Period-voltage-driven quantum phase transition of superconducting charge qubits

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In this paper, we employ an experimentally feasible scheme to realize a uniaxial spin model in superconducting charge qubits connected in parallel to a common superconducting inductance. Furthermore, we demonstrate that this nonlinear spin model with nonzero parallel field can give rise to a second-order phase transition under periodic modulation. It is shown that, when the amplitude of the periodically driven gate voltage in our proposal is varied, this second-order phase transition occurs from a normal to a deformed phase in the resonant case. Furthermore, in this system the adiabatic and cyclic conditions needed to generate the geometric phase can be easily implemented, and therefore this predicted phase transition could be detected in experiments by measuring the ground-state geometric phase.

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Since experiments have demonstrated that a superconducting charge qubit (SCCQ) can display good quantum coherence properties for a long time,[1–3] it has been regarded as a promising solid-state system to process quantum information and implement quantum computing. It has been shown that, when the charging energy is much larger than the Josephson energy, the SCCQ can play the essential role of an artificial two-level atom near the degeneracy point.[4–7] However, it should be noticed that this artificial atom can be controlled by both current, voltage, and external magnetic flux, whereas the natural atom is driven only by using microwave photons that excite electrons from one state to another. Recently, time-dependent electromagnetic fields have been used to control the coupling between superconducting flux qubits in terms of two different theoretical approaches, which offer a remarkable way to implement any logic gate as well as to measure flux qubit states tomographically.[8–12] For SCCQs, coupling between qubits can also be successfully achieved using variable-frequency magnetic fields.[13] Here we use a periodically driven gate voltage to manipulate a quantum phase transition (QPT) for many SCCQs. Moreover, we consider that all charge qubits work at the optimal point, and that the qubits are mostly immune to charge noise produced by uncontrollable charge fluctuations.[2]

When a many-body quantum system is driven by a controllable parameter, the ground-state energy has a structural change at a critical value of this parameter. This phenomenon is called a QPT, and it has attracted considerable attention in the modern theoretical and experimental communities.[14] In condensed matter physics, the QPT has been a key concept in the study of electrical and magnetic properties. Recently, concepts of quantum information such as entanglement and the geometric phase have also been used to characterize quantum critical phenomena theoretically.[15–26] Here we mainly focus on the uniaxial spin model in an arbitrary field, whose Hamiltonian can be written as $H=-(1/N)S_x^2+h_xS_x+h_zS_z$ with $h_z \geqslant 0$. It has been demonstrated that this model can exhibit a first-order phase transition at $h_x=0$ when the parallel field h_x is varied, and, moreover, for $h_x \neq 0$ no phase transition can be found when the perpendicular field h_z is varied.[26–28] However, it should be noticed that this model as well as the Lipkin-Meshkov-Glick model[29–31] have long been of primarily theoretical interest. Experimental observation of the phase transition has not been achieved successfully; this remains an interesting and open problem in solid-state physics due to the highly developed fabrication techniques required.

In this paper, we demonstrate that the above-mentioned uniaxial spin model with $h_x \neq 0$ can exhibit a second-order QPT under periodic modulation of the perpendicular field h_z. This technique may be regarded as an additional way to control QPTs. Moreover, we employ an experimentally feasible scheme to realize this Hamiltonian in SCCQs connected in parallel to a common superconducting inductance. In such a system, this second-order phase transition occurs from the normal to the deformed phase as a function of the amplitude of the periodically driven gate voltage in the resonant case. Finally, it is also given that for a lower frequency of a driven gate voltage the ground-state geometric phase, which is, remarkably, of the first order, can be generated naturally, and is a good witness to demonstrate this QPT.

