Spin current and rectification in one-dimensional electronic systems

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We demonstrate that spin current can be generated by an ac voltage in a one-channel quantum wire with strong repulsive electron interactions in the presence of a nonmagnetic impurity and uniform static magnetic field. In a certain range of voltages, the spin current can exhibit a power dependence on the ac voltage bias with a negative exponent. The spin current expressed in units of $\hbar/2$ per second can become much larger than the charge current in units of the electron charge per second. The spin current generation requires neither spin-polarized particle injection nor time-dependent magnetic fields.

I. INTRODUCTION

The pioneering paper by Christen and Büttiker[1] has stimulated much interest to rectification in quantum wires and other mesoscopic systems. Most attention was focused on the simplest case of Fermi liquids.[2,3] Recently, this research was extended to strongly interacting systems where Luttinger liquids are formed.[4–7] One of the topics of current interest is the rectification effect in Luttinger liquids in a magnetic field.[3,6,7] In the presence of a magnetic field, both spin and charge currents can be generated. So far, however, only charge currents in Luttinger liquids have been studied. In this paper, we show that a dc spin current can be generated by an ac voltage bias in a single-channel quantum wire.

In recent years, many approaches to the generation of spin currents in quantum wires were put forward. Typically, both spin and charge currents are generated, and the spin current expressed in units of $\hbar/2$ per second is smaller than the electric current in units of e per second (e is the electron charge). Such a situation naturally emerges in partially polarized systems, since each electron carries the charge e and its spin projection on the z axis is $\pm\hbar/2$. A proposal on how to obtain a spin current *exceeding* the charge current in a quantum wire was published by Sharma and Chamon[8] who considered a Luttinger liquid in the presence of a time-dependent magnetic field in a region of the size of an electron wavelength. In a very different physical context, a spin current without charge current was predicted for edge modes in the quantum Hall effect in graphene.[9] Pure spin currents can also flow in open circuits, which cannot support charge currents.[10] In this paper, we show that the generation of a dc spin current exceeding the charge current is also possible in closed circuits without time-dependent magnetic fields. The spin current can be generated in a spatially asymmetric Luttinger liquid system in the presence of an ac bias. Interestingly, in a certain interval of low voltages, the dc spin current grows as a negative power of the ac voltage when the voltage decreases.

The paper is organized as follows. The next section contains a qualitative discussion. We briefly address the simplest case of noninteracting electrons, discuss its differences from the most interesting case of strong electron interaction, and estimate at what conditions the effect can be observed. Section III contains the details of the bosonization procedure which we use to treat the electron-electron interaction. In Sec. IV, we calculate analytically the spin and charge rectification currents in the presence of a weak asymmetric potential. Numerical results for a simple model with strong asymmetric potential are discussed in Appendix A. Appendices B and C contain technical details of the perturbation theory employed in Sec. IV.

II. MODEL AND PHYSICS OF THE PROBLEM

The rectifying quantum wire is sketched in Fig. 1. It consists of a one-dimensional conductor with a scatterer in the center of the system at $x=0$. The scatterer creates an asymmetric potential $U(x) \neq U(-x)$. The size of the scatterer $a_U \sim 1/k_F$ is of the order of the electron wavelength. A spin current can be generated only if time-reversal symmetry is broken. Thus, we assume that the system is placed in a uniform magnetic field **H**. The field defines the S_z direction of the electron spins. If the wire is sufficiently narrow, then the effect of the magnetic field on the kinetic energy of electrons can be neglected and the field enters the problem only via its interaction with the spins. At its two ends, the wire is connected to nonmagnetic electrodes, labeled by $i=1,2$. The left electrode, $i=1$, is controlled by an ac voltage source, while the right electrode, $i=2$, is kept on ground.

The magnetic field **H** breaks the symmetry between the two orientations of the electron spin. In a uniform wire, this would not result in a net spin current since the conductances
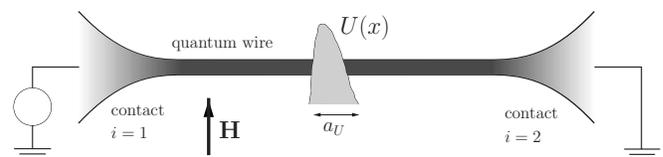

FIG. 1. Sketch of the one-dimensional conductor connected to two electrodes on both ends. Currents are driven through a voltage bias V that is applied on the left electrode while the right electrode is kept on ground. The system is magnetized by the field **H**. Electrons are backscattered off the asymmetric potential $U(x)$. $U(x) \neq 0$ in the region of size $a_U \sim 1/k_F$.
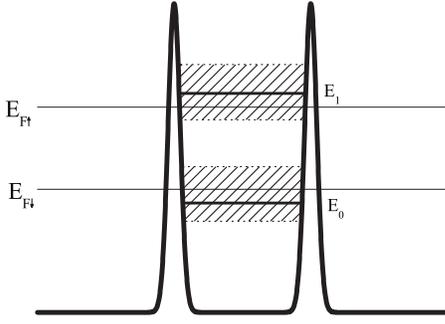
FIG. 2. Double-well potential with quasistationary levels. The transmission coefficient is maximal in the shaded regions. The narrow potentials $u_1(x)$ and $u_2(x)$ are centered at the positions $x=0$ and $x=a$ ($a < k_F^{-1}$), respectively, and are modeled by δ functions in Eq. (A1).

of the spin-up and -down channels would be the same,[17] e^2/h, and the spin currents of the spin-up and -down electrons would be opposite. In the presence of a potential barrier, such a cancellation does not occur.[18] In a system with strong electron interaction, the spatial asymmetry of the wire leads to an asymmetric I-V curve,[4] $I(V) \neq I(-V)$. Thus, an ac voltage bias generates spin and charge dc currents, I_s^r and I_c^r.

As we will see, the problem is most interesting in the case of strong electron interaction. Before addressing that more difficult case, let us discuss what happens in the absence of electron interaction. By noninteracting system, we mean a wire in which electron-electron interaction is completely screened by the gates. In such situation, the charge density in the wire is not fixed but depends on the gate potential and the electrochemical potentials of the leads. The leads define the chemical potentials μ_L and μ_R of the left- and right-moving electrons, which are injected from the right and left reservoirs, respectively. In what follows, we will assume that the chemical potential $\mu_L = 0$ and μ_R oscillates between $+eV$ and $-eV$. Thus, in the presence of the magnetic field **H**, the Fermi energies counted from the band bottom equal $E_F^L(S_z) = E_F + 2S_z\mu H$ for the left-moving electrons and $E_F^R(S_z) = E_F + \mu_R + 2S_z\mu H$ for the right-moving electron, where $S_z = \pm 1/2$ is the electron spin projection, E_F the Fermi level in the absence of the magnetic field and voltage bias, and μ the electron magnetic moment.

As we will see, in a strongly interacting system, the form of the potential barrier $U(x)$ plays little role. However, it is crucial in the noninteracting case. Let us choose $U(x)$ in the form of the double potential barrier so that quasistationary levels E_n, $n=0,1,\dots$, are present (Fig. 2). Thus, in the noninteracting case, we consider a resonant tunneling diode.[11] The spin and charge currents as functions of the chemical potential μ_R are

$$I_c(\mu_R) = I(\mu_R, 1/2) + I(\mu_R, -1/2), \qquad (1)$$

$$I_s(\mu_R) = \frac{\hbar}{2e}[I(\mu_R, 1/2) - I(\mu_R, -1/2)], \qquad (2)$$

where e is the electron charge,

$$I(\mu_R, S_z) = \frac{e}{h} \int_{E_F^L(S_z)}^{E_F^R(S_z)} dE\, T(E), \qquad (3)$$

and $T(E)$ is the transmission coefficient. The transmission coefficient is small far from the energies of the quasistationary levels $E = E_n$ and increases as E approaches E_n. Obviously, if one applies a dc voltage $eV = \mu_R$, then the charge current, expressed in units of e per second, exceeds the spin current in units of $\hbar/2$ per second. The situation changes in the presence of an ac bias. The dc currents generated by an ac voltage bias can be estimated as $I_{c/s}^r = [I_{c/s}(V) + I_{c/s}(-V)]/2$. Let us now assume that the magnetic field is tuned such that the Fermi level of the spin-down electrons $E_F - \mu H$ is close to the quasistationary level E_0 and exceeds E_0, while the Fermi level of the spin-up electrons $E_F + \mu H$ is close to E_1 and lies below E_1. Also, let eV be smaller than the distances $|E_0 - E_F + \mu H|$ and $|E_1 - E_F - \mu H|$ between the Fermi levels and quasistationary levels in the absence of the voltage bias. In addition, we assume that $T(E_F + \mu H) = T(E_F - \mu H)$. From the energy dependence of the transmission coefficient $T(E)$ near the resonant levels, one finds that $I(eV, 1/2) > I(eV, -1/2)$ and $|I(-eV, 1/2)| < |I(-eV, -1/2)|$. Hence, $I_s^r > \frac{\hbar}{2e}I_c^r$. By an appropriate choice of parameters, one can produce any ratio of the spin and charge rectification currents.

