

# Angle-dependent tunneling through quantum dots coupled to noncollinearly oriented magnetic leads and subject to magnetic fields

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The quantum rate equation approach is applied to studying the angle-dependent electronic current  $I(\theta)$  through a quantum dot coupled to two magnetic leads with noncollinear magnetizations and subject to a perpendicular magnetic field. The analytic expressions for  $I(\theta)$  are obtained in free and Coulomb blockade regimes. It is found that the current exhibits different angle dependence in both the regimes and deviates significantly from  $I(\theta) \propto \sin^2 \theta/2$ . The perpendicular magnetic field applied on the quantum dot plays a spin-flip effect in electronic transport and promotes greatly the electronic current at  $\theta = \pi$ .

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## I. INTRODUCTION

Spin-dependent electronic transport in quantum-dot (QD) systems has attracted great interest since the 1990s,<sup>1-3</sup> as it opened up new application prospects for microelectronics and spintronics. Strong Coulomb interactions have important consequences for electronic transport on the nanometer length scale in low-dimensional systems, such as quantum dots, nanojunctions, and other artificial nanoscale devices,<sup>1,4-9</sup> due to the smallness of such systems. Coulomb blockade of the single-electron tunneling is a fundamental physical phenomenon of the QD system at low temperatures.<sup>4,10,11</sup> The spin degree of freedom plays an important role in the transport if the source and drain leads attached to the QD are magnetic because the electronic current will be spin polarized. Magnetization configurations of the two leads are very important, since they affect the polarization of the electronic current and so the magnitude of current. Many experimental and theoretical works have been devoted to studying parallel and antiparallel magnetization configurations.<sup>4,12-18</sup> Up to now, only a few works have been made to study the transport properties of the QD coupled to noncollinear magnetic leads.<sup>19-24</sup> Fransson<sup>21</sup> found that the current of the system in the strongly coupled regime is a nonmonotonic function of the angle between the magnetization directions in the two leads, which was attributed to the spin-dependent energy shift of the QD state. The transport through QDs coupled to leads with noncollinear magnetizations in the sequential tunneling regime was seriously analyzed by Braun *et al.*,<sup>24</sup> while transport in the Coulomb blockade regime was considered by Weymann and Barnas.<sup>22</sup>

Recently, Gorelik *et al.*<sup>4</sup> studied the spin-polarized electronic transport through a magnetic single-electron transistor with a central QD subject to an external magnetic field perpendicular to the magnetizations in two leads. They proposed a new phenomenon of Coulomb promotion of spin-dependent tunneling, which arises from combined effects of spin-flip processes induced by the magnetic field and Coulomb correlations on the QD. However, only the collinear magnetization case was taken into account there. The study of the angle-dependent electronic current in the noncollinear magnetization case is highly desirable, in which new phenomena may emerge due to the combined effects of noncollinearly oriented magnetic leads and the magnetic field ap-

plied to the QD. The physical picture of the present QD system compared to multilayers is significantly different. Because of the smallness of the QD, its conductive states are zero-dimensionally confined. The QD can accommodate only a few electrons so that the Coulomb interaction plays a blockade role, leading to single-electron tunneling behavior.

Transport problems in mesoscopic systems can be investigated by many methods, such as the quasiclassical Boltzmann equation approach,<sup>25-28</sup> the Kubo formula,<sup>25,26</sup> the density-matrix (rate equation) approach,<sup>4,29-32</sup> and the Green's function approach.<sup>27,33,34</sup> In this paper we shall extend the density-matrix approach<sup>4</sup> to the case of the QD coupled to noncollinearly oriented magnetic leads and subject to a perpendicular magnetic field (see Fig. 1). Angle-dependent electronic currents  $I(\theta)$  exhibit different behaviors in the free regime, where two electrons can exist in the QD energy level, and in the Coulomb blockade regime, where at most one electron exists in the QD energy level. In the absence of an external magnetic field,  $I_F(\theta)$  in the free regime varies monotonically from  $\theta=0$  to  $\pi$ , similar to the normal spin-valve effect in magnetic multilayers,<sup>35-37</sup> while  $I_{CB}(\theta)$  in the Coulomb blockade regime has a nonmonotonic variation as a result of the competition of the spin-valve effect and spin splitting of the QD state energies. In the presence of an external magnetic field, the spin flip of electrons through the QD significantly promotes electronic tunneling in the antiparallel magnetization configuration, no matter whether the dot is in the free or Coulomb blockade regime. We also compare the present theoretical results with those obtained earlier in the literature. In special cases of collinear magnetization orientations ( $\theta=0$  or  $\pi$ ), our analytic formula for the angle-

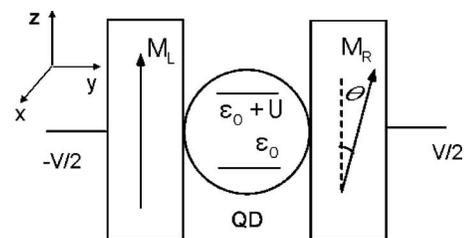


FIG. 1. Sketch of the nanomagnetic tunnel junction of a QD coupled to two leads with magnetizations  $M_L$  and  $M_R$  at an angle of  $\theta$ .  $V$  is the bias voltage applied to the junction.

