T_c suppression and resistivity in cuprates with out of plane defects

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Recent experiments introducing controlled disorder into optimally doped cuprate superconductors by both electron irradiation and chemical substitution have found unusual behavior in the rate of suppression of the critical temperature T_c vs increase in residual resistivity. We show here that the unexpected discovery that the rate of T_c suppression vs resistivity is stronger for out of plane than for in plane impurities may be explained by consistent calculation of both T_c and resistivity if the potential scattering is assumed to be nearly forward in nature. For realistic models of impurity potentials, we further show that significant deviations from the universal Abrikosov-Gor'kov T_c suppression behavior may be expected for out of plane impurities.

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I. INTRODUCTION

The destruction of superconductivity by disorder has been traditionally used to probe the nature of the superconducting state. In classic superconductors, pair breaking is caused only by magnetic impurities,[1] and the functional form of the T_c suppression, when plotted vs impurity concentration or change in normal state resistivity, is known to follow the universal curve predicted by Abrikosov and Gor'kov (AG).[2] In unconventional superconductors, ordinary nonmagnetic impurities are also expected to break pairs, and in the simplest approximation where the impurities are treated as pointlike (delta-function) potential scatterers, the T_c suppression also follows the AG form.

As in so many other respects, the experimental situation in the cuprates agrees qualitatively with the simplest notions of what should happen to d-wave superconductors in the presence of disorder but differs in some important details. For example, when Zn is substituted for Cu in the Cu-O planes, T_c is suppressed rapidly as expected for a d-wave superconductor. Here, "rapidly" means that the disorder-induced scattering rate required to destroy superconductivity is on the order of the gap scale rather than the Fermi energy E_F, as would be expected in an s-wave system. Nevertheless, the initial slope of the T_c vs $\Delta\rho$ curve found in experiment is a factor of 2–3 smaller than the universal AG curve.[3] This discrepancy has been attributed to scattering in higher angular momentum channels by several authors, who modeled the scattering potential with a separable form describing scattering in both s-wave and single higher ℓ-wave channels.[4,5] This is a simple and tractable way of including the finite range of the scatterers qualitatively but is neither consistent by the microscopy of screened impurities in the cuprates nor capable of treating the limit of extreme forward scattering, claimed to be of relevance in the cuprate case.[6–8]

A further paradox was reported recently by Fujita *et al.*,[13] who showed that the rate of suppression $dT_c/d\rho$ is significantly higher for out of plane cation substituents in $Bi_2Sr_2CuO_6$ (Bi-2201) and $La_{2-x}Sr_xCuO_4$ (LSCO) than for Zn in the same system. Since Zn is thought to be a near-unitary scatterer in these materials, this is somewhat mysterious at first sight. If one accounts for the fact that the out of plane defects are poorly screened and may act primarily as forward scatterers, however, the increase of resistivity with defect concentration will be slowed and one may be able to understand this discrepancy. It is the primary purpose of this paper to correlate, within simple models, the slower rates of both T_c suppression and resistivity increase in the case of near-forward scatterers to see if light can be shed on this puzzle.

Other types of deviations from traditional AG behavior have been observed and require explanation. In the past several years, electron irradiation studies have appeared, which are able to suppress superconductivity to zero in a controlled fashion in contrast to early studies which studied only the initial slope of the T_c suppression by disorder.[9] Rullier-Albenque *et al.*[10] used 2.5 MeV electron irradiation to create defects throughout $YBa_2Cu_3O_7$ (YBCO-123) and $YBa_2Cu_3O_{6.6}$ samples. In neither sample was an AG-type behavior for T_c observed. In the optimally doped sample, on the contrary, a remarkable linear behavior in T_c vs ρ was observed down to and including samples of vanishingly small T_c. Because the nearly "normal" samples are expected to have a small superfluid stiffness, this effect was interpreted provisionally in terms of a phase fluctuation model proposed by Emery and Kivelson.[11] Deviations from AG theory at small superfluid stiffness are to be expected even within the framework of mean field theory, however, as pointed out by Franz *et al.*,[12] who studied the problem numerically and included the self-consistent suppression of the order parameter around each impurity site. These authors reported positive curvature tails in T_c vs impurity concentration n_i for large n_i, in contrast to the negative curvature in the AG plot.