Note that the rectification effect for noninteracting electrons is possible even if the potential $U(x)$ is symmetric. The asymmetry of the system, necessary for rectification, is introduced by the applied voltage bias. The charge density injected into the wire from the leads is proportional to $\mu_L + \mu_R$ and hence is different for the opposite signs of the voltage. If the injected charge density were independent of the voltage sign, i.e., μ_R oscillated between $eV/2$ and $-eV/2$ and $\mu_L = -\mu_R$ oscillated between $-eV/2$ and $eV/2$, then the rectification effect would be impossible for noninteracting electrons. This follows from Eqs. (1)–(3) and the fact that the transmission coefficient $T(E)$ is independent of the direction of the incoming wave for noninteracting particles.[12] In the presence of electron repulsion, both the asymmetry of the potential and the voltage dependence of the injected charge contribute to the rectification current. It turns out that in the case of strong electron interaction, the rectification effect due to the asymmetry of the potential barrier dominates.

The above example is based on a special form of the potential barrier in the wire and assumes that the magnetic field and chemical potentials are tuned in order to obtain the desired effect. As shown below, in the presence of strong repulsive electron interaction, no tuning is necessary and no quasistationary states are needed to obtain the spin current which is greater than the charge current. In fact, the spin rectification effect is possible even for weak asymmetric potentials $U(x)$. This can be understood from the following toy model (a related model for rectification in a two-dimensional electron gas was studied in Ref. 13): Let there be no uniform magnetic field **H** and no asymmetric potential $U(x)$. Instead, both right↔left and spin-up↔spin-down symmetries are broken by a weak coordinate-dependent magnetic field $B_z(x) \neq B_z(-x)$, which is localized in a small region of size
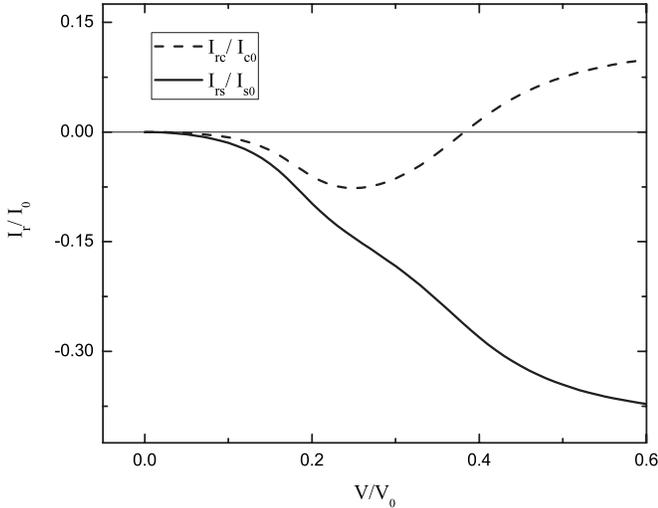
FIG. 3. Normalized charge rectification current I_c/I_{c0} and spin rectification current I_s/I_{s0} versus applied voltage V/V_0 for noninteracting electrons with $E_F=400\epsilon_0$, $\mu H=75\epsilon_0$, $u_1=50\epsilon_0 a$, and $u_2=-50\epsilon_0 a$, where $\epsilon_0=\hbar^2/ma^2$ (see Fig. 2 and Appendix A). $I_{c0}=50e\epsilon_0/\hbar$, $I_{s0}=25\epsilon_0$, and $V_0=50\epsilon_0/e$ are arbitrary reference currents and voltage.

$\sim 1/k_F$ (we do not include the components $B_{x,y}$ in the toy model). Let us also assume that the spin-up and -down electrons do not interact with the electrons of the opposite spin. Then, the system can be described as the combination of two spin-polarized one-channel wires with opposite spin-dependent potentials $\pm \mu B_z(x)$, where μ is the electron magnetic moment. According to Ref. 4, an ac bias generates a rectification current in each of those two systems and the currents are proportional to the cubes of the potentials $(\pm \mu B_z)^3$. Thus, $I_\uparrow^r = -I_\downarrow^r$. Hence, no net charge current $I^r=I_\uparrow^r+I_\downarrow^r$ is generated in the leading order. At the same time, there is a nonzero spin current in the third order in B_z. A similar effect is present in a more realistic Luttinger liquid model considered below.

The main focus of this paper is on the case of weak asymmetric potentials. A simple model with strong impurities is studied in Appendix A. In Figs. 3 and 4, we have represented the results from a numerical evaluation for the spin and charge currents $I_{s,c}^r$ for the potential shown in Fig. 2. Figure 3 shows the noninteracting case. In Fig. 4, we represent the case of strong electron interaction. We have chosen parameters (explained in the figure captions) such that I_c^r is smaller than I_s^r for a range of the applied voltage. Further information on the numerical approach is given in Appendix A.

Transport in a strongly interacting system in the presence of a strong asymmetric potential $U(x)$ is a difficult problem which cannot be solved analytically and is sensitive to a particular choice of the potential. As we have mentioned, Appendix A contains the numerical analysis of a simple model of interacting electrons with a strong potential barrier. On the other hand, the interacting problem can be solved analytically in the limit of a weak potential $U(x)$ with the help of the bosonization and Keldysh techniques (Sec. III). We will see that the rectification current exhibits a number of
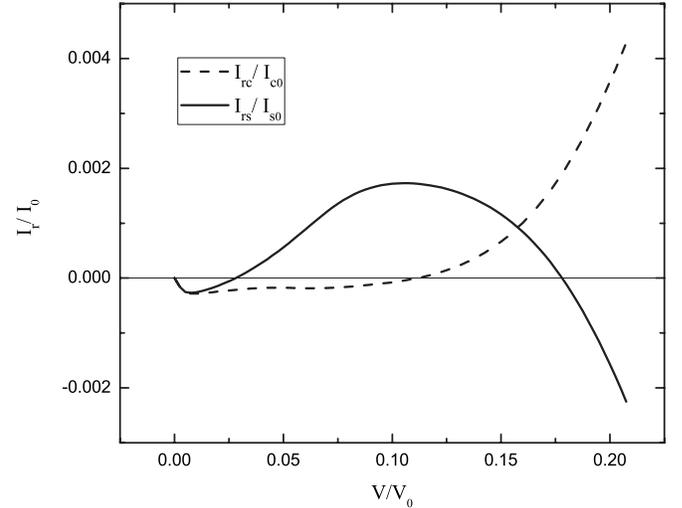
FIG. 4. Normalized charge rectification current I_c/I_{c0} and spin rectification current I_s/I_{s0} versus applied voltage V/V_0 for interacting electrons with $E_F=100\epsilon_0$, $\mu H=25\epsilon_0$, $\gamma=12.6\epsilon_0 a/e$, $u_1=25\epsilon_0 a$, and $u_2=50\epsilon_0 a$, where $\epsilon_0=\hbar^2/ma^2$ (see Fig. 2 and Appendix A). $I_{c0}=50e\epsilon_0/\hbar$, $I_{s0}=25\epsilon_0$, and $V_0=50\epsilon_0/e$ are arbitrary reference currents and voltage.

universal features, independent of the form of the potential $U(x)$. In particular, in a wide interval of interaction strength, the spin rectification current can exceed the charge rectification current for an arbitrary shape of the asymmetric potential barrier.

