Diffusion and transfer of entanglement in an array of inductively coupled flux qubits

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(Received 7 February 2007; revised manuscript received 12 June 2007; published 3 August 2007)

A theoretical scheme to generate multipartite entangled states in a Josephson planar-designed architecture is reported. This scheme improves the one published by Migliore *et al.* [Phys. Rev. B **74**, 104503 (2006)] since it speeds up the generation of W entangled states in an $M\times N$ array of inductively coupled Josephson flux qubits by reducing the number of necessary steps. In addition, the same protocol is shown to be able to transfer the W state from one row to the other.

DOI: 10.1103/PhysRevB.76.052501 PACS number(s): 03.67.Mn, 03.67.Lx, 85.25.Dq

I. INTRODUCTION

In the past few years, condensed-matter architectures based on Josephson-junction qubits have appeared to be promising candidates for quantum information processors.[1,2] These solid-state systems can be scaled up to a large number of qubits and satisfy DiVincenzo's requirements for quantum computing,[3] i.e., state preparation, manipulation, and readout. Among them, qubits on the basis of the superconducting quantum interference devices (SQUIDs) offer the possibility of realizing switchable (inductive) interbit couplings,[4–6] and therefore provide promising platforms to generate multipartite entanglements among "macroscopic" quantum systems in deterministic ways. Remarkable experimental achievements with flux qubits include the realization of complex single-qubit operation schemes,[7] the generation of entangled states[8,9] in systems of coupled flux qubits,[10] while the direct measurement of the entanglement in Josephson architectures has been performed via state tomography[11] for two superconducting phase qubits.

Within such frameworks, we proposed a scheme for the generation of a W entangled state in a chain of N spatially separated flux qubits by exploiting their sequential couplings with one of them playing the role of an entanglement mediator.[12] We remind that the N-partite W entangled state is a natural generalization to the N-qubit state of the tripartite W state $|W\rangle_3=(|100\rangle+|010\rangle+|001\rangle)/\sqrt{3}$, i.e., the totally symmetric (apart from possible phase factors) quantum superposition of N two-state systems where only one of them is in its excited state. In Ref. 12, the success of this scheme relies on the possibilities of both preparing the initial state of the qubits and tuning the coupling energy and/or the interaction time between each qubit and the mediator, provided the time necessary for the desired quantum processes is short enough with respect to the decoherence time.

The protection against noise is evidently one of the central issues in quantum information technology and the reduction of the duration spent for specific quantum operations is important for it. In this Brief Report, we improve the scheme proposed in Ref. 12 by analyzing the dynamics of an *array* of flux qubits, which can be selectively coupled in pairs, for instance, by exploiting the tunable flux transformer proposed by Castellano *et al.*[4] and demonstrate how it is possible to *diffuse* a W state prepared in one row to two or more rows with a few steps. Such a scheme helps reduce the time for the generation of multipartite entanglement. Furthermore, we show that the same protocol also provides us with a way to shift, or *transfer*, the W state from row to row. We emphasize that W states are promising candidates for the experimental realization of quantum information processing in multiparticle systems since they possess entanglement robustness against local operation even under qubit loss.

II. ARRAY OF QUBITS

The idea presented in this Brief Report is based on the theoretical proposal in Ref. 12 to generate a W state in a chain of rf SQUID or persistent current (3JJ) qubits. We first recapitulate its essential idea within the present setup illustrated in Fig. 1. Here, in order to minimize the susceptibility to external noise of a large-inductance rf SQUID, as proposed by Mooij *et al.*,[13] we consider the planar array sketched in Fig. 1 constituted by $M\times N$ spatially separated, and consequently not directly interacting, tunable 3JJ qubits, that is, a superconducting loop containing three Josephson junctions, two of equal size (i.e., with $E_{J,1}=E_{J,2}=E_J$) and the third one smaller by a factor α (i.e., with $E_{J,3}=\alpha E_J$, $\alpha<1$). This parameter may be adjusted, for instance, by substituting the third junction with a dc SQUID behaving as an effective Josephson junction with tunable Josephson energy, $E_{J,3}\equiv E_{J,3}(\phi_c)$, ϕ_c being an additional control flux threading the dc-SQUID loop. In such conditions, by applying an external flux ϕ_x close to a half-integer number of flux quanta, $\phi_0=h/2e$, and choosing $\alpha\approx 0.8$, the potential energy of the total system forms a double well which permits two stable configurations of minimum energy corresponding to two persistent currents $\pm I_p\approx\pm 2\pi\alpha E_J/\phi_0$ in the loop. This fact allows to engineer a two-state quantum system (qubit) whose effective Hamiltonian, on the basis of the two energy eigenstates $|0_{mn}\rangle$ and $|1_{mn}\rangle$ of the (m,n) qubit of the array (which at $\phi_x=\phi_0/2$ are maximal superpositions of the two persistent current states $|L_{mn}\rangle$ and $|R_{mn}\rangle$), reads

