Emergence of supersymmetry at a critical point of a lattice model

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(Received 22 June 2007; revised manuscript received 6 July 2007; published 6 August 2007)

Supersymmetry is a symmetry between a boson and a fermion. Although there is no apparent supersymmetry in nature, its mathematical consistency and appealing properties have led many people to believe that supersymmetry may exist in nature in the form of a spontaneously broken symmetry. In this paper, we explore an alternative possibility by which supersymmetry is realized in nature, that is, supersymmetry dynamically emerges in the low-energy limit of a nonsupersymmetric condensed matter system. We propose a (2+1)-dimensional lattice model which exhibits an emergent space-time supersymmetry at a quantum critical point. It is shown that there is only one relevant perturbation at the supersymmetric critical point in the ϵ expansion and the critical theory is the two copies of the Wess-Zumino theory with four supercharges. Exact critical exponents are predicted.

DOI: 10.1103/PhysRevB.76.075103 PACS number(s): 05.30.−d, 11.30.Pb, 64.60.Ak

I. INTRODUCTION

Poincare invariance is the underlying space-time symmetry of relativistic quantum field theories. There are only two mathematically consistent ways of extending the symmetry in nontrivial ways due to a no-go theorem.[1] One is a conformal symmetry and the other, a supersymmetry. A conformal symmetry combines the Poincare invariance with scale invariance. It is realized in the long-distance limit of a massless theory at which all finite length scales are scaled out. A supersymmetry is a symmetry between a boson and a fermion. Generators of supersymmetry Q_α, which are called supercharges, are spinors and they satisfy the commutation relations

$$\{Q_\alpha, Q_\beta\} = 2P_\mu \Gamma^\mu_{\alpha\beta} + Z_{\alpha\beta}, \quad [Q_\alpha, P_\mu] = 0, \tag{1}$$

where α and β are spinor indices, $\Gamma^\mu_{\alpha\beta}$ are constants, P_μ is the energy-momentum operator and $Z_{\alpha\beta}$ are central charges. Because supercharges are fermionic operators, a boson is transformed into a fermion (and vice versa) under supersymmetry transformations. Therefore, the number of bosonic modes is equal to the number of fermionic modes in supersymmetric theories. The second commutation relation in Eq. (1) implies $[P_\mu P^\mu, Q_\alpha]=0$, and masses of supersymmetric partners are identical. Since bosons and fermions contribute to quantum effective actions with quantum corrections of the opposite signs and they have same masses, the effects of quantum fluctuations of bosons and fermions are canceled with each other in supersymmetric theories. Because of this, quantum corrections are highly constrained by kinematics. If there are enough supersymmetries, there is no quantum correction at all (nonrenormalization) for some quantities. Due to the nonrenormalization property, supersymmetry has been proposed as a way of stabilizing the hierarchy of vastly different mass scales present in the standard model. It may also play an important role in the unification of the gauge interactions. On the other hand, many supersymmetric theories have been studied as toy models where a supersymmetry enables one to understand strong coupling physics rigorously.[2,3]

Despite the unique mathematical consistency and beautiful properties of supersymmetry, nature does not exhibit supersymmetry at low-energy scales. If nature is supersymmetric, it should be spontaneously broken. If that is the case, supersymmetry will become manifest at a high-energy scale. An alternative way of finding supersymmetry in nature may be to go to a low energy in condensed matter systems, relying on the principle of emergence.

A new symmetry can emerge in the low-energy limit although a microscopic model does not respect the symmetry. For example, a low-energy effective theory can have the full Poincare invariance although the underlying lattice explicitly breaks rotational symmetry in a condensed matter system. Even a gauge symmetry[4] and a general covariance[5,6] can be emergent. The emergence of a new symmetry can be a characteristic of a new state of many-body systems.[7] Searching for new states of matter in condensed matter systems is becoming an important research avenue as new materials which cannot be understood in conventional theories are synthesized[8,9] and highly controllable correlated many-body systems can be fabricated in cold atom systems.[10,11] Therefore, it would be of interest to find a condensed matter system which shows an emergent supersymmetry.