dependent electronic current is reduced to those obtained recently by Gorelik *et al.*<sup>4</sup> In the Coulomb blockade regime, it is found that the  $I_{CB}(\theta)$  peak appears at a certain angle  $\theta$  between 0 and  $\pi$  rather than just at  $\theta=0$ , whether the QD is subject to a magnetic field or not. In the absence of an external magnetic field, this phenomenon can be explained by the reason mentioned above: i. e., the competition of the spin-valve effect and spin-dependent shift of the QD state energies. In the presence of a perpendicular magnetic field, although the  $I_{CB}(\theta)$  behavior is qualitatively unchanged,  $I_{CB}(\theta)$  is promoted near  $\theta=\pi$  and reduced near  $\theta=0$  due to the magnetic-field-induced spin flip of electrons through the QD.

## II. THEORY AND ANALYTICAL RESULTS

Consider a magnetic single-electron-transistor device shown in Fig. 1, in which a QD, subject to an external magnetic field  $h_x$  along the  $x$  direction, is located between two spin-polarized leads with magnetizations at an angle of  $\theta$  in the  $z$ - $y$  plane. For the dot of nanometer size, only one electron energy level needs to be considered on the dot. This electron level may be doubly occupied by electrons with different spins in the free regime, but can accommodate one electron at most in the Coulomb blockade regime. The main effect of the external magnetic field perpendicular to the magnetizations in the leads is to induce coherent spin-flip dynamics on the dot so as to significantly affect the transport of electrons through the device. Assuming  $h_x$  is small enough, we neglect its effect on lead magnetization and the Zeeman splitting of the energy level on the dot.<sup>4</sup> By choosing the spin quantization axis along the  $z$  axis, the system Hamiltonian can be written as  $H=H_L+H_R+H_d+H_T$  with<sup>4,20,21,34</sup>

$$\begin{aligned} H_L &= \sum_{\alpha s} \varepsilon_{L\alpha s} a_{L\alpha s}^\dagger a_{L\alpha s}, \\ H_R &= \sum_{\beta s} \varepsilon_{R\beta s} a_{R\beta s}^\dagger a_{R\beta s}, \\ H_d &= \sum_s \varepsilon a_s^\dagger a_s - U a_\uparrow^\dagger a_\downarrow^\dagger a_\uparrow a_\downarrow + \sum_{s,s'} \mu h_x a_s^\dagger \tau_x^{ss'} a_{s'}, \\ H_T &= \sum_{\alpha s} t_L a_s^\dagger a_{L\alpha s} + \sum_{\beta s} t_R a_s^\dagger \left( a_{R\beta s} \cos \frac{\theta}{2} - i a_{R\beta \bar{s}} \sin \frac{\theta}{2} \right) + \text{H.c.} \end{aligned} \quad (1)$$

Here  $H_L$  ( $H_R$ ) is the noninteracting electron Hamiltonian in the left (right) lead, and  $a_{L\alpha s}^\dagger$  and  $a_{L\beta s}$  ( $a_{R\beta s}^\dagger$  and  $a_{R\alpha s}$ ) are the corresponding creation and annihilation operators for electrons. In the light of the Stoner theory, the electronic energy in the ferromagnetic lead is given by  $\varepsilon_{L\alpha s} = \varepsilon_{L\alpha} - sM_L$  ( $\varepsilon_{R\beta s} = \varepsilon_{R\beta} - sM_R$ ) with  $\alpha$  ( $\beta$ ) denoting the level in the left (right) leads,  $s=1$  ( $-1$ ) for the spin-up (down) electron,  $\bar{s}=-s$ , and  $M_L$  ( $M_R$ ) representing the magnetization of the left (right) lead.  $H_d$  is the QD Hamiltonian in which  $a_s^\dagger$  ( $a_s$ ) is the creation (annihilation) operator on the QD with energy  $\varepsilon$  and spin  $s$ ,  $U$  is the Coulomb energy, and  $\tau_x^{ss'}$  is the Pauli matrix.

$H_T$  stands for the tunneling of electrons between the dot and leads with hopping coefficients  $t_L$  and  $t_R$ .

