Observation of an extrinsic critical velocity using matter wave interferometry

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We report an experiment that uses a superfluid helium quantum interference device to probe the initial onset of the motion of a single vortex line driven by axial flow in a macroscopic channel. When the superfluid velocity reaches a temperature independent critical value ($v_c \sim 1$ mm/s) periodic 2π phase slippage occurs with a frequency of the order of a few Hz. As the axial flow velocity increases, the frequency increases, possibly stepwise.

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The state of superfluid helium is characterized by a complex order parameter whose phase gradient is proportional to the superfluid velocity:[1] $v_s = \hbar/m\nabla\phi$. Anderson[2] introduced the (now widely accepted) mechanism for energy dissipation in such a quantum fluid. He pointed out that if a quantized vortex line passes across a channel containing axial superflow, the vortex could grow in size at the expense of the channel's flow energy. A vortex completely crossing the channel leads to a 2π decrease in the order-parameter phase difference between the two ends of the channel, which corresponds to a decrement in the axial flow velocity. This description of the simplest dissipation process is referred to as a 2π phase slip.

In macroscopic channels (i.e., transverse dimensions greater than ~ 1 μm) the onset of dissipation (i.e., vortex crossing) usually occurs abruptly due to an instability and growth of preexisting pinned vorticity. For such extrinsic[3] critical velocities vortex line elements pinned at the channel walls are induced to grow in size, either traversing the flow or continuously spooling downstream.[4,5]

Although the growth of quantum turbulence[6] and the associated dissipation in large channels has been studied for over five decades, there is not yet a complete picture that describes the initial instability and the subsequent dynamics of the quantum vortices. One of the main difficulties in probing and understanding the nature of extrinsic processes in superfluid ^{4}He has been that the onset of dissipation is typically detected by a change in some macroscopic property such as the pressure head across the ends of the flow passage[7] or by the attenuation of second sound[8] or propagating ion beams.[9] These techniques rely on some minimum amount of quantized turbulence to raise the signal of interest above the instrumental noise. Therefore, these methods are not sufficiently sensitive to detect the very initial onset of dissipation, which would involve the motion of a single vortex filament. Recently, small solid hydrogen particles have been used successfully as tracers in superfluid ^{4}He allowing the direct observation of vortex cores.[10]

In conducting an experiment to directly measure the phase gradient associated with superfluid ^{4}He flow,[11] we have unexpectedly come across an interesting phenomenon related to the initial onset of a single vortex motion. We place a flow tube in one arm of a matter wave interferometer and directly monitor the order-parameter phase difference across the tube's ends. When a vortex initially pinned at the wall moves transversely across the tube the interferometer registers a phase change. This provides us with a new way to detect the motion of a single vortex line in a macroscopic passage.

Our basic apparatus is shown in Fig. 1 (and described in more detail in Ref. 11). The flow tube of inner length $l \approx 2.5$ cm and cross sectional area $\sigma \approx 3.78 \times 10^{-2}$ cm^2 contains a heater at one end and a strong thermal coupling (to a temperature regulated bath) at the opposite end. When power $\dot{Q}$ is applied to the heater, a countercurrent heat flow results along the tube. The induced superfluid velocity is given by

$$v_s = \frac{\rho_n}{\rho_s \rho s T \sigma}\dot{Q}, \tag{1}$$

where T is the temperature, s is the entropy per unit mass, ρ is the total fluid density, and ρ_s and ρ_n are superfluid and normal fluid densities, respectively. Since the superfluid velocity is proportional to the phase gradient of the superfluid order parameter, a finite v_s means there exists a phase difference across the ends of the tube

$$\Delta\phi = \frac{m_4}{\hbar}\frac{\rho_n}{\rho_s \rho s T}\frac{l}{\sigma}\dot{Q}, \tag{2}$$

where $\hbar$ is Planck's constant divided by 2π and m_4 is the atomic mass of helium. Our superfluid interferometer measures this phase difference. If a single vortex filament moves transversely across the tube, $\Delta\phi$ (between the ends of the

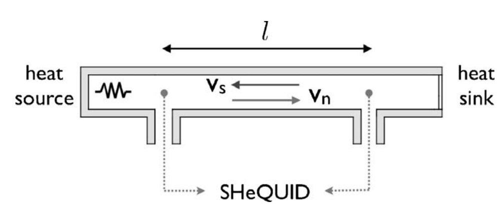

FIG. 1. Schematic of our experimental configuration. The inside of the flow tube is filled with superfluid ^{4}He and the entire apparatus is immersed in a bath of liquid helium. A resistive heater and a thin Cu sheet serve as a heat source and a sink. The tube is made of Stycast 1266 (insulating) to minimize the heat loss through the walls. The tube is connected to a superfluid helium quantum interference device (SHeQUID)[12] at two locations (separated by distance l) enabling us to monitor the phase difference $\Delta\phi$ that exists between them. Here, v_s and v_n indicate superfluid and normal fluid velocities, respectively.