It has been suggested that supersymmetry can emerge in the low-energy limit of a nonsupersymmetric theory.[12–14] In (1+1) dimensions, supersymmetry emerges at the tricritical point of the dilute Ising model.[15] Emergent supersymmetries play important roles in realizing lattice versions of supersymmetric field theories in various dimensions.[16] For example, the $\mathcal{N}=1$ (3+1)-dimensional super Yang-Mills theory can emerge without an underlying supersymmetry although the notion of the emergent supersymmetry is rather obscure in this case due to the opening of a mass gap caused by confinement. Supersymmetric field theories can also be realized in lattices by fine tunings of bare parameters[17] or by a dynamical mechanism where some supersymmetries which are already present in lattices guarantee the emergence of continuum supersymmetries without fine tuning.[18] In this paper, we construct a (2+1)-dimensional [(2+1)D] lattice model where supersymmetry may dynamically emerge at a quantum critical point without any lattice supersymmetry.

It is noted that nonrelativistic supersymmetries have been considered in condensed matter systems.[19,20] In nonrelativistic systems, supercharges are scalars (not spinors) and the anticommutator of supercharges generates only energy (not momentum), that is, $\{Q,Q^\dagger\}=H$, where H is a Hamiltonian. In such systems, supersymmetries play the roles as in $(0+1)$-dimensional quantum mechanical systems.[21] The present work concerns an emergence of a full space-time supersymmetry in a (2+1)D relativistic system where the relativity is also emergent out of a nonrelativistic microscopic system. In this case, the algebra of supercharges generates the translations in both time and space through the energy-momentum operator as in Eq. (1).

II. MICROSCOPIC MODEL AND LOW-ENERGY EFFECTIVE THEORY

The microscopic system is a mixture of fermions and bosons. The Hamiltonian is composed of three parts,

$$H=H_f+H_b+H_{fb}, \tag{2}$$

where

$$H_f=-t_f\sum_{\langle i,j\rangle}(f_i^\dagger f_j+\text{H.c.}),$$

$$H_b=t_b\sum_{\langle I,J\rangle}(e^{i(\theta_I-\theta_J)}+\text{H.c.})+\frac{U}{2}\sum_I n_I^2,$$

$$H_{fb}=h_0\sum_I e^{i\theta_I}(f_{I+\mathbf{b}_1}f_{I-\mathbf{b}_1}+f_{I-\mathbf{b}_2}f_{I+\mathbf{b}_2}+f_{I-\mathbf{b}_1+\mathbf{b}_2}f_{I+\mathbf{b}_1-\mathbf{b}_2})+\text{H.c.} \tag{3}$$

Here, H_f describes spinless fermions with nearest neighbor hopping on the honeycomb lattice, H_b describes bosons with nearest neighbor hopping and an on-site repulsion on the triangular lattice which is dual to the honeycomb lattice, and H_{fb} couples the fermions and bosons. The lattice structure is shown in Fig. 1(a). f_i is the fermion annihilation operator and $e^{-i\theta_I}$ the lowering operator of n_I which is conjugate to the angular variable θ_I. i,j and I,J are site indices for the honeycomb and triangular lattices, respectively. $t_f,t_b>0$ are the hopping energies for the fermions and bosons, respectively, and U is the on-site boson repulsion energy. Note that the boson hopping is frustrated. This will play a crucial role for the emergent supersymmetry as will be shown later. $\mathbf{b}_1$ and $\mathbf{b}_2$ are vectors which connect a site on the triangular lattice to the neighboring honeycomb lattice sites, as is shown in Fig. 1(a). h_0 is the pairing interaction strength associated with the process where two fermions in the f-wave channel around a hexagon are paired and become a boson at the center of the hexagon, and vice versa. In this sense, the boson can be regarded as a Cooper pair made of two spinless fermions in the f-wave function, as is shown in Fig. 1(b). This model has a global U(1) symmetry under which the fields transform as $f_i\to f_i e^{i\varphi}$ and $e^{-i\theta_I}\to e^{-i\theta_I}e^{i2\varphi}$.

First, we identify low-energy modes of the fermions and bosons in the absence of the coupling H_{fb}. At zero chemical potential, the fermions are half filled and their energy spectrum is given by $e_{\mathbf{k}}^f=\pm t_f\sqrt{(1+\cos k_1+\cos k_2)^2+(\sin k_1-\sin k_2)^2}$, where $k_1=ak_x$, $k_2=a\frac{-k_x+\sqrt{3}k_y}{2}$ with a the lattice spacing. There exist two Fermi points at $\mathbf{k}_A=\frac{2\pi}{a}(\frac{1}{3},\frac{1}{\sqrt{3}})$ and $\mathbf{k}_B=\frac{2\pi}{a}(\frac{2}{3},0)$ as shown in Fig. 1(c). Since the energy dispersion is linear near the Fermi points, the low-energy excitations are described by two Dirac fermions,