II. TOY MODEL

To understand physically how some of these effects might arise, we consider a simple model of forward scattering by impurities in which all effects can be calculated analytically. We will assume that in plane impurities are to be described by δ-function-like isotropic potentials, and out of plane scatterers are to be described by an extended potential in the plane; this simply assumes that the screened Coulomb potential created by out of plane impurities produces a "footprint" sensed by quasiparticles moving in the CuO_2 planes.

As proposed by Kee,[6] we interpolate between these cases by considering a weak scattering potential $V_{\vec{k}\vec{k}'}$, which exists only at the Fermi surface $|\vec{k}|=k_F$ and cuts off unless $\vec{k}$ and $\vec{k}'$ are sufficiently close,

$$V(\phi,\phi') = \begin{cases} v_0 & \text{if } |\phi-\phi'| < \phi_c \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

The diagonal and off-diagonal self-energies in this model are then

$$\Sigma_0(\phi,\omega_n) = -\Gamma \int_{\phi-\phi_c}^{\phi+\phi_c} d\phi' \frac{i\tilde{\omega}_n}{\sqrt{\tilde{\omega}_n^2 + \tilde{\Delta}_0^2 f(\phi')^2}}, \tag{2}$$

$$\Sigma_1(\phi,\omega_n) = \Gamma \int_{\phi-\phi_c}^{\phi+\phi_c} d\phi' \frac{\tilde{\Delta}_0 f(\phi')}{\sqrt{\tilde{\omega}_n^2 + \tilde{\Delta}_0^2 f(\phi')^2}}, \tag{3}$$

where $\Gamma=\pi n_i N_0 v_0^2$, with N_0 the density of states at the Fermi level. In Eqs. (2) and (3), the renormalized Matsubara frequencies and gap magnitudes are $i\tilde{\omega}_n = i\omega_n - \Sigma_0$ and $\tilde{\Delta}_0=\Delta_0+\Sigma_1$, and we assume a d-wave form of the unrenormalized order parameter, $f(\phi)=\cos 2\phi$. Note that we work in units where Boltzmann's constant $k_B=1$. The critical temperature is determined as usual from the linearized gap equation

$$\Delta_0 = V_d N_0 2\pi T_c \sum_{\omega_n>0}^{\omega_c} \frac{1}{2\pi} \int d\phi' f(\phi')^2 \frac{\tilde{\Delta}_0}{\tilde{\omega}_n}, \tag{4}$$

where the d-wave pairing interaction is given by $(V_d N_0)^{-1} = (1/2)\ln(2e^{\gamma}\omega_c/\pi T_{c0})$, and T_{c0} is T_c in the absence of impurities. If we now take $\phi_c \ll 1$ and $T_c \lesssim T_{c0}$, we find to leading order $\tilde{\Delta}_0/\tilde{\omega}_n = \Delta_0/\omega_n$, i.e., there is no pair breaking by pure forward scattering, analogous to Anderson's theorem in an s-wave superconductor with isotropic nonmagnetic impurities.

Since we are only interested in calculating the critical temperature, we can neglect quadratic variations of the order parameter and therefore perform the angular integrations in the definition of the self-energies. After solving the resulting set of equations for the ratio $\tilde{\Delta}_0/\tilde{\omega}_n$ that enters Eq. (4), we can write

$$1 = V_d N_0 \pi T_c \sum_{\omega_n>0}^{\omega_c} \frac{1}{\omega_n + 2\Gamma\phi_c - \Gamma \sin(2\phi_c)}. \tag{5}$$