Rectification is a nonlinear transport phenomenon. Thus, it cannot be observed at low voltages at which the I-V curve is linear and hence symmetric. In Luttinger liquids, the I-V curve is nonlinear at $eV>k_BT$, where T is the temperature.[14] We will concentrate on the limit of the zero temperature which corresponds to the strongest rectification. We expect qualitatively the same behavior at $T \sim V$. At higher temperatures, the charge and spin rectification effects disappear. Since the temperatures of the order of millikelvins can be achieved with dilution refrigeration, the rectification effect is possible even for the voltages as low as $V \lesssim 1\mu$V.

In this paper, we focus on the low-frequency ac bias. We define the rectification current as the dc response to a low-frequency square voltage wave of amplitude V:

$$I_s^r(V) = [I_s(V) + I_s(-V)]/2, \tag{4}$$

$$I_c^r(V) = [I_c(V) + I_c(-V)]/2. \tag{5}$$

The above dc currents are expressed via the currents of spin-up and -down electrons: $I_c^r=I_\uparrow^r+I_\downarrow^r$, $I_s^r=(\hbar/2e)[I_\uparrow^r-I_\downarrow^r]$. The spin current exceeds the charge current if the signs of $I_\uparrow^r$ and $I_\downarrow^r$ are opposite. Equations (4) and (5) for the dc do not contain the frequency ω of the ac bias. They are valid as long as the frequency

$$\omega < eV/\hbar. \tag{6}$$

Indeed, as shown below, the rectification current is determined by electron backscattering off the asymmetric poten-

tial. Hence, one can neglect the time dependence of the ac voltage in Eqs. (4) and (5) if the period of the ac bias exceeds the duration τ of one backscattering event. The time $\tau \sim \tau_{\text{travel}} + \tau_{\text{uncertainty}}$ includes two contributions. τ_{travel} is the time of the electron travel across the potential barrier. $\tau_{\text{uncertainty}}$ comes from the uncertainty of the energy of the backscattered particle. If the barrier amplitude $U(x) < E_F$ and the barrier occupies a region of size $a_U \sim 1/k_F$, then $\tau_{\text{travel}} \sim 1/k_F v \sim \hbar/E_F$, where $v \sim \hbar k_F/m$ is the electron velocity. The energy uncertainty $\sim eV$ translates into $\tau_{\text{uncertainty}} \sim \hbar/eV$. Thus, for $eV < E_F$, one obtains condition (6). The same condition can be derived with the approach of Appendix A of Ref. 15 and emerges in a related problem.[16] Note that for realistic voltages, the low-frequency condition (6) allows rather high frequencies. Even for $V \sim 1 \ \mu\text{V}$, the maximal $\omega \sim 1$ GHz.

There remains the question of the asymmetric impurity: We require a potential $U(x)$ that is localized within $\sim 1/k_F$. A possible realization is to generate two different (symmetric) local potentials by two gates within a distance $\sim 1/k_F$ or an electric potential created by an asymmetric gate of size $\sim 1/k_F$ placed at the distance $\sim 1/k_F$ from the wire. Electron densities of $\rho \sim 10^{11}$ cm^{-2} are possible nowadays in two-dimensional electron gases, yielding $1/k_F$ up to several 10 nm. Confinement in a one-dimensional wire will reduce the electron density further so that this number may increase further. Modern techniques allow placing electric gates of widths of ~ 20 nm at distances of ~ 20–50 nm. A realization of an asymmetric potential in this way is, therefore, within the reach. Alternatively, in the case of shorter electron wavelength, it should be possible to place an asymmetrically shaped scanning tunneling microscope tip close to the wire. An applied bias would yield an asymmetric scattering potential. With such a tip, the asymmetry cannot be directly tuned, but most of our predictions are not sensitive to the precise shape of the potential. Certainly, an asymmetric potential may simply emerge by chance due to the presence of two point impurities of unequal strengths at the distance $\sim 1/k_F$.

III. BOSONIZATION AND KELDYSH TECHNIQUE

At $\omega < V$, the calculation of the rectification currents reduces to the calculation of the stationary contributions to the dc I-V curves $I_s(V)$ and $I_c(V)$ that are even in the voltage V. We assume that the Coulomb interaction between distant charges is screened by the gates. This will allow us to use the standard Tomonaga-Luttinger model with short range interactions.[14] Electric fields of external charges are also assumed to be screened. Thus, the applied voltage reveals itself only as the difference of the electrochemical potentials E_1 and E_2 of the particles injected from the left and right reservoirs. We assume that one lead is connected to the ground so that its electrochemical potential $E_2 = E_F$ is fixed. The electrochemical potential of the second lead $E_1 = E_F + eV$ is controlled by the voltage source (see Fig. 1). Since the Tomonaga-Luttinger model captures only low-energy physics, we assume that $eV < E_F$, where E_F is of the order of the bandwidth. Rectification occurs due to backscattering off the asymmetric potential $U(x)$. We will assume that the asym-

metric potential is weak, $U(x) < E_F$. This will enable us to use perturbation theory.

We assume that the magnetic field **H** couples only to the electron spin, and we neglect the correction $-e\mathbf{A}/c$ to the momentum in the electron kinetic energy. Indeed, for a uniform field, one can choose $\mathbf{A} \sim y$, where the y axis is orthogonal to the wire and y is small inside a narrow wire. As shown in Ref. 18, such a system allows a formulation within the bosonization language and, in the absence of the asymmetric potential, can be described by a quadratic bosonic Hamiltonian

$$H_0 = \sum_{\nu,\nu'=L,R} \sum_{\sigma,\sigma'=\uparrow,\downarrow} \int dx (\partial_x \phi_{\nu\sigma}) \mathcal{H}_{\nu\sigma,\nu'\sigma'} (\partial_x \phi_{\nu'\sigma'}), \quad (7)$$

where σ is the spin projection and $\nu = R, L$ labels the left- and right-moving electrons, which are related to the boson fields $\phi_{\nu\sigma}$ as $\psi_{\nu\sigma}^\dagger(x) \sim \eta_{\nu\sigma}^\dagger e^{\pm i[k_{F\nu\sigma}x + \phi_{\nu\sigma}(x)]}$ with $\pm$ for $\nu = R, L$. The operators $\eta_{\nu\sigma}^\dagger$ are the Klein factors adding a particle of type (ν, σ) to the system, and $k_{F\nu\sigma}/\pi$ is the density of (ν, σ) particles in the system. The densities of the spin-up and -down electrons are different since the system is polarized by the external magnetic field. The 4×4 matrix $\mathcal{H}$ describes the electron-electron interactions. In the absence of spin-orbit interactions, $L \leftrightarrow R$ parity is conserved and we can introduce the quantities $\phi_\sigma = \phi_{L\sigma} + \phi_{R\sigma}$ and $\Pi_\sigma = \phi_{L\sigma} - \phi_{R\sigma}$ such that the Hamiltonian decouples into two terms depending on ϕ_σ and Π_σ only. In the absence of the external field, this Hamiltonian would further be diagonalized by the combinations $\phi_{c,s} \propto \phi_\uparrow \pm \phi_\downarrow$, and similarly for $\Pi_{c,s}$, expressing the spin and charge separation. Here, this is no longer the case because of the external magnetic field. If we focus on the ϕ fields only [as Π will not appear in the operators describing backscattering off $U(x)$], the fields diagonalizing the Hamiltonian, $\tilde{\phi}_{c,s}$, have a more complicated linear relation to $\phi_{\uparrow,\downarrow}$, which we can write as

$$\begin{pmatrix} \phi_\uparrow \\ \phi_\downarrow \end{pmatrix} = \begin{pmatrix} \sqrt{g_c}[1 + \alpha] & \sqrt{g_s}[1 + \beta] \\ \sqrt{g_c}[1 - \alpha] & -\sqrt{g_s}[1 - \beta] \end{pmatrix} \begin{pmatrix} \tilde{\phi}_c \\ \tilde{\phi}_s \end{pmatrix}, \quad (8)$$