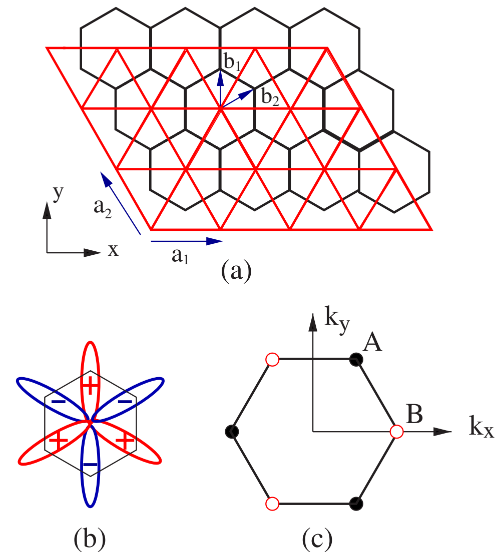

FIG. 1. (Color online) (a) The lattice structure in the real space. Fermions are defined on the honeycomb lattice, while bosons live on the dual triangular lattice. $\mathbf{a}_1$ and $\mathbf{a}_2$ are the lattice vectors with length a, and $\mathbf{b}_1$ and $\mathbf{b}_2$ are two independent vectors which connect a site on the triangular lattice to the nearest neighbor sites on the honeycomb lattice. (b) The phases of a fermion pair in real space. (c) The first Brillouin zone in momentum space. A and B indicate two inequivalent points with momenta $\mathbf{k}_A=\frac{2\pi}{a}(\frac{1}{3},\frac{1}{\sqrt{3}})$ and $\mathbf{k}_B=\frac{2\pi}{a}(\frac{2}{3},0)$ where the low-energy modes are located. ψ_1, ϕ_2 are located at $\mathbf{k}_A$ and ψ_2, ϕ_1 at $\mathbf{k}_B$.

$$\mathcal{L}_f=i\sum_{n=1}^{2}\bar{\psi}_n\left(\gamma_0\partial_\tau+c_f\sum_{i=1}^{2}\gamma_i\partial_i\right)\psi_n. \tag{4}$$

Here, ψ_1 (ψ_2) denotes the two-component complex fermion at momentum $\mathbf{k}_A$ ($\mathbf{k}_B$). $\partial_\mu=(\partial_\tau,\partial_x,\partial_y)$ are the derivatives in imaginary time and the spatial directions. $\gamma_0\equiv\sigma_3$, $\gamma_1\equiv\sigma_1$, and $\gamma_2\equiv\sigma_2$ with σ_μ the Pauli matrices. $\bar{\psi}_n\equiv-i\psi_n^\dagger\gamma_0$ and $c_f\sim t_f a$ is the Fermi velocity.

To obtain a low-energy theory for the bosons, we introduce a soft boson field $\Phi_I=|\Phi_I|e^{-i\theta_I}$ and the potential $V(\Phi)=u_2|\Phi|^2+u_4|\Phi|^4$ which gives a finite amplitude to the soft boson field. In energy-momentum space, the boson action becomes

$$\mathcal{S}_b=\int dk\left(\frac{1}{2U}k_0^2+e_{\mathbf{k}}^b+u_2\right)|\Phi_k|^2 + u_4\int dk_1dk_2dq\,\Phi_{k_2-q}^*\Phi_{k_1+q}^*\Phi_{k_1}\Phi_{k_2}, \tag{5}$$

where $\int dk\equiv\int\frac{dk_0dk_xdk_y}{(2\pi)^3}$ is the energy-momentum integration and $e^b_{\mathbf{k}}=2t_b[\cos k_1+\cos k_2+\cos(k_1+k_2)]$. Since the boson hopping has the wrong sign, $\mathbf{k}=(0,0)$ is not the minimum of $e^b_{\mathbf{k}}$; rather, two minima occur at $\mathbf{k}_A$ and $\mathbf{k}_B$ where the nodal points of the fermions are located. Therefore, we have two low-energy boson modes. We introduce ϕ_1 and ϕ_2 to represent the low-energy modes near the $\mathbf{k}_B$ and $\mathbf{k}_A$ points, respectively. Note that the ϕ_1 (ϕ_2) boson carries the same momentum as the ψ_2 (ψ_1) fermion. With this convention, we will see that only those bosons and fermions which carry the same index $(n=1,2)$ interact with each other at low energies. Expanding $e^b_{\mathbf{k}}$ near the two minima, we obtain the effective Lagrangian for the low-energy bosons,