It is worthwhile noting at this point that an expansion in powers of ϕ_c applied to Eq. (5) as performed in Ref. 6 is appropriate except when $T_c \to 0$. Following this approach would lead to the incorrect conclusion that the T_c suppression is linear all the way to $T_c=0$ in the forward scattering $\phi_c \to 0$ limit. Instead, it is clear that expression (5) may be summed exactly, leading to a modified AG result of the form

$$\ln \frac{T_c}{T_{c0}} = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{\Gamma(\phi_c - \sin\phi_c \cos\phi_c)}{\pi T_c}\right), \tag{6}$$

where the pair breaking parameter in the forward scattering limit $(\phi_c \ll 1)$ is of the order of ϕ_c^3:

$$\frac{\Gamma(\phi_c - \sin\phi_c\cos\phi_c)}{\pi T_c} \approx \frac{2\Gamma\phi_c^3}{3\pi T_c}. \tag{7}$$

Thus, we see that the effective pair breaking rate will decrease dramatically as the scattering becomes more forward in nature. This is indeed consistent with the conclusions of earlier works,[4] which concluded that anisotropic scattering slows T_c suppression for fixed impurity concentration.

It remains, however, to calculate the dependence of the normal state resistivity on the impurity scattering within the same framework if one wishes to compare directly to experiments where pair breaking is measured by the increase in residual resistivity. For an isotropic two-dimensional Fermi surface, this may be written as

$$\rho = \frac{2m^2}{e^2 p_F^2} \int d\phi' [1 - \cos(\phi' - \phi)] n_i |V(\phi,\phi')|^2, \tag{8}$$

leading us to the expression

$$\rho = \frac{\hbar}{e^2}\frac{2\Gamma}{E_F}(2\phi_c - 2\sin\phi_c) \tag{9}$$

so that T_c may be expressed directly in terms of the impurity-induced resistivity as

$$\ln \frac{T_c}{T_{c0}} = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + Q(\phi_c)\rho\frac{e^2 E_F}{4\pi\hbar T_c}\right), \tag{10}$$

where the factor $Q(\phi_c)$ is given as

$$Q(\phi_c) = (\phi_c - \sin\phi_c\cos\phi_c)/(\phi_c - \sin\phi_c). \tag{11}$$

It ranges from $Q(0)=4$ in the forward scattering limit to $Q(\pi)=1$ in the case of isotropic scattering, and its dependence on ϕ_c is shown in Fig. 1. The factor $Q(\phi_c)$ is also directly related to the initial slope of the T_c suppression that can be derived from Eq. (10) as

$$\frac{T_c}{T_{c0}} \approx 1 - \frac{\pi}{8} Q(\phi_c)\rho\left(\frac{E_F}{T_{c0}}\frac{e^2}{\hbar}\right). \tag{12}$$

We see that although the rate of T_c suppression (or resistivity increase) is certainly much slower for forward scatterers than in the isotropic scattering case, the dependence on the resistivity is stronger for the former class of defects—within our model, about a factor of 4. This may indeed explain the results of Fujita *et al.*,[13] since in these experiments, out of plane (more forward scattering) and in plane (isotropic) impurities are studied in separate samples. In Fig. 2, we show the suppression of T_c as a function of resistivity for different values of ϕ_c as given by Eq. (10). While the initial slope is indeed seen to increase as the scattering is made more anisotropic, approaching the value of 4 as $\phi_c \to 0$, we see also that within the toy model, there is no deviation from

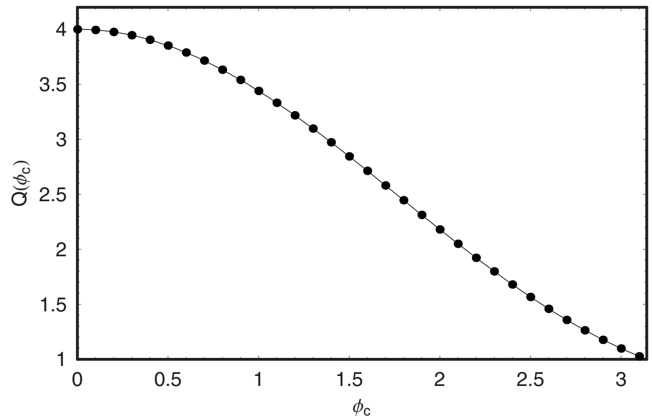

FIG. 1. The factor $Q(\phi_c)$ as a function of the maximum scattering angle ϕ_c. It reflects the decrease of the initial slope of T_c suppression as a function of resistivity if one approaches the forward scattering limit ($\phi_c \rightarrow 0$).

the *form* of the AG curve, as exhibited explicitly in Eq. (6).