and which corresponds to the matrix $\hat{A}^T$ of Ref. 18. The normalization has been chosen such that the propagator of the $\tilde{\phi}$ fields with respect to the Hamiltonian [Eq. (7)] is evaluated to $\langle \tilde{\phi}_{c,s}(t_1) \tilde{\phi}_{c,s}(t_2) \rangle = -2 \ln(i(t_1-t_2)/\tau_c + \delta)$, where $\delta > 0$ is an infinitesimal quantity and $\tau_c \sim \hbar/E_F$ the ultraviolet cutoff time. For noninteracting electrons without a magnetic field, $g_c = g_s = 1/2$. $g_c < 1/2$ ($> 1/2$) for repulsive (attractive) interactions. The interaction constants depend on microscopic details and the magnetic field. The dimensionless parameter which controls the interaction strength is the ratio of the potential and kinetic energies of the electrons. This ratio grows as the charge density decreases, and hence lower electron densities correspond to stronger repulsive interaction. In the absence of the magnetic field, terms in Eq. (7) in the form of $\exp(\pm 2i\sqrt{g_s}\phi_s)$ may become relevant and open a spin gap for $g_s < 1/2$. In our model, they can be neglected since they are suppressed by the rapidly oscillating factors $\exp(\pm 2i[k_{F\uparrow} - k_{F\downarrow}]x)$. It is convenient to model the

leads as the regions near the right and left ends of the wire without electron interaction.[17]

Backscattering off the impurity potential $U(x)$ is described by the following contribution to the Hamiltonian[14] $H = H_0 + H'$:

$$H' = \sum_{n_\uparrow, n_\downarrow} U(n_\uparrow, n_\downarrow) e^{in_\uparrow \phi_\uparrow(0) + in_\downarrow \phi_\downarrow(0)}, \qquad (9)$$

where the fields are evaluated at the impurity position $x = 0$ and $U(n_\uparrow, n_\downarrow) = U^*(-n_\uparrow, -n_\downarrow)$, since the Hamiltonian is Hermitian. The fields Π do not enter the above equation due to the conservation of the electric charge and the z projection of the spin. The Klein factors are not written because they drop out in the perturbative expansion. $U(n_\uparrow, n_\downarrow)$ are the amplitudes of backscattering of $n_\uparrow$ spin-up and $n_\downarrow$ spin-down particles with $n_\sigma > 0$ for $L \rightarrow R$ and $n_\sigma < 0$ for $R \rightarrow L$ scattering. $U(n_\uparrow, n_\downarrow)$ can be estimated as[14] $U(n_\uparrow, n_\downarrow) \sim k_F \int dx U(x) e^{in_\uparrow 2k_{F\uparrow} x + in_\downarrow 2k_{F\downarrow} x} \sim U$, where U is the maximum of $U(x)$. In the case of a symmetric potential, $U(x) = U(-x)$, the coefficients $U(n_\uparrow, n_\downarrow)$ are real.

The spin and charge currents can be expressed as

$$I_{s,c} = L_{s,c}^1 + R_{s,c}^1 = L_{s,c}^2 + R_{s,c}^2, \qquad (10)$$

where $L_{s,c}^i$ and $R_{s,c}^i$ denote the current of the left and right movers near electrode i, respectively (see Fig. 2). For a clean system [$U(x) = 0$], the currents obey[17] $R_{s,c}^1 = R_{s,c}^2$, $L_{s,c}^1 = L_{s,c}^2$ and $I_c = 2e^2 V/h$, $I_s = 0$. With backscattering off $U(x)$, particles are transferred between L and R in the wire, and hence $R_s^2 = R_s^1 + dS_R/dt$ and $R_c^2 = R_c^1 + dQ_R/dt$, where Q_R and S_R denote the total charge and the z projection of the spin of the right-moving electrons.[16] The currents $L_{s,c}^2$ and $R_{s,c}^1$ are determined by the leads (i.e., the regions without electron interaction in our model[17]) and remain the same as in the absence of the asymmetric potential. Thus, the spin and charge currents can be represented as $I_c = 2e^2 V/h + I_c^{bs}$ and $I_s = I_s^{bs}$, where the backscattering current operators are[4,5,14]

$$\hat{I}_c^{bs} = d\hat{Q}_R/dt = i[H, \hat{Q}_R]/\hbar$$
$$= \frac{-ie}{\hbar} \sum_{n_\uparrow, n_\downarrow} (n_\uparrow + n_\downarrow) U(n_\uparrow, n_\downarrow) e^{in_\uparrow \phi_\uparrow(0) + in_\downarrow \phi_\downarrow(0)}, \qquad (11)$$

$$\hat{I}_s^{bs} = d\hat{S}_R/dt = -\frac{i}{2} \sum_{n_\uparrow, n_\downarrow} (n_\uparrow - n_\downarrow) U(n_\uparrow, n_\downarrow) e^{in_\uparrow \phi_\uparrow(0) + in_\downarrow \phi_\downarrow(0)}. \qquad (12)$$

The calculation of the rectification currents reduces to the calculation of the currents [Eqs. (11) and (12)] at two opposite values of the dc voltage.

To find the backscattered current, we use the Keldysh technique.[19] We assume that at $t = -\infty$, there is no backscattering in the Hamiltonian [$U(x) = 0$], and then the backscattering is gradually turned on. Thus, at $t = -\infty$, the numbers N_L and N_R of the left- and right-moving electrons are conserved separately: The system can be described by a partition function with two chemical potentials $E_1 = E_F + eV$ and $E_2 = E_F$ conjugated with the particle numbers N_R and N_L. This initial

state determines the bare Keldysh Green's functions.

We will consider only the zero temperature limit. It is convenient to switch[16] to the interaction representation $H_0 \rightarrow H_0 - E_1 N_R - E_2 N_L$. This transformation induces a time dependence in the electron creation and annihilation operators. As a result, each exponent in Eq. (9) is multiplied by $\exp(ieVt[n_\uparrow + n_\downarrow]/\hbar)$.

In the Keldysh formulation,[19] the backscattering currents [Eqs. (11) and (12)] are evaluated as

$$I_{c,s}^{bs} = \langle 0 | \mathcal{S}(-\infty, 0) \hat{I}_{c,s}^{bs} \mathcal{S}(0, -\infty) | 0 \rangle, \qquad (13)$$

where $|0\rangle$ is the ground state for the Hamiltonian H_0, Eq. (7), and $\mathcal{S}(t, t')$ the evolution operator for H' from t' to t in the interaction representation with respect to H_0. The result of this calculation depends on the elements of the matrix [Eq. (8)], which describe the low-energy degrees of freedom and depend on the microscopic details. Several regimes are possible[5] at different values of the parameters $g_s > 0$, $g_c > 0$, α, and β. In this paper, we focus on one particular regime in which the main contribution to the rectification current comes from backscattering operators $U(1, 0)$, $U(0, -1)$, and $U(-1, 1)$.

IV. RECTIFICATION CURRENTS

In order to calculate the current [Eq. (13)], we will expand the evolution operator in powers of $U(n, m)$. Such perturbative approach is valid only if $U < E_F$. The details of the perturbative calculation are discussed in Appendix C.

The currents [Eq. (13)] can be estimated using a renormalization group procedure.[14] As we change the energy scale E, the backscattering amplitudes $U(n_\uparrow, n_\downarrow)$ scale as

$$U(n, m; E) \sim U(n, m)(E/E_F)^{z(n,m)}, \qquad (14)$$

where the scaling dimensions are

$$\begin{aligned} z(n, m) &= n^2[g_c(1 + \alpha)^2 + g_s(1 + \beta)^2] + m^2[g_c(1 - \alpha)^2 \\ &+ g_s(1 - \beta)^2] + 2nm[g_c(1 - \alpha^2) - g_s(1 - \beta^2)] - 1 \\ &= n^2 A + m^2 B + 2nm C - 1. \end{aligned} \qquad (15)$$

The renormalization group (RG) stops at the scale of the order $E \sim eV$. At this scale, the backscattering current can be represented as $I_{c,s}^{bs} = V r_{c,s}(V)$, where the effective reflection coefficient $r_{c,s}(V)$ is given by the sum of contributions of the form[4,14]

$$(\text{const}) U(n_1, m_1; E = eV) U(n_2, m_2; eV) \cdots U(n_p, m_p; eV). \qquad (16)$$

Such a perturbative expansion can be used as long as

$$U(n, m; E = eV) < E_F \qquad (17)$$

for every (n, m). This condition defines the RG cutoff voltage V^* such that

$$U(n_0, m_0; E = eV^*) = E_F \qquad (18)$$

for the most relevant operator $U(n_0, m_0)$. The RG procedure cannot be continued to lower-energy scales $E < V^*$.