$$\mathcal{L}_b=\sum_{n=1}^{2}\left[|\partial_\tau\phi_n|^2+c_b^2\sum_{i=1}^{2}|\partial_i\phi_n|^2+m^2|\phi_n|^2\right]+\lambda_1\kappa\sum_{n=1}^{2}|\phi_n|^4+\lambda_2\kappa|\phi_1|^2|\phi_2|^2, \tag{6}$$

where $c_b\sim\sqrt{t_bUa}$ is the boson velocity which is, in general, different from the fermion velocity c_f. Although both the fermions and bosons have the "relativistic" energy spectra, there is no Lorentz symmetry if the velocities are different. The Lorentz symmetry requires the velocities of all massless particles to be identical. As will be shown later, the Lorentz symmetry will emerge in the low-energy limit through quantum corrections. m is the boson mass and the coupling constants λ_1 and λ_2 are made dimensionless by introducing a mass scale κ. Note that momentum conservation does not allow an interaction such as $\phi_2^*\phi_2^*\phi_1\phi_1$.

We can obtain the interaction between the low-energy fermions and bosons by rewriting H_{fb} in energy-momentum space and keeping only the low-energy modes. The resulting interaction Lagrangian is

$$\mathcal{L}_{fb}=h\kappa^{1/2}\sum_{n=1}^{2}(\phi_n^*\psi_n^T\varepsilon\psi_n+\text{c.c.}), \tag{7}$$

where ε is the 2×2 antisymmetric matrix with $\varepsilon_{12}=-\varepsilon_{21}=1$. Terms such as $\phi_2^*\psi_1^T\varepsilon\psi_1$ or $\phi_2^*\psi_1^T\varepsilon\psi_2$ are not allowed because they do not satisfy momentum conservation.

III. RENORMALIZATION GROUP ANALYSIS

Now, we perform a one-loop renormalization group (RG) analysis in $4-\epsilon$ dimensions for the low-energy effective theory given by

$$\mathcal{L}=\mathcal{L}_f+\mathcal{L}_b+\mathcal{L}_{fb}. \tag{8}$$

We use the dimensional regularization scheme where the number of fermion components and the traces of gamma matrices are fixed.[23] Maintaining the same number of fermionic and bosonic modes in $4-\epsilon$ dimension is important because supersymmetry requires that the number of modes is the same for the bosons and fermions. If there is a gauge symmetry, more sophisticated regularization scheme is necessary to preserve both gauge symmetry and supersymmetry because the number of components of gauge boson should depend on the dimension of space-time.[24] Since there is no gauge symmetry in the present model, the simple dimensional regularization scheme can maintain supersymmetry.[25] Of course, the present model has no supersymmetry. The point is that it is convenient to use a regularization scheme which can maintain supersymmetry in probing an emergent supersymmetry in the low-energy limit.

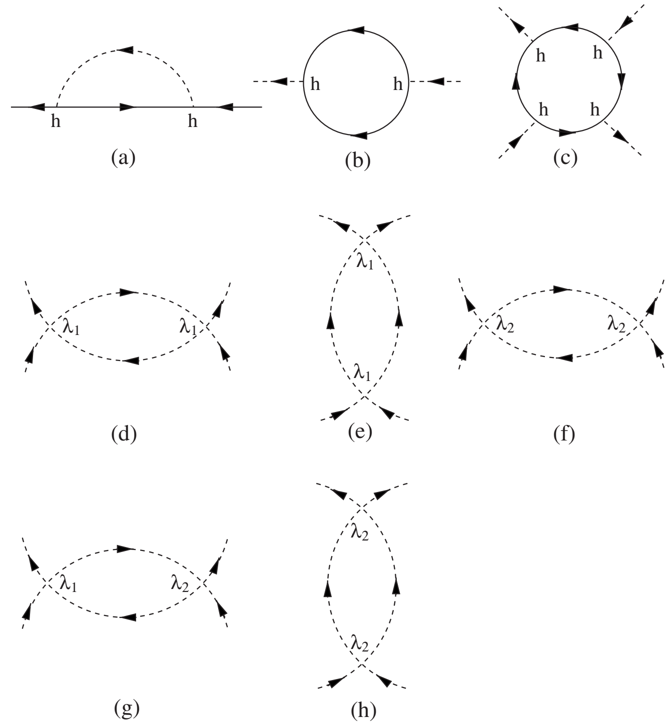

FIG. 2. One-loop diagrams. (a) and (b) are the self-energy corrections of fermions and bosons respectively. (c)–(f) contribute to the vertex correction of λ_1, while (f), (g), and (h) contribute to λ_2.