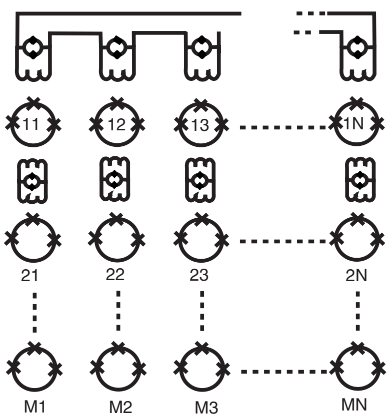

FIG. 1. Schematic illustration of an $M\times N$ array of Josephson flux qubits. An inductive qubit-qubit coupling is realized by means of a superconducting switch, namely, a transformer with variable flux-transfer function $\mathcal{R}(\phi_{cx}^{(mn)})$, as proposed in Ref. 4. It is possible to control the flux-transfer function, and therefore the inductive-coupling constant, by modulating the critical current of the inner dc SQUID of each transformer via an externally applied magnetic flux $\phi_{cx}^{(mn)}$. Each individual coupling between a pair of qubits is effectively turned on by adjusting the control fluxes of the relevant "switches" with all the other switches kept off.

$$H_{mn}=\frac{1}{2}\hbar\omega_{mn}\sigma_z^{(mn)},\quad \omega_{mn}=\sqrt{\Delta_{mn}^2(\phi_c)+\epsilon_{mn}^2(\phi_x)},\qquad (1)$$

where $m=1,\ldots,M$, $n=1,\ldots,N$, and $\sigma_z^{(mn)}=|1_{mn}\rangle\langle 1_{mn}|-|0_{mn}\rangle\langle 0_{mn}|$ is a Pauli operator for the (m,n)th qubit. The energy spacing, corresponding to a transition frequency ω_{mn} typically in the range of microwaves, can be tuned by properly selecting both the tunneling frequency, Δ_{mn}, between $|L_{mn}\rangle$ and $|R_{mn}\rangle$, and $\epsilon_{mn}(\phi_x)=2I_p(\phi_x-\phi_0/2)/\hbar$, both depending on the system parameters.

In the following discussion, we assume that all the qubits have a common energy gap $\omega=\omega_{mn}$, $\forall(m,n)$.

The qubits are coupled with each other, as depicted in Fig. 1. The coupled dynamics of the total system is described by the Hamiltonian

$$H=H_0+\sum_{m,k=1}^{M}\sum_{n,\ell=1}^{N}H'_{mn,k\ell},\quad H_0=\sum_{m=1}^{M}\sum_{n=1}^{N}H_{mn},\qquad (2)$$

where

$$H'_{mn,k\ell}=g_{mn,k\ell}(\sigma_+^{(mn)}\sigma_-^{(k\ell)}+\sigma_-^{(mn)}\sigma_+^{(k\ell)})\qquad (3)$$

is the rotating-wave coupling between qubits (m,n) and (k,ℓ) with $\sigma_+^{(mn)}=|1_{mn}\rangle\langle 0_{mn}|$ and $\sigma_-^{(mn)}=|0_{mn}\rangle\langle 1_{mn}|$ the raising and lowering operators for qubit (m,n), respectively.

The coupling constants $g_{11,1n}=2\left(\frac{\xi_{11,1n}}{L}\right)\frac{\phi_0^2}{2}$ $(n=2,\ldots,N)$ between the first and the nth qubit in the first row can be turned on and off via controlling the magnetic fluxes $\phi_{cx}^{(mn)}$ externally applied to the (m,n) qubit, as shown in Fig. 1. Analogously, we may control the coupling constants $g_{mn,(m+1)n}=2\left(\frac{\xi_{mn,(m+1)n}}{L}\right)\frac{\phi_0^2}{2}$ (with $m=1,\ldots,M-1$; $n=1,\ldots,N$) between the mth and the $(m+1)$th qubits in the nth column. Here, $\xi_{11,1n}=r_{11}\mathcal{R}(\phi_{cx}^{(11)})\mathcal{R}(\phi_{cx}^{(1n)})r_{1n}$ and $\xi_{mn,(m+1)n}=r_{mn}\mathcal{R}(\phi_{cx}^{(m+1)n)})r_{(m+1)n}$, r_{mn} being the flux transforming ratio between the arm of the transformer and the qubit (m,n). In the following calculations, we work in the interaction picture with respect to H_0.