Figure 1 shows our proposed solid-state device where many identical SCCQs are connected in parallel to a common superconducting inductance. If we choose a material where the superconducting energy gap is larger than the

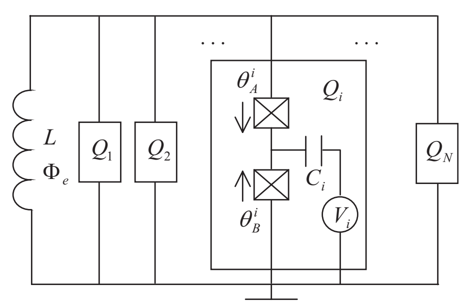

FIG. 1. Schematic diagram of implementation of the uniaxial spin model of superconducting charge qubits connected in parallel to a common superconducting inductance L. The ith qubit is connected by two Josephson junctions with phase drops θ_A^i and θ_B^i, and is biased by a voltage V^i through the gate capacitor C^i.

single-electron charging energy, which can suppress the quasiparticle tunneling at low temperatures, the Hamiltonian for the ith SCCQ is given by[4–7]

$$H_i = 4E_C^i(n^i - n_g^i)^2 - 2E_J^i \cos(\pi\Phi_e/\Phi_0)\cos\theta^i \quad (1)$$

with $E_C^i = e^2/2C^i$ being the charging energy and $n_g^i = C^iV^i/2e$ being the total dimensionless gate charge, where C^i is the gate capacitance, V^i is the controllable gate voltage, n^i is the number of excess Cooper pairs on the island while its conjugate variable is the phase difference θ^i of two Josephson junctions, E_J^i is the Josephson energy, Φ_e is the external magnetic flux through the common superconducting inductance L, and $\Phi_0 = \pi\hbar/e$ is the flux quantum. When the charging energy is much larger than the Josephson energy, the relevant physics for the SCCQ near the degeneracy point can be captured by considering only two charge eigenstates such as $|0\rangle$ and $|1\rangle$,[4–7] which correspond to zero and one extra Cooper pair in the superconducting island of each qubit. In the representation of these eigenstates, we can obtain a two-state Hamiltonian

$$H_i = -2E_C^i(1-2n_g^i)\sigma_z^i - E_J^i\cos(\pi\Phi_e/\Phi_0)\sigma_x^i, \quad (2)$$

where the Pauli matrixes σ_l^i $(l=z,x)$ are given by $\sigma_z^i = |0\rangle^{i\,i}\langle 0| - |1\rangle^{i\,i}\langle 1|$, $\sigma_x^i = |0\rangle^{i\,i}\langle 1| + |1\rangle^{i\,i}\langle 0|$.

Because the common superconducting inductance L for coupling the SCCQs has a large value ($L \sim 10$ nH),[32–34] if the circuit is not too large, the inductance of the circuit except for L can be neglected. Therefore, the whole Hamiltonian for Fig. 1 can be written as

$$H = \sum_i H_i + \frac{1}{2}LI^2 \quad (3)$$

with $I = \Sigma_i I_i$ being the total persistent current through the common superconducting inductance L, which can be reduced to the Hamiltonian[32–34]

$$H = \sum_{i=1}^{N}(\varepsilon\sigma_z^i + \bar{E}_J\sigma_x^i) - \sum_{j>i=1}^{N}\lambda\sigma_x^i\sigma_x^j, \quad (4)$$

where the bias energy is $\varepsilon = E_c(CV/e-1)/2$, the tunneling energy is $\bar{E}_J = -E_J\cos(\pi\Phi_e/\Phi_0)$, and the long-range interaction constant among qubits is

$$\lambda = L(\pi^2E_J^2/\Phi_0^2)\sin^2(\pi\Phi_e/\Phi_0). \quad (5)$$

It should be noticed that in another proposal the interbit coupling term $\sigma_y^i\sigma_y^j$ has also been achieved.[5] However, in that device two conditions should be met: (i) the eigenfrequency ω_{LC} of the LC circuit is much greater than the quantum manipulation frequencies, that is, the allowed number N of qubits is limited since ω_{LC} scales with $1/\sqrt{N}$; (ii) the phase conjugate to the total charge on the qubit capacitors fluctuates weakly. In our scheme, a common inductance is used to couple all SCCQs and both dc and ac supercurrents can flow through the inductance. Therefore, these two limitations do not apply to our approach; namely; the number N of qubits can be any required value.