One expects that the leading contribution to the back-scattering current emerges in the second order in $U(n,m)$ if the above condition is satisfied. The leading contribution to the *rectification current* may, however, emerge in the third order. Indeed, the second order contributions to the charge current were computed in Ref. 14. The spin current can be found in exactly the same way. The result is

$$I_{c,s}^{bs(2)}(V) \sim \sum (\text{const}) |U(n,m)|^2 |V|^{2z(n,m)+1} \, \text{sgn}(V). \quad (19)$$

If the (unrenormalized) $U(n,m)$ were independent of the voltage, the above current would be an odd function of the bias and hence would not contribute to the rectification current. The backscattering amplitudes depend[14] on the charge densities $k_{F\nu\sigma}$ though, which, in turn, depend on the voltage in our model.[4] The voltage-dependent corrections to the amplitudes are linear in the voltage at low bias $eV \ll E_F$. Hence, the second order contributions to the rectification currents scale as $U^2 |V|^{2z(n,m)+2}$ (see Appendix B). The additional factor of V makes the second order contribution smaller than the leading third order contribution [Eq. (23)] at sufficiently high impurity strength $U \ll E_F$ (as shown in Appendix B, U/E_F must exceed $[V/E_F]^{1+z(1,0)-z(0,1)-z(1,-1)}$). Note that the second order contribution to the rectification current is nonzero even for a symmetric potential $U(x)$ and emerges solely due to the voltage dependence of the injected charge density (cf. Sec. II). The leading third order contribution emerges solely due to the asymmetry of the scatterer.

The main third order contribution comes from the three backscattering operators most relevant in the renormalization group sense [small $z(n,m)$, Eq. (15)]. They are identified in Appendix B. Under conditions (B6)–(B8), (B12), (B14), and (B15), the most relevant operator is $U(1,0)$, the second most relevant $U(0,-1)$, and the third most relevant $U(-1,1)$. The cutoff voltage V^* is determined by the scaling dimension $z(1,0)$, $eV^* \sim E_F(U/E_F)^{1/[1-A]}$. The leading nonzero third order contributions to the spin and charge currents come from the product of the above three operators in the Keldysh perturbation theory (see Appendix B). This leads to

$$I_{c,s}^{bs} \sim U^3 V^{2(A+B-C-1)}. \quad (20)$$

This contribution dominates the spin rectification current at

$$E_F(U/E_F)^{1/[2+2C-2B]} \equiv eV^{**} > eV > eV^*, \quad (21)$$

as is clear from the comparison with the leading second order contribution $I_{s,2}^{bs} \sim U^2 V^{2A}$, Eq. (B10). Interestingly, the current [Eq. (20)] grows as the voltage decreases in the regime (B6)–(B8), (B12), (B14), and (B15).

However, does the current [Eq. (20)] actually contribute to the rectification effect? In general, Eq. (20) is the sum of odd and even functions of the voltage and only the even part is important for us. One might naively expect that such a contribution has the same order of magnitude for the spin and charge currents. A direct calculation shows, however, that this is not the case and the spin rectification current is much greater than the charge rectification current.

In order to calculate the prefactors in the right hand side of Eq. (20), one has to employ the Keldysh formalism. The details are explained in Appendix C. Here, let

us shortly summarize the essential steps: The third order Keldysh contribution reduces to the integral of $P(t_1,t_2,t_3) = \langle T_c \exp(i\phi_\uparrow(t_1) + ieVt_1/\hbar)\exp(-i\phi_\downarrow(t_2) - ieVt_2/\hbar) \times \exp(i[-\phi_\uparrow(t_3) + \phi_\downarrow(t_3)]) \rangle$ over $(t_1 - t_3)$ and $(t_2 - t_3)$, where T_c denotes time ordering along the Keldysh contour $-\infty \to 0 \to -\infty$ and the angular brackets denote the average with respect to the ground state of the noninteracting Hamiltonian [Eq. (7)]. The integration can be performed analytically as discussed in Appendix C. One finds

$$I_c^{bs} = \frac{16e\tau_c^2}{\pi\hbar^3} \, \text{sgn}(eV) \left| \frac{eV\tau_c}{\hbar} \right|^{a+b+c-2} \Gamma(1-a)\Gamma(1-b)$$

$$\times \Gamma(2-a-b-c)\Gamma(a+b-1)\sin\frac{\pi a}{2}\sin\frac{\pi b}{2}$$

$$\times \sin\frac{\pi(a+b)}{2}\sin\pi(a+b+c)$$

$$\times \text{Re}[U(1,0)U(-1,1)U(0,-1)], \quad (22)$$

$$I_s^{bs} = \frac{16\tau_c^2}{\pi\hbar^2}\sin\frac{\pi a}{2}\sin\frac{\pi b}{2} \left| \frac{eV\tau_c}{\hbar} \right|^{a+b+c-2} \Gamma(a+b-1)$$

$$\times \Gamma(2-a-b-c)\Gamma(1-a)\Gamma(1-b)$$

$$\times \left\{ \text{Im}[U(1,0)U(-1,1)U(0,-1)]\cos\frac{\pi(a+b+c)}{2} \right.$$

$$\times \left[\sin\frac{\pi c}{2} + \cos\frac{\pi(a-b)}{2}\sin\frac{\pi(a+b+c)}{2} \right.$$

$$\left. + \sin\frac{\pi(a+b)}{2}\cos\frac{\pi(a+b+c)}{2} \right]$$

$$+ \frac{1}{2}\text{Re}[U(1,0)U(-1,1)U(0,-1)]\sin\frac{\pi(a-b)}{2}$$

$$\left. \times \sin\pi(a+b+c)\text{sgn}(eV) \right\}, \quad (23)$$

where $a = 2A - 2C$, $b = 2B - 2C$, $c = 2C$, and $\tau_c \sim \hbar/E_F$ is the ultraviolet cutoff time. The charge current [Eq. (22)] is an *odd* function of the voltage and hence does *not* contribute to the rectification effect. The spin current [Eq. (23)] is a sum of an *even* and odd functions and hence determines the spin rectification current

$$I_s^r = \frac{16\tau_c^2}{\pi\hbar^2}\sin\frac{\pi a}{2}\sin\frac{\pi b}{2}\cos\frac{\pi(a+b+c)}{2} \left| \frac{eV\tau_c}{\hbar} \right|^{a+b+c-2}$$

$$\times \Gamma(a+b-1)\Gamma(2-a-b-c)\Gamma(1-a)\Gamma(1-b)$$

$$\times \text{Im}[U(1,0)U(-1,1)U(0,-1)]$$

$$\times \left[\cos\frac{\pi(a-b)}{2}\sin\frac{\pi(a+b+c)}{2} \right.$$

$$\left. + \sin\frac{\pi c}{2} + \sin\frac{\pi(a+b)}{2}\cos\frac{\pi(a+b+c)}{2} \right]. \quad (24)$$

It is nonzero if $\text{Im}[U(1,0)U(-1,1)U(0,-1)] \neq 0$, which is satisfied for asymmetric potentials. The leading contribution to the charge rectification currents comes from other terms in
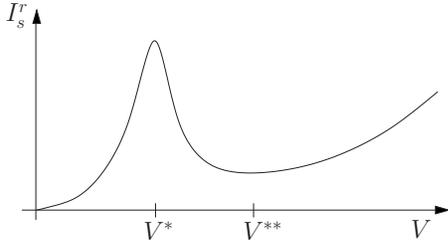
FIG. 5. Qualitative representation of the spin rectification current. The spin current exceeds the charge current and follows a power-law dependence on the voltage with a negative exponent in the interval of voltages $V^* < V < V^{**}$.

the perturbation expansion. Thus, we expect that in the region (B6)–(B8), (B12), (B14), and (B15), the spin rectification current exceeds the charge rectification current in an appropriate interval of voltages [Eq. (21)]. The difference between the spin and charge rectification currents can be easily understood from the limit $A=B$. In that case, the charge current changes its sign under the transformation $U(1,0) \leftrightarrow U(0,-1)$, $V \rightarrow -V$. Since $U(1,0)$ and $U(0,-1)$ enter the current only in the combination $U(1,0)U(0,-1)$, this means that the charge current must be an odd function of the voltage bias. A similar argument shows that at $A=B$, the spin rectification current is an even function of the voltage in agreement with Eq. (23).