If the inter-row couplings are turned off, $g_{mn,(m+1)n}=0$, the system is essentially the one analyzed in Ref. 12. It is, therefore, possible to generate an N-partite W state in the first row, among qubits $(1,1),\ldots,(1,N)$, as follows:

(1) We prepare the initial state $|\Psi_0\rangle=|1_{11}\cdots 0_{1N}\rangle\otimes|0_{21}\cdots 0_{2N}\rangle\otimes\cdots\otimes|0_{M1}\cdots 0_{MN}\rangle$, with only the qubit (1,1) in the excited state and the rest of the qubits of the array in their own ground state.

(2) The coupling $g_{11,12}$ is turned on during a proper time interval $0<t<\tau_1$ with the other couplings turned off.

(3) The coupling $g_{11,12}$ is turned off at $t=\tau_1$ and $g_{11,13}$ is on during $\tau_1<t<\tau_1+\tau_2$.

(4) In this way, qubit (1,1) is coupled with $(1,2),\ldots,(1,N)$ one by one.

It is shown in Ref. 12 that, by exploiting the knowledge of the coupling constants $g_{11,1(n+1)}$ when the relevant interaction is turned on, by setting the interaction times τ_n so that

$$\sin\theta_n=\frac{1}{\sqrt{N-n+1}},\quad \cos\theta_n=\sqrt{\frac{N-n}{N-n+1}},\qquad (4)$$

with $\theta_n=g_{11,1(n+1)}\tau_n/\hbar$ $(n=1,\ldots,N-1)$, a W state

$$|w_1\rangle=|W\rangle_1\otimes|0_{21}0_{22}\cdots 0_{2N}\rangle\otimes\cdots\otimes|0_{M1}0_{M2}\cdots 0_{MN}\rangle,\qquad (5)$$

with

$$|W\rangle_m=\frac{1}{\sqrt{N}}\big(|1_{m1}0_{m2}\cdots 0_{mN}\rangle-i|0_{m1}1_{m2}\cdots 0_{mN}\rangle-\cdots-i|0_{m1}0_{m2}\cdots 1_{mN}\rangle\big),\qquad (6)$$

is generated in the first row $m=1$ of the array.

We underline that rapid-single-flux-quantum Josephson-junction based logic circuits[14,15] make it possible to produce flux pulses characterized by rise and fall times $t_{r/f}$ of the order of 10 ps. Therefore, we are able to obtain switching times much shorter than the duration of any step in our scheme, typically less than or of the order of the inverse of coupling energy $\hbar/g_j\approx 2$ ns.

III. DIFFUSION AND TRANSFER OF W STATE

Let us now discuss schemes for spreading the W state [Eq. (5)] prepared in the first row of the array to other rows, as well as for transferring it from row to row, by making use of the switchable inductive coupling $g_{mn,(m+1)n}$ between each qubit of the mth row $(m=1,\ldots,M-1)$ and the corresponding one in the $(m+1)$th row. To this end, we consider the following "collective" step by step scheme:

(1) Each of the N qubits in the first row, already prepared in the W state [Eq. (5)], is put in inductive interaction with the corresponding one in the second row during $0<t<\tau_{\rm I}$ by turning on the couplings $g_{11,21}=g_{12,22}=\cdots=g_{1N,2N}=g_{\rm I}\neq 0$, while other qubits evolve freely.

(2) At $t=\tau_{\rm I}$, the couplings $g_{11,21},g_{12,22},\ldots,g_{1N,2N}$ are turned off, and $g_{21,31}=g_{22,32}=\cdots=g_{2N,3N}=g_{\rm II}\neq 0$ are turned

on for time interval $\tau_{\rm I}<t<\tau_{\rm I}+\tau_{\rm II}$ in order to couple the qubits in the second row with the corresponding ones in the third.

(3) Similarly, the interactions between adjacent rows are successively switched on and off.

Then, by properly selecting the interaction times (by turning on and off the coupling constants), it is possible to transfer the W state from row to row or to diffuse it to multirows.