In the ϵ expansion, the above Lagrangian contains all the relevant and marginal terms. A four fermion interaction has the scaling dimension $D=6-2\epsilon+O(\epsilon^2)$ and can be ignored for a small ϵ. In the following, we do not consider the four fermion interaction. However, in principle, the four fermion interaction can become important in (2+1)D due to a strong interaction, in which case one needs to tune a microscopic four fermion interaction term to reach the fixed points we will discuss in the following. The boson mass is always a relevant perturbation and we tune it to zero in order to examine the RG flow of the other couplings in the massless subspace. At the one loop level, there are eight diagrams which are shown in Fig. 2. Each diagram contributes to the quantum effective action as follows:

$$\delta\mathcal{L}^{(a)}=\frac{4h^2}{(4\pi)^2c_f^2\epsilon}\sum_{n=1}^{2}i\bar{\psi}_n\left(f_0\gamma_0\partial_\tau+c_ff_1\sum_{i=1}^{2}\gamma_i\partial_i\right)\psi_n,$$

$$\delta\mathcal{L}^{(b)}=\frac{4h^2}{(4\pi)^2c_f^2\epsilon}\sum_{n=1}^{2}\left(|\partial_\tau\phi_n|^2+c_f^2\sum_{i=1}^{2}|\partial_i\phi_n|^2\right),$$

$$\delta\mathcal{L}^{(c)} = \frac{16h^4}{(4\pi)^2 c_f^2 \epsilon} \sum_{n=1}^{2} |\phi_n|^4,$$

$$\delta\mathcal{L}^{(d)} = -\frac{16\lambda_1^2}{(4\pi)^2 c_b^2 \epsilon} \sum_{n=1}^{2} |\phi_n|^4,$$

$$\delta\mathcal{L}^{(e)} = -\frac{4\lambda_1^2}{(4\pi)^2 c_b^2 \epsilon} \sum_{n=1}^{2} |\phi_n|^4,$$

$$\delta\mathcal{L}^{(f)} = -\frac{\lambda_2^2}{(4\pi)^2 c_b^2 \epsilon} \sum_{n=1}^{2} |\phi_n|^4 - \frac{2\lambda_2^2}{(4\pi)^2 c_b^2 \epsilon} |\phi_1|^2 |\phi_2|^2,$$

$$\delta\mathcal{L}^{(g)} = -\frac{16\lambda_1\lambda_2}{(4\pi)^2 c_b^2 \epsilon} |\phi_1|^2 |\phi_2|^2,$$

$$\delta\mathcal{L}^{(h)} = -\frac{2\lambda_2^2}{(4\pi)^2 c_b^2 \epsilon} |\phi_1|^2 |\phi_2|^2, \tag{9}$$

where $f_0 = \frac{4}{\alpha(\alpha+1)^2}$ and $f_1 = \frac{4(2\alpha+1)}{3\alpha(\alpha+1)^2}$ with $\alpha = c_b / c_f$. From the renormalized quantum effective action, the beta functions are obtained to be

$$\frac{dh}{dl} = \frac{\epsilon}{2} h - \frac{1}{(4\pi c_f)^2} \left(2 + \frac{16 c_f^3}{c_b (c_f + c_b)^2} \right) h^3,$$

$$\frac{d\lambda_1}{dl} = \epsilon \lambda_1 - \frac{1}{(4\pi)^2} \left(\frac{20\lambda_1^2 + \lambda_2^2}{c_b^2} + \frac{8h^2\lambda_1}{c_f^2} - \frac{16h^4}{c_f^2} \right),$$

$$\frac{d\lambda_2}{dl} = \epsilon \lambda_2 - \frac{1}{(4\pi)^2} \left(\frac{4\lambda_2^2 + 16\lambda_1\lambda_2}{c_b^2} + \frac{8h^2\lambda_2}{c_f^2} \right),$$

$$\frac{dc_f}{dl} = \frac{32h^2 c_f (c_b - c_f)}{3(4\pi)^2 c_b (c_b + c_f)^2}, \quad \frac{dc_b}{dl} = -\frac{2h^2 c_b (c_b^2 - c_f^2)}{(4\pi c_b c_f)^2}, \tag{10}$$

where the scaling parameter l increases in the infrared.