III. REALISTIC MODEL

To confirm the above intuition and get a sense of the range of behavior possible in real systems, we now consider a more realistic model consisting of randomly distributed out of plane impurities with Yukawa potentials $V_i = V_0 \exp(-\kappa r_i)/r_i$, where r_i is the distance from a dopant atom to the lattice site i in the plane. $1/\kappa$ gives the screening length of the impurity potential, and the forward scattering case ($\kappa \rightarrow 0$) as well as the isotropic scattering case ($\kappa \rightarrow \infty$) are included with this specific choice of the impurity potential. The Fourier components of the screened Coulomb potential can be written as

$$|V(\vec{k},\vec{k}')|^2 = \frac{|V_0|^2}{|\vec{k}-\vec{k}'|^2+\kappa^2}. \tag{13}$$

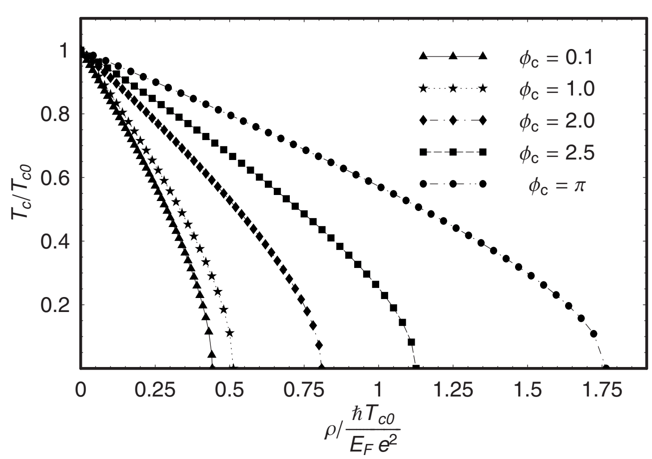

FIG. 2. T_c suppression as a function of resistivity calculated within the toy model. We find a decrease of the initial slope of the T_c suppression as a function of resistivity if we approach the forward scattering limit.

We use a square lattice to mimic the copper oxide plane and include up to next-nearest hopping terms in numerical computations. The electronic band is taken to be $\epsilon_k = -2t(\cos k_x + \cos k_y) - 4t' \cos k_x \cos k_y - \mu$, with $t' = -0.3t$ and $\mu = -1.1t$ in order to mimic the known shape of hole-doped cuprate Fermi surfaces near optimal doping. The T_c suppression is calculated by solving the gap equation self-consistently:

$$\Delta_{\vec{k}} = \frac{1}{\Lambda\beta}\sum_{\omega_n,\vec{k}'} V^d_{\vec{k},\vec{k}'}\frac{\tilde{\Delta}_{k'}}{\tilde{\omega}_n^2+\epsilon_{k'}^2}, \tag{14}$$

with an effective electron-electron pairing interaction $V^d_{\vec{k},\vec{k}'} = V^d f(\vec{k}) f(\vec{k}')$. The choice of the symmetry function $f(\vec{k}) = (1/2)(\cos k_x - \cos k_y)$ leads to a $d_{x^2-y^2}$ symmetry of the order parameter. The diagonal and off-diagonal parts of the self-energy are given by

$$\Sigma_0(\vec{k},\omega) = n_i \sum_{\vec{k}'} |V(\vec{k},\vec{k}')|^2 \frac{\tilde{\omega}}{\tilde{\omega}^2 - \epsilon_{k'}^2 - \tilde{\Delta}_{k'}^2}, \tag{15}$$

