

Magnetic flux in mesoscopic rings: Quantum Smoluchowski regime

J. Dajka, L. Machura, Sz. Rogoziński, