}^{(0)}$, $u_{yz}^{(0)}$, and $u_{zx}^{(0)}$ are also homogeneous. The analogous consideration results in homogeneity of all components of the strain tensor $v_{ij}^{(0)}$ for the nonmagnetic layer.

The boundary conditions for the components of the strains follow from the first boundary condition of the set of equations (11) for the displacement vectors. They have the form

$$(u_{ij}^{(0)} - u_{ij}^{(00)})|_{z_0} = v_{ij}^{(0)}|_{z_0}, \quad i,j = x,y. \quad (18)$$

At $i=j=x$ and $i=j=y$, this condition states the continuity of the relative elongations along the x and y axes, respectively, whereas at $i=x$ and $j=y$, it states the continuity of the shearing strains between the xOz and yOz planes at the boundary of the adjacent layers. In view of the homogeneity of the strains, the boundary conditions in Eqs. (18) as well as the boundary conditions for the components of the stresses, the latter conditions of the set of equations (11), transform to the relationships between $u_{ij}^{(0)}$ and $v_{ij}^{(0)}$ and between $\sigma_{iz}^{(0u)}$ and $\sigma_{iz}^{(0v)}$ in the bulk of the corresponding layers:

$$u_{ij}^{(0)} - u_{ij}^{(00)} = v_{ij}^{(0)}, \quad i,j = x,y,$$

$$\sigma_{iz}^{(0u)} = \sigma_{iz}^{(0v)}, \quad i = x,y,z. \quad (19)$$

Here, the spontaneous magnetostriction strains $u_{ij}^{(00)}$ corresponding to the orientation of the magnetization vector in the process of the MS preparation $\mathbf{M}_{00}$ have the form

$$u_{ij}^{(00)} = -\gamma(3K_1 M_{00i} M_{00j} - \lambda_1 M_{00}^2 \delta_{ij})/6\mu_1 K_1. \quad (20)$$

Note that there are no constraints on the orientation of the vector $\mathbf{M}_{00}$ here yet. Using all six equations of the set of equations (19), we express $v_{ij}^{(0)}$ in terms of $u_{ij}^{(0)}$:

$$v_{ij}^{(0)} = u_{ij}^{(0)} - u_{ij}^{(00)}, \quad i,j = x,y,$$

$$v_{zz}^{(0)} = [\gamma M_{0z}^2 + (\lambda_1 + 2\mu_1)u_{zz}^{(0)} - (\lambda_2 - \lambda_1)(u_{xx}^{(0)} + u_{yy}^{(0)}) + (\kappa_2 K_2 - \kappa_1 K_1)\Delta T + \lambda_2(u_{xx}^{(00)} + u_{yy}^{(00)})]/(\lambda_2 + 2\mu_2),$$

$$v_{iz}^{(0)} = \mu_1 u_{iz}^{(0)}/\mu_2 + \gamma M_{0i} M_{0z}/2\mu_2, \quad i = x,y. \quad (21)$$

We derive the components $u_{ij}^{(0)}$ by the minimization of the ground state energy of the MS with respect to $u_{ij}^{(0)}$. It is enough to consider the energy accounting for the sum of the ferromagnetic layer volume V_1 and nonmagnetic layer volume V_2:

$$U^{(0)} = \int_{V_1} \mathcal{H}_1(\mathbf{M}_0, u_{ij}^{(0)}) dV + \int_{V_2} \mathcal{H}_2(v_{ij}^{(0)}) dV. \quad (22)$$

Because the variables $\mathbf{M}_0$, $u_{ij}^{(0)}$, and $v_{ij}^{(0)}$ are homogeneous and the layers are plane parallel to each other, the problem reduces to the minimization of the expression

$$\mathcal{H}^{(0)} = [\mathcal{H}_1(\mathbf{M}_0, u_{ij}^{(0)}) l_1 + \mathcal{H}_2(v_{ij}^{(0)}) l_2]/l \quad (23)$$

with respect to $u_{ij}^{(0)}$ using the relationships between $v_{ij}^{(0)}$ and $u_{ij}^{(0)}$ described by Eqs. (21). Solving the set of the six equations $\partial\mathcal{H}^{(0)}/\partial u_{ij}^{(0)}=0$, we obtain

$$u_{ii}^{(0)} = -\frac{\gamma l_1}{2\bar{\mu} l}\left(M_{0i}^2 - \frac{\lambda_1 M_0^2}{3K_1}\right) + \frac{\Delta L \mu_2 l_2 l_1}{S_0 \bar{\mu} l}\gamma M_{0z}^2 - \frac{\Delta L(\lambda_1 + 2\mu_1)\mu_2 l_2 l_1}{3S_0 K_1 \bar{\mu} l}\gamma M_0^2 + \frac{\bar{\kappa}\Delta T}{3} + \frac{\mu_2 l_2}{\bar{\mu} l} u_{ii}^{(00)} - \frac{2\Delta L \mu_2 l_2 \mu_1 l_1}{S_0 \bar{\mu} l}(u_{xx}^{(00)} + u_{yy}^{(00)}), \quad i = x,y,$$

$$u_{zz}^{(0)} = -\frac{\gamma l_1}{2\bar{\mu} l}\left(M_{0z}^2 - \frac{\lambda_1 M_0^2}{3K_1}\right) - \frac{(S_0 + 2\Delta L \lambda_1 l_1)\mu_2 l_2}{S_0(\lambda_1 + 2\mu_1)\bar{\mu} l}\gamma M_{0z}^2 + \frac{2\Delta L \mu_2 l_2 \lambda_1 l_1}{3S_0 K_1 \bar{\mu} l}\gamma M_0^2 + \frac{\bar{\kappa}\Delta T}{3} - \frac{3K_1 K_2 \Delta\kappa \Delta T \mu_2 l_2}{S_0} - \frac{3K_2 \lambda_1 \mu_2 l_2}{S_0}(u_{xx}^{(00)} + u_{yy}^{(00)}),$$

