Light-induced Hall effect in semiconductors with spin-orbit coupling

Xi Dai[1,2] and Fu-Chun Zhang[2]
[1]*Institute of Physics, Chinese Academy of Sciences, Beijing, China*
[2]*Department of Physics and Center of Theoretical and Computational Physics, The University of Hong Kong, Hong Kong, China*
(Received 11 October 2006; revised manuscript received 17 April 2007; published 27 August 2007)

We show that optically excited electrons by a circularly polarized light in a semiconductor with spin-orbit coupling subject to a weak electric field will carry a Hall current transverse to the electric field. This light-induced Hall effect is a special type of anomalous Hall effect and is a result of quantum interference of the light and the electric field and can be viewed as a physical consequence of the spin current induced by the electric field. The light-induced Hall conductivity is calculated for the p-type GaAs bulk material and the n-type and p-type quantum well structures.

DOI: 10.1103/PhysRevB.76.085343 PACS number(s): 72.25.−b, 71.70.Ej, 72.20.My, 75.47.−m

I. INTRODUCTION

The phase coherent semiconductor spintronic device is an important candidate for quantum devices, which allows the storage, manipulation, and transport of quantum information.[1] Due to the quantum nature of a spin system, a single electron with spin $1/2$ is an ideal qubit for quantum computing and an ideal unit for data storage. Therefore, the study of spin transport is important for the future development of spintronic techniques. Circularly polarized light[2–4] may be used to manipulate and detect the spin of an electron. In that process, the absorption of the light may induce a spin polarized charge current, which is called circular photogalvanic effect.[5] The photogalvanic effect was first proposed almost three decades ago[6] and has been detected in both bulk materials and semiconductor quantum well structures.[7]

In this paper, we propose another effect for a broad class of semiconductors with spin-orbit coupling, which we shall call light-induced Hall effect. In this effect, optically excited electrons by a circularly polarized light in a semiconductor subject to a weak static electric field will carry a Hall current transverse to the electric field. The light-induced Hall effect is a special type of anomalous Hall effect. Different from the photogalvanic effect, which occurs only in the gyrotropic materials, the light-induced Hall effect shows up also in some nongyrotropic compounds, such as GaAs with the bulk zinc-blende structure.[1] The effect may be viewed as a response of the local spin Hall current to the circularly polarized light.[8–17] In the materials with the structure inversion asymmetry, the effect is expected to be more pronounced when the incident light is normal to the sample so that the photogalvanic effect[2] vanishes. We estimate the effect quantitatively by calculating the Hall photocurrent for three different systems: the p-type GaAs bulk material and the n-type and p-type GaAs quantum well structures.

We begin with a more detailed discussion of the light-induced Hall effect. Let us consider a semiconductor with an incident circularly polarized light along the z axis with $\vec{e}_p$ as its Poynting unit vector and a weak external static electric field $\vec{E}$ along the x axis. Similar to the ordinary Hall effect, a transverse electric current along the y direction will be generated in addition to the current along the x direction. The schematic plot of the light-induced Hall effect is illustrated in Fig. 1. The transverse current in this case is entirely induced by the circularly polarized light through the optical transition from the valence band to the conduction band, and its direction and magnitude can be determined by $\mathbf{J}_{\rm Hall}=\sigma_{xy}\lambda\mathbf{E}\times\mathbf{e}_P$, where σ_{xy} is the light-induced Hall conductivity and $\lambda=\pm1$ is the helicity of the light. From the symmetry point of view, the circularly polarized light in the light-induced Hall effect plays a similar role to the magnetic field in the ordinary Hall effect, which breaks the time reversal symmetry. However, unlike the ordinary Hall effect, the light-induced Hall effect is purely a quantum effect induced by the spin-orbit coupling. As will be shown below, the light-induced Hall effect is induced by the Berry curvature of the band structure in the k space.

The light-induced Hall effect may be understood as a quantum interference effect between the light and the static electric field. As discussed by Murakami *et al.*[8] and by Sinova *et al.*,[9] when an electron (or a hole) moving parallel or antiparallel to the y axis is accelerated along the x direction due to the electric field, the electron spin will tilt upward or downward in the z direction, thus generating a nonzero spin current $j_y^z=\hbar/4(v_y\sigma^z+\sigma^z v_y)$. The spin Hall effect has generated a lot of research interest recently.[8–17] In the presence of a right-handed circularly polarized light, the electrons in the GaAs bulk or quantum well will be pumped from the valence band to the conduction band. Within the dipole approximation, only the electron with a total angular momentum along the z axis $J_z=-3/2$ ($J_z=-1/2$) will absorb a photon and tran-