There are two solutions for $\beta_h = 0$. One is the unstable solution with $h=0$ and the other, the stable one with a finite h. At $h=0$, the bosons and fermions are decoupled. The fermion system consists of noninteracting Dirac fermions. The RG flow of the boson couplings in the subspace of $m=h=0$ is shown in Fig. 3. In the subspace of $m=h=0$, there are three fixed points, that is, the Gaussian (GA) fixed point with $(h^*, \lambda_1^*, \lambda_2^*) = (0,0,0)$, the Wilson-Fisher (WF) fixed point with $(h^*, \lambda_1^*, \lambda_2^*) = \left(0, \frac{(4\pi c_b)^2 \epsilon}{20}, 0\right)$, and the O(4) fixed point with $(h^*, \lambda_1^*, \lambda_2^*) = \left(0, \frac{(4\pi c_b)^2 \epsilon}{24}, \frac{(4\pi c_b)^2 \epsilon}{12}\right)$. Because the linear term of the beta function at the O(4) fixed point accidentally vanishes along a direction, at higher order in ϵ, there occurs a stable fourth fixed point between the WF fixed point and the O(4) fixed point.[22] However, the fixed points in the $m=h=0$ plane are all unstable because the pairing interaction h is relevant.

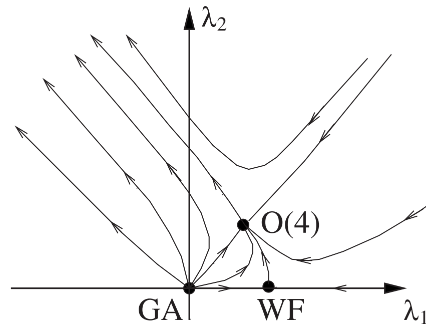

FIG. 3. The schematic RG flow of the bosonic couplings in the subspace of $m=h=0$.

Once we turn on the pairing interaction, h flows to a finite value with $h^2 = \frac{(4\pi c_f)^2 \epsilon}{2} \left(2 + \frac{16 c_f^3}{c_b (c_f + c_b)^2} \right)^{-1}$, and the boson and fermion velocities begin to flow as can be seen from the last two equations in Eq. (10). Because the pairing interaction mixes the velocities of the boson and fermion, the difference of the velocities exponentially flows to zero in the low-energy limit as is shown in Fig. 4(a). The line of $c_b = c_f$ is critical, and the value of the velocity in the infrared limit is a nonuniversal value. This implies that the bosons and fermions have the same energy dispersion and Lorentz symmetry emerges at low energies due to quantum fluctuations. Now, we consider the flow of h, λ_1, and λ_2 with a fixed $c_b = c_f = c$. The RG flow is displayed in Fig. 4(b). In the following, we will use units where $c=1$. With a nonzero h, the system flows to a stable fixed point,

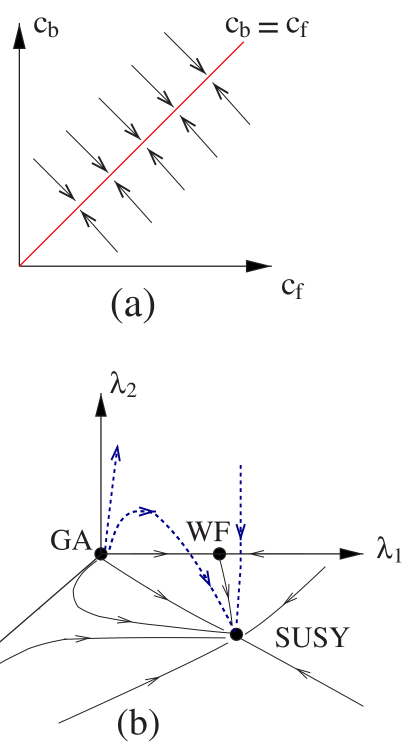

FIG. 4. (Color online) Schematic RG flows of (a) the velocities with $h \neq 0$ and (b) λ_1, λ_2, and h in the subspace of $m=0$. In (b), the solid lines represent the flow in the plane of (h, λ_1) and the dashed lines, the flow outside the plane.

$$(h^*,\lambda_1^*,\lambda_2^*) = \left(\sqrt{\frac{(4\pi)^2\epsilon}{12}},\frac{(4\pi)^2\epsilon}{12},0\right). \tag{11}$$