$$u_{xy}^{(0)} = -\frac{\gamma M_{0x} M_{0y} l_1}{2\bar{\mu} l} + \frac{\mu_2 l_2}{\bar{\mu} l} u_{xy}^{(00)},$$

$$u_{iz}^{(0)} = -\frac{\gamma M_{0i} M_{0z}}{2\mu_1}, \quad i = x,y, \quad (24)$$

where

$$\Delta L = \lambda_1 \mu_2 - \lambda_2 \mu_1, \quad \Delta\kappa = \kappa_2 - \kappa_1, \quad \bar{\mu} = (\mu_1 l_1 + \mu_2 l_2)/l,$$

$$\bar{\kappa} = 3[\kappa_1 K_1(\lambda_2 + 2\mu_2)\mu_1 l_1 + \kappa_2 K_2(\lambda_1 + 2\mu_1)\mu_2 l_2]/S_0,$$

$$S_0 = 3K_1(\lambda_2 + 2\mu_2)\mu_1 l_1 + 3K_2(\lambda_1 + 2\mu_1)\mu_2 l_2. \quad (25)$$

Substituting the equilibrium strains $u_{ij}^{(0)}$ determined by Eqs. (24) into Eqs. (21), we can get explicit expressions for the equilibrium strains $v_{ij}^{(0)}$ of the nonmagnetic layer in terms of the MS parameters.

Let us consider several limiting cases. If the investigation of the MS is carried out at the temperature and orientation of the magnetization the same as those during the preparation of the MS, the equilibrium strains of the layers have also to be the same. Indeed, at $\mathbf{M}_0=\mathbf{M}_{00}$ and $\Delta T=0$, we obtain from Eqs. (24) and (21) $u_{ij}^{(0)}=u_{ij}^{(00)}$ and $v_{ij}^{(0)}=0$ [in Fig. 1(a), these strains correspond to dashed lines for the ferromagnetic layer and dotted-dashed lines for the nonmagnetic layers]. At the limiting case $l_2=0$, the MS becomes a homogeneous ME ferromagnet, for which the equilibrium strains have the form

$$u_{ij}^{(0)}=-\gamma(3K_1M_{0i}M_{0j}-\lambda_1M_0^2\delta_{ij})/6\mu_1K_1+\kappa_1\Delta T\delta_{ij}/3. \tag{26}$$

In this expression, the former term corresponds to the spontaneous magnetostriction strains of the ME ferromagnet, whereas the latter term corresponds to its thermal expansion. We also consider the specific limiting case corresponding to the absence of the magnetization ($\mathbf{M}=0$) or ME coupling ($\gamma=0$) in the MS. In this case, the strains result only from the thermal expansion and Eqs. (24) reduce to the corresponding equations obtained in Ref. 19 for a layered capacitor consisting of alternating conductive and dielectric layers with different elastic parameters.

Formulas of Eqs. (24) and (21) are the solution of the equations of the elasticity theory from the common set of equations (7) of the ME ground state. These formulas represent the dependence of the equilibrium strains of the ferromagnetic and nonmagnetic layers on the magnetization vector $\mathbf{M}_0$ and temperature T. It remains to solve the first equation of the set of equations (7) and to derive the dependence of the equilibrium orientation of $\mathbf{M}_0$ on the magnetic field $\mathbf{H}$. Substituting $u_{ij}^{(0)}$ and $v_{ij}^{(0)}$ into $\mathbf{H}_0^{(e)}$, we obtain

$$\mathbf{H}_0^{(e)}=\mathbf{H}+\mathbf{H}_{0m}+\left[\frac{2S_2l_1}{S_0\bar{\mu}l}\gamma^2M_0^2+\frac{(S_0-2\Delta L\mu_1l_1)\mu_2l_2}{S_0\mu_1\bar{\mu}l}\gamma^2M_{0z}^2-\frac{2\gamma\bar{\kappa}\Delta T}{3}\right]\mathbf{M}_0+[(S_0-2\Delta L\mu_1l_1)\gamma^2M_0^2M_{0z}-2S_1\gamma^2M_{0z}^3$$
$$+6K_1K_2\mu_1\bar{\mu}l\Delta\kappa\Delta T\gamma M_{0z}]\frac{\mu_2l_2\mathbf{l}}{S_0\mu_1\bar{\mu}l}-\{[S_0u_{xx}^{(00)}-2\Delta L\mu_1l_1(u_{xx}^{(00)}+u_{yy}^{(00)})]M_{0x}\mathbf{i}+[S_0u_{yy}^{(00)}-2\Delta L\mu_1l_1(u_{xx}^{(00)}+u_{yy}^{(00)})]M_{0y}\mathbf{j}$$
$$+S_0u_{xy}^{(00)}(M_{0y}\mathbf{i}+M_{0x}\mathbf{j})-3K_2\lambda_1\bar{\mu}l(u_{xx}^{(00)}+u_{yy}^{(00)})M_{0z}\mathbf{l}\}\frac{2\gamma\mu_2l_2}{S_0\bar{\mu}l}, \tag{27}$$

where

$$S_1=3K_1(\lambda_2+\mu_2)\mu_1l_1+3K_2(\lambda_1+\mu_1)\mu_2l_2,$$
$$S_2=(\lambda_1+\mu_1)(\lambda_2+2\mu_2)\mu_1l_1+(\lambda_2+\mu_2)(\lambda_1+2\mu_1)\mu_2l_2. \tag{28}$$