Following Gurvitz and Prager,<sup>29</sup> the evolution of the whole system in the free regime can be described by the many-body wave function

$$\begin{aligned} |\Psi(t)\rangle &= \left\{ b_0 + \sum_{\alpha s s'} b_{\alpha s' [s]} a_s^\dagger a_{L\alpha s'} \right. \\ &+ \sum_{\alpha < \alpha' s s'} b_{\alpha \alpha' s s'} a_s^\dagger a_{L\alpha s}^\dagger a_{L\alpha' s'} \\ &+ \sum_{\alpha \beta s s'} b_{\alpha \beta s' (s)} a_{R\beta s}^\dagger a_{L\alpha s'} \\ &+ \sum_{\alpha < \alpha', \beta s s' s'' s'''} b_{\alpha \alpha' \beta s' s'' (s') [s]} a_s^\dagger a_{R\beta s'}^\dagger a_{L\alpha s''} a_{L\alpha' s'''} \\ &+ \dots \left. \right\} |0\rangle. \end{aligned} \quad (2)$$

Here the ‘‘vacuum state’’  $|0\rangle$  denotes that all the levels in the two leads are initially filled with electrons up to the Fermi energy, and  $b(t)$  are the time-dependent probability amplitudes for finding the system in the corresponding states described above with the initial condition  $b_0(0)=1$  and all the other  $b(0)=0$ . For brevity, we generally write  $b(t)$  as  $b$ , and subscripts  $s$ ,  $(s)$ , and  $[s]$  of  $b$  stand for the electron spin in the left lead, right lead, and QD, respectively. If the QD is occupied by two electrons, the QD spin subscripts of  $b$  must be  $[s\bar{s}]$  so as to be omitted. Substituting Eqs. (1) and (2) into the Schrödinger equation  $i\frac{d}{dt}|\Psi(t)\rangle = H|\Psi(t)\rangle$  results in an infinite set of coupled differential equations for  $b(t)$ . Using the Laplace transform

$$\tilde{b}(E) = \int_0^\infty e^{iEt} b(t) dt, \quad (3)$$

together with the initial conditions, we get an infinite set of algebraic equations for  $\tilde{b}(E)$ ,<sup>29,31,32</sup>

$$(E + i\Gamma_L/2)\tilde{b}_0 = i,$$

$$\left[ E + \varepsilon_{L\alpha s} - \varepsilon + \mu h_i \tau_i^{ss} + i\Gamma_L^{\bar{s}}/2 + \left( i\Gamma_R^s \cos^2 \frac{\theta}{2} + i\Gamma_R^{\bar{s}} \sin^2 \frac{\theta}{2} \right) / 2 \right] \tilde{b}_{\alpha s [s]} + \mu h_i \tau_i^{s\bar{s}} \tilde{b}_{\alpha s [\bar{s}]} - t_L \tilde{b}_0 = 0,$$

$$\left[ E + \varepsilon_{L\alpha \bar{s}} - \varepsilon + \mu h_i \tau_i^{s\bar{s}} + i\Gamma_L^s/2 + \left( i\Gamma_R^s \cos^2 \frac{\theta}{2} + i\Gamma_R^{\bar{s}} \sin^2 \frac{\theta}{2} \right) / 2 \right] \tilde{b}_{\alpha \bar{s} [s]} + \mu h_i \tau_i^{s\bar{s}} \tilde{b}_{\alpha \bar{s} [\bar{s}]} = 0,$$

$$[E + \varepsilon_{L\alpha s} + \varepsilon_{L\alpha' s} - 2\varepsilon - U + i\Gamma_R/2] \tilde{b}_{\alpha \alpha' s s} - t_L \tilde{b}_{\alpha s [\bar{s}]} = 0,$$

$$[E + \varepsilon_{L\alpha s} + \varepsilon_{L\alpha' \bar{s}} - 2\varepsilon - U + i\Gamma_R/2]\tilde{b}_{\alpha\alpha' s\bar{s}} - t_L \tilde{b}_{\alpha s [s]} = 0. \quad (4)$$

Here  $\tilde{b}(E)$  has been simply written as  $\tilde{b}$ , and  $\Gamma_L^s = 2\pi\rho_{Ls}|t_L|^2$  ( $\Gamma_R^s = 2\pi\rho_{Rs}|t_R|^2$ ) is the line width function due to coupling between the dot and the left (right) lead with  $\rho_{Ls}$  ( $\rho_{Rs}$ ) the density of states in the left (right) lead. In addition, we have defined  $\Gamma_L = \Gamma_L^\uparrow + \Gamma_L^\downarrow$ ,  $\Gamma_R = \Gamma_R^\uparrow + \Gamma_R^\downarrow$ ,  $\Gamma^\uparrow = \Gamma_L^\uparrow + \Gamma_R^\uparrow$ ,  $\Gamma^\downarrow = \Gamma_L^\downarrow + \Gamma_R^\downarrow$ , and  $\Gamma = \Gamma_L + \Gamma_R = \Gamma^\uparrow + \Gamma^\downarrow$ . In this work we have assumed that both  $k_B T$  (temperature) and  $\Gamma$  are much smaller than bias voltage  $V$  and  $|V-U|$ , and the level position of the quantum dot is just at the Fermi level.<sup>4</sup>