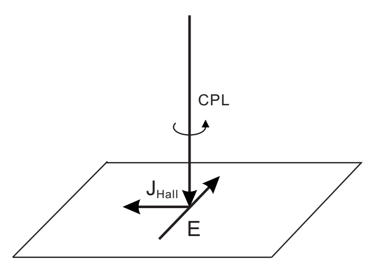

FIG. 1. The Hall photocurrent generated by the circularly polarized light and static electric field. The circularly polarized light and the static electric field are applied along the z and x directions, respectively, and the Hall photocurrent is generated along the y direction.

sits into the conduction band with $S_z=-1/2$ ($S_z=1/2$). Therefore, if the electron spin in the valence band tilts upward or downward, the corresponding transition rate to the conduction band will then be enhanced or suppressed due to the transition selection rule. As a consequence, there will be an imbalance of the photoexcited conduction electron density in the k space along the y direction, which leads to a spin polarized current along the y direction.

The light-induced Hall effect can also be viewed as an optical response of systems carrying a pure spin current. The possible physical consequence induced by the spin current is a highly interesting issue in the field of spintronics. The light-induced Hall effect generated by the spin current can then be either used to detect the existence of a spin current or to design the new type of quantum devices. The spin current generated by spin Hall effect is very difficult to detect. Up to now, the only way to measure the spin current flowing through the sample is to measure the spin accumulation at the edges generated by the spin current.[18,19] Furthermore, it is also complicated to abstract a quantitative information of the spin current from the measurement of the spin accumulation.[20] The charge current can be measured relatively easier by detecting the magnetic field built up around the current, for instance, the light-induced Hall effect should shed light on the new methods of measuring spin current.

It is also very interesting to compare the light-induced Hall effect with the familiar anomalous Hall effect in ferromagnetic metals, where a large anomalous Hall current is generated perpendicular to both applied electric field **E** and the magnetization **M** of the metal. Although from the basic symmetry point of view, the light-induced Hall effect is similar to other anomalous Hall effect such as in the ferromagnetic metals, it has two main distinguished features. Firstly, the spin polarization of the conduction electrons is induced as a by-product of the optical transition process and is not a physical origin of the transverse photocurrent. Secondly, unlike the anomalous Hall effect in other systems, the spin polarization in the present case is generated by the nonequilibrium processes, which has never been discussed in detail before.

II. SEMICLASSICAL ANALYSES

In this section, we will use semiclassical approach to examine a related but more general question: What will be the optical response of a system carrying a pure spin current? By the pure spin current, we mean the charge current vanishes. We shall consider a p-type GaAs sample and the optical transition between the heavy hole valence band and the conduction bands, unless specifically indicated otherwise. The Hamiltonians for the conduction band and valence band near the Γ point are given by

$$H_v(\vec{P}) = \frac{P^2}{2m}\left(\gamma_1 + \frac{5\gamma_2}{2}\right) - \frac{\gamma_2}{m}(\mathbf{S}\cdot\mathbf{P})^2, \tag{1}$$

$$H_c(\vec{P}) = \frac{P^2}{2m_c}, \tag{2}$$

with $\gamma_1=6.92$ and $\gamma_2=2.1$ for bulk GaAs. $\mathbf{S}=(S_x,S_y,S_z)$ are the spin-3/2 operators, and m_c is the effective mass for the conduction band.

In a semiclassical approach, the spin current operator of a charged particle in the twofold degenerate heavy hole bands can be expressed as $j_y^z(k)=\frac{3}{2}\hbar\sigma_z\mathbf{v}_y$, with $\mathbf{v}_y=\frac{\partial H_k}{\partial \mathbf{k}_y}=\frac{\hbar\mathbf{k}_y}{m_{hh}}$. The spin current of the heavy holes is then

$$J_y^z = \sum_k \mathrm{Tr}[\rho_k j_y^z(k)],$$

where ρ_k is a 2×2 reduced density matrix of momentum $\hbar k$. In the diagonal representation of ρ_k, the spin current is given by

$$J_y^z = \sum_{\mathbf{k}} \frac{3\hbar^2\mathbf{k}_y}{2m_{hh}}(a_k - b_k)\cos 2\theta_k, \tag{3}$$

where a_k and b_k are the two eigenvalues of the matrix ρ_k, and the unitary matrix U_k satisfies

$$U_k^+\rho_k U_k = \begin{pmatrix} a_k & 0 \\ 0 & b_k \end{pmatrix},$$

with

$$U_k = \begin{pmatrix} \cos\theta_k e^{-i\phi_k/2} & \sin\theta_k e^{-i\phi_k/2} \\ \sin\theta_k e^{i\phi_k/2} & -\cos\theta_k e^{i\phi_k/2} \end{pmatrix}.$$