To illustrate the mechanism, let us look at a 3×3 array ($M=N=3$) for the sake of simplicity. In this case, the W state prepared in the first row [Eq. (5)] reads

$$|w_1\rangle = |W\rangle_1 \otimes |0_{21}0_{22}0_{23}\rangle \otimes |0_{31}0_{32}0_{33}\rangle. \tag{7}$$

By switching on the inductive couplings $g_{11,21}=g_{12,22}=g_{13,23}=g_{\rm I}\neq 0$ for a time period $\tau_{\rm I}$, the W state $|w_1\rangle$ in Eq. (7) is driven into

$$\begin{aligned}|\Psi_{\rm I}\rangle = \frac{1}{\sqrt{3}}\big(&\cos\theta_{\rm I}|1_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle - i\cos\theta_{\rm I}|0_{11}1_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle - i\cos\theta_{\rm I}|0_{11}0_{12}1_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \\ &- i\sin\theta_{\rm I}|0_{11}0_{12}0_{13}\rangle \otimes |1_{21}0_{22}0_{23}\rangle - \sin\theta_{\rm I}|0_{11}0_{12}0_{13}\rangle \otimes |0_{21}1_{22}0_{23}\rangle - \sin\theta_{\rm I}|0_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}1_{23}\rangle\big) \otimes |0_{31}0_{32}0_{33}\rangle,\end{aligned} \tag{8}$$

and this is further converted into

$$\begin{aligned}|\Psi_{\rm I}\rangle = \frac{1}{\sqrt{3}}\big(&\cos\theta_{\rm I}|1_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \otimes |0_{31}0_{32}0_{33}\rangle - i\cos\theta_{\rm I}|0_{11}1_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \otimes |0_{31}0_{32}0_{33}\rangle \\ &- i\cos\theta_{\rm I}|0_{11}0_{12}1_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \otimes |0_{31}0_{32}0_{33}\rangle - i\sin\theta_{\rm I}\cos\theta_{\rm II}|0_{11}0_{12}0_{13}\rangle \otimes |1_{21}0_{22}0_{23}\rangle \otimes |0_{31}0_{32}0_{33}\rangle \\ &- \sin\theta_{\rm I}\cos\theta_{\rm II}|0_{11}0_{12}0_{13}\rangle \otimes |0_{21}1_{22}0_{23}\rangle \otimes |0_{31}0_{32}0_{33}\rangle - \sin\theta_{\rm I}\cos\theta_{\rm II}|0_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}1_{23}\rangle \otimes |0_{31}0_{32}0_{33}\rangle \\ &- \sin\theta_{\rm I}\sin\theta_{\rm II}|0_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \otimes |1_{31}0_{32}0_{33}\rangle + i\sin\theta_{\rm I}\sin\theta_{\rm II}|0_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \otimes |0_{31}1_{32}0_{33}\rangle \\ &+ i\sin\theta_{\rm I}\sin\theta_{\rm II}|0_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \otimes |0_{31}0_{32}1_{33}\rangle\big)\end{aligned} \tag{9}$$

after the second step with the couplings $g_{21,31}=g_{22,32}=g_{23,33}=g_{\rm II}\neq 0$ turned on while others are off, where $\theta_{\rm I/II}=g_{\rm I/II}\tau_{\rm I/II}/\hbar$.

Equation (8) clearly shows that the tuning $\sin\theta_{\rm I}=\cos\theta_{\rm I}=1/\sqrt{2}$ realizes a *one-step diffusion* of the W state up to the second row, namely, from the tripartite to a hexapartite W state,

$$\begin{aligned}|W_2\rangle = \frac{1}{\sqrt{6}}\big(&|1_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle - i|0_{11}1_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle - i|0_{11}0_{12}1_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle - i|0_{11}0_{12}0_{13}\rangle \otimes |1_{21}0_{22}0_{23}\rangle \\ &- |0_{11}0_{12}0_{13}\rangle \otimes |0_{21}1_{22}0_{23}\rangle - |0_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}1_{23}\rangle\big) \otimes |0_{31}0_{32}0_{33}\rangle.\end{aligned} \tag{10}$$

Alternatively, if we select a different interaction strength so as to satisfy $\sin\theta_{\rm I}=1$, another W state,

$$|w_2\rangle = -i|0_{11}0_{12}0_{13}\rangle \otimes |W\rangle_2 \otimes |0_{31}0_{32}0_{33}\rangle, \tag{11}$$

is established, that is, the tripartite W state in the first row is shifted, or transferred, to the second row after the one-step diffusion.