Next we introduce the density matrix of the system. The dynamics of the system can be described by the time evolution of the total density matrix, which contains a number of degrees of freedom in the dot and the leads. Since the leads are treated as reservoirs of noninteracting electrons, it is the dynamics of the dot degrees of freedom that determines the transport behavior. By integrating out the degrees of freedom of the leads we can get a reduced density matrix for the dot degrees of freedom only. As the QD consists of only one energy level, in the Hilbert space, the density matrix can be defined as

$$\rho_d = \begin{pmatrix} \rho_0 & 0 & 0 & 0 \\ 0 & \rho_\uparrow & \rho_{\uparrow\downarrow} & 0 \\ 0 & \rho_{\downarrow\uparrow} & \rho_\downarrow & 0 \\ 0 & 0 & 0 & \rho_2 \end{pmatrix}, \quad (5)$$

where the diagonal elements  $\rho_0$ ,  $\rho_\uparrow$ ,  $\rho_\downarrow$ , and  $\rho_2$  are the probabilities of the dot level being empty, occupied by a spin-up and spin-down electron, and by two electrons, respectively, and the complex off-diagonal elements  $\rho_{\uparrow\downarrow}$  and  $\rho_{\downarrow\uparrow}$  are responsible for coherent quantum effects in the electron transport.<sup>29</sup> Owing to the conservation of probability, there exists the completeness relation

$$\text{tr}(\rho_d) = \rho_0 + \rho_\uparrow + \rho_\downarrow + \rho_2 = 1. \quad (6)$$

The matrix elements of the density matrix of the system can be written as

$$\rho_i = \rho_i^{(0)} + \rho_i^{(1)} + \dots,$$

with subscript  $i=0, \uparrow, \downarrow$ , and 2, where

$$\rho_0^{(0)} = |b_0|^2, \quad \rho_0^{(1)} = \sum_{\alpha\beta s s'} |b_{\alpha\beta s'(s)}|^2, \quad \dots,$$

$$\rho_s^{(0)} = \sum_{\alpha s'} |b_{\alpha s' [s]}|^2, \quad \rho_s^{(1)} = \sum_{\alpha < \alpha', \beta s' s'' s'''} |b_{\alpha\alpha' \beta s'' s''(s')} [s]}|^2, \quad \dots,$$

$$\begin{aligned} \rho_2^{(0)} &= \sum_{\alpha < \alpha' s s'} |b_{\alpha\alpha' s s'}|^2, & \rho_2^{(1)} \\ &= \sum_{\alpha < \alpha' < \alpha'' \beta s' s'' s'''} |b_{\alpha\alpha' \alpha'' \beta s' s'' s''(s)}|^2, & \dots, \end{aligned}$$

and

$$\rho_{s\bar{s}} = \rho_{s\bar{s}}^{(0)} + \rho_{s\bar{s}}^{(1)} + \dots,$$

where

$$\rho_{s\bar{s}}^{(0)} = \sum_{\alpha s'} b_{\alpha s' [s]}^* b_{\alpha s' [\bar{s}]},$$

$$\rho_{s\bar{s}}^{(1)} = \sum_{\alpha < \alpha', \beta s' s'' s'''} b_{\alpha\alpha' \beta s'' s''(s')}^* [s] b_{\alpha\alpha' \beta s' s''(s')}^* [\bar{s}], \quad \dots \quad (7)$$

Here superscript  $n$  in  $\rho_i^{(n)}$  is the number of electrons in the right lead. The current flowing through the system is  $I(t) = e dN_R(t)/dt$ , where  $N_R(t)$  is the number of electrons accumulated in the right lead—i.e.,

$$I(t) = \sum_n n \left[ \frac{d\rho_0^{(n)}(t)}{dt} + \frac{d\rho_\uparrow^{(n)}(t)}{dt} + \frac{d\rho_\downarrow^{(n)}(t)}{dt} + \frac{d\rho_2^{(n)}(t)}{dt} \right]. \quad (8)$$

By means of the inverse Laplace transform, from Eqs. (4) and (7), we get the time evolution of  $\rho_i^{(n)}$  with  $i=0, \uparrow, \downarrow, 2, \uparrow\downarrow, \downarrow\uparrow$ , and  $n=0, 1, 2, \dots$ <sup>29</sup> After lengthy algebra,<sup>29</sup> from Eq. (8) we obtain the electronic current in the free regime as

$$I_f(t)/e = \Gamma_R \rho_2 + (\Gamma_R^\uparrow \rho_\uparrow + \Gamma_R^\downarrow \rho_\downarrow) \cos^2 \frac{\theta}{2} + (\Gamma_R^\downarrow \rho_\uparrow + \Gamma_R^\uparrow \rho_\downarrow) \sin^2 \frac{\theta}{2}. \quad (9)$$