The system has a time reversal symmetry in the absence of the circularly polarized light, and we have

$$\theta_{-k} = \pi/2 - \theta_k, \quad \phi_{-k} = -\phi_k, \quad a_{-k} = a_k, \quad b_{-k} = b_k. \tag{4}$$

When the photon frequency ω of the incident light is switched on, an electron in the heavy hole bands with the momentum and energy satisfying $\epsilon_k^{hh}+\epsilon_k^c+E_g=\hbar\omega$ will be excited to the conduction band, where $\epsilon_k^{hh}=\frac{\hbar^2k^2}{2m_{hh}}$ and $\epsilon_k^c=\frac{\hbar^2k^2}{2m_c}$ are the hole and conduction electron dispersions, respectively, $m_{hh}=\frac{m}{\gamma_1-2\gamma_2}$ is the hole effective mass, and E_g is the energy gap of GaAs. If the light is fully right-handed polarized, the only allowed transition will be from $|-3/2,k\rangle$ state in the heavy hole band to the $|-1/2,k\rangle$ state in the conduction band. Assuming the above transition rate to be Γ, we obtain the following electric current in the conduction band:

$$J_y^e(\omega) = \sum_{\mathbf{k}} \frac{e\Gamma\tau}{m_c}\mathbf{k}_y(a_k - b_k)\sin^2\theta_k\,\delta(\epsilon_k^{hh} + \epsilon_k^c + E_g - \hbar\omega), \tag{5}$$

where τ is the momentum relaxation time for the electrons in the conduction band. In the above derivation, we have only included the electric current carried by the electrons in the conduction band and neglect the contribution from the valence band. This approximation is justified for the effective mass is much heavier and the momentum relaxation time is much shorter in the valence band.

Integrating out the photon energy $\hbar\omega$ and using Eq. (4), we have

$$\int \hbar J_y^e(\omega)d\omega = \sum_{\mathbf{k}} \frac{e\Gamma\tau}{m_c}\mathbf{k}_y(a_k - b_k)\sin^2\theta_k = \frac{e\Gamma\tau m_{hh}}{3m_c\hbar}J_y^z. \tag{6}$$

Therefore, the light-induced charge Hall current is proportional to the spin current in the absence of the light. For the n-type sample, we obtain a very similar formula by using the same analysis,

$$\int \hbar J_y^e(\omega)d\omega = \frac{e\Gamma\tau}{\hbar}J_y^z. \tag{7}$$

Since the charge current is directly caused by the spin current, the effect can be viewed as a direct physical consequence of the spin current. Namely, by using this effect, one could directly measure the spin current in the bulk. So far, we have assumed a finite spin current in the system. The spin current can be generated for different mechanisms, for example, by the spin dependent impurity scattering (extrinsic spin Hall effect), the spin-orbit coupling (intrinsic spin Hall effect), or the spin injection from magnetic junctions. In the next section, we will examine the light-induced Hall effect, where the spin current is generated by an intrinsic spin Hall effect.

III. LIGHT-INDUCED HALL CONDUCTIVITY

In what follows, we will discuss the light-induced Hall effect in three different systems, namely, the three-dimensional (3D) hole system described by the Luttinger model, the two-dimensional (2D) hole gas described by the Luttinger model with the confinement potential along the z direction, and the 2D electron gas described by the Rashba model.

A. Light-induced Hall conductivity in 3D Luttinger model

In this section, we study the light-induced Hall effect in the GaAs bulk material. The Hamiltonian is given by Eqs. (1) and (2). This Hamiltonian can be easily diagonalized. To calculate the interband transition rate in the presence of the static electric field, we use a nonlinear response theory, where we will take into account of the second-order correction combining the electric field $\mathbf{E}$ and the intensity of the light I. This high-order response term can be obtained by the following way. First, we switch off the light field and obtain the approximate wave function to the first-order of the static electric field $\mathbf{E}$. Then, we switch on the light field and use the $\mathbf{E}$ dependent wave function to calculate the transition rate. Following Refs. 10 and 23, the electric field is included in the Hamiltonian through the vector potential $\mathbf{A}=\mathbf{E}t$ and the momentum $\mathbf{P}$ in Eqs. (1) and (2) is replaced by $\mathbf{P}-e\mathbf{E}t$.

