Quantitative model for the I_CR product in *d*-wave Josephson junctions

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We study theoretically the Josephson effect in *d*-wave superconductor/diffusive normal metal/insulator/diffusive normal metal/*d*-wave superconductor (D/DN/I/DN/D) junctions. This model aims to describe practical junctions in high-T_C cuprate superconductors, in which the product of the critical Josephson current (I_C) and the normal state resistance (R) (the so-called I_CR product) is very small compared to the prediction of the standard theory for clean *d*-wave superconductor/insulator/*d*-wave superconductor (DID) junctions. We show that the I_CR product in D/DN/I/DN/D junctions can be much smaller than that in DID junctions. The proposed theory describes the behavior of I_CR products quantitatively in high-T_C cuprate junctions.

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Josephson effect in high-T_C superconductors has attracted much attention[1,2] because of its potential applications in future technologies.[3] In particular, applications in electronics, such as the single-flux quantum devices, are extremely promising, since the operating frequency is proportional to the product of the critical Josephson current (I_C) and the normal state resistance (R) (the so-called I_CR product) which is approximately proportional to the superconductivity critical temperature.[4] However, almost all experimental data for high-T_C Josephson junctions fabricated so far have shown I_CR values much smaller than those predicted by the standard theory, irrespective of what kind of junctions they are.[5,6] This strongly suggests that the interfaces in the cuprates are intrinsically pair breaking. Several models were proposed which treat this issue.[7–10] However, the situation has not been described so far in a quantitative manner.

One possibility addressed in the present Brief Report and not investigated before is that superconductivity is destroyed near the interface in *d*-wave superconductor/insulator/*d*-wave superconductor (DID) junctions, where diffusive normal metal (DN) regions are induced. Thus, DID junctions turn into *d*-wave superconductor/diffusive normal metal/insulator/diffusive normal metal/*d*-wave superconductor (D/DN/I/DN/D) junctions (see Fig. 1). In these junctions, I_CR product can be much smaller than that in DID junctions. In this Brief Report, we will explore this possibility and provide a quantitative model which is compared to the experimental data.

The Josephson effect is a phase-sensitive phenomenon and thus depends strongly on a superconducting pairing symmetry.[1,2,11] In DID junctions, nonmonotonic temperature dependence of critical current[12–16] occurs due to the formation of midgap Andreev resonant states (MARSs) at the interface.[17] The MARSs stem from sign change of pair potentials of *d*-wave superconductors.[18] It was also predicted that MARSs strongly enhance the Josephson current at low temperatures.[13] On the other hand, in Josephson junctions with DN, the role of the MARSs changes.

In superconductor/diffusive normal metal/superconductor (S/DN/S) junctions, Cooper pairs penetrate into the DN as a result of the proximity effect, providing the Josephson coupling.[19–23] Scattering of electrons by impurities in the DN layer makes superconducting coherence length shorter and thus suppresses the Josephson current. In D/DN/D junctions, the Josephson current is suppressed by the MARSs,[25–28] in contrast to DID junctions, because MARSs compete with proximity effect.[29,30] Therefore, I_CR product in D/DN/I/DN/D junctions can be much smaller than that in DID junctions.

In the present Brief Report, we calculate Josephson current in D/DN/I/DN/D junctions as a model of the actual DID (e.g., grain boundary) junctions. We show that I_CR product in D/DN/I/DN/D junctions can be much smaller than that in DID junctions and clarify the conditions with which the I_CR product is most enhanced in D/DN/I/DN/D junctions. Our theory can explain the above mentioned general trend of the high-T_C Josephson junctions quantitatively, in contrast to previous theoretical models of high-T_C cuprate junctions. The obtained results may provide useful information for fabrication of high-T_C Josephson junctions.

Let us formulate the model for a D/DN/I/DN/D junction.

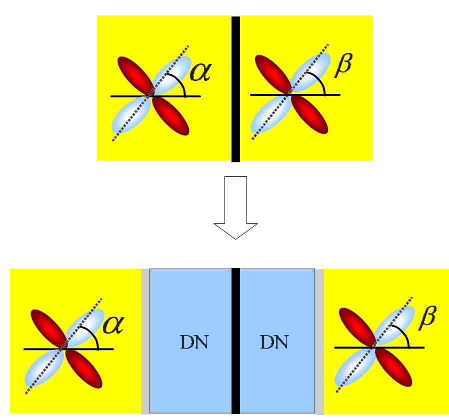

FIG. 1. (Color online) Schematic illustration of the model for the D/DN/I/DN/D junction.