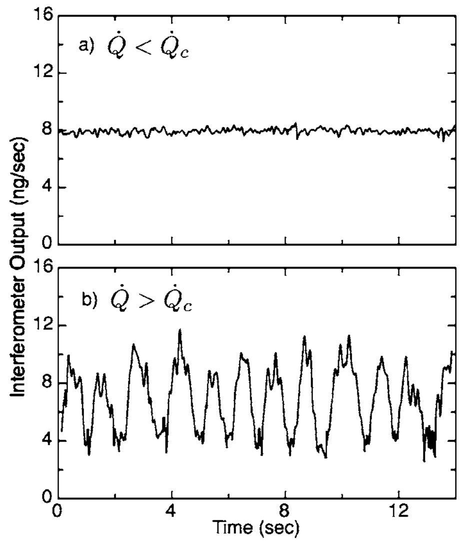

FIG. 2. Interferometer output [$\propto\cos(\Delta\phi/2)$] as a function of time when (a) $\dot{Q}<\dot{Q}_c$ and (b) $\dot{Q}>\dot{Q}_c$. These data are taken at $T_\lambda - T \approx 16$ mK. At this temperature, $\dot{Q}_c \approx 130$ μW, and the phase difference $\Delta\phi$ at $\dot{Q}_c$ is on the order of $250\times 2\pi$.

tube) will change by 2π. Since our interferometer can detect phase changes as small as $(3\times 10^{-2})2\pi$ in one second, the instrument is sensitive to even a very small transverse motion of a single vortex.

In operation, the interferometer output signal is proportional to $\cos(\Delta\phi/2)$. As we raise the power $\dot{Q}$, the phase difference $\Delta\phi$ increases proportionally as expected from Eq. (2), and the interferometer signal varies cosinusoidally[11] with $\Delta\phi$. For a fixed $\dot{Q}$, $\Delta\phi$ is constant in time and therefore the signal from our interferometer is constant in time [see Fig. 2(a)]. However, at some critical power $\dot{Q}_c$ [or corresponding critical velocity v_c obtained from Eq. (1)], the interferometer signal suddenly begins to oscillate in time [see Fig. 2(b)]. These time oscillations have the same peak-to-peak amplitude as that of the data shown in Fig. 2 of Ref. 11 (obtained at the same temperature), which indicates a periodic 2π variation (in time) of $\Delta\phi$. This is the unmistakable signature of well-behaved 2π phase slippage.

To gain insight into the vortex dynamics responsible for the phase slippage, we have measured the frequency of the interferometer output oscillation as a function of heater power at various temperatures. Some data are plotted in Fig. 3 against the axial superfluid velocity[13] calculated from $\dot{Q}$ using Eq. (1). It is apparent that in addition to the sudden onset of the phase variation at v_c, there are transitions to different oscillation frequencies creating plateaus. These data sets exhibit considerable hysteresis suggesting that the signal arises from an instability in pinned vorticity. In the operating temperature range of our current interferometer ($5\text{ mK}\leqslant T_\lambda - T \leqslant 25$ mK), we find that the values of v_c are temperature independent with a mean of 1 mm/s and a standard deviation of 0.2 mm/s.