By substituting this expression for the effective field $\mathbf{H}_0^{(e)}$ into the first equation of the set of equations (7), we obtain the explicit form of the equation for the equilibrium orientation of the magnetization $\mathbf{M}_0$. It is difficult to solve the obtained equation in the general case in view of the cumbersome expression for $\mathbf{H}_0^{(e)}$. Let us consider the MS in which the magnetization vector $\mathbf{M}_{00}$ in the process of the MS preparation lies in the plane of the layers parallel to the x axis. In this case, the spontaneous magnetostriction strains $u_{ij}^{(00)}$ have the form

$$u_{xx}^{(00)}=-(\lambda_1+\mu_1)\gamma M_0^2/3K_1\mu_1,$$
$$u_{yy}^{(00)}=u_{zz}^{(00)}=-\lambda_1\gamma M_0^2/6K_1\mu_1,$$
$$u_{ij}^{(00)}=0,\quad i\neq j, \tag{29}$$

and the effective magnetic field $\mathbf{H}_0^{(e)}$ has the form

$$\mathbf{H}_0^{(e)}=\mathbf{H}+\mathbf{H}_{0m}+\left[\frac{2(\lambda_1+\mu_1)}{3K_1\mu_1}\gamma^2M_0^2+\frac{(S_0-2\Delta L\mu_1l_1)\mu_2l_2}{S_0\mu_1\bar{\mu}l}\gamma^2M_{0z}^2-\frac{2\gamma\bar{\kappa}\Delta T}{3}\right]\mathbf{M}_0-\frac{\mu_2l_2}{\mu_1\bar{\mu}l}\gamma^2M_0^2M_{0y}\mathbf{j}-\frac{2S_1\mu_2l_2}{S_0\mu_1\bar{\mu}l}\gamma^2M_{0z}^3\mathbf{l}+\frac{6K_1K_2\Delta\kappa\Delta T\mu_2l_2}{S_0}\gamma M_{0z}\mathbf{l}. \tag{30}$$

The term within the square brackets is proportional to $\mathbf{M}_0$ and vanishes after substituting $\mathbf{H}_0^{(e)}$ in the first equation of the set of equations (7). However, we should need it below when we consider the equations of motion. Three terms following this term describe three types of the effective magnetic anisotropy resulting from the ME (first and second terms) and thermoelastic (third term) interaction between the MS layers. The first term describes the xOz easy-plane anisotropy, the second one describes the xOy easy-plane anisotropy, and the third term describes the anisotropy along the z axis.

In the process of remagnetization of the MS in the plane of the layers xOy along the easy direction ($\mathbf{H}\parallel Ox$), the equilibrium equation has the form

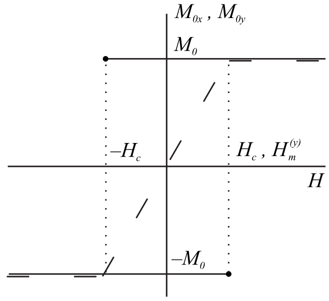

FIG. 2. Magnetization curves of the MS at the orientation of the magnetic field **H** in the plane of the layers along the x (solid lines) and y (dashed lines) axes.

$$M_{0y}\left(H+\frac{\mu_2 l_2}{\mu_1\bar{\mu}l}\gamma^2M_0^2M_{0x}\right)=0. \tag{31}$$

The dependence of the projection M_{0x} on the value of the external magnetic field **H** is shown in Fig. 2 by solid lines, and the coercivity H_c is given by

$$H_c=\frac{\mu_2 l_2}{\mu_1\bar{\mu}l}\gamma^2M_0^3. \tag{32}$$

In the process of remagnetization of the MS in the plane of the layers xOy along the hard direction ($\mathbf{H}\parallel Oy$), the equilibrium equation has the form

$$M_{0x}\left(H-\frac{\mu_2 l_2}{\mu_1\bar{\mu}l}\gamma^2M_0^2M_{0y}\right)=0. \tag{33}$$

The dependence of the projection M_{0y} on the value of the external magnetic field **H** corresponding to this equation is shown in Fig. 2 by the dashed line. The field $H=H_m^{(y)}$ corresponding to the magnetic saturation coincides in magnitude with the field H_c for the previous case. In the process of remagnetization of the MS perpendicular to the plane of the layers ($\mathbf{H}\parallel Oz$), the rotation of the vector $\mathbf{M}_0$ in the xOz plane takes place, which is described by the equation

$$M_{0x}\left(H-4\pi M_{0z}-\frac{2S_1\mu_2 l_2}{S_0\mu_1\bar{\mu}l}\gamma^2M_{0z}^3+\frac{6K_1K_2\Delta\kappa\Delta T\mu_2 l_2}{S_0}\gamma M_{0z}\right)=0. \tag{34}$$

In contrast to the two previous cases, this is the nonlinear equation in the projection of the magnetization M_{0z}. However, due to the smallness of the nonlinear term as compared with $4\pi M_{0z}$, the dependence of M_{0z} on H differs little in shape from the inclined line similar to the dashed line in Fig. 2. The magnetic saturation field $H_m^{(z)}$ in this case differs greatly from the field $H_m^{(y)}$ and it is determined by the formula

$$H_m^{(z)}=4\pi M_0+\frac{2S_1\mu_2 l_2}{S_0\mu_1\bar{\mu}l}\gamma^2M_0^3-\frac{6K_1K_2\Delta\kappa\Delta T\mu_2 l_2}{S_0}\gamma M_0. \tag{35}$$

It is seen from this formula that the thermoelastic magnetic anisotropy may result in decreasing or increasing magnetic saturation field $H_m^{(z)}$ depending on the sign of the product of $\gamma\Delta\kappa\Delta T$.