We assume that the DN layer has a length L much larger than the mean free path and is characterized by the resistance R_d. The DN/D interfaces located at $x=\pm L$ have the resistance R'_b, while the DN/I interface at $x=0$ has the resistance R_b. We model infinitely narrow insulating barriers by the delta function $U(x)=H'\delta(x+L)+H\delta(x)+H'\delta(x-L)$. The resulting transparencies of the interfaces T_m and T'_m are given by $T_m=4\cos^2\phi/(4\cos^2\phi+Z^2)$ and $T'_m=4\cos^2\phi/(4\cos^2\phi+Z'^2)$, where $Z=2H/v_F$ and $Z'=2H'/v_F$ are dimensionless constants, v_F is Fermi velocity, and ϕ is the injection angle measured from the interface normal. In the following, we assume $Z\gg 1$. The schematic illustration of the model is shown in Fig. 1. The pair potential along the quasiparticle trajectory with the injection angle ϕ is given by $\Delta_L=\Delta\cos[2(\phi-\alpha)]\exp(-i\Psi)$ and $\Delta_R=\Delta\cos[2(\phi-\beta)]$ for the left and the right superconductors, respectively. Here, Ψ is the phase difference across the junction, and α and β denote the angles between the normal to the interface and the crystal axes of the left and right d-wave superconductors, respectively. The lobe direction of the pair potential and the direction of the crystal axis are chosen to be the same.

We parametrize the quasiclassical Green's functions G and F with a function Φ_ω:[19,20]

$$G_\omega=\frac{\omega}{\sqrt{\omega^2+\Phi_\omega\Phi^*_{-\omega}}},\quad F_\omega=\frac{\Phi_\omega}{\sqrt{\omega^2+\Phi_\omega\Phi^*_{-\omega}}},\tag{1}$$

where ω is the Matsubara frequency. In the DN layers, the Green's functions satisfy the Usadel equation[24]

$$\xi^2\frac{\pi T_C}{\omega G_\omega}\frac{\partial}{\partial x}\left(G^2_\omega\frac{\partial}{\partial x}\Phi_\omega\right)-\Phi_\omega=0,\tag{2}$$

where $\xi=\sqrt{D/2\pi T_C}$ is the coherence length, D is the diffusion constant, and T_C is the transition temperature of superconducting electrodes. To solve the Usadel equation, we apply the generalized boundary conditions derived in Refs. 25 and 26 at $x=\pm L$ and the boundary conditions in Ref. 22 at $x=0$.

The Josephson current is given by

$$\frac{eIR}{\pi T_C}=i\frac{RTL}{2R_dT_C}\sum_\omega\frac{G^2_\omega}{\omega^2}\left(\Phi_\omega\frac{\partial}{\partial x}\Phi^*_{-\omega}-\Phi^*_{-\omega}\frac{\partial}{\partial x}\Phi_\omega\right),\tag{3}$$

where T is temperature and $R\equiv 2R_d+R_b+2R'_b$ is the normal state resistance of the junction. In the following, we focus on the I_CR value as a function of temperature and clarify the cases when I_CR is enhanced. Below, $\Delta(0)$ denotes the value of Δ at zero temperature. Note that it is realistic to choose small magnitude of Z' and R'_b and large Thouless energy E_{Th} ($\equiv D/L^2$) compared to $\Delta(0)$ because thin DN regions could be naturally formed due to the degradation of superconductivity near the interface.

In Fig. 2, we show I_CR value for $Z'=1$, $R_d/R_b=0.1$, and $(\alpha,\beta)=(0,0)$ with various $E_{Th}/\Delta(0)$ and R_d/R'_b. I_CR increases with $E_{Th}/\Delta(0)$ and R_d/R'_b because proximity effect is enhanced. As E_{Th} increases, the slope of the I_CR decreases.

Figure 3 shows I_CR value for $E_{Th}/\Delta(0)=1$, $R_d/R_b=0.1$, and $(\alpha,\beta)=(0,0)$ with various Z' and R_d/R'_b. As Z' increases, the slope of the I_CR increases. The peculiar effect is that I_CR increases with Z', indicating that proximity effect is enhanced by the increase of Z'. This stems from the sign change of the pair potential.[25,26] For the case of d-wave symmetry with $\alpha=\beta=0$, injection angles of a quasiparticle can be separated into two regions: $\phi_+=\{\phi|0\leqslant|\phi|<\pi/4\}$ and $\phi_-=\{\phi|\pi/4\leqslant|\phi|\leqslant\pi/2\}$. The signs of pair potential for ϕ_+ and that for ϕ_- are opposite. As a result, the sign change of pair potentials suppresses the proximity effect in the DN and hence Josephson currents. As Z' increases, the contribution from ϕ_+ dominates over that from ϕ_-. Therefore, I_CR increases with Z'.