The second step further diffuses or transfers the W state to the third row. Indeed, Eq. (9) shows that one choice, $\sin\theta_{\rm I}=\sqrt{2/3}$, $\cos\theta_{\rm I}=1/\sqrt{3}$, and $\sin\theta_{\rm II}=\cos\theta_{\rm II}=1/\sqrt{2}$, yields a complete W state all over the 3×3 array,

$$\begin{aligned}|W_3\rangle = \frac{1}{3}\big(&|1_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \otimes |0_{31}0_{32}0_{33}\rangle - i|0_{11}1_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \otimes |0_{31}0_{32}0_{33}\rangle \\ &- i|0_{11}0_{12}1_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \otimes |0_{31}0_{32}0_{33}\rangle - i|0_{11}0_{12}0_{13}\rangle \otimes |1_{21}0_{22}0_{23}\rangle \otimes |0_{31}0_{32}0_{33}\rangle \\ &- |0_{11}0_{12}0_{13}\rangle \otimes |0_{21}1_{22}0_{23}\rangle \otimes |0_{31}0_{32}0_{33}\rangle - |0_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}1_{23}\rangle \otimes |0_{31}0_{32}0_{33}\rangle \\ &- |0_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \otimes |1_{31}0_{32}0_{33}\rangle + i|0_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \otimes |0_{31}1_{32}0_{33}\rangle \\ &+ i|0_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \otimes |0_{31}0_{32}1_{33}\rangle\big),\end{aligned} \tag{12}$$

namely, the W state [Eq. (5)] is diffused from the three qubits to the nine ones with only two steps; more generally, the W state prepared in the first row is diffused all over the $M\times N$ array via $N-1$ steps. Another choice, $\sin\theta_{\rm I}=\sin\theta_{\rm II}=1$, generates

$$|w_3\rangle = -|0_{11}0_{12}0_{13}\rangle \otimes |0_{21}0_{22}0_{23}\rangle \otimes |W\rangle_3, \tag{13}$$

transferring the W state [Eq. (5)] in the first row to the third.

IV. CONCLUSIONS

In summary, we have extended the theoretical scheme for the generation of a W entangled state proposed in Ref. 12 to a scheme for an $M\times N$ array of qubits. A remarkable feature is that an entanglement realized as a W state in one row of the array is diffused to two rows and the number of qubits involved in the W state is doubled after a single step. The entanglement is further diffused all over the array by repeating similar processes to yield an $(M\times N)$-qubit W state. This procedure would facilitate the generation of a large-scale multipartite W state with fewer steps, and as a result, would help save time for its generation. Furthermore, we have demonstrated the possibility of transferring the W state prepared in one row to another at will. We have illustrated these schemes in the context of the inductively coupled flux qubits, but they are also applicable to other systems, provided the qubit-qubit couplings are controllable.

The experimental realization of our proposal is possible, although it has to face various technological challenges. The fabrication of the proposed circuit is not difficult. Moreover, to exploit tunable qubits and flux transformers allows both to strongly reduce the single-qubit parameter disorder and at the same time to select common energy gaps with a disorder of a few percent[16] and to control the duration of interqubit couplings. In addition, state preparation can be accurately realized with well-defined procedures including relaxation and single-qubit rotations. A delicate point concerns the system time scales against undesired effects due to the coupling of each qubit with bosonic baths, traceable back to the presence of many unavoidable noise sources. For instance, the effective impedance characterizing the dissipative electronic circuitry coupled to the single qubit progressively degrades its coherent evolution. The consequent impact on the single-qubit decoherence rates and on the performance of a gate of two inductively coupled qubit has been studied, bringing to light that, by carefully engineering the environmental impedances, the bipartite systems are characterized by rates in the range $10^{-7}-10^{-6}$ s.[17] Since the observed relaxation and decoherence times for a single flux qubit are in the range $1-10\ \mu$s,[2,15,18] the passage to a bipartite system speeds up the decoherence process. Thus, the present scheme, shortening significantly the generation time of a multipartite W state, provides an effective way to anticipate the occurrence of coherence loss effects. We assume that the eigenfrequency ω of a Josephson qubit is of the order of 10 GHz and that the inductive qubit-qubit coupling constant is of the order of 0.5 GHz.[2,6] Under these conditions, the length of a generic step (during which only a fraction of a Rabi oscillation takes place) is of the order of 2 ns. This means that the W state [Eq. (12)] for the 3×3 array is generated approximately after 8 ns, which is much shorter than the 16 ns required for the generation of a nine-partite W state by the scheme discussed in Ref. 12. Extending this argument to a larger array (for instance, to a 5×5 array), we find that the generation of an entangled state of 25 qubits (which requires eight steps) is roughly compatible with the currently observed relaxation times, $\approx$300 ns, characterizing two coupled qubits. We wish to emphasize that this estimation is suitable in our case, since, during each step, we deal with noninteracting bipartite systems of inductively coupled qubits. If we access only one qubit at a step to entangle multiple qubits, such a large-scale entanglement cannot be established within such limited time.

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