Let us estimate the order of magnitude of the effects resulting from the ME coupling and thermal expansion of the layers. If we assume that the elastic moduli of both types of the layers are much the same, then, as one can see from Eqs. (30) or (24), the magnitudes $\gamma M_0^2/\mu$ and $\Delta\kappa\Delta T$ should be compared with each other. Let us choose nickel and palladium as materials of the layers (their real elastic moduli differ by a factor of 2, but we need the order of magnitude). The nickel parameters are $\gamma M_0^2=7.65\times10^7$ erg/cm^3 at $M_0=500$ Gs and $\mu=7.5\times10^{11}$ dyn/cm^2. The difference between their coefficients of thermal expansion is $\Delta\kappa\propto10^{-6}$ K^{-1}. At the temperature change $\Delta T=50$ K, we obtain

$$\gamma M_0^2/\mu\propto\Delta\kappa\Delta T\propto10^{-5}, \tag{36}$$

that is, the effects resulting from the ME and thermoelastic interaction between the MS layers can be of the same order of magnitude.

IV. SPECTRUM OF MAGNETOELASTIC WAVES

We investigate the spectrum of plane ME waves propagating against a background of periodic inhomogeneities of the MS resulting from its initial structure as well as from its periodically inhomogeneous ME ground state considered in the previous section of this paper. We assume that the magnetic field **H** is applied along the MS axis z and sufficiently large so that the equilibrium magnetization $\mathbf{M}_0$ is directed along the MS axis too [$H>H_m^{(z)}$, where $H_m^{(z)}$ is given by Eq. (35)]. We restrict our consideration to the waves propagating along the z axis. Using usual linearization of the Landau-Lifshitz equation, we obtain from Eqs. (8) the set of coupled ME equations of motion for the ferromagnetic layer,

$$\dot{m}_x=-\omega_{0y}m_y+g\alpha M_0\partial^2m_y/\partial z^2-g\gamma M_0^2\partial u_y/\partial z,$$

$$\dot{m}_y=\omega_{0x}m_x-g\alpha M_0\partial^2m_x/\partial z^2+g\gamma M_0^2\partial u_x/\partial z,$$

$$\rho_1\ddot{u}_i=\mu_1\partial^2u_i/\partial z^2+\gamma M_0\partial m_i/\partial z,\quad i=x,y, \tag{37}$$

where

$$\omega_{0i}=g(H-4\pi M_0)+2g\gamma M_0(u_{ii}^{(0)}-u_{zz}^{(0)}),\quad i=x,y, \tag{38}$$

and the equations of motion for the nonmagnetic layer,

$$\rho_2\ddot{v}_i=\mu_2\partial^2v_i/\partial z^2,\quad i=x,y. \tag{39}$$

As one can see from Eqs. (38), the partial frequencies ω_{0i}, where $i=x,y$, depend on the equilibrium ME strains $u_{ij}^{(0)}$ of the ferromagnetic layer. The expressions for $u_{ij}^{(0)}$ have the form of Eqs. (24) in the general case. The equilibrium strains depend not only on the equilibrium orientation of the vector $\mathbf{M}_0$ in the external field but also on the orientation of the magnetization vector $\mathbf{M}_{00}$ during the MS preparation. We choose $\mathbf{M}_{00}\parallel Ox$ as in Sec. III. In this case, the partial frequencies ω_{0x} and ω_{0y} have the form

$$\omega_{0x} = g\left[H - 4\pi M_0 + \frac{\gamma^2 M_0^3}{\mu_1} - \frac{2S_1\mu_2 l_2}{S_0\mu_1\bar{\mu} l}\gamma^2 M_0^3 + \frac{6K_1K_2\Delta\kappa\Delta T\mu_2 l_2}{S_0}\gamma M_0\right],$$

$$\omega_{0y} = g\left[H - 4\pi M_0 + \frac{\gamma^2 M_0^3}{\mu_1} - \frac{(2S_1 - S_0)\mu_2 l_2}{S_0\mu_1\bar{\mu} l}\gamma^2 M_0^3 + \frac{6K_1K_2\Delta\kappa\Delta T\mu_2 l_2}{S_0}\gamma M_0\right], \quad (40)$$

where S_0 and S_1 are given by Eqs. (25) and (28), respectively. The first and second terms in each expressions of Eqs. (40) take into account the influence of the external and magnetodipole fields, respectively. The third term is the ME contribution, which is characteristic to the continuum ferromagnet too.[20,21] The fourth and fifth terms describe the contribution of the inhomogeneous ME ground state to the dynamic features of the system.

For the calculation of the spectrum of ME waves, we use the method based on Floquet's theorem (see, for example, Refs. 22 and 23). By assuming that $\mathbf{m}(z,t),\mathbf{u}(z,t) \propto \exp[i(kz-\omega t)]$, we obtain the dispersive equation of ME waves corresponding to the set of equations (37) in the ferromagnetic layer in the form

$$[(\omega_{0x} + Dk^2)(\omega^2 - v_1^2k^2) + Bv_1^2k^2][(\omega_{0y} + Dk^2)(\omega^2 - v_1^2k^2) + Bv_1^2k^2] - \omega^2(\omega^2 - v_1^2k^2)^2 = 0, \quad (41)$$

where $D = g\alpha M_0$ is the exchange stiffness, $B = g\gamma^2 M_0^3/\mu_1$ is the ME parameter, and $v_1^2 = \mu_1/\rho_1$ is the velocity of elastic waves in the ferromagnetic layer. The equation is of fourth degree in k^2 and we denote its four solutions by k_n^2, $n = 1,\ldots,4$. Thus, the eigenfunctions of the ferromagnetic layer can be written as

$$m_i(z) = \sum_{n=1}^{4} (a_{in+}e^{ik_nz} + a_{in-}e^{-ik_nz}), \quad i = x,y,$$

$$u_i(z) = \sum_{n=1}^{4} (A_{in+}e^{ik_nz} + A_{in-}e^{-ik_nz}), \quad i = x,y, \quad (42)$$