By using the collective spin operators $S_z = \Sigma_{i=1}^N\sigma_z^i$ and $S_x = \Sigma_{i=1}^N\sigma_x^i$, the Hamiltonian (4) can be rewritten as

$$H = -(\lambda/2)S_x^2 + \bar{E}_JS_x + \varepsilon S_z, \quad (6)$$

which is identical to the uniaxial model in arbitrary field ($h_x = \bar{E}_J$ and $h_z = \varepsilon$). However, it should be noticed that here the effective parallel and perpendicular fields h_x and h_z can be controlled by the external magnetic flux Φ_e and gate voltage V, independently. It has been demonstrated that when $\bar{E}_J$ is varied a first-order phase transition occurs at $2\bar{E}_J/N\lambda = 0$ for $1 > 2\varepsilon/N\lambda \geq 0$, whereas no transition happens for $2\varepsilon/N\lambda \geq 1$. Also, for $2\bar{E}_J/N\lambda \neq 0$ no QPT has been found when ε is varied.[26–28] However, in the present paper we will demonstrate that a second-order phase transition can occur when $2\bar{E}_J/N\lambda \neq 0$ in terms of periodic modulation. Furthermore, this predicted phase transition can be achieved using current experimental techniques in solid-state systems. To show our proposal, the gate voltage is chosen as

$$V = \bar{V} + V_0\cos(\omega t), \quad (7)$$

where V_0 and ω are the amplitude and the frequency of this periodically driven gate voltage, respectively, and $\bar{V}$ is the static gate voltage. If the SCCQs work at their optimal point ($C\bar{V} = e$), where the qubits can be mostly immune from charge noise produced by uncontrollable charge fluctuations,[2] the Hamiltonian (6) can be reduced to a time-dependent collective Hamiltonian

$$H(t) = -\frac{\bar{\lambda}}{N}S_x^2 + \bar{E}_JS_x + \bar{\varepsilon}\cos(\omega t)S_z, \quad (8)$$

where $\bar{\varepsilon} = E_cCV_0/2e$ and $\bar{\lambda} = N\lambda/2$. If a rotation of coordinates such that $x \to z$ and $z \to x$ is implemented, the Hamiltonian (8) can be rewritten as $H_r(t) = -(\bar{\lambda}/N)S_z^2 + \bar{E}_JS_z + \bar{\varepsilon}\cos(\omega t)S_x$, which can be transformed to a time-independent Hamiltonian by using the unitary operator $R(t) = \exp[-i\omega tS_z]$. In terms of this unitary transformation $|\Psi_n(t)\rangle = R^+(t)|\psi_n(t)\rangle$ and the well-known rotating-wave approximation, the time-dependent Schrödinger equation $id|\psi_n(t)\rangle/dt = H_r(t)|\psi_n(t)\rangle$ can be reduced to the equation $id|\Psi_n(t)\rangle/dt = H_R|\Psi_n(t)\rangle$ with

$$\begin{aligned} H_R &= R^+(t)H(t)R(t) + iR(t)dR^+(t)/dt \\ &= -(\bar{\lambda}/N)S_z^2 + \Delta S_z + (\bar{\varepsilon}/2)S_x \end{aligned} \quad (9)$$

with $\Delta = (\bar{E}_J - \omega)$ being the detuning parameter. By using the rotation of coordinates again, the required time-independent Hamiltonian is given by

$$H_R = -\frac{\bar{\lambda}}{4N}(S_+^2 + S_-^2 + S_+S_- + S_-S_+) + \frac{\Delta}{2}(S_+ + S_-) + \frac{\bar{\varepsilon}}{2}S_z, \quad (10)$$

where $S_\pm = S_x \pm iS_y$. The expression (10) enables us to conveniently employ the Holstein-Primakoff transformation of angular momentum operators defined as $S_+ = b^+\sqrt{N - b^\dagger b}$, $S_- = \sqrt{N - b^\dagger b}\,b$, and $S_z = (b^\dagger b - N/2)$ with $[b, b^\dagger] = 1$,[35] which can approximately lead to the ground-state energy dominating the QPT.