By summing  $d\rho_i^{(n)}/dt$  over  $n$ ,  $d\rho_i/dt$  can be obtained as

$$\frac{d\rho_0}{dt} = -\Gamma_L \rho_0 + (\Gamma_R^\uparrow \rho_\uparrow + \Gamma_R^\downarrow \rho_\downarrow) \cos^2 \frac{\theta}{2} + (\Gamma_R^\downarrow \rho_\uparrow + \Gamma_R^\uparrow \rho_\downarrow) \sin^2 \frac{\theta}{2},$$

$$\begin{aligned} \frac{d\rho_\uparrow}{dt} &= \left( \Gamma_L^\uparrow - \Gamma_L^\downarrow - \Gamma_R^\uparrow \cos^2 \frac{\theta}{2} - \Gamma_R^\downarrow \sin^2 \frac{\theta}{2} \right) \rho_\uparrow + \left( \Gamma_R^\uparrow \sin^2 \frac{\theta}{2} \right. \\ &\quad \left. + \Gamma_R^\downarrow \cos^2 \frac{\theta}{2} \right) \rho_2 + i\mu h_x (\rho_{\uparrow\downarrow} - \rho_{\downarrow\uparrow}), \end{aligned}$$

$$\begin{aligned} \frac{d\rho_\downarrow}{dt} &= \left( \Gamma_L^\downarrow - \Gamma_L^\uparrow - \Gamma_R^\downarrow \cos^2 \frac{\theta}{2} - \Gamma_R^\uparrow \sin^2 \frac{\theta}{2} \right) \rho_\downarrow + \left( \Gamma_R^\downarrow \sin^2 \frac{\theta}{2} \right. \\ &\quad \left. + \Gamma_R^\uparrow \cos^2 \frac{\theta}{2} \right) \rho_2 + i\mu h_x (\rho_{\downarrow\uparrow} - \rho_{\uparrow\downarrow}), \end{aligned}$$

$$\frac{d\rho_2}{dt} = \Gamma_R^\uparrow \rho_\downarrow + \Gamma_R^\downarrow \rho_\uparrow - \Gamma_R \rho_2,$$

$$\frac{d\rho_{\uparrow\downarrow}}{dt} = i\mu h_x (\rho_\downarrow - \rho_\uparrow) - \frac{\Gamma}{2} \rho_{\uparrow\downarrow},$$

$$\frac{d\rho_{\downarrow\uparrow}}{dt} = i\mu h_x (\rho_\uparrow - \rho_\downarrow) - \frac{\Gamma}{2} \rho_{\downarrow\uparrow}.$$

In the steady state with  $d\rho_i/dt=0$ , the steady current given in Eq. (9) in the free regime becomes

$$I_f = e(\Gamma_L \rho_0 + \Gamma_L^\uparrow \rho_\downarrow + \Gamma_L^\downarrow \rho_\uparrow), \quad (10)$$

which provides a clear physical picture for the electron transport through the system and can be understood by the following argument. For the empty level of the dot with probability  $\rho_0$ , the tunneling current is given by  $e\Gamma_L \rho_0$ . For the dot level occupied by a spin-up (spin-down) electron with probability  $\rho_\uparrow$  ( $\rho_\downarrow$ ), only the spin-down (spin-up) electron can enter the dot so that the tunneling current is given by  $e\Gamma_L^\downarrow \rho_\uparrow$  ( $\Gamma_L^\uparrow \rho_\downarrow$ ). If the dot level is doubly occupied by electrons, no electron can enter the dot and so no term of  $\rho_2$  appears in Eq. (10). As a result, the total tunneling current is the sum of those in the empty- and single-occupied cases. We wish to point out here that  $\rho_0$ ,  $\rho_\uparrow$ , and  $\rho_\downarrow$  each are a function of  $\theta$ ,  $\Gamma_L^s$ , and  $\Gamma_R^s$ , which can be determined from the set of

differential equations above with  $d\rho_i/dt=0$ . Substituting their expressions into Eq. (10), we finally obtain

$$I_f = \frac{e\Gamma_L \Gamma_R}{\Gamma} \left[ \frac{16\mu^2 h_x^2 + \Gamma^2 - P_L^2 \Gamma \Gamma_L - P_R^2 \Gamma \Gamma_R \cos^2 \theta}{16\mu^2 h_x^2 + \Gamma^2 - (P_L \Gamma_L + P_R \Gamma_R \cos \theta)^2} \right], \quad (11)$$

where  $P_L = (\Gamma_L^\uparrow - \Gamma_L^\downarrow)/\Gamma_L$  and  $P_R = (\Gamma_R^\uparrow - \Gamma_R^\downarrow)/\Gamma_R$  are polarizations of the left and right leads, respectively.