The voltage dependence of the spin rectification current is illustrated in Fig. 5. Expression (20) describes the current in the voltage interval $V^{**} > V > V^*$. In this interval, the current *increases* as the voltage decreases in the regime (B6)–(B8), (B12), (B14), and (B15). At lower voltages, the perturbation theory breaks down. The current must decrease as the voltage decreases below V^* and eventually reach 0 at $V=0$. At higher voltages, $E_F > eV > eV^{**}$, the second order rectification current [Eq. (19)] dominates. The leading second order contribution $I_s^r \sim |U(1,0)|^2 V^{2z(1,0)+2}$ grows as the voltage increases. The charge rectification current has the same order of magnitude as the spin current.

The Tomonaga-Luttinger model cannot be used for the highest voltage region $E_F \sim eV$. It is easier to detect charge currents than spin currents. However, the measurement of the spin current can be reduced to the measurement of charge currents: Let us split the right end of the wire into two branches and place them in opposite strong magnetic fields so that only electrons with one spin orientation can propagate in each branch. If both branches are grounded, they still inject exactly the same charge and spin currents into the wire as one unpolarized lead. However, the current generated in the wire will split between two branches into the currents of spin-up and spin-down electrons. If they are opposite, then pure spin current is generated.

V. CONCLUSIONS

In this paper, we have shown that rectification in quantum wires in a uniform magnetic field can lead to a spin current that largely exceeds the charge current. The paper focuses on the regime of low voltages and weak asymmetric potentials

in which the perturbation theory provides quantitatively exact predictions. Qualitatively, the same behavior is expected up to $eV, U \sim E_F$. The spin rectification effect is solely due to the properties of the wire and does not require time-dependent magnetic fields or spin-polarized injection as from magnetic electrodes. The currents are driven by the voltage source only. In an interval of low voltages, the spin current grows as the voltage decreases. In contrast to some other situations, the z component of the total spin is conserved and hence the dc spin current is constant throughout the system.

APPENDIX A: HIGH POTENTIAL BARRIER

In this appendix, we first briefly consider the model of noninteracting electrons, Sec. II, and then a simple Hartree-type model for strongly interacting electrons.

1. Model without interaction

We consider noninteracting electrons in the presence of the potential

$$U(x) = u_1 \delta(x) + u_2 \delta(x - a). \tag{A1}$$

The transmission coefficient can be found from elementary quantum mechanics,

$$T(E) = \frac{1}{(1 - 2s_1 s_2 \sin^2 ka)^2 + (s_1 + s_2 + s_1 s_2 \sin 2ka)^2}, \tag{A2}$$

where $E = \hbar^2 k^2/2m$ and $s_i = mu_i/k\hbar^2$. The spin and charge rectification currents can be computed from Eqs. (1) and (2). Figure 3 shows their voltage dependence for a certain choice of u_1, u_2, the voltage bias V, and the magnetic field H.

2. Model with interaction

It is difficult to find a general analytic expression for the current in the regime when both the electron interaction and potential barrier are strong. If all characteristic energies, U, eV, $\hbar^2/[ma^2]$, and the typical potential energy of an electron E_P, are of the order of E_F, then one can estimate the spin and charge rectification currents with dimensional analysis: $I_c^r \sim eE_F/\hbar$, $I_s^r \sim E_F$.

To obtain a qualitative picture of the interaction effects in the case of a high potential barrier [Eq. (A1)], we restrict our discussion to a simple model in the spirit of the zero-mode approximation.[20] We assume that electrons move in a self-consistent Hartree-type field. In our ansatz, the self-consistent field takes three different constant values V_L, V_M,

and V_R on the left of the potential barrier, between two δ-function scatterers, and on the right of the potential barrier. In the spirit of the Luttinger liquid model, we assume that the constants V_L, V_M, and V_R are proportional to the average charge density in the respective regions, e.g., $V_M = \frac{\gamma}{a}\int_0^a dx\rho(x)$, where γ is the interaction constant.

A result is shown in Fig. 4. We see that the voltage dependence of the spin and charge rectification currents exhibits a behavior similar to that of the noninteracting case.

APPENDIX B: ESTIMATION OF HIGHER PERTURBATIVE ORDERS

In this appendix, we compare contributions to the rectification currents from different orders of perturbation theory. We focus on the regime when the third order contribution dominates. This appendix contains five sections and has the following structure: (1) We introduce a parametrization for the scaling dimensions [Eq. (15)]. (2) We discuss the operators most relevant in the RG sense. (3) We determine at what conditions the second order contribution to the rectification current dominates. Appendix B 3 also contains a lemma which is important in Appendix B 4. (4) We determine at what conditions the third order contribution to the current dominates. (5) We estimate the voltages and currents at which the spin rectification current can exceed the charge rectification current in realistic systems.

As shown in Refs. 4 and 5, there are two effects leading to rectification in Luttinger liquids, which are here very shortly summarized: the *density-driven* and the *asymmetry-driven* rectification effects. The former appears at second order in U. It appears because the backscattering potential depends on the particle densities in the system, which, in turn, are modified by the external voltage bias. The leading order backscattering currents are of the form[14] $I^{bs}(V) \sim \mathrm{sgn}(V)U^2|V|^\alpha$, so that the rectification currents $I^r = [I^{bs}(V) + I^{bs}(-V)]$ vanish. Due to the density dependence, however, an expansion of U to linear order in V cancels the $\mathrm{sgn}(V)$, and we obtain a rectification current $I^r \sim U^2|V|^{\alpha+1}$.

The asymmetry-driven rectification effect appears at third order in U. It is due solely to the spatial asymmetry of the potential $U(x)$: Due to backscattering off U, screening charges accumulate close to the impurity. These create an electrostatic nonequilibrium backscattering potential $W(x)$ for incident particles, leading to an effective potential $\bar{U}(x) = U(x) + W(x)$. The spatial distribution of charges follows from the shape of $U(x)$ and the applied voltage bias. An asymmetric $U(x)$ leads to different electrostatic potentials for positive or negative bias and hence to rectification. If we expand the current, $I^{bs} \sim \bar{U}^2 \sim U^2 + UW + \cdots$, the asymmetry appears first at order UW. Since the charge density in the vicinity of the impurity is modified by the modification of the particle current through backscattering, W itself is (self-consistently) related to the backscattering current as $W \sim I^{bs}$. Hence, $W \sim U^2$, so that the asymmetric rectification effect appears first at third order in U, $I^r \sim UW \sim U^3$.

The main result of this paper are expressions for the currents that result from the perturbation theory at third order in the impurity potential U. In this appendix, we show that the considered contribution indeed dominates the second and other third order expressions in the region defined by Eqs. (B6)–(B8), (B12), (B14), and (B15). In addition, we give the proof that higher perturbative orders $N \geq 4$ cannot exceed these values in the considered range of the system parameters g_c, g_s, α, and β. Unless we want to emphasize the correct dimensions, we set $E_F = 1$, $e = 1$, and $\hbar = 1$ in this appendix. We assume that $U < E_F$ and $eV < E_F$.

An important observation is the following: In Eq. (16), $\Sigma_i n_i = \Sigma_i m_i = 0$. This follows from the fact that in the absence of backscattering, the numbers of right and left movers with different spin orientations are conserved.