$$\Sigma_1(\vec{k},\omega) = n_i \sum_{\vec{k}'} |V(\vec{k},\vec{k}')|^2 \frac{\tilde{\Delta}_{k'}}{\tilde{\omega}^2 - \epsilon_{k'}^2 - \tilde{\Delta}_{k'}^2}, \tag{16}$$

where we can neglect quadratic variations of $\tilde{\Delta}_k$ since we are only interested in the region near T_c. To compare the suppression of T_c to the increase of the normal conducting resistivity, we have to calculate the resistance within the same model. For the transport rate $\tau^{-1}(\vec{k})$, we use an approximation introduced by Ziman[14] that expands the quasiparticle scattering rate by an additional scattering-in term,

$$\frac{1}{\tau_k} = \frac{2\pi}{\hbar} n_i \int \frac{dk_x' dk_y'}{(2\pi/a)^2} \delta(\epsilon_{k'}) \left(1 - \frac{\vec{v}_F(\vec{k}) \cdot \vec{v}_F(\vec{k}')}{|\vec{v}_F(\vec{k})||\vec{v}_F(\vec{k}')|}\right) |V_{kk'}|^2, \tag{17}$$

and that has proven to be very accurate even for highly anisotropic transport rates.[15] In the expression for the transport rate, we have replaced the full T matrix by the single impurity scattering potential $V_{kk'}$. The conductivity can then be calculated from

$$\rho^{-1} = \sigma_{xx} = e^2 \int \frac{d^2k}{(2\pi)^2} \tau_k v_{F,x}^2 \delta(\epsilon_k) \tag{18}$$

assuming a cubic symmetry of the transport tensor.

In Fig. 3, the suppression of T_c as a function of resistivity is shown for the tight binding model discussed in this section and $T_{c0} = 0.01t$. As we have already seen for the toy model, the initial rate of T_c suppression vs resistivity increases with increasing range of the impurity potential $1/\kappa$ and seems to be a robust feature of the T_c suppression due to forward scattering processes. However, the particular shape of the T_c suppression curve depends on the details of the considered model, e.g., the band structure, the doping strength, or the functional form of the scattering potential. In Fig. 4, it is shown that by approaching the weak coupling limit $T_{c0} \ll E_F$ due to a lowering of T_{c0}, we find a more linear depen-

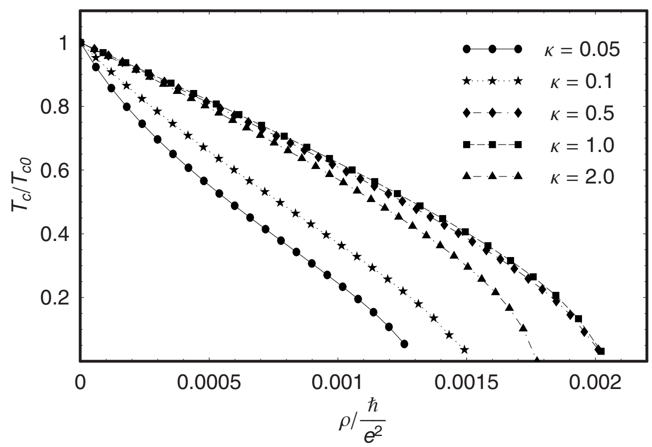

FIG. 3. T_c suppression as a function of resistivity calculated within a tight binding model. $1/\kappa$ gives the range of the scattering potential, where $\kappa \ll 1$ corresponds to the forward scattering and $\kappa \gg 1$ to the isotropic scattering case. The nonmonotonic behavior of the curves for different κ results from the competing effects of a reduction of the downward curvature with a simultaneous decrease of the initial slope for decreasing κ. The critical temperature of the pure sample is chosen to be $T_{c0}=0.01t$.

dence of the critical temperature on the resistivity, reminiscent of the linearity found in the electron irradiation experiments. At present, we do not have an analytical understanding of the origins of this quasilinearity.