Data such as those shown in Fig. 3 shed new light on the initial instability process of trapped vorticity in the flow of superfluid ^{4}He and may yield further insights to the mecha-

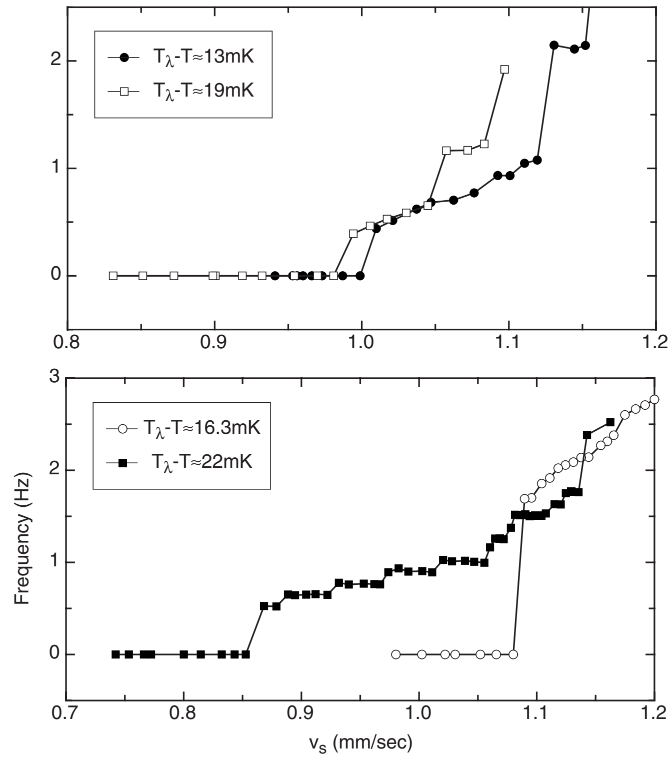

FIG. 3. Frequency of interferometer output oscillation as a function of power input. Zero frequency means that the interferometer signal [$\propto\cos(\Delta\phi/2)$] is constant in time. Plots are separated for clarity.

nisms that lead to the growth of quantized turbulence. Since the superfluid interferometer operates near 2 K, a regime with a large background of normal fluid, the technique reported here has the simplifying feature that vortex motion is damped and therefore will be less chaotic. Hopefully for this regime vortex dynamics simulations can reveal a process wherein a trapped vortex filament periodically crosses the tube when the flow velocity reaches speeds $\sim$1 mm/s. Initial attempts at such simulations[14] show transverse vortex crossing but additional work is needed to simulate the complete vortex dynamics in our particular experimental geometry.

It may be meaningful to compare our observed critical velocity (or critical heat current) to values reported in experiments investigating nonlinear regimes with heat flux very close to T_λ. The investigation of ^{4}He properties in the presence of finite heat flux close to T_λ has drawn much interest since superfluid is considered to be a clean system suited for studying nonlinear critical phenomena. However, if the flow reaches the critical velocity for vortex crossing and the dissipation sets in, the superfluid is no longer such an ideal testing ground. Day *et al.*[15] report a breakdown of Fourier's law ($\vec{Q}=-\kappa\nabla T$) when the heat current reaches a critical value $\dot{Q}_c$, which corresponds to counterflow superfluid velocities of about 1–2 mm/s. The authors attribute this effect to a divergent transport coefficient (in the presence of finite heat flux) predicted by the renormalization group theory.[16,17] However, since those velocities are very close to the vortex instability onset observed in our present experiment, it might be important to consider how vortices driven by the superflow should or should not affect the critical phase transitions near T_λ.

In summary, the unprecedented sensitivity and novel nature of a superfluid matter wave interferometer has allowed us to directly monitor the order parameter of superfluid ^{4}He flowing in a macroscopic passage. We have observed the initial onset of the motion of a single vortex filament and found that at a velocity of about 1 mm/s the phase difference begins to oscillate in time, indicative of simple periodic phase slippage in the tube. There are many interesting questions that remain. For example, once the heat input exceeds the critical value and the amplitude of the interferometer signal starts to oscillate, it keeps oscillating for as long as we can continuously observe it (which is ~20 s at these temperatures).[18] If we could observe it for a longer period of time, would it continue indefinitely? Can the finite slope in observed plateaus in Fig. 3 give any clues about the vortex dynamics (e.g., crossing time, etc.) and can smaller steps within a plateau (seen in the $T_\lambda - T \approx 22$ mK plot in Fig. 3, for example) be explained with a simple model? One can try to answer these questions with simulation, theory, and more experiments, and we think that there is a tremendous amount of fundamental physics to be uncovered here. It is unfortunate that this discovery (exciting though it may be) is tangential to our main body of work and therefore cannot be pursued to the extent that it deserves. We hope the technique and the results presented here will give some insights to active researchers in the field of quantum turbulence leading to an enhanced understanding of vortex instability processes in superfluid helium flow.

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