where $a_{in\pm}$ and $A_{in\pm}$, $n = 1,\ldots,4$, are the unknown amplitudes of the spin and elastic components of the ME wave in the ferromagnetic layer, respectively. The substitution of Eqs. (42) into Eqs. (37) gives the following relationships between the elastic and magnetic amplitudes and between different projections of the magnetic amplitudes of the ME wave in the ferromagnetic layer:

$$A_{in\pm} = \frac{\mp ik_n\gamma M_0 a_{in\pm}}{\rho_1(\omega^2 - v_1^2k_n^2)}, \quad i = x,y, \quad n = 1,\ldots,4,$$

$$a_{yn\pm} = \frac{-i\omega(\omega^2 - v_1^2k_n^2)a_{xn\pm}}{(\omega_{0y} + Dk_n^2)(\omega^2 - v_1^2k_n^2) + Bv_1^2k_n^2}, \quad n = 1,\ldots,4. \quad (43)$$

By assuming that $\mathbf{v}(z,t) \propto \exp[i(kz-\omega t)]$, we obtain the dispersive equation of the elastic wave corresponding to the set of equations (39) in the form

$$\omega^2 - v_2^2k^2 = 0, \quad v_2^2 = \mu_2/\rho_2. \quad (44)$$

Denoting the solutions of this equation by $k_{5\pm} = \pm\omega/v_2$, we write the eigenfunctions of the nonmagnetic layer in the form

$$v_i(z) = A_{i5+}e^{ik_5z} + A_{i5-}e^{-ik_5z}, \quad i = x,y, \quad (45)$$

where $A_{i5\pm}$ are the unknown amplitudes of the elastic wave in the nonmagnetic layer.

The boundary conditions for the magnetic and elastic components on the surfaces of the layers have the form of Eqs. (12). Setting down these conditions for the x and y components of $\mathbf{m}$, $\mathbf{u}$, and $\mathbf{v}$, we obtain

$$dm_i/dz|_{z_0} = 0,$$

$$u_i|_{z_0} = v_i|_{z_0},$$

$$(\mu_1 du_i/dz + \gamma M_0 m_i)|_{z_0} = \mu_2 dv_i/dz|_{z_0}, \quad i = x,y, \quad (46)$$

where z_0 is the coordinate of the boundary between the ferromagnetic and nonmagnetic layers. By substituting Eqs. (42) and (45) into Eqs. (46) at $z_0 = 0$ and l_1 and taking into account the periodicity condition

$$\mathbf{m}(z+l) = \exp(iKl)\mathbf{m}(z), \quad \mathbf{u}(z+l) = \exp(iKl)\mathbf{u}(z), \quad (47)$$

where K is the wave number of waves propagating in the MS, we obtain a set of 12 homogeneous linear equations (for example, in terms of $A_{xn\pm}$, $n = 1,\ldots,4$, and $A_{x5\pm}$, $A_{y5\pm}$). The condition of consistency of this linear set is the equality to zero of its determinant. By computing this determinant with the MAPLE program, we obtain the equation for the dispersion law $K(\omega)$ in the form

$$P_2\cos^2 Kl + P_1\cos Kl + P_0 = 0, \quad (48)$$

where

$$P_2 = \sum_{\substack{i,j=1\\ i<j}}^{4} P_{ij}^{(2)}S_iS_j = P_{12}^{(2)}S_1S_2 + P_{13}^{(2)}S_1S_3 + P_{14}^{(2)}S_1S_4 + P_{23}^{(2)}S_2S_3 + P_{24}^{(2)}S_2S_4 + P_{34}^{(2)}S_3S_4,$$

$$P_1 = \sum_{\substack{i,j,k=1\\ i\neq j,k\\ j<k}}^{4} (P_{ijk}^{(11)} C_5 C_i S_j S_k + P_{ijk}^{(12)} S_5 S_i C_j C_k) + \sum_{\substack{i,j,k=1\\ i<j<k}}^{4} P_{ijk}^{(13)} S_5 S_i S_j S_k + \sum_{i=1}^{4} P_i^{(14)} S_5 S_i,$$

$$P_0 = \left[\sum_{\substack{i,j,k,l=1\\ i,j,k\neq l\\ i<j<k}}^{4} (P_{ijkl}^{(01)} C_i C_j C_k S_l + P_{ijkl}^{(02)} S_i S_j S_k C_l) + \sum_{\substack{i,j=1\\ i\neq j}}^{4} P_{ij}^{(03)} C_i S_j \right] C_5 S_5 + \left[\sum_{\substack{i,j,k,l=1\\ i,j\neq k,l\\ i<j,k<l}}^{4} P_{ijkl}^{(04)} C_i C_j S_k S_l + \sum_{\substack{i,j=1\\ i<j}}^{4} (P_{ij}^{(05)} C_i C_j + P_{ij}^{(06)} S_i S_j) \right] S_5^2$$
$$+ \sum_{\substack{i,j,k,l=1\\ i,j\neq k,l\\ i<j,k<l}}^{4} P_{ijkl}^{(07)} C_i C_j S_k S_l + (P_{1234}^{(08)} C_1 C_2 C_3 C_4 + P_{1234}^{(09)} S_1 S_2 S_3 S_4 + 1) S_5^2. \tag{49}$$