We assume that the electric field is switched on at time $t=0$ and obtain the first-order time-dependent wave function $|m,k,t\rangle^E$ for such a system in terms of the instantaneous eigenstates,

$$|m,\mathbf{k},t\rangle^E = \exp\left\{-i\int_0^t \frac{dt'\varepsilon_m(\mathbf{k},t')}{\hbar}\right\} \times\left\{|m,\mathbf{k},t\rangle + i\sum_{n\neq m}\frac{|n,\mathbf{k},t\rangle(f_{n,k}-f_{m,k})\mathbf{\Omega}_{nm}(\mathbf{k},t)\cdot e\mathbf{E}}{[\varepsilon_n(\mathbf{k},t)-\varepsilon_m(\mathbf{k},t)]} \times\left[1-\exp\left\{-i\int_0^t\frac{[\varepsilon_n(\mathbf{k},t')-\varepsilon_m(\mathbf{k},t')]dt'}{\hbar}\right\}\right]\right\}, \tag{8}$$

where $|m,k,t\rangle$ is the instantaneous eigenstates of the Hamiltonian $H_v(\hbar\mathbf{k}-e\mathbf{E}t)$, which satisfies

$$H_v(\hbar\mathbf{k}-e\mathbf{E}t)|n,\mathbf{k},t\rangle = \varepsilon_n(\mathbf{k},t)|n,\mathbf{k},t\rangle, \tag{9}$$

and $f_{n,k}$ is the Fermi distribution function and $\Omega_{nm}(\mathbf{k},t)=\langle n,\mathbf{k},t|\frac{\partial}{\partial\mathbf{k}}|m,\mathbf{k},t\rangle$ is the Berry curvature of the Bloch states.

We then switch on the circularly polarized light. The optical transition rate can be obtained by solving the time-dependent Schrödinger equation perturbatively. After a lengthy derivation, which is given in the Appendix, we can show that the optical transition rate in the presence of the electric field can be obtained by applying the Fermi golden rule to the system by using the above wave function. The result is given by

$$\Gamma_{mk,\alpha} \approx \left\{|\langle\alpha,\mathbf{k}|W|m,\mathbf{k}\rangle|^2 + 2\sum_{n\neq m} \times\mathrm{Re}\left[i\frac{W_{\alpha n}(\mathbf{k})\mathbf{\Omega}_{nm}(\mathbf{k},0)W_{m\alpha}(\mathbf{k})\cdot e\mathbf{E}(f_{n,k}-f_{m,k})}{[\varepsilon_n(\mathbf{k})-\varepsilon_m(\mathbf{k})]}\right]\right\} \times f_{m,k}(1-f_{\alpha,k})\delta(\hbar\omega-\varepsilon_\alpha(\mathbf{k})+\varepsilon_m(\mathbf{k})). \tag{10}$$

The matrix $\hat{W}$ describes the coupling between the electrons in the solid and the right-handed circularly polarized light in the dipole approximation with only two nonzero matrix elements to be $\hat{W}_{1/2,3/2}=g$ and $\hat{W}_{-1/2,1/2}=g/\sqrt{3}$, where $g=dE_{rad}$ with d the effective dipole induced by the light and E_{rad} the amplitude of the electric field of the light. Assuming the power density of the light to be 100 mW/mm^2 and $d=4.8\times10^{-29}$ C m,[24] we estimate the coupling energy of electron and the light to be 2.6038×10^{-6} eV.

Within the simplest relaxation time approximation, we can express the Hall photocurrent as the summation of the electron and hole currents,

$$\langle\mathbf{j}_{total}\rangle = \sum_{\mathbf{k}}\sum_{m\alpha}[e\mathbf{v}^e_{\alpha\alpha}(\mathbf{k})\Gamma_{mk,\alpha}\tau_e - e\mathbf{v}^h_{mm}(\mathbf{k})\Gamma_{mk,\alpha}\tau_h], \tag{11}$$

where $\mathbf{v}_k^e$ and $\mathbf{v}_k^h$ are the velocity operators, and τ_e and τ_h are the relaxation times for the electron and hole, respectively. For circularly polarized light propagating normal to the xy plane, the contribution from the first term in Eq. (10) vanishes after integrating over $\vec{k}$. Therefore, in the present case, the total charge current, which is found along the y direction, is linearly proportional to the static external electric field. Similar to the ordinary Hall effect, we can express the transverse charge current in terms of the electric field, which reads $J_y=\sigma_{xy}^{ph}E_x$, with

$$\sigma_{xy}^{ph}=\sum_{m\alpha}\sum_{\mathbf{k}}[ev_{\alpha\alpha,x}^{e}(\mathbf{k})\Gamma_{mk,\alpha}\tau_e-ev_{mm,x}^{h}(\mathbf{k})\Gamma_{mk,\alpha}\tau_h]\times\left\{2\sum_{n\neq m}\text{Re}\, i\frac{W_{\alpha n}(\mathbf{k})\mathbf{\Omega}_{nm}(\mathbf{k},0)W_{m\alpha}(\mathbf{k})\cdot\mathbf{e}_y(f_{n,k}-f_{m,k})}{[\varepsilon_n(\mathbf{k})-\varepsilon_m(\mathbf{k})]}\times f_{m,k}(1-f_{\alpha,k})\delta(\hbar\omega-\varepsilon_\alpha(\mathbf{k})+\varepsilon_m(\mathbf{k}))\right\}. \quad (12)$$