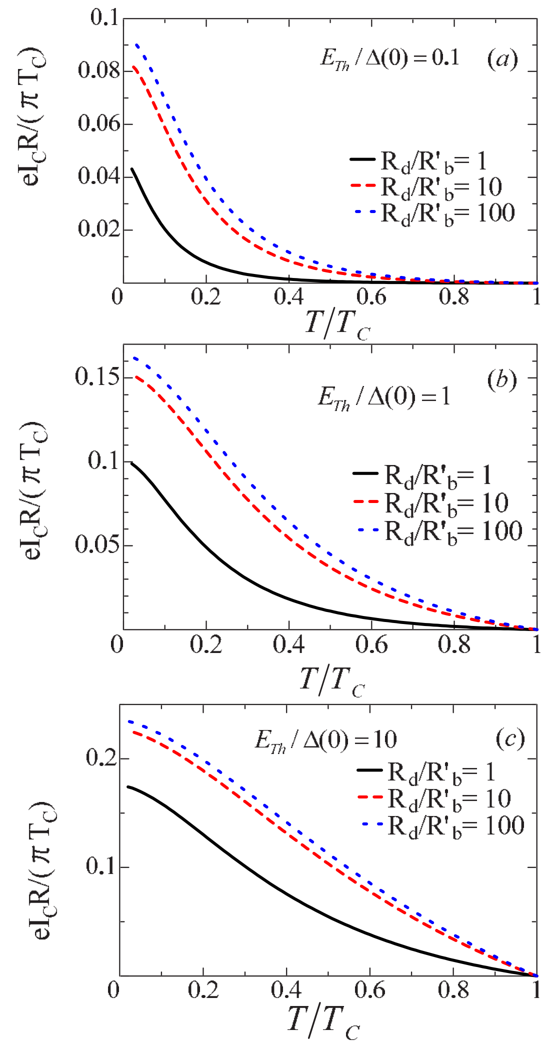

FIG. 2. (Color online) I_CR value for $Z'=1$, $R_d/R_b=0.1$, and $(\alpha,\beta)=(0,0)$.

In Fig. 4, we plot the I_CR value for $E_{Th}/\Delta(0)=1$, $Z'=1$, and $(\alpha,\beta)=(0,0)$ with various R_d/R_b and R_d/R'_b. I_CR increases with R_d/R_b due to the enhancement of the proximity effect.

Figure 5 displays I_CR value for $E_{Th}/\Delta(0)=0.1$, $Z'=0.1$, $R_d/R_b=0.1$, and $R_d/R'_b=10$ with various α and β. The formation of MARSs suppresses the proximity effect. Therefore, I_CR decreases with the increase of α and β.[25–28] In the actual junctions, there is inevitable roughness at the interface and hence the effective values of α and β at the interface

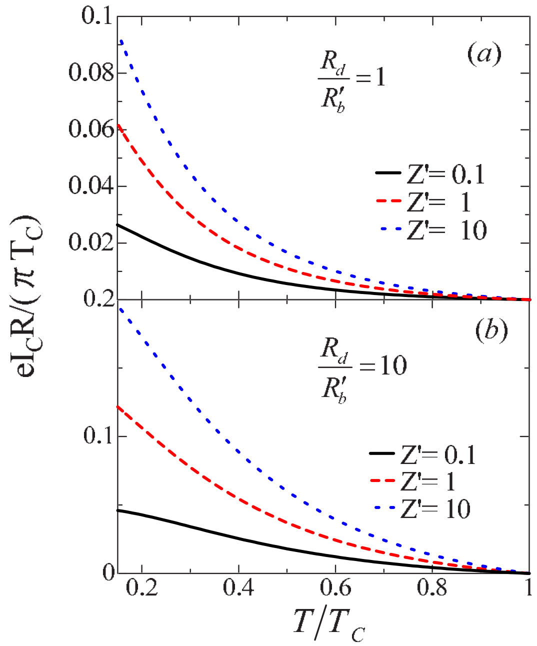

FIG. 3. (Color online) $I_C R$ value for $E_{Th}/\Delta(0)=1$, $R_d/R_b=0.1$, and $(\alpha,\beta)=(0,0)$.

become random even if junctions with $\alpha=\beta=0$ are fabricated. This provides the mechanism of suppression of the $I_C R$ product.

Finally, we compare the present theory with the experimental data of Ref. 6 and with the theory for clean DID junctions by Tanaka and Kashiwaya (TK)[13] using Eq. (46) in Ref. 13. The temperature dependences of I_C are plotted in Fig. 6 taking $\alpha=\beta=0$ and $R=0.375\ \Omega$ for theoretical plots.