In the Coulomb blockade regime, in which double occupation of electrons on the dot level is prohibited, the same calculated procedure as above can be performed provided  $\rho_2=0$  is taken into account. The tunneling current in the Coulomb blockade regime can also be obtained as

$$I_{CB} = e\Gamma_L \Gamma_R \left[ \frac{16\mu^2 h_x^2 + (1 - P_R^2 \cos^2 \theta) \Gamma_R^2}{16\mu^2 h_x^2 (\Gamma_L + \Gamma) + 2(1 - P_L P_R \cos \theta) \Gamma_L \Gamma_R^2 + (1 - P_R^2 \cos^2 \theta) \Gamma_R^3} \right]. \quad (12)$$

This current can be also written as  $I_{CB} = e\Gamma_L \rho_0^c$  where  $\rho_0^c$  is the probability of the empty dot level in the Coulomb blockade regime. It stems from the fact that in this case the electron can pass through the dot only when the dot level is empty.  $\rho_0^c$  in the Coulomb blockade regime must be different from  $\rho_0$  in the free regime. It can be shown that, if one takes  $\theta=0$  or  $\pi$ , Eqs. (11) and (12) reduce to the expressions for  $I_f$  and  $I_{CB}$  obtained by Gorelik *et al.*,<sup>4</sup> the latter being just the special result of the former in collinear magnetization configurations.

### III. RESULTS AND DISCUSSIONS

Equations (11) and (12) are the main results in this work. In what follows we make some discussions of the analytical results and give their graphical presentation. We wish to show the angle dependence of the tunneling conductance and the effect of the external magnetic field on the tunnel magnetoresistance (TMR). The left and right leads are assumed to have the same polarization—i.e.,  $P_L = P_R = P$ . The asymmetry of the linewidth functions is described by  $\chi = (\Gamma_L - \Gamma_R)/\Gamma$  and the magnitude of the magnetic field applied to the QD described by  $\eta = 2\mu h_x/\Gamma$ . The angle-dependent TMR ratio is defined as

$$\text{TMR} \equiv [I_\parallel - I(\theta)]/I_\parallel, \quad (13)$$

where  $I(\theta)$  and  $I_\parallel$  are the tunneling currents with the lead magnetizations at angle  $\theta$  and parallel to each other ( $\theta=0$ ), respectively.

#### A. Free regime

In the free regime, where  $U$  is smaller than the bias voltage applied to the junction, the QD energy level may be occupied by two electrons. In this case the Coulomb interac-

tion plays little role in the tunneling current. The  $\theta$  dependences of the tunneling current and TMR ratio exhibit a normal spin-valve effect, as shown in Figs. 2(a) and 2(b). The current arrives at its maximum and minimum, respectively, in the parallel and antiparallel magnetization configurations, exhibiting a monotonic change in between. If the two ferromagnetic leads are made of the identical half metal ( $P=1$ ) and there is no external magnetic field ( $h_x=0$ ), from Eq. (11), we have  $I_f(\theta=\pi)=0$  and  $I_f(\theta=0)=e\Gamma_L \Gamma_R/\Gamma$ , so that the TMR ratio is equal to 1 for  $\theta=\pi$ , as shown by the solid line in Figs. 2(a) and 2(b). It is interesting to see that the  $\theta$  dependence of the TMR ratio greatly deviates from the ideal shape of  $\sin^2 \theta/2$ . Such a deviation comes from the appearance of  $\cos \theta$  in the denominator in Eq. (11). Physically, it stems from the fact that in the quantum-dot spin-valve system there exist spin accumulation on the QD and an exchange field effect, making the TMR effect less pronounced compared with a single magnetic tunnel junction.<sup>24</sup> As a result, the calculated results shown in Fig. 2 are consistent with those obtained by Braun *et al.*<sup>24</sup> and by Wetzels *et al.*<sup>38</sup>

We next discuss the effect of the magnetic field applied to the QD on  $I_f(\theta)$  and TMR ratio. Figure 3 shows the  $\theta$  dependence of  $I_f$  for different magnetic fields  $\eta$ . It is found that  $I_f(\theta=\pi)$  increases with  $\eta$ , even though the qualitative behavior of  $I_f(\theta)$  remains unchanged. It is the spin-flip effect induced by the external magnetic field that promotes the current through the device with antiparallel magnetization. For  $P_L = P_R = 1$ , Eq. (11) is reduced to  $I_f(\theta=0) = e\Gamma_L \Gamma_R/\Gamma$  and  $I_f(\theta=\pi) = (e\Gamma_L \Gamma_R/\Gamma) \eta^2 / (\eta^2 + \Gamma_L \Gamma_R/\Gamma)$ . In this case,  $I_f(\pi)$  increases rapidly with magnetic field while  $I_f(0)$  is independent of  $\eta$ .

#### B. Coulomb blockade regime

In the Coulomb blockade regime, where  $U$  is greater than the bias voltage, the QD energy level can be occupied by