It is known that in the standard way of expanding boson operators of the Holstein-Primakoff transformation we should suppose $b^\dagger b/N \ll 1$, which indicates that the quantum system is located in the ground state in the thermodynamic limit or fully polarized in the z direction. In order to describe the collective behavior of the Hamiltonian (10) induced by the long-range corrections, we can either perform a rotation to bring the z axis along the semiclassical spin direction,[36] or shift the boson operator by setting[37]

$$d^\dagger = b^\dagger - \sqrt{N}\beta. \tag{11}$$

Here we choose the latter procedure in which the condition $b^\dagger b/N \ll 1$ can be satisfied automatically. By means of the new boson operator $d^\dagger$, the Hamiltonian (10) can be expanded as a power series in $1/\sqrt{N}$ by[38]

$$H_R = NH_0 + N^{1/2}H_1 + N^0 H_2 + \cdots \tag{12}$$

with

$$H_0 = \frac{1}{2(1+\eta^2)^2}\left(-2\bar{\lambda}\eta^2 - \frac{\bar{\varepsilon}}{2}(1-\eta^4) + 2\Delta\eta(1+\eta^2)\right), \tag{13}$$

$$H_1 = \frac{(d^\dagger + d)}{2(1+\eta^2)^{3/2}}[-2\bar{\lambda}\eta(1-\eta^2) + \Delta(1-\eta^4) + \bar{\varepsilon}\eta(1+\eta^2)], \tag{14}$$

$$\begin{aligned} H_2 = \frac{1}{2(1+\eta^2)}\Bigg\{ & [\bar{\varepsilon}(1+\eta^2) - \bar{\lambda}(1-6\eta^2) \\ & + \Delta\eta(1+\eta^2)(4+\eta^2)]\Delta d^\dagger d \\ & + \left[-\frac{\bar{\lambda}(1-4\eta^2)}{2} + \Delta\eta\left(1+\frac{\eta^2}{2}\right)(1+\eta^2)\right] \\ & \times [(d^\dagger)^2 + d^2] - \bar{\lambda}(1-2\eta^2)/2 + \Delta\eta(1+\eta^2)\Bigg\}, \end{aligned} \tag{15}$$

where $\beta = \eta/\sqrt{1+\eta^2}$.

The first term (NH_0) of the expansion Hamiltonian (12) is the Hartree-Bogoliubov ground-state energy.[39] By means of minimizing this ground-state energy we can cancel the second term ($N^{1/2}H_1$), which indicates that the free energy should be fixed at the minimum value in any quantum system. A simple study of the minimized ground-state energy shows that the parameter η is an odd function of Δ for all $\Delta \neq 0$. However, for $\Delta = 0$ ($\bar{E}_J = \omega$), namely, the resonant case, a second-order phase transition can occur. According to the resonant condition $\cos(\pi\Phi_e/\Phi_0) = -\omega/E_J$ resulting from $H_1 = 0$, the parameter η can be immediately obtained by $\eta = \sqrt{(2\bar{\lambda} - \bar{\varepsilon})/(2\bar{\lambda} + \bar{\varepsilon})}$ for $V_0 \leqslant (V_0)_c$ and $\eta = 0$ for $V_0 \geqslant (V_0)_c$, where the critical gate voltage is given by

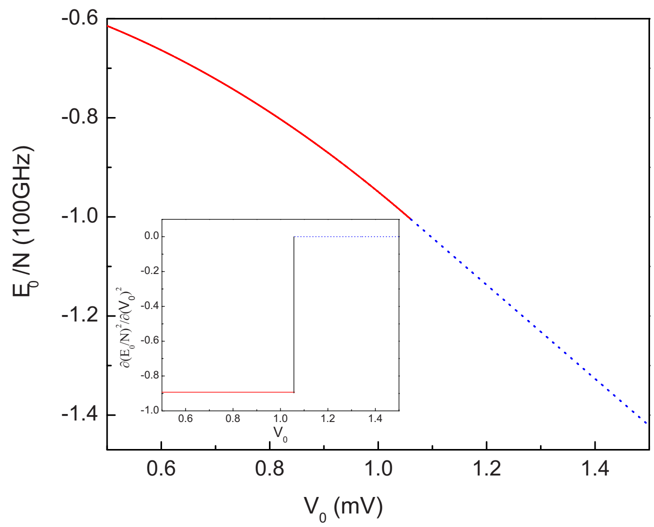

FIG. 2. (Color online) Scaled ground-state energy E_0/N versus gate voltage V_0. The relative parameters are chosen as $E_J = 9.1$ GHz, $E_C = 152$ GHz, $L = 10$ nH, and $N = 2000$, respectively. In order to obtain the adiabatic condition the frequency ω satisfies $\cos(\pi\Phi_e/\Phi_0) = -\omega/E_J \simeq 0$. The corresponding critical gate voltage is calculated as $(V_0)_c = 1.06$ mV.