For the 3D Luttinger model, the Hamiltonian in the absence of the external fields can be solved analytically. We finally obtain a simple analytic expression for the light-induced Hall conductivity at low temperatures,

$$\sigma_{xy}^{ph}=\sum_{m=L,H}\frac{3\pi^2+2}{16\hbar\omega m_c\pi}\frac{\alpha I e^2\tau m_0\hbar(f_{\bar{m},k_m}-f_{m,k_m})}{(\hbar\omega-E_g)\gamma_2\mu_m},$$

where α is the optical absorption coefficient, which is around $10^4\ \text{cm}^{-1}$ for GaAs, k_m satisfies the equation $\hbar\omega-\varepsilon_c(\mathbf{k}_m)+\varepsilon_m(\mathbf{k}_m)=0$, τ is the momentum relaxation time of the conduction electron, m_0 is the free electron mass, m_c is the effective mass of the conduction electron, and μ_m is the effective optical mass defined as $\mu_m^{-1}=m_c^{-1}+m_v^{-1}$. If we choose the typical experimental parameters for GaAs as $\alpha=10^4\ \text{cm}^{-1}$, $\tau_e=10^{-12}$ s, $I=100\ \text{mW/mm}^2$, $E_g=1.42$ eV, and $\hbar\omega=1.67$ eV, we obtain the light-induced anomalous Hall conductivity to be $7.5805\times10^{-3}\ \Omega^{-1}\ \text{m}^{-1}$.

B. Light-induced Hall conductivity in quantum well structure

We now calculate the light-induced Hall conductivity defined in Eq. (12) in the quantum well structure. In this case, we consider the applied circularly polarized light to be normal to the plane to eliminate the photogalvanic current. In the absence of the $\vec{E}$ field, the theory predicts a pure spin current within the plane[21,22] with a null charge current. In the presence of the $\vec{E}$ field, the calculation is similar to the difference of the replacement of the eigenstates of the subbands by Eq. (12). Below, we calculate the light-induced Hall conductivity for both p-type and n-type quantum well samples numerically. The Hamiltonian of the GaAs quantum well structure can be written as

$$H_{v,well}(\mathbf{P})=H_v(\mathbf{P})+V(z)+\lambda_v(\mathbf{P}\times\mathbf{S}), \quad (13)$$

$$H_{c,well}(\mathbf{P})=H_c(\mathbf{P})+V(z)+\lambda_c(\mathbf{P}\times\sigma), \quad (14)$$

where $H_v(\mathbf{P})$ and $H_c(\mathbf{P})$ are the Hamiltonians and λ_v and λ_c are the effective Rashba couplings for the valence (v) and conduction (c) bands, respectively. $\mathbf{S}$ and σ are the spin-3/2 and spin-1/2 matrices for valence and conduction bands, respectively. We choose the confinement potential $V(z)=+\infty$ for $|z|>L$, and $V(z)=0$ otherwise. As presented in our previous paper,[17] we only consider the structural inversion symmetry breaking by including the Rashba coupling in our Hamiltonians and neglect the nonsymmetric terms in $V(z)$. Using the numerical techniques presented in detail in Ref. 17, we first obtain the dispersions for the e1, HH1, and LH1 subbands, which are plotted in Fig. 1. In the calculations, we choose $\gamma_1=7.0$, $\gamma_2=1.9$, and $m_e=0.067m_0$, where m_0 is the bare electron mass. We then calculate the light-induced Hall conductivity for both the p-type and n-type quantum well structures with the following parameters: $\tau_e=5.8863\times10^{-11}$ s, $I=100\ \text{mW/mm}^2$, and $g=2.6038\times10^{-6}$ eV. The carrier density is chosen to be $9.2807\times10^{10}\ \text{cm}^{-2}$ for the n-type case and $2.6261\times10^{11}\ \text{cm}^{-2}$ for the p-type case. The results are shown in Figs. 2 and 3. In the present study, we only include the transition between the HH1, LH1, and e1 subbands. The different behavior of the light-induced Hall effect between the n-type and p-type samples is quite clear as shown in these figures. In the n-type sample, the contribution to the Hall conductivity from the HH1-e1 transition has an opposite sign to that of the HH1-e1 transition. While in the p-type sample, the contributions to the Hall conductivity from the above two transitions have the same sign. This interesting asymmetric behavior of n-type and p-type samples can be understood in the following way. In the n-type sample, the Fermi surface lies within the subband e1 and the electron spin tilts out of the plane in the presence of an

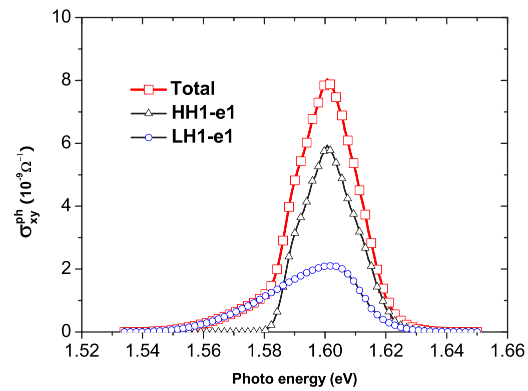

FIG. 2. (Color online) The light-induced Hall conductivity in a p-type GaAs quantum well as a function of photon energy. See the tex for the parameters.