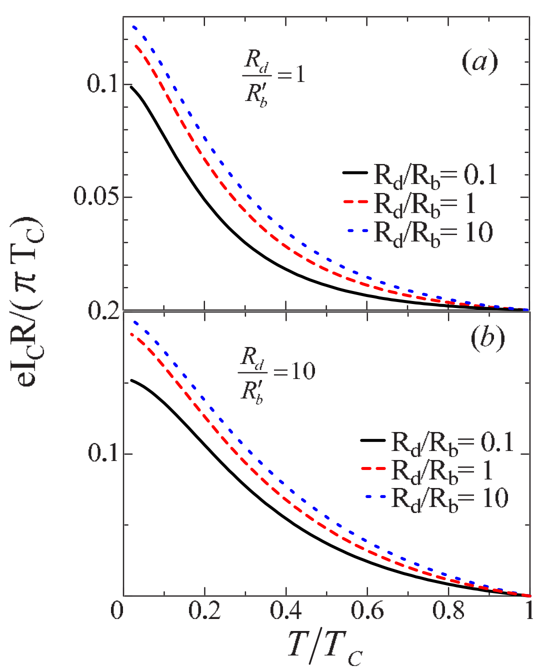

FIG. 4. (Color online) $I_C R$ value for $E_{Th}/\Delta(0)=1$, $Z'=1$, and $(\alpha,\beta)=(0,0)$.

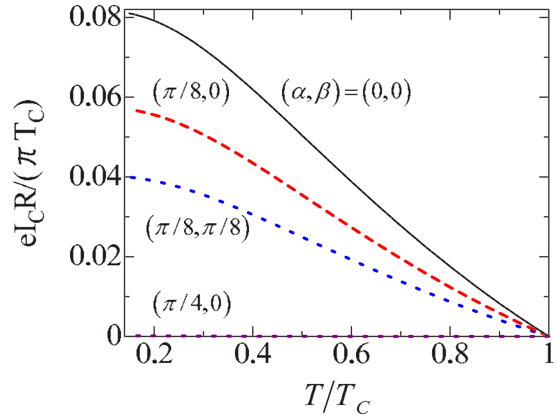

FIG. 5. (Color online) $I_C R$ value for $E_{Th}/\Delta(0)=0.1$, $Z'=0.1$, $R_d/R_b=0.1$, and $R_d/R'_b=10$.

We choose $E_{Th}/\Delta(0)=3$, $Z'=0.1$, $R_d/R_b=0.01$, and $R_d/R'_b=100$ in the present theory, and the barrier parameter $Z=10$ in the TK theory. As shown in this figure, the present theory can explain the experimental results quantitatively, while the discrepancy between the TK theory and the data is rather large, about an order of magnitude. Note that in the TK theory, the $I_C R$ is not sensitive to the choice of Z parameter. To estimate the realistic size of the DN region, we can take $\Delta(0)=10$ meV and $D=10^{-3}\ \mathrm{m}^2/\mathrm{s}$, and then obtain the length of the DN region $L=4.7$ nm. This length estimate is reasonable for the ramp-edge junctions described in Ref. 6.

In summary, we have studied the Josephson current in D/DN/I/DN/D junctions as a model of high-T_C superconductor junctions. We have shown that the $I_C R$ product in D/DN/I/DN/D junctions can be much smaller than that in DID junctions and have found the conditions when the $I_C R$ in D/DN/I/DN/D junctions is largest. The requirements for the large magnitude of $I_C R$ product are no roughness at the

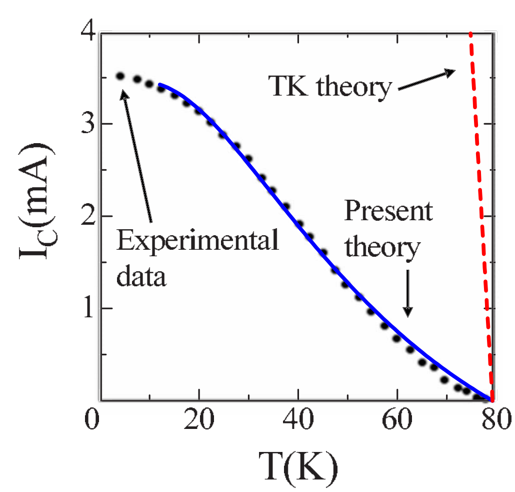

FIG. 6. (Color online) Comparison between the present theory (solid line), experimental data (Ref. 6) (dotted line), and the TK theory (Ref. 13) (broken line). I_C is plotted as a function of temperature, taking $R=0.375\ \Omega$ and $\alpha=\beta=0$ for theoretical plots. We choose $E_{Th}/\Delta(0)=3$, $Z'=0.1$, $R_d/R_b=0.01$, and $R_d/R'_b=100$ in the present theory, and $Z=10$ in the TK theory.

interfaces, large magnitudes of Z', R_d/R_b, R_d/R'_b, and E_{Th}, and $(\alpha,\beta)=(0,0)$. Note that these parameters are not easily controllable in real junctions: small magnitude of Z' and R'_b and large E_{Th} are realistic for naturally formed DN layers, and hence the only tunable parameter is R_b. Our theory can explain the experimental results on the quantitative level, in contrast to the previous treatment of nondiffusive DID junctions.

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