$$(V_0)_c = \frac{2\pi^2 eNL(E_J^2 - \omega^2)}{E_c C\Phi_0^2}. \tag{16}$$

It is very necessary to check whether this critical gate voltage can be reached with current experimental techniques. As in Ref. 40, the Josephson and charging energies are chosen as $E_J = 9.1$ GHz and $E_C = 152$ GHz ($C = 798$ aF), respectively. For large inductance $L = 10$ nH and lower frequency ω [$\cos(\pi\Phi_e/\Phi_0) = -\omega/E_J \simeq 0$], the long-range interaction constant can be evaluated as $\lambda \simeq 0.2$ GHz. If the number of qubits is considered to be $N = 2000$, the critical gate voltage is given by $(V_0)_c = 1.06$ mV, which can be easily achieved experimentally. It should be pointed out that a rigorous QPT can only be realized in the thermodynamic limit $N \to \infty$, but some of the basic features of the phase transition can still be demonstrated with a finite particle number, which is of primary interest and a hot topic in modern physics.[29–31]

With the help of the parameter η, the scaled Hartree-Bogoliubov ground-state energy can be derived from $E_0/N = -\bar{\lambda}/4 - \bar{\varepsilon}^2/16\bar{\lambda}$ for $V_0 \leqslant (V_0)_c$ and $E_0/N = -\bar{\varepsilon}/4$ for $V_0 \geqslant (V_0)_c$. This ground-state energy and its second-order derivative with respect to V_0 as a function of V_0 are plotted in Fig. 2, which clearly illustrates a *second-order* phase transition in contrast to the well-known first-order phase transition.[26] For the case of $V_0 \leqslant (V_0)_c$, the energy of the first term in the resonant Hamiltonian $H_R = -(\bar{\lambda}/N)S_x^2 + (\bar{\varepsilon}/2)S_z$ dominates, which means that both quantum tunneling and macroscopic collective excitation occur; however, for the case of $V_0 \geqslant (V_0)_c$ the energy of the second term is dominant, which implies that the quantum tunneling is suppressed and therefore no collective excitation happens. Therefore, similar to the behavior in the Lipkin-Meshkov-Glick model,[29–31] we can call the phase when $V_0 \leqslant (V_0)_c$ the deformed phase and the phase when $V_0 \geqslant (V_0)_c$ the normal phase. On the other hand, if the third term $(N^0 H_2)$ of the expansion Hamiltonian (12) in the resonance condition $\Delta=0$ is diagonalized by means of the Bogoliubov transformation, the excited-state energy in the normal phase can be given by $E_\varepsilon^2=\{[\bar{\varepsilon}(1+\eta_0^2)-\bar{\lambda}(1-6\eta_0^2)]^2-\bar{\lambda}^2(1-4\eta_0^2)^2\}/4(1+\eta_0^2)^2$. When the amplitude of the periodically driven gate voltage approaches the critical value $(V_0)_c$, this excited-state energy vanishes as

$$E_\varepsilon[V_0 \to (V_0)_c] \sim |V_0-(V_0)_c|^{1/2}, \tag{17}$$

whose corresponding characteristic length scale is $l \sim |V_0-(V_0)_c|^{-\upsilon}$ with $\upsilon=1/2$. Therefore, the dynamical critical exponent can be derived from $E_\varepsilon \sim |V_0-(V_0)_c|^{z\upsilon}$ by $z=1$, which shows the universality principle of the QPT.[14]

In the rest of this paper, we discuss how to observe this QPT experimentally. It has been shown that in our proposal the frequency of a periodically driven gate voltage can be chosen as a very low value $[\cos(\pi\Phi_e/\Phi_0)=-\omega/E_J\simeq 0]$ so that the adiabatic condition can be well satisfied. Thus, the GP, which describes a phase factor of the wave functions depending only on the geometry of the path when a time-dependent quantum system undergoes an adiabatic and cyclic evolution,[41] can be generated naturally. For the time-dependent Hamiltonian $H_r(t)$, the ground-state GP can be calculated as follows:

$$\gamma_0 = i\int_0^{2\pi}\langle\psi_0|R^\dagger(t)\frac{d}{d\varphi}R(t)|\psi_0\rangle d\varphi = 2\pi\langle\Psi_0|J_z|\Psi_0\rangle, \tag{18}$$

where $|\Psi_0\rangle=|0\rangle$ is the vacuum state of the boson operator b. However, for the time-dependent Hamiltonian $H(t)$ the ground-state GP becomes

$$\Gamma_0 = 2\pi\langle\Psi_0|J_x|\Psi_0\rangle = \begin{cases} \dfrac{\pi N\sqrt{4\bar{\lambda}^2-\bar{\varepsilon}^2}}{2\bar{\lambda}}, & V_0 \leqslant (V_0)_c, \\ 0, & V_0 \geqslant (V_0)_c. \end{cases} \tag{19}$$

Figure 3 shows the scaled ground-state GP and its first-order derivative with respect to V_0 as a function of V_0, which indicates that this predicted QPT characterized by the nonanalyticity of the ground-state GP is, remarkably, of the first order. Thus, we argue that this QPT can be observed directly by measuring the abrupt change of the ground-state GP, which awaits experimental validation. The critical behavior for the scaled ground-state GP is given by

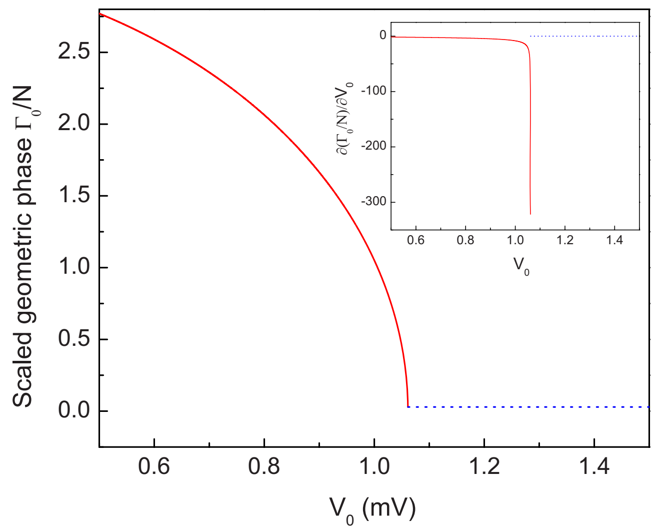

FIG. 3. (Color online) Ground-state geometric phase versus gate voltage V_0 with the same parameters as those in Fig. 2.

$$\Gamma_0/N[V_0 \to (V_0)_c] \sim |V_0-(V_0)_c|, \tag{20}$$

whose geometric critical exponent is $z=2$.

In conclusion, we have predicted a second-order QPT in the uniaxial spin model by means of periodic modulation, which may be regarded as an additional way to control the QPT. Moreover, we have employed an experimentally feasible scheme to realize this prediction in SCCQs connected in parallel to a common superconducting inductance. In such a system, this second-order phase transition occurs from the normal to the deformed phase as a function of the amplitude of the periodically driven gate voltage in the resonant case. Since all charge qubits work at the optimal point, the qubits can be mostly immune from charge noise produced by uncontrollable charge fluctuations. Furthermore, in our proposed time-dependent solid-state quantum system, the adiabatic and cyclic conditions needed to generate the GP can be easily implemented. In experiments, this predicted QPT can be detected by measuring the ground-state GP, which is remarkably of the first order.

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[1] Y. Yu, S. Han, X. Chu, S.-I. Chu, and Z. Wang, Science **296**, 889 (2002).
[2] D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, Science **296**, 886 (2002).
[3] I. Chiorescu, Y. Nakamura, C. J. P. M. Harmans, and J. E. Mooij, Science **299**, 1869 (2003).
[4] Y. Nakamura, Y. A. Pashkin, and J. S. Tsai, Nature (London) **398**, 786 (1999).
[5] Y. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. **73**, 357 (2001).
[6] J. Q. You and F. Nori, Phys. Today **58**(11), 42 (2005).
[7] A. M. Zagoskin, S. Ashhab, J. R. Johansson, and F. Nori, Phys.