Here, $C_n = \cos k_n l_1$, $S_n = \sin k_n l_1$, $n = 1, \ldots, 4$, $C_5 = \cos k_5 l_2$, and $S_5 = \sin k_5 l_2$. The coefficients $P_{ij}^{(2)}$, $P_{i(jk)}^{(1n)}$, and $P_{ij(kl)}^{(0n)}$ are polynomials that depend on all parameters of the MS and the frequency ω, but do not contain the trigonometric functions and the wave number K. In general, Eq. (48) contains several thousand terms and one can dissect it with numerical methods only. In the case of the absence of the ME coupling ($\gamma = 0$), Eq. (48) breaks into two independent equations corresponding to elastic waves in the elastic MS,

$$\cos Kl - \cos k_u l_1 \cos k_v l_2 + \frac{1}{2}\left(\frac{\mu_1 k_u}{\mu_2 k_v} + \frac{\mu_2 k_v}{\mu_1 k_u} \right) \sin k_u l_1 \sin k_v l_2 = 0, \tag{50}$$

where $k_u = \omega / v_1$ and $k_v = \omega / v_2$, and to the localized spin-wave modes in the ferromagnetic layers,

$$\omega_n = \omega_0 + g\alpha M_0 k_n^2, \tag{51}$$

where $k_n = \pi n / l_1$, $n = 0, 1, 2, \ldots$, and $\omega_0 = g(H - 4\pi M_0)$ is the ferromagnetic resonance (FMR) frequency.

The result of the numerical calculation of the general dispersion law $\omega(K)$ following from Eq. (48) is shown in Fig. 3 in the extended zone scheme. We assume that the MS consists of the alternating nickel and palladium layers and use the following values of the parameters:[24,25] $H_0 = 10$ kOe, $\alpha = 5 \times 10^{-12}$ cm^2, $M_0 = 500$ Gs, $E_1 = 9K_1\mu_1/(3K_1 + \mu_1) = 2 \times 10^{12}$ dyn/cm^2, $\mu_1 = 7.5 \times 10^{11}$ dyn/cm^2, $\rho_1 = 8.9$ g/cm^3, $E_2 = 9K_2\mu_2/(3K_2 + \mu_2) = 1.2 \times 10^{12}$ dyn/cm^2, $\mu_2 = 5 \times 10^{11}$ dyn/cm^2, $\rho_2 = 12$ g/cm^3, $\kappa_1 = 1.3 \times 10^{-5}$ K^{-1}, $\kappa_2 = 1.2 \times 10^{-5}$ K^{-1}, $l_1 = 150$ Å, $l_2 = 100$ Å, and $\Delta T = -50$ K. To make the graph more descriptive, we assumed that the dimensionless parameter of the ME coupling $\eta = \gamma^2 M_0^2 / \mu_1$ is equal to 0.5, the value that is one order greater than the real (for Ni, for example, $\eta \approx 0.027$).

In Fig. 3, the dispersion law is shown only for the resonance polarizations of the ME waves. Usual gaps (forbidden zones) in the spectrum at the Brillouin zone boundaries of the MS appear at $K = \pi m / l$, where $m = 1, 2, \ldots$. They are due to the interaction of the elastic component of ME waves with the periodic MS. One of these gaps corresponding to the boundary of the first Brillouin zone $Kl = \pi$ is shown in Fig. 3. The virtual dispersion law of spin waves in the ferromagnetic layers in the absence of the ME coupling is shown by the dashed curve; the localized frequencies ω_n and their wave numbers k_n corresponding to Eq. (51) are situated on this virtual curve. ME resonances exist in the vicinity of crossings of the elastic wave dispersion law with the localized frequencies of spin waves ω_n ($n = 0, 1, 2, \ldots$). There are two such ME resonances in Fig. 3 that correspond to the frequency of magnetostatic oscillations ($n = 0$) and the frequency of the first spin-wave mode ($n = 1$). The ME resonances in the MS were investigated theoretically for the first time in Ref. 15. Note that the wave numbers corresponding to the Brillouin zone boundaries are multiplied by $1/l$ and the corresponding frequencies increase linearly with increasing the number m. The wave numbers of spin-wave oscillations are multiplied by $1/l_1$ and the corresponding frequencies increase quadratically with increasing the number n. Therefore, the sequence of gaps corresponding to the Bril-

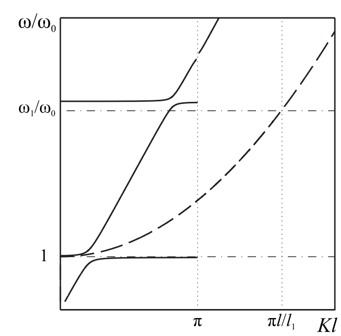

FIG. 3. The dispersion law of the ME waves in the MS (solid curve) in the extended zone scheme (the value of the dimensionless parameter of the ME coupling η at this graph is taken one order greater than the real one). The dispersion law of spin waves in the ferromagnetic layers (dashed curve) and the frequencies of the spin-wave oscillations (dotted-dashed lines) are also shown.

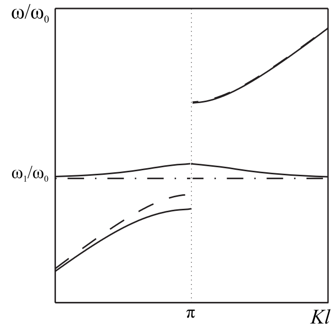

FIG. 4. The modification of the dispersion law of the ME waves (solid curve) at the coincidence of the spin-wave mode frequency (dotted-dashed line) with the gap (forbidden zone) corresponding to the first Brillouin zone boundary. The dispersion law of elastic waves in the MS at the absence of the ME interaction is shown by the dashed curve.

louin zones and to the ME resonances may be situated on the dispersive curve arbitrarily relative to each other depending on the parameters of the system. For example, at other parameters of the MS or values of the magnetic field, the first spin-wave mode may be not in the first Brillouin zone but in the second or third zones.