IV. CONCLUSIONS

We have investigated the disorder-induced reduction of the critical temperature in high T_c compounds due to out of plane disorder, assuming that the out of plane defects act primarily as elastic forward scatterers. These calculations may be important not only to understand experiments where this type of disorder is varied systematically but also to understand T_c's in systems which are intrinsically disordered by the doping process. Calculating both the normal state resistivity as well as the T_c suppression in both a simple toy model and for more realistic scattering potentials and bands, we found that although the effect of forward scattering on both quantities is smaller than for isotropic scattering, the T_c suppression as a function of resistivity is stronger for forward than for isotropic scatterers. In the case of the toy model, we showed that the suppression is AG-like, with modified pair breaking parameter. The basic physics of the more rapid initial suppression of T_c vs ρ was confirmed numerically using more realistic Yukawa-type impurity potentials within a tight binding model.

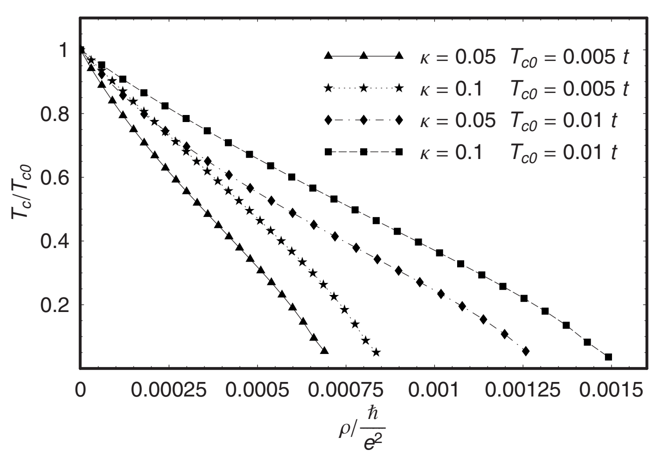

FIG. 4. T_c suppression as a function of resistivity in the forward scattering limit for two different values of T_{c0}. The upward curvature that is visible especially for very low values of κ diminishes for lower values of T_{c0}, and the T_c suppression as a function of resistivity becomes more linear.

Our result stands in apparent contradiction to earlier work comparing T_c vs ρ for anisotropic potentials scatterers,[5] where only s- and d-wave components of the scattering potential were retained. These authors concluded that the greater the ratio of potentials V_d/V_s, the weaker would be the suppression of T_c vs ρ. While this model is mathematically consistent, it is physically unrealistic and does not yield generic results. As shown in the Appendix, more realistic potentials may be expanded in Fermi surface harmonics, but to obtain the proper (weaker) enhancement of the resistivity with disorder, it is important to retain the p-wave component as well. If this is done for a generic potential, the result of this paper is obtained.

This point leads us to an important conclusion regarding T_c suppression experiments making use of Zn to replace Cu. It has been recognized for some time that the T_c suppression rate vs residual resistivity for Zn is smaller than predicted by AG theory for pointlike scatterers in a d-wave superconductor[3] by about a factor of 3 in optimally doped YBCO, and even smaller in other materials such as LSCO. We have now argued that the "conventional" explanation, based on highly anisotropic scattering by Zn impurities, is unlikely to be correct. It seems, therefore, that even in the optimally doped cuprates, the deviation must be ascribed to effects of strong correlations or strong coupling superconductivity. Within Eliashberg theory,[18] the AG pair-breaking parameter is renormalized $\Gamma \to \Gamma/(1+\lambda)$, where λ is the dimensionless coupling, while the disorder-induced resistivity change is unrenormalized to leading order. The physical origin of this effect is not clear in the cuprate context, however, Kulic and Oudovenko[5] argue that renormalizations of the scattering vertex due to strong interactions within the t-J model introduce significant suppression of the transport rate induced by impurities. While this cannot explain the effects of forward potential scattering for out of plane scatterers as discussed here, since the renormalization in their model is independent of the anisotropy of the scattering, it may be part of the solution to the Zn problem. Another perspective on the same physics to explain this effect may involve the low-energy spin fluctuations known to be induced by disorder in strongly correlated systems.[16] No theoretical work which considers both T_c and residual ρ is available at this writing; it is difficult to predict *a priori* which quantity will be more strongly influenced by correlations.