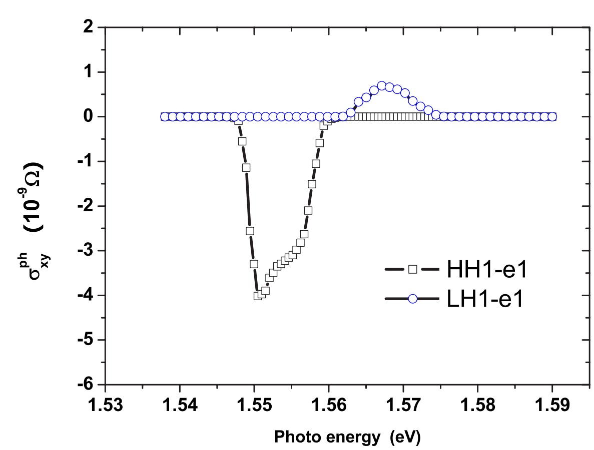

FIG. 3. (Color online) The light-induced Hall conductivity in a n-type GaAs quantum well as a function of photon energy. See the text for the parameters.

in-plane electric field. For definiteness, we consider a right-handed circularly polarized light. According to the selection rule, the only allowed process in the HH1-e1 transition is from $|S_z=-3/2\rangle$ in the valence band to $|s_z=-1/2\rangle$ in the conduction band, and that of the LH1-e1 transition is from $|S_z=-1/2\rangle$ in the valence band to $|s_z=1/2\rangle$ in the conduction band. Thus, as the electron spin tilts out of the plane, the two transition rates will be shifted in the opposite directions, which contributes to the Hall conductivity with opposite signs.

From Eq. (12), we know that the strength of the light-induced Hall effect is determined by the optical coupling matrix and the Berry curvature of the Bloch states at the manifold in the k space which satisfies the energy conservation. The physical consequence of the Berry curvature in k space was first found in the anomalous Hall effect and later in the spin Hall effect. Therefore, the light-induced Hall effect we proposed here may also be viewed as a another physical consequence of the Berry curvature in the momentum space.

IV. CONCLUSION

Finally, we remark the role of the disorder in the light-induced Hall effect. It has been established that the spin Hall effect in the linear k-dependent Rashba systems vanishes in the thermodynamic limit due to the vertex correction.[14,16] Therefore, in such systems, the spin current induced by spin Hall effect may only exist in the mesoscopic scale. Since the light-induced Hall effect is generated by the direct optical absorption modulated by the static electric field, the light-induced Hall effect only requires the spin current in the scale of the light wavelength, which is in the mesoscopic scale for GaAs. Therefore, unlike the spin Hall effect, for systems such as the n-type GaAs quantum well structure described by the Rashba model, we expect that the vertex correction will not play a vital role in our theory for the light-induced Hall effect.

In summary, we have proposed another effect, the light-induced Hall effect, in this paper. This effect is generated by the modulation of the optical transition rate in the k space induced by an in-plane static electric field. The effect can be viewed as the quantum interference effect between the light field and the static electric field and can be thought as the physical consequence generated by the nonzero spin current at the mesoscopic scale and the electric field. The effect can also be viewed as the physical effect reflecting the Berry curvature of the Bloch state in the k space. We have also calculated the light-induced Hall conductivity for three different semiconductor systems and made the quantitative predictions, which are observable.

ACKNOWLEDGMENTS

This work was supported in part by RGC of HKSAR via RGC705405 and RGC702105 and also by the Lu-Xin Energy Group, China.

APPENDIX

In this appendix, we present a detailed calculation of the interband transition rate in the presence of a radiation field of intensity I and a static electric field $\mathbf{E}$ along the x direction and derive Eq. (10) in the text.

The nonlinear response theory is needed to include the correction to the second order of the product of parameters $\mathbf{E}$ and I. This high-order correction term can be obtained by the following way. We first switch off the light field and obtain the wave function accurate to the first order of the static electric field $\mathbf{E}$. Then, we switch on the light field and use the wave function of the system in the presence of the static electric field to calculate the transition rate.