The nonzero h is a consequence of strong pairing fluctuations at the critical point. This is crucial in obtaining Lorentz symmetry and supersymmetry as will be discussed in the following. At the fixed point, λ_2 vanishes and there is no coupling between the two sets of low-energy modes, (ϕ_n,ψ_n) with $n=1,2$. Physically, this implies that the Bose condensates which carry the different momenta $\mathbf{k}_A$ and $\mathbf{k}_B$ develop independently in the condensed phase. At the critical point, $\lambda_1 = h^2$ and the theory becomes invariant under the transformation,

$$\delta_{\xi_n}\phi_n = -\bar{\psi}_n\xi_n, \quad \delta_{\xi_n}\phi_n^* = \bar{\xi}_n\psi_n,$$

$$\delta_{\xi_n}\psi_n = i\partial\!\!\!/\phi_n^*\xi_n - \frac{h}{2}\phi_n^2\varepsilon\bar{\xi}_n^T,$$

$$\delta_{\xi_n}\bar{\psi}_n = i\bar{\xi}_n\partial\!\!\!/\phi_n - \frac{h}{2}\phi_n^{*2}\xi_n^T\varepsilon, \tag{12}$$

where $\partial\!\!\!/ = \gamma_\mu\partial_\mu$ and ξ_n is a two-component spinor of Grassmann variables which parametrizes the transformation. This is a supersymmetry because the bosons and fermions are mixed under the transformation. Since the two sets of modes (ϕ_n,ψ_n) are decoupled at the critical point, the supersymmetry transformations are independent for $n=1$ and 2. That is why the spinor ξ_n has the index n. Here, $\delta_{\xi_n}\bar{\psi}_n \neq -i(\delta_{\xi_n}\psi_n)^\dagger\gamma_0$ because we are using the imaginary time formalism. The supersymmetry leads to conserved supercurrents,

$$J_\mu^n = \partial\!\!\!/\phi_n\gamma_\mu\psi_n + i\frac{h}{2}\phi_n^2\gamma_\mu\epsilon\bar{\psi}_n^T,$$

$$\bar{J}_\mu^n = \bar{\psi}_n\gamma_\mu\partial\!\!\!/\phi_n^* + i\frac{h}{2}\phi_n^{*2}\psi_n^T\epsilon\gamma_\mu. \tag{13}$$

Here, the spinor indices in J_μ^n and $\bar{J}_\mu^n$ are suppressed. For each n, there are four independent supercharges, $Q_\alpha^n = \int dx^2 J_{0\alpha}^n$ and $\bar{Q}_\alpha^n = \int dx^2\bar{J}_{0\alpha}^n$, in (2+1)D. The supercharges satisfy the commutation relations such as Eq. (1). This corresponds to $\mathcal{N}=2$ supersymmetry in each sector of n, that is, there are twice as much supercharges as the minimum number of supercharges in (2+1)D. The resulting super-Poincare invariance is an emergent symmetry because the microscopic model has neither Poincare symmetry nor supersymmetry. Each set made of one complex boson and one two-component fermion forms a chiral multiplet of the supersymmetry. This critical theory is the $\mathcal{N}=2$ Wess-Zumino theory[26] with two copies of chiral multiplets.[27] The critical exponents calculated in the one-loop level,

$$\eta_\phi = \eta_\psi = \epsilon/3, \tag{14}$$

match those of the Wess-Zumino theory in the leading order of ϵ. In the supersymmetric theory, the one-loop result is exact as will be explained later. It is of note that the critical exponent is independent of regularization scheme although the values of the couplings depend on regularization scheme.

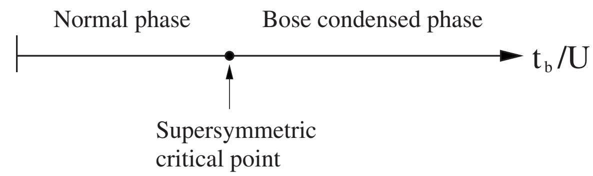

FIG. 5. The schematic phase diagram as a function of the ratio of the boson hopping t_b to the on-site boson repulsion energy U for a generic value of h_0.

The schematic phase diagram of the Hamiltonian [Eq. (2)] for a generic value of h_0 is shown in Fig. 5 in the parameter space of t_b/U. There is a second order phase transition between the normal phase for small t_b/U and the Bose condensed phase for large t_b/U. In the normal phase, the fermions are gapless, while a gap opens in the Bose condensed phase. The critical point is described by the supersymmetric Wess-Zumino theory although both the normal and the Bose condensed phases are nonsupersymmetric.