We summarize the situation comparing the current theory to experiments in Fig. 5. Since we are primarily interested in order-of-magnitude physics, we discuss only the initial

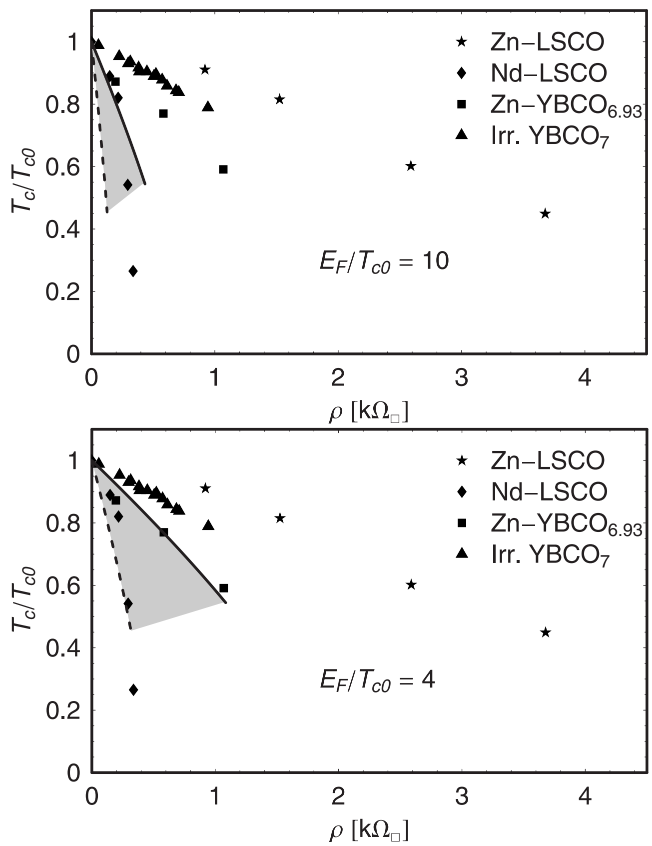

FIG. 5. Comparison of experimental data on T_c suppression in optimally doped cuprates. Diamonds, Nd-LSCO (Ref. 13); squares, Zn-YBCO (Ref. 17); triangles, electron-irradiated YBCO (Ref. 10); stars, Zn-LSCO (Ref. 13). Solid lines, toy model result for isotropic scatterers; dashed lines, for purely forward scatterers. Top (bottom) panel, $E_F/T_{c0}=10(4)$.

slopes of T_c suppression measured in various experiments and compare to the toy model result [Eq. (12)]. To obtain a fit, one must make an assumption about the parameter E_F/T_{c0} which enters this expression. A reasonable choice for the high-T_c materials is $E_F/T_{c0}=10$, and this choice puts most of the data on Zn-YBCO about a factor of 3 higher in slope, as found by previous authors. The range of T_c suppression initial slopes within the current approach is then shown in the figure in gray, ranging from the isotropic result to the extreme forward scattering result. For comparison, a set of curves is also given for the unphysical case $E_F/T_{c0}=4$. While these parameters should not be taken too seriously in a quantitative sense, it seems clear that (a) a hitherto unaccounted for physical mechanism is required to explain the data on Zn-substituted samples and (b) the effect of out of plane scatterers within the present model can only account for a factor of 4 or so increase in the magnitude of the slope. Thus, our work has explained qualitatively the paradoxical result that the out of plane scatterers generically reduce T_c more quickly than in plane relative to residual resistivity, and there is still a quantitative question remaining regarding the magnitude of the suppression in both cases.