Following Ref. 10, we use a gauge where the electric field appears in the Hamiltonian through the vector potential $\mathbf{A}=\mathbf{E}t$, and the momentum $\mathbf{P}$ in Eqs. (1) and (2) is replaced by $\mathbf{P}-e\mathbf{E}t$. In this gauge, the Hamiltonian of the system in the finite electric field and zero radiation field is time dependent, given by $H_0(\hbar\mathbf{k}-e\mathbf{E}t)$. Denoting its instantaneous eigenstate to be $|n,\mathbf{k},t\rangle$, we have

$$H_0(\hbar\mathbf{k}-e\mathbf{E}t)|n,\mathbf{k},t\rangle=\varepsilon_n(\mathbf{k},t)|n,\mathbf{k},t\rangle, \tag{A1}$$

with $\varepsilon_n(\mathbf{k},t)$ the corresponding instantaneous eigenenergy and $H_0=H_c+H_V$. Then, the time-dependent wave function accurate to the first order in E is given by

$$|m,\mathbf{k},t\rangle^E=\exp\left\{-i\int_0^t\frac{dt'\,\varepsilon_{k,m}(t')}{\hbar}\right\}\left\{|m,\mathbf{k},t\rangle+i\sum_{n\neq m}\frac{|n,\mathbf{k},t\rangle(f_{n,k}-f_{m,k})\mathbf{\Omega}_{nm}(\mathbf{k},t)\cdot e\mathbf{E}}{[\varepsilon_n(\mathbf{k},t)-\varepsilon_m(\mathbf{k},t)]}\times\left[1-\exp\left\{-i\int_0^t\frac{[\varepsilon_n(\mathbf{k},t')-\varepsilon_m(\mathbf{k},t')]dt'}{\hbar}\right\}\right]\right\}, \tag{A2}$$

where $f_{n,k}$ is the Fermi distribution function. It is straightforward to see that the wave function in Eq. (A2) satisfies the Schrödinger equation below to the first order in the electric field E,

$$i\hbar\frac{\partial}{\partial t}|m,\mathbf{k},t\rangle^E=H_0(\hbar\mathbf{k}-e\mathbf{E}t)|m,\mathbf{k},t\rangle^E. \tag{A3}$$

We next include an interaction term given below between the system and the light,

$$H_I(t)=\sum_{n,\alpha}\frac{W_{n\alpha}|n\rangle\langle\alpha|(e^{i\omega t}+e^{-i\omega t})}{2}+\text{H.c.} \tag{A4}$$

The total wave function can be written in the form

$$|m,\mathbf{k},t\rangle^{E,L}=|m,\mathbf{k},t\rangle^E+|\phi(t)\rangle, \tag{A5}$$

where $|\phi(t)\rangle$ is the correction to the wave function due to the interaction with the light. The Schrödinger equation for the total wave function can be written, accurate to the leading nonzero order in the light field, as

$$i\hbar\frac{\partial}{\partial t}|m,\mathbf{k},t\rangle^{E,L}=i\frac{\partial}{\partial t}|m,\mathbf{k},t\rangle^E+H_I(t)|m,\mathbf{k},t\rangle^E+H_0(\hbar\mathbf{k}-e\mathbf{E}t)\times|\phi(t)\rangle. \tag{A6}$$

We thus obtain an equation for $|\phi(t)\rangle$ as follows:

$$i\hbar\frac{\partial}{\partial t}|\phi(t)\rangle = H_I(t)|m,\mathbf{k},t\rangle^E + H_0(\hbar\mathbf{k}-e\mathbf{E}t)|\phi(t)\rangle. \quad (A7)$$

We expand $|\phi(t)\rangle$ in terms of the eigenstates of H_v in the absence of the electric field,

$$|\phi(t)\rangle = \sum_\alpha C_{mk,\alpha}(t)e^{-i\varepsilon_\alpha(k)t}|\alpha,\mathbf{k}\rangle, \quad (A8)$$

where C can be found from Eq. (A7) as

$$C_{mk,\alpha}(t) = -\frac{i}{\hbar}\int_0^t \langle\alpha,\mathbf{k}|H_I(t')|m,\mathbf{k},t'\rangle^E e^{i\varepsilon_\alpha(k)t'}dt'.$$