Although the evidences for the emergent supersymmetry, that is, the supersymmetric relation of the couplings and the scaling dimensions, are obtained based on the calculation to leading order in ϵ, we expect that the same conclusion holds to all orders in ϵ, at least for small ϵ for the following reason. The present model contains the supersymmetric Wess-Zumino theory in the sense that the supersymmetric Wess-Zumino theory can be reached at least by fine tunings of the bare parameters. Therefore, the Wess-Zumino theory should appear as a fixed point of this model in any case in the parameter region of $\lambda_1 \sim h^2 \sim \epsilon$, although we do not know *a priori* whether it is a stable or unstable fixed point. Since the fixed point which was identified as the Wess-Zumino theory at leading order in ϵ is the only fixed point which exists in that parameter range, and the higher-order terms in the ϵ expansion cannot generate a new fixed point near the generic stable fixed point, the fixed point obtained to leading order in ϵ should correspond to the Wess-Zumino fixed point to all orders in ϵ. In other words, if the fixed point obtained to leading order in ϵ were not the supersymmetric Wess-Zumino fixed point, another fixed point which corresponds to the Wess-Zumino theory should have appeared to leading order in ϵ.

Note that the supersymmetry transformations in Eq. (12) mix bosons and fermions which carry different global U(1) charges, where the U(1) charges of the fermions and bosons are given by $Q_\psi=1$ and $Q_\phi=2$, respectively. Therefore, the supercharges should carry a non-trivial U(1) charge which is called R charge. At the supersymmetric critical point, the super-Poincare symmetry is enlarged to an even bigger symmetry, that is, the superconformal symmetry which includes the generators of the super-Poincare group, and additional fermionic generators and the R charge.[28] The additional fermionic generators arise from the commutator of the supercharges and the conformal generators. The R charge which enters in the superconformal algebra is related to the global U(1) charge as $R=-\frac{Q}{3}$. The factor of 3 in the definition of the R charge is due to the cubic superpotential of the Wess-

Zumino theory.[29] Due to unitarity, there exists a constraint on the R charge and scaling dimension of an operator. In (2+1) dimensional, the superconformal symmetry puts a lower bound on a scaling dimension as $D_{\mathcal{O}} \geqslant |R_{\mathcal{O}}|$, where $D_{\mathcal{O}}$ and $R_{\mathcal{O}}$ are the scaling dimension and the R charge of an operator $\mathcal{O}$, respectively. The equality is saturated for a chiral primary field, and gives the exact anomalous dimensions for the fundamental fields with $\eta_\phi = \eta_\psi = 1/3$.[30] Note that these coincide with the values obtained by putting $\epsilon = 1$ in the one-loop results of Eq. (14). Nonchiral primary fields generally receive radiative corrections, and the critical exponent for the order parameter is calculated to be $\nu_\phi = \frac{1}{2} + \frac{\epsilon}{4} + \mathcal{O}(\epsilon^2)$ at leading order in ϵ.[14]

IV. CONCLUSION

In conclusion, we found a (2+1)D nonsupersymmetric lattice model whose quantum critical point is described by the supersymmetric Wess-Zumino theory in an ϵ expansion. The supersymmetric critical point describes the generic second order phase transition between a normal phase and a Bose condensed phase of the Bose-Fermi mixed system. The exact anomalous scaling dimensions are predicted. In principle, the boson can dynamically arise as a Cooper pair in a system which has only fermions as microscopic degrees of freedom. In such a case, the critical point describes a superconducting phase transition. Therefore, supersymmetry can emerge at a critical point of a pure fermionic system. It is of interest to examine such possibility in the future. Finally, although the supersymmetric theory describes the generic second order phase transition in $4-\epsilon$ dimension for a small ϵ, in (2+1) dimensional, we cannot exclude other possibilities. For example, the supersymmetric critical point may arise only as a multicritical point due to an occurrence of other supersymmetry-breaking relevant perturbation, or the critical point itself may disappear due to a first order phase transition.

ACKNOWLEDGMENTS

The author thanks M. P. A. Fisher, S. Sachdev, X.-G. Wen, and C. Wu for helpful discussions, M. Hermele for pointing out the O(4) fixed point, and J. Alicea for proofreading the paper and making helpful suggestions. This research was supported by the National Science Foundation under Grant No. PHY05-51164.

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