Rev. Lett. **97**, 077001 (2006).
[8] C. Rigetti, A. Blais, and M. Devoret, Phys. Rev. Lett. **94**, 240502 (2005).
[9] Y. X. Liu, L. F. Wei, J. S. Tsai, and F. Nori, Phys. Rev. Lett. **96**, 067003 (2006).
[10] Y. X. Liu, C. P. Sun, and F. Nori, Phys. Rev. A **74**, 052321 (2006).
[11] P. Bertet, C. J. P. M. Harmans, and J. E. Mooij, Phys. Rev. B **73**, 064512 (2006).
[12] A. O. Niskanen, Y. Nakamura, and J. S. Tsai, Phys. Rev. B **73**, 094506 (2006).
[13] X. He, Y. Liu, J. You, and F. Nori, arXiv:quant-ph/0703147, Phys. Rev. A (to be published).
[14] S. Sachdev, *Quantum Phase Transitions* (Cambridge University Press, Cambridge, U.K., 1999).
[15] A. Osterloh, L. Amico, G. Falci, and R. Fazio, Nature (London) **416**, 608 (2002).
[16] T. J. Osborne and M. A. Nielsen, Phys. Rev. A **66**, 032110 (2002).
[17] G. Vidal, J. I. Latorre, E. Rico, and A. Kitaev, Phys. Rev. Lett. **90**, 227902 (2003).
[18] J. Vidal, G. Palacios, and R. Mosseri, Phys. Rev. A **69**, 022107 (2004).
[19] J. Vidal, R. Mosseri, and J. Dukelsky, Phys. Rev. A **69**, 054101 (2004).
[20] T. Barthel, S. Dusuel, and J. Vidal, Phys. Rev. Lett. **97**, 220402 (2006).
[21] A. C. M. Carollo and J. K. Pachos, Phys. Rev. Lett. **95**, 157203 (2005).
[22] S. L. Zhu, Phys. Rev. Lett. **96**, 077206 (2006).
[23] F. Plastina, G. Liberti, and A. Carollo, Europhys. Lett. **76**, 182 (2006).
[24] G. Chen, J. Li, and J. Q. Liang, Phys. Rev. A **74**, 054101 (2006).
[25] G. Chen, Z. D. Chen, J. Q. Li, and J.-Q. Liang, Phys. Rev. B **75**, 212508 (2007).
[26] J. Vidal, Phys. Rev. A **73**, 062318 (2006).
[27] J. Vidal, J. M. Arias, J. Dukelsky, and J. E. García-Ramos, Phys. Rev. C **73**, 054305 (2006).
[28] J. M. Arias, J. Dukelsky, J. E. García-Ramos, and J. Vidal, Phys. Rev. C **75**, 014301 (2007).
[29] S. Dusuel and J. Vidal, Phys. Rev. Lett. **93**, 237204 (2004).
[30] F. Leyvraz and W. D. Heiss, Phys. Rev. Lett. **95**, 050402 (2005).
[31] G. Chen and J.-Q. Liang, New J. Phys. **8**, 297 (2006).
[32] J. Q. You, J. S. Tsai, and F. Nori, Phys. Rev. Lett. **89**, 197902 (2002).
[33] J. Q. You, J. S. Tsai, and F. Nori, Phys. Rev. B **68**, 024510 (2003).
[34] J. Q. You, X. B. Wang, T. Tanamoto, and F. Nori, Phys. Rev. A **75**, 052319 (2007).
[35] T. Holstein and H. Primakoff, Phys. Rev. **58**, 1098 (1949).
[36] S. Dusuel and J. Vidal, Phys. Rev. B **71**, 224420 (2005).
[37] C. Emary and T. Brandes, Phys. Rev. E **67**, 066203 (2003).
[38] A. Klein and E. R. Marshalek, Rev. Mod. Phys. **63**, 375 (1991).
[39] A. Dzhioev, Z. Aouissat, A. Storozhenko, A. Vdovin, and J. Wambach, Phys. Rev. C **69**, 014318 (2004).
[40] Yu A. Pashkin, T. Yamamoto, O. Astafiev, Y. Nakamura, D. V. Averin, and J. S. Tsai, Nature (London) **421**, 823 (2003).
[41] M. V. Berry, Proc. R. Soc. London, Ser. A **392**, 45 (1984).