1. Parametrization of scaling dimensions

According to Eq. (15),

$$z(n,m) = n^2 A + m^2 B + 2nmC - 1, \tag{B1}$$

where

$$A = [g_c(1 + \alpha)^2 + g_s(1 + \beta)^2], \tag{B2}$$

$$B = [g_c(1 - \alpha)^2 + g_s(1 - \beta)^2], \tag{B3}$$

$$C = [g_c(1 - \alpha^2) - g_s(1 - \beta^2)]. \tag{B4}$$

Since g_c and g_s are positive, A and B are also positive. C can have any sign. It satisfies the inequality

$$|C| < \sqrt{AB}. \tag{B5}$$

Indeed, $AB - C^2 = 4g_c g_s(1 - \alpha\beta)^2 > 0$. Any values of $A, B > 0$ and $-\sqrt{AB} < C < \sqrt{AB}$ are possible. For example, one can set $\alpha = \beta = (\sqrt{A} - \sqrt{B})/(\sqrt{A} + \sqrt{B})$, $g_c = (\sqrt{A} + \sqrt{B})^2[1 + C/\sqrt{AB}]/8$, and $g_s = (\sqrt{A} + \sqrt{B})^2[1 - C/\sqrt{AB}]/8$.

2. Most relevant operators

Depending on the values of A, B, and C, many different possibilities for relative importance of different backscattering operators $U(m,n)$ exist. In this paper, we focus on the situation when the most relevant operator is $U(1,0)$, the second most relevant operator is $U(0,-1)$, and the third most relevant operator is $U(-1,1)$ [certainly, the scaling dimensions of the operators $U(n,m)$ and $U(-n,-m)$ are always the same]. We will also assume that the operator $U(1,0)$ is *relevant* in the RG sense, i.e., $z(1,0) < 0$. The analysis of the situation in which $U(0,-1)$ is the most relevant operator, $U(1,0)$ is the second most relevant, and $U(-1,1)$ is the third most relevant follows exactly the same lines. Similarly, little changes if $U(1,1)$ is the third most relevant operator.

The scaling dimensions of the three aforementioned operators are $A - 1$, $B - 1$, and $A + B - 2C - 1$. The following inequality must be satisfied in order for these operators to be most relevant backscattering operators: $A - 1 < B - 1 < A + B - 2C - 1 <$ (all other scaling dimensions). Hence,

$$B > A > 2C. \tag{B6}$$

Since $z(1,0) < 0$,

$$A < 1. \tag{B7}$$

When are all other operators less relevant? We must consider three classes of operators: (1) $U(1,1)$, (2) $U(n,0)$ and $U(0,n)$ with $|n| > 1$, and (3) all other operators.

(1) Since $z(1,1) = A+B+2C-1$, one finds

$$C > 0. \tag{B8}$$

(2) $\quad z(0,n) = Bn^2 - 1 > z(n,0) = An^2 - 1 \geqslant 4A - 1 > A + B - 2C - 1$. Thus,

$$3A + 2C > B. \tag{B9}$$

(3) $\quad z(n,m) - (A+B-2C-1) = An^2 + Bm^2 + 2Cnm - (A+B-2C) \geqslant An^2 + Bm^2 - C(n^2+m^2) - A - B + 2C = (A-C)(n^2-1) + (B-C)(m^2-1) > 0$, since $B-C > A-C > 0$ in accordance with Eq. (B6), $|n|, |m| \geqslant 1$, and either $|n|$ or $|m|$ exceeds 1. Thus, case (3) gives no new restriction on A, B, and C.

3. Second order contribution to the current

When is the second order contribution to the rectification current dominant? Any operator $U(n,m)$ can be represented as $\widetilde{U}(n,m) + VU_1(n,m) + \cdots$, where $U_1 \sim \widetilde{U}/E_F$. Any second order contribution to the current which contains $\widetilde{U}$ only is an odd function of the voltage bias. Indeed, any such contribution is proportional to $\widetilde{U}(n,m)\widetilde{U}^*(n,m) = \widetilde{U}(n,m)\widetilde{U}(-n,-m)$. The transformation $\widetilde{U} \leftrightarrow \widetilde{U}^*$, $V \to -V$ changes the sign of the current. At the same time, the transformation $\widetilde{U} \leftrightarrow \widetilde{U}^*$ cannot change the second order current at all. Hence, it is odd in the voltage. The same argument applies to any perturbative contribution which contains only $\widetilde{U}$, if every operator $\widetilde{U}(n,m)$ enters in the same power as $\widetilde{U}^*(n,m)$. In particular, if only two operators $\widetilde{U}(n,0)$ and $\widetilde{U}(0,m)$ and their conjugate enter then the resulting current, the contribution is odd.

Thus, all second order contributions to the rectification current must contain U_1. As is clear from Eq. (19), the leading second order contribution is proportional to the square of the most relevant operator, $|U(1,0)|^2$. It scales as

$$I_2 \sim VU^2 V^{2z(1,0)+1} \sim U^2 V^{2A}. \tag{B10}$$

In this section, we discuss at what conditions this contribution dominates for all $V > V^*$ [see Eq. (18)]. Since $U(1,0)$ is the most relevant operator, its renormalized amplitude $U(1,0;E=V)$ exceeds the renormalized amplitude of all other operators on every energy scale. At the same time, it remains lower than 1 (i.e., E_F) for $V > V^*$. This certainly means that the renormalized amplitudes are smaller than 1 for all other operators too. Hence, the product of any operators is smaller than the product of any two of them and that product cannot exceed $U^2(1,0;E)$. This guarantees that the second order current [Eq. (B10)] exceeds any second or higher order contribution which contains any operator $VU_1(n,m)$. Thus, we have to compare I_2 with higher order contributions to the rectification current which contain $\widetilde{U}$ only. Every such contribution is at least third order and contains at least one operator less relevant than $U(0,-1)$ [if it

contains $U(\pm1,0)$ and $U(0,\pm1)$ only, then it must contain VU_1 as discussed above]. Thus, any rectification current contribution with $\widetilde{U}$ only cannot exceed $U^3 V^{2z(1,0)+z(1,-1)+1}$. Comparison with Eq. (B10) at $V \sim V^*$ leads to the condition

$$B > 2C + 1. \tag{B11}$$

4. Third order contribution to the current

The most interesting question is different. When does the third order contribution dominate the rectification current? We will focus on the third order contribution I_3 proportional to $U(1,0)U(0,-1)U(-1,1)$ at $V \sim V^*$. Note that this contribution is proportional to $\sim V^{2(A+B-C-1)}$ and hence scales as a negative power of the voltage if

$$A + B < C + 1. \tag{B12}$$

At $V \sim V^*$, $U \sim V^{1-A}$. Thus,

$$I_3(V = V^*) \sim V^{2B-A-2C+1}. \tag{B13}$$

We need to compare I_3, Eq. (B13), with the following types of contributions: (1) those containing at least three different operators [we treat a pair of $U(n,m)$ and $U(-n,-m) = U^*(n,m)$ as one operator], (2) those containing only one type of operators, and (3) those containing two types of operators.

Cases (1) and (2) are easy.

(1) I_3 contains the product of the three most relevant operators and hence always exceeds the product of any other three different operators at any energy scale $E_F > V > V^*$. Any contribution with three different operators is the product of three different operators times perhaps some other combination of operators which cannot exceed 1 at $E_F > V > V^*$. Hence, it is smaller than I_3.

(2) Any contribution to the rectification current with only one type of operators must contain VU_1. As discussed in the previous section, the leading contribution of such type emerges in the second order. It is I_2, Eq. (B10). At $V \sim V^*$, $I_2(V=V^*) \sim V^2$. The condition $I_2(V^*) < I_3(V^*)$ means that

$$2B < A + 2C + 1. \tag{B14}$$

(3) We have to consider three possibilities: (3.1) one operator has the form $U(n,0)$ and the second operator has the form $U(k,m)$, $m \neq 0$ or one operator has the form $U(0,m)$ and the other one has the form $U(n,k)$, $n \neq 0$; (3.2) both operators have the form $U(n_i,0)$ or both operators have the form $U(0,m_i)$; and (3.3) both operators have the form $U(n_i,m_i)$ with n_i, $m_i \neq 0$.