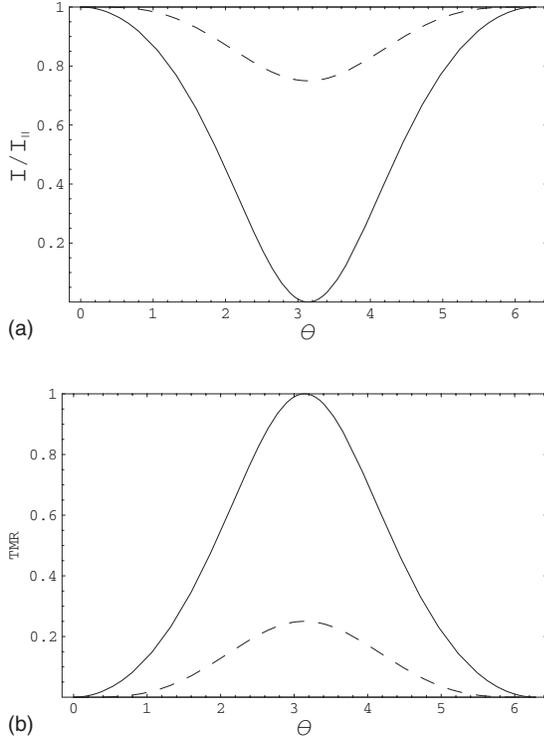


FIG. 2. Angle dependence of the electronic current (a) and TMR ratio (b) in the free regime for  $\eta=0$  and  $\chi=0$  with lead polarizations:  $P=1$  (solid line), 0.5 (dashed line), and 0 (horizontal line).

one electron at most. In this case the Coulomb interaction plays an important role in the electronic current through the QD. Figure 4 shows the  $\theta$  dependence of  $I_{CB}$  for  $P=1$  (solid line), 0.95 (dot-dashed line),  $P=0.5$  (dashed line), and  $P=0$  (dotted line) in the absence of  $h_x$ . It is found that  $I_{CB\parallel}=I_{CB}(\theta=0)$  is no longer maximal and  $I_{CB}(\theta)$  is greater than  $I_{CB\parallel}$  in a wide range of  $\theta$ , exhibiting a maximum at a certain value of  $\theta$ . In reality, in the absence of  $h_x$ , Eq. (12) is reduced to

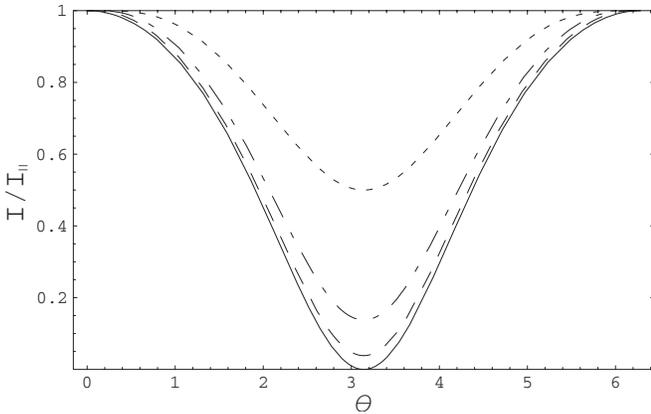


FIG. 3. Angle dependence of the electronic current in the free regime for  $P=1$  and  $\chi=0$  with external magnetic fields:  $\eta=0$  (solid line), 0.1 (dashed line), 0.2 (dot-dashed line), and 0.5 (dotted line).

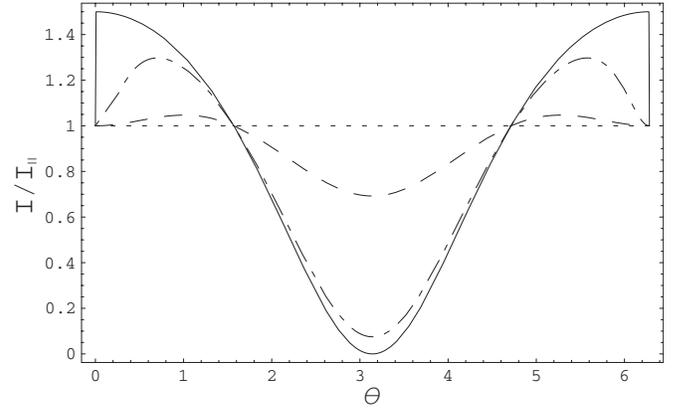


FIG. 4. Angle dependence of the electronic current in the Coulomb blockade regime for  $\eta=0$  and  $\chi=0$  with lead polarizations  $P=1$  (solid line), 0.95 (dot-dashed line), 0.5 (dashed line), and 0 (dotted line).

$$I_{CB} = \frac{e\Gamma_L\Gamma_R}{2\Gamma_L + \Gamma_R} \frac{1}{1 - \beta(\theta, P)}, \quad (14)$$

with

$$\beta(\theta, P) = \frac{2\Gamma_L}{2\Gamma_L + \Gamma_R} \frac{P^2 \cos \theta (1 - \cos \theta)}{1 - P^2 \cos^2 \theta}. \quad (15)$$