Using Eq. (A2) for $|m,k,t\rangle$ and noting that all the time dependences in $|m,k,t'\rangle$, $\varepsilon_n(\mathbf{k},t')$, $|n,k,t'\rangle$, and $\varepsilon_m(k,t')$ in the right-hand side of Eq. (A2) are slowly varying in time within the period of the external radiation field for the weak static electric field, we can thus take the small t limit in all these functions. We then obtain

$$C_{mk,\alpha}(t) = -\frac{i}{\hbar}\int_0^t (e^{i\omega t'}+e^{-i\omega t'})dt'\Bigg\{\langle\alpha,k|W|m,k\rangle + i\sum_n \frac{(f_{n,k}-f_{m,k})\langle\alpha,\mathbf{k}|W|n,\mathbf{k}\rangle\mathbf{\Omega}_{nm}(\mathbf{k},0)\cdot\vec{E}}{[\varepsilon_n(\mathbf{k})-\varepsilon_m(\mathbf{k})]}\Bigg\} \times e^{-i[\varepsilon_m(k)-\varepsilon_\alpha(k)]t'}[1-e^{-i[\varepsilon_n(k)-\varepsilon_m(k)]t'}]. \quad (A9)$$

The transition rate is given by

$$\Gamma_{mk,\alpha} = |dC_{mk,\alpha}(t)/dt|^2. \quad (A10)$$

After some straightforward algebra, we obtain the transition rate given by Eq. (10) in the main text.

[1] D. Awschalom, D. Loss, and N. Samarth, *Semiconductor Spintronics and Quantum Computation* (Springer, New York, 2002).
[2] S. Ganichev and W. Prettl, J. Phys.: Condens. Matter **15**, R935 (2003).
[3] E. N. Bulgakov and A. F. Sadreev, JETP Lett. **73**, 505 (2001).
[4] V. N. Ritus, Sov. Phys. JETP **24**, 1041 (1967).
[5] B. Sturman and V. Fridkin, *The Photovoltaic and Photorefractive Effects in NonCentrosymmetric Materials* (Gordon and Breach, New York, 1992).
[6] E. Ivechenko and G. Pikus, Pis'ma Zh. Eksp. Teor. Fiz. **27**, 640 (1978) [Sov. Phys. JETP **27**, 640 (1978)]; V. Belinicher, Phys. Lett. **66A**, 213 (1978).
[7] V. Asnin, A. Bakun, A. Danishevskii, E. Ivchenko, G. Pikus, and A. Rogachev, Solid State Commun. **30**, 565 (1979); S. D. Ganichev, E. L. Ivchenko, S. N. Danilov, J. Eroms, W. Wegscheider, D. Weiss, and W. Prettl, Phys. Rev. Lett. **86**, 4358 (2001).
[8] S. Murakami, N. Nagaosa, and S. C. Zhang, Science **301**, 1348 (2003); Phys. Rev. B **69**, 235206 (2004).
[9] J. Sinova, D. Culcer, Q. Niu, N. A. Sinitsyn, T. Jungwirth, and A. H. MacDonald, Phys. Rev. Lett. **92**, 126603 (2004).
[10] J. Hu, B. Bernevig, and C. Wu, Int. J. Mod. Phys. B **17**, 5991 (2003).
[11] S. Q. Shen, Phys. Rev. B **70**, 081311 (2004).
[12] H. Engel, E. Rashba, and B. Halperin, arXiv:cond-mat/0603306.
[13] E. I. Rashba, Phys. Rev. B **68**, 241315(R) (2003).
[14] J. Inoue, G. E. W. Bauer, and L. W. Molenkamp, Phys. Rev. B **67**, 033104 (2003).
[15] S. Q. Shen, M. Ma, X. C. Xie, and F. C. Zhang, Phys. Rev. Lett. **92**, 256603 (2004).
[16] E. G. Mishchenko, A. V. Shytov, and B. I. Halperin, Phys. Rev. Lett. **93**, 226602 (2004).
[17] X. Dai, Z. Fang, Y. G. Yao, and F. C. Zhang, Phys. Rev. Lett. **96**, 086802 (2006).
[18] V. Sih, R. C. Myers, Y. K. Kato, W. H. Kato, W. H. Lau, A. C. Gossard, and D. D. Awschalom, Nat. Phys. **1**, 31 (2005).
[19] J. Wunderlich, B. Kaestner, J. Sinova, and T. Jungwirth, Phys. Rev. Lett. **94**, 047204 (2005).
[20] J. M. Luttinger, Phys. Rev. **102**, 1030 (1956); R. Winkler, H. Noh, E. Tutuc, and M. Shayegan, Phys. Rev. B **65**, 155303 (2002).
[21] R. D. R. Bhat, F. Nastos, Ali Najmaie, and J. E. Sipe, Phys. Rev. Lett. **94**, 096603 (2005); R. D. R. Bhat and J. E. Sipe, *ibid.* **85**, 5432 (2000).
[22] J. Li, X. Dai, S. Q. Shen, and F. C. Zhang, Appl. Phys. Lett. **88**, 162105 (2006).
[23] E. Fradkin, *Field Theories of Condensed Matter Systems* (Perseus, Cambridge, 1991), Chap. 9.
[24] P. Y. Yu and M. Cardona, *Fundamentals of Semiconductors* (Springer, New York, 1996).