The final experimental result we have attempted to discuss is the fascinating linear T_c vs ρ suppression measured on electron-irradiated single crystals of optimally doped YBCO by Rullier-Albenque *et al.*[10] The deviation from the AG pair breaking result was attributed by these authors to phase fluctuations according to a model put forward in Ref. 11. This is plausible for the underdoped sample where phase fluctuations are expected to be strong and for the highly irradiated optimally doped sample where the superfluid density is also small. We note, however, that within this scenario, it is puzzling and must be regarded as accidental that the initial T_c slope in the optimally doped sample, due to pair breaking, is the same as the final slope before superconductivity disappears. It is therefore equally likely, in our opinion, that the quasilinearity found by these authors is due to the effects of out of plane defects created by the electron irradiation, as discussed here, and/or the effects of order parameter suppression as discussed in Ref. 12.

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APPENDIX: FOURIER EXPANSION OF THE IMPURITY POTENTIAL

In previous works, the effect of forward scattering on the critical temperature has been studied by including higher angular momenta of the impurity potential when calculating the self-energies. In this appendix, we show the link between the results of our toy model and an approach, where the impurity potential is decomposed in its relevant Fourier components. Particularly, we want to point out the importance of the p-wave component of the impurity potential for calculating the normal conducting resistivity that has been neglected in previous works and that is the key in understanding the stronger suppression of the critical temperature as a function of resistivity in the case where forward scattering is the dominant scattering process.

Starting with the square of the impurity potential that is given in Eq. (1), we can expand it in a Fourier series of its two arguments ϕ and ϕ',

$$|V(\phi,\phi')|^2 = v_0^2 V_0 + v_0^2 \sum_{k=1}^{\infty} V_k[\cos(k\phi)\cos(k\phi') + \sin(k\phi)\sin(k\phi')], \tag{A1}$$

with $V_0=\phi_c/\pi$ and $V_k=2\sin(k\phi_c)/k\pi$. Using this expansion to calculate the self-energies near T_c, we notice that the integration projects out only the s- and the d-wave part of the impurity potential, leading to

$$\Sigma_0 = 2\pi\Gamma V_0 \tag{A2}$$

and

$$\Sigma_1(\phi,\omega_n) = \pi\Gamma V_2 \frac{\tilde{\Delta}_0 \cos(2\phi)}{\tilde{\omega}_n}. \tag{A3}$$

Solving for $\tilde{\Delta}_0/\tilde{\omega}_n$ and performing the Matsubara frequency summation in the gap equation, we find

$$\ln \frac{T_c}{T_{c0}} = \psi\left(\frac{1}{2}\right) - \psi\left(\frac{1}{2} + \frac{\Gamma(2V_0 - V_2)}{2T_c}\right). \quad \text{(A4)}$$

This result is in agreement with the results of Kulic and Oudovenko, and their parameters Γ_s and Γ_d can now be directly compared to the toy model parameter ϕ_c:

$$\Gamma_s = 4\Gamma\phi_c, \quad \Gamma_d = 2\Gamma \sin(2\phi_c). \quad \text{(A5)}$$

To calculate the normal conducting resistivity, one has to weight the quasiparticle scattering probability by its impact on the resistivity leading to Eq. (8). It is obvious that in the case of pure forward scattering, the increase of resistivity with impurity concentration is much slower than for isotropic scattering, a fact that has been taken into account by the factor of $1-\cos(\phi-\phi')$ that projects not only the s-wave part but also the p-wave part of the impurity potential out of the Fourier expansion. The resistivity can then be written as

$$\rho = \frac{4\pi\hbar}{e^2 E_F}\Gamma\left(V_0 - \frac{1}{2}V_1\right). \quad \text{(A6)}$$

Solving this equation for Γ and inserting it in the expression for the T_c suppression lead to Eq. (12), where $Q(\phi_c)$ can be expressed by the s-, p-, and d-wave parts of the impurity potential as $Q(\phi_c)=(2V_0-V_2)/(2V_0-V_1)$. The dependence of $Q(\phi_c)$ on the maximum scattering angle ϕ_c is shown in Fig. 1. It shows that the effect of impurity scattering on the resistivity is drastically reduced in the forward scattering limit due to the p-wave character of the scattering-in term while the effect on the T_c reduction is not as strong, since the only anisotropic component which plays a role in the latter case is the d wave.

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