It then follows that  $\beta(\theta, P)=0$  at  $\theta=0$ , and so  $I_{CB}(\theta=0) = e\Gamma_L\Gamma_R/(2\Gamma_L + \Gamma_R)$ , independent of  $P$ . Since  $\beta(\theta, P) > 0$  for  $0 < \theta < \pi/2$ , we have  $I_{CB}(\theta) > I_{CB\parallel}$  in this  $\theta$  range, which is consistent with the numerical result shown in Fig. 4. From Eqs. (14) and (15), it is found that the maximal  $I_{CB}(\theta)$  appears at  $\cos \theta_c = (1 - \sqrt{1 - P^2})/P^2$  for  $0 \leq P \leq 1$ . Here  $\cos \theta_c$  is close to 1/2 for small  $P$  and equal to 1 at  $P=1$ , increasing with  $P$ . As a result, in Fig. 4 the  $I_{CB}(\theta)$  maximum moves towards the left (the direction of decreasing  $\theta$ ) with increasing  $P$ . For  $P=0$ , the leads are nonmagnetic and so  $I_{CB}$  is  $\theta$  independent. The situation of  $P=1$  is somewhat special, in which the maximal  $I_{CB}$  appears just at  $\theta=0$ . Therefore there is a jump of  $I_{CB}$  at  $\theta=0$  from  $I_{CB\parallel} = e\Gamma_L\Gamma_R/(2\Gamma_L + \Gamma_R)$  to its maximum  $e\Gamma_L\Gamma_R/(\Gamma_L + \Gamma_R)$ , increased by half if  $\Gamma_L = \Gamma_R$  is taken, as shown in Fig. 4. The behavior of  $I_{CB}(\theta) > I_{CB\parallel}$  for  $0 < \theta < \pi/2$  is quite different from that in the free regime where  $I_{F\parallel}$  is always maximal. As a result, such an anomaly of  $I_{CB}(\theta)$  must come from the Coulomb correlations on the QD. For ferromagnetic leads, the levels in the QD experience a spin split which may result in a decrease of the tunneling current.<sup>12</sup> The largest spin splitting of the QD levels is caused by a parallel magnetic alignment of the leads, whereas an antiparallel alignment gives the smallest spin split. Such a competitive factor of  $I_{CB}(\theta)$  decreasing with  $\theta$  increased from 0 to  $\pi$  may lead to a nonmonotonic behavior of  $I_{CB}(\theta)$  shown in Fig. 4. The present results in the absence of an external magnetic field are qualitatively consistent with those obtained by Fransson.<sup>21</sup>

In the presence of a magnetic field applied to the QD, there is a spin-flip effect, which makes the change of  $I_{CB}(\theta)$  become small, as shown in Fig. 5. For the half-metallic

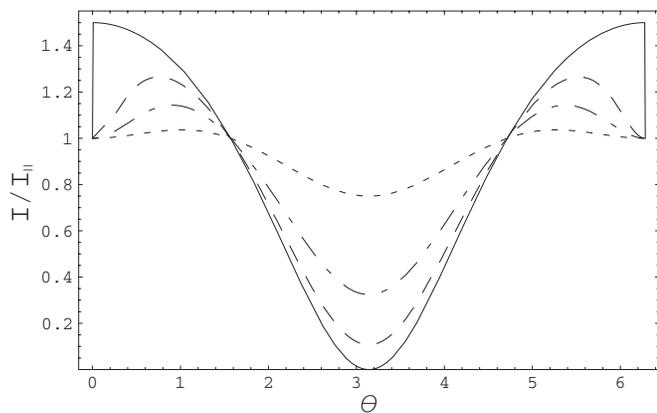


FIG. 5. Angle dependence of the electronic current in the Coulomb blockade regime for  $P=1$  and  $\chi=0$  with external magnetic fields:  $\eta=0$  (solid line), 0.1 (dashed line), 0.2 (dot-dashed line), and 0.5 (dotted line).

leads, owing to the spin flip on the QD, the electronic current is no longer equal to 0 at  $\theta=\pi$ . Just like in the free regime, the external magnetic field promotes the current near  $\theta=\pi$  in the Coulomb blockade regime. Such a promotion of  $I_{CB}(\theta=\pi)$  is even stronger than that of  $I_F(\theta=\pi)$ . On the other hand,  $I_{CB}(\theta)$  for  $\theta<\pi/2$  decreases as  $\eta$  increases, indicating

that the spin flip induced by  $\eta$  is unfavorable to the spin-split effect on the QD level.

In summary, based on the quantum rate equation, we have studied the angle dependence of the electronic current through a QD coupled to two magnetic leads with noncollinear magnetizations in the free and Coulomb blockade regimes. In the absence of external magnetic field, the angle-dependent electronic current in the free regime varies monotonically from the parallel to antiparallel alignment, while in the Coulomb blockade regime it varies nonmonotonically due to the competition of the spin-valve effect and spin splitting of the QD-state energies. The external magnetic field results in a spin flip of the electron on the QD so as to significantly promote the electronic current at  $\theta=\pi$  in both the free and Coulomb blockade regimes, even though its angle dependence remains qualitatively unchanged. The present calculated results indicate that there is a large departure of  $I_F(\theta)$  from  $\sin^2 \theta/2$  behavior. In the collinear magnetization configuration, the present analytic expressions for  $I_F$  and  $I_{CB}$  reduce to those obtained by Gorelik *et al.*<sup>4</sup>

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