The special interest represents the coincidence of some ME resonance with the gap at the Brillouin zone boundary of the MS. We consider this situation here. In Fig. 4, the modification of the dispersion law at the coincidence of the frequency of the first spin-wave mode ω_1 with the gap corresponding to the first Brillouin zone boundary is shown. When calculating this graph, in contrast to Fig. 3, we use the value of the ME coupling close to the real ($\eta=0.03$) with the aim of representing the real relationship of scales of the Brillouin and ME gaps in the spectrum. The coincidence of the ME resonance with the Brillouin gap was obtained by increasing the thickness of the nonmagnetic layer: $l_2=140$ Å instead of 100 Å in Fig. 3; other parameters of the MS are the same (in the experiment, this coincidence may be obtained by changing the magnitude of the magnetic field). The frequencies of elastic and spin waves in the absence of the ME coupling are shown in Fig. 4 by dashed and dotted-dashed curves, respectively. The modification of the dispersion law in the presence of the ME coupling is shown by solid curves. One can see that the presence of the ME coupling results in both the modification of the initial dispersion curve and the broadening of the localized level $\omega=\omega_1$ into the permitted zone placed inside the forbidden zone.

For the long waves, when $Kl, k_n l_1, k_5 l_2 \ll 1$, $n=1,\ldots,4$, the dispersion equation (48) takes the simple form

$$[(\omega_{0x}-B+P)(\omega^2-v^2K^2)+Pv^2K^2][(\omega_{0y}-B+P)\times(\omega^2-v^2K^2)+Pv^2K^2]-\omega^2(\omega^2-v^2K^2)^2=0, \tag{52}$$

where

$$P=\frac{g\gamma^2M_0^3}{\mu_1}\frac{\mu_2 l_1}{\mu_2 l_1+\mu_1 l_2}.$$

Velocity of elastic waves is described by some effective parameter v depending on the elastic parameters of both layers:

$$v^2=\frac{\mu_1\mu_2 v_1^2 v_2^2 l^2}{(\mu_2 l_1+\mu_1 l_2)(v_2^2\mu_1 l_1+v_1^2\mu_2 l_2)}. \tag{53}$$

Equation (52) is analogous in its form to the equation for the dispersion law of coupled magnetostatic spin and elastic waves in a homogeneous ME ferromagnet and transforms to it in the limiting case $l_1=l$, $l_2=0$. Indeed, in this limiting case, Eq. (52) coincides with Eq. (41) if the exchange stiffness D in the latter equation is equal to zero.

Thus, the ME waves in the MS do not have the dispersion resulting from the exchange coupling. Far from the frequency of the ME resonance, their dispersion is determined by the elastic parameters of both layers of the MS and in the vicinity of the frequency of the ME resonances the dispersion is determined by both the elastic and ME parameters.

The formula for the FMR frequency follows from Eq. (52) at $K=0$:

$$\omega=g\left[H-4\pi M_0+\frac{\gamma^2M_0^3}{\mu_1}\frac{\mu_2 l_1}{\mu_2 l_1+\mu_1 l_2}-\frac{\gamma^2M_0^3}{\mu_1}\frac{2S_1\mu_2 l_2}{S_0\bar{\mu}l}+\frac{6K_1K_2\Delta\kappa\Delta T\mu_2 l_2}{S_0}\gamma M_0\right]^{1/2}\left[H-4\pi M_0+\frac{\gamma^2M_0^3}{\mu_1}\frac{\mu_2 l_1}{\mu_2 l_1+\mu_1 l_2}-\frac{\gamma^2M_0^3}{\mu_1}\frac{(2S_1-S_0)\mu_2 l_2}{S_0\bar{\mu}l}+\frac{6K_1K_2\Delta\kappa\Delta T\mu_2 l_2}{S_0}\gamma M_0\right]^{1/2}. \tag{54}$$

The comparison of the expressions in the brackets of Eq. (54) with the corresponding partial frequencies, Eqs. (40), shows that the fourth and fifth terms in each of these expressions are wholly determined by the ME ground state calculated in Sec. III. Thus, these terms are absent in the formula for the FMR frequency in Refs. 16 and 17, where effects of the ME ground state were not taken into account. The third term in each bracket of Eq. (54) has a different origin. It is seen from the formulas for the partial frequencies, Eqs. (40), that its base is the isotropic ME term $B=\gamma^2M_0^3/\mu_1$ being also in a homogeneous ferromagnet and leading to the ME gap in the spin-wave spectrum at the intrinsic magnetic field equal to zero[20,21] [in our case, this situation corresponds to $H=H_m^{(z)}$, where $H=H_m^{(z)}$ is determined by Eq. (35)]. Because of the dynamic interaction of waves with the initial periodic structure, the difference of the terms $P-B=-\gamma^2M_0^3l_2/(\mu_2 l_1$

$+\mu_1 l_2$) adds to this term [see the coefficients of Eq. (52)] and the term P appears in each bracket of Eq. (54) describing the FMR frequency. Namely, this term determines the gap in the spin-wave spectrum in the MS at the intrinsic magnetic field equal to zero. This term returns to its initial form ($P \to B$) in the limiting case of a homogeneous ferromagnet ($l_1 \to l, l_2 \to 0$). Note that neither the ME ground state nor the initial spontaneous strains $u_{ij}^{(00)}$ were taken into account in Refs. 16 and 17. That is why the formula for the FMR frequency of the MS in these papers contains only the term corresponding to the difference $B-P$.

V. CONCLUSION

The ME ground state that comes from the elastic interaction between the layers of the MS consisting of alternating layers of a ferromagnet and a nonmagnetic dielectric as well as coupled ME waves propagating in this MS against a background of the obtained ground state are studied. The MS is assumed to be infinite both in the plane of the layers xOy and along the MS axis. We assume that the method of the MS preparation corresponds to the absence of the elastic stresses resulting from the interaction between the MS layers at the preparation temperature $T=T_0$ and the orientation of the magnetization vectors $\mathbf{M}=\mathbf{M}_{00}$ in the plane of the magnetic layers. We also assume that each ferromagnetic layer has passed into the state with the spontaneous magnetostriction strains corresponding to the initial orientation of the magnetization vector $\mathbf{M}_{00}$ before the joining of the layers. As the accidental deviations from the initial equilibrium state are possible during the real process of the MS preparation, the MSs annealed at the temperature T_0, apparently, correspond more to this initial ideal state.