(3.1). Let us assume that one operator has the form $U(n,0)$ and the second one is $U(k,m)$. The case of the operators $U(0,m)$ and $U(n,k)$ can be considered in exactly the same way. We must have the same number of operators $U(k,m)$ and $U(-k,-m)$ in the perturbative contribution since the sum of the second indices $\pm m$ must be 0. [The other cases are covered in (3.3).] From the analysis of the sum of the first indices, one concludes that the operators $U(n,0)$ and $U(-n,0)$ also enter in the same power. It follows from the previous section that the perturbative contribution must con-

tain at least one U_1 operator and hence is smaller than I_2. Hence, it is also smaller than I_3.

(3.2) We will focus on the case when both operators have the form $U(n_i,0)$. The case when both operators have the form $U(0,m_i)$ is very similar and does not lead to a new restriction on A, B, and C. The scaling dimensions of the operators $U(n,0)$ are An^2-1. Operators with greater n are less relevant. Since the contribution contains two different operators, it must be at least third order [we treat $U(n,0)$ and $U(-n,0)$ as the same operator]. At least one of the two operators must have $|n_i|>1$ [otherwise, all operators are $U(\pm 1,0)$]. Thus, the contribution cannot exceed $U^2(1,0;E=V)U(2,0;E=V)\sim U^3V^{6A-2}$. The comparison with $I_3 \sim U^3V^{2A+2B-2C-2}$ at $E_F>V>V^*$ yields

$$B < 2A + C. \tag{B15}$$

Note that the above condition is stronger than Eq. (B9).

(3.3) This case is easy: the contribution must be at least third order again. Both operators $U(n_i,m_i)$ are less relevant than $U(1,0)$ and $U(0,-1)$ and no more relevant than $U(1,-1)$. Thus, the contribution is automatically smaller than I_3 at any energy scale $E_F>V>V^*$.

We now have a full set of conditions at which the third order contribution dominates at $V\sim V^*$ and the spin rectification current scales as a negative power of the voltage. These are Eqs. (B6)–(B8), (B12), (B14), and (B15).

The above analysis shows that I_3 exceeds any contribution to the spin rectification current which does not contain VU_1 in the whole region $E_F>V>V^*$. I_2 dominates the remaining contributions for any $V>V^*$. The contributions become equal, $I_2=I_3$, at $V=V^{**}=U^{1/[2+2C-2B]}$. In the interval of voltages $V^{**}>V>V^*$, the spin rectification current is dominated by I_3. At $V>V^{**}$, the spin and charge rectification currents are dominated by I_2.

5. Numerical estimates

In order to get a feeling about the magnitude of the effect, let us consider a particular choice of parameters $A=B=7/12$, $C=7/24$, $eV\sim 0.01E_F$, and $eV^*\sim 10^{-4}E_F$. For such A, B, and C, the scaling dimensions of the three most relevant operators are the same. The inequalities (B7), (B12), (B14), and (B15) are satisfied. The equality $A=B=2C$ corresponds to a limiting case of Eq. (B6). One finds that $U\sim 0.01E_F$ and $eV^{**}\sim 0.1E_F$. Repeating the arguments of the previous section, one can estimate the leading correction to I_3 as $\delta I\sim (eV/E_F)^{7/12}I_3\ll I_3$. The spin rectification current is the difference of two opposite electric currents of the spin-up and -down electrons times $\hbar/[2e]$. Even if E_F is as low as ~ 0.1 meV, this still corresponds to the voltage V of the order of microvolts and the currents[21] [Eq. (24)] of spin-up and -down electrons of the order of picoamperes, i.e., within the ranges probed in experiments with semiconductor heterostructures. Certainly, the current increases if E_F or V^* is increased.

APPENDIX C: EXPLICIT EVALUATION OF THE THIRD ORDER CURRENTS

The charge or spin currents in the third order in the potentials U are evaluated from the following perturbative expression:

$$I_{c,s}^{bs(3)}(V) = \frac{(-i)^3}{2!} \sum (n_\uparrow \pm n_\downarrow) \int_{C_K} \frac{dt_1 dt_2}{\hbar^2} \langle T_c \hat{U}(n_\uparrow, n_\downarrow; 0)$$
$$\times \hat{U}(m_\uparrow, m_\downarrow; t_1) \hat{U}(l_\uparrow, l_\downarrow; t_2) \rangle, \tag{C1}$$

where the sum runs over indices satisfying $n_\sigma + m_\sigma + l_\sigma = 0$ for $\sigma = \uparrow, \downarrow$, C_K is the Keldysh contour $-\infty \to 0 \to \infty$, T_c is the time order on C_K, and we omitted a constant prefactor. The operators $\hat{U}$ are given by

$$\hat{U}(n_\uparrow, n_\downarrow; t) = U(n_\uparrow, n_\downarrow) e^{i(n_\uparrow + n_\downarrow)teV/\hbar} e^{in_\uparrow \phi_\uparrow(t) + in_\downarrow \phi_\downarrow(t)}. \tag{C2}$$

The most relevant expressions are those arising from the combinations $U(1,0)U(0,-1)U(-1,1)$ and $U(-1,0)U(0,1)U(1,-1)$ (see Appendix B). The third order contributions to the current contain correlation functions of the form

$$P(t_1,t_2,t_3) = \langle T_c e^{\pm i[\phi_\uparrow(t_1) - \phi_\downarrow(t_2) - \phi_\uparrow(t_3) + \phi_\downarrow(t_3)]} \rangle. \tag{C3}$$

We evaluate the correlation functions within the quadratic model described by Eq. (7) and use relations (8) and $\langle \tilde{\phi}_{c,s}(t_1) \tilde{\phi}_{c,s}(t_1) \rangle = -2 \ln(i(t_1-t_2)/\tau_c + \delta)$, with an infinitesimal $\delta > 0$ and $\tau_c \sim \hbar/E_F$ the ultraviolet cutoff time. This leads to

$$P(t_1,t_2,t_3) = [iT_c(t_1-t_3)/\tau_c + \delta]^{2C-2A}[iT_c(t_2-t_3)/\tau_c + \delta]^{2C-2B}$$
$$\times [iT_c(t_1-t_2)/\tau_c + \delta]^{-2C}, \tag{C4}$$

where $T_c(t_i-t_j)=(t_i-t_j)$, if time t_i stays later than t_j on the Keldysh contour, and otherwise, $T_c(t_i-t_j)=(t_j-t_i)$. Expression (C4) is independent of the $\pm$ signs in Eq. (C3).

The spin and charge current contributions, proportional to $U(1,0)U(0,-1)U(-1,1)$, are complex conjugate to those proportional to $U(-1,0)U(0,1)U(1,-1)=U^*(1,0)U^*(0,-1)U^*(-1,1)$. Thus, it is sufficient to calculate only the contributions of the first type. In the case of the charge current, their calculation reduces to the calculation of the following two integrals over the Keldysh contour:

$$\int dt_1 dt_3 P(t_1,0,t_3)\exp(ieVt_1/\hbar) \tag{C5}$$

and

$$\int dt_2 dt_3 P(0,t_2,t_3)\exp(-ieVt_2/\hbar). \tag{C6}$$

One of the times t_1 and t_2 is zero since the current operator is taken at $t=0$ in Eq. (13). The two integrals can be evaluated in exactly the same way. We will consider only the first

integral. We find eight integration regions. They correspond to $2 \times 2 = 4$ possibilities for the branches of the Keldysh contour on which t_1 and t_3 are located and two possible relations $|t_1| > |t_3|$ or $|t_3| > |t_1|$. In all eight cases, we first integrate over t_3. The integral reduces to the Euler B function. Then, we integrate over t_1. This yields a Γ function. Finally, we obtain Eq. (22).

The spin current contains three contributions proportional to $U(1,0)U(0,-1)U(-1,1)$. Two of them reduce to the integrals (C5) and (C6). The third contribution is proportional to

$$\int dt_1 dt_2 P(t_1, t_2, 0) \exp(ieV[t_1 - t_2]/\hbar). \tag{C7}$$

Again, we have eight integration regions determined by the choice of the branches of the Keldysh contour and the relations $|t_1| > |t_2|$ and $|t_2| > |t_1|$. In each region, it is convenient to introduce new integration variables: $\tau = |t_1 - t_2|$ and $t = \min(t_1, t_2)$. The integration over t reduces to a B function. The integration over τ produces an additional Γ-function factor. Finally, one obtains Eq. (23).

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[21] Note a large numerical factor in Eq. (24).