The general equations of motion for the magnetization and displacements break into sets of the static and dynamic equations. The static equations describe the ME ground state in the external field $\mathbf{H}$ at the temperature T which are different from the initial field $\mathbf{H}=0$ and temperature T_0. The dynamic equations describe ME waves propagating against a background of both the initial periodically inhomogeneous parameters of the MS and the periodically inhomogeneous ME ground state corresponding to the solution of the set of the static equations.

As the result of the solution of the static problem, the three new terms of the effective magnetic anisotropy arise spontaneously in the magnetic part of the effective MS energy. Two of them result from the ME interaction between the MS layers and represent the xOz easy-plane anisotropy and the xOy easy-plane anisotropy. Their magnitudes are proportional to the square of the ME coupling parameter γ. The third term results from the thermomagnetic interaction between the MS layers and represents the anisotropy with the axis along the z coordinate. The magnitude of this anisotropy is proportional to the first degree of the ME coupling parameter γ, the difference of the thermal expansion coefficients of the ferromagnetic and nonmagnetic layers $\Delta\kappa$, and the difference of the investigation and preparation temperatures ΔT. Depending on the sign of these values, this anisotropy may have both positive and negative signs, with which it adds to the shape anisotropy resulting from the demagnetizing field of the ferromagnetic layer $-4\pi M_{0z}$. All three terms of the anisotropy depend on the relationship between thicknesses of the magnetic and nonmagnetic layers. The obtained ME ground state exhibits the following properties. At $\mathbf{H}=0$ and $T=T_0$, the vector $\mathbf{M}$ is oriented in the plane of the layers along the x axis and the elastic stresses resulting from the interaction between the MS layers vanish. The elastic strains in the nonmagnetic layer $v_{ij}^{(0)}$ also vanish and the elastic strains in the ferromagnetic layer take the form of $u_{ij}^{(0)}=u_{ij}^{(00)}$, where $u_{ij}^{(00)}$ are the spontaneous strains due to the initial orientation of the magnetization $\mathbf{M}_{00}$. At remagnetizing of the MS by the homogeneous rotation of the magnetic moment in the magnetic field $\mathbf{H}$ directed in the plane of the layers along the x axis, the projection $M_{0x}(H)$ exhibits a rectangular hysteresis loop with the coercivity H_c proportional to γ^2 and l_2/l. At application of $\mathbf{H}$ in the plane of the layers along the y axis, the projection $M_{0y}(H)$ follows the anhysteretic magnetization curve $M_{0y}/M_0=H/H_m^{(y)}$, where the magnetic saturation field $H_m^{(y)}$ coincides in magnitude with H_c for the previous case. At application of the field along the MS axis z, the projection M_{0z} follows the nonlinear anhysteretic magnetization curve determined by Eq. (34). The magnetic saturation field $H_m^{(z)}$ differs in this case from $4\pi M_0$ by the value of the sum of fields of the ME and thermoelastic effective anisotropies. Estimations show that both of these effective anisotropies may have the same order of magnitude for temperature changing by 50 °C.

The spectrum of ME waves propagating against a background of the periodically inhomogeneous ME state of the MS is studied for the particular case when the magnetic field $\mathbf{H}$, static part of the magnetization $\mathbf{M}_0$, and wave vector are directed along the normal to the surface of the layers. The method based on Floquet's theorem is used to obtain the ME wave spectrum. The analytical form of the dispersion law containing several thousand terms is computed with the MAPLE program. Numerical investigation shows that the obtained spectrum corresponds qualitatively to the spectrum that was obtained first in Ref. 15 and then in Refs. 16 and 17: besides usual gaps (forbidden zones) situated at the Brillouin zone boundaries of the MS at $K=\pi m/l$, $m=1,2,\ldots$, the ME gaps appear in the vicinities of crossings of the elastic wave dispersion law with the localized frequencies of spin waves $\omega_n=\omega_0+g\alpha M_0 k_n^2$ at $k_n=\pi n/l_1$, $n=0,1,2,\ldots$. However, taking into account the ME ground state arising from the elastic interaction between the layers that was calculated in the first part of our paper leads to the essentially different dependences of the coefficients in the dispersion law on all parameters of the MS, external magnetic field, and temperature. The coincidence of the ME gap with the gap at the Brillouin zone boundary of the MS also was not considered earlier. In this case, the ME coupling leads to the modification of the dispersion law of elastic waves at the Brillouin zone boundaries as well as to the broadening of the localized spin-wave level into the permitted zone placed inside the forbidden zone. For the long waves (at $K \ll \pi/l$) near the vicinity of the first ME resonance corresponding to $n=0$, the equation for the dispersion law is analogous to the equation for the dispersion law of coupled magnetostatic and elastic waves in the homogeneous ferromagnet: the dispersion that is due to the exchange interaction is absent in the ME waves in the MS. The FMR frequency that came from the general dispersion law at $K=0$ is modified in the general case by all three terms of the effective magnetic anisotropies resulting from the ME ground state as well as by the isotropic ME term. The latter term leads to the gap in the spin-wave spectrum at H corresponding to the equal to zero magnitude of the intrinsic magnetic field. In contrast to the ME gap in the homogeneous ferromagnet, this gap in the MS is proportional to the relation of the ferromagnetic layer thickness l_1 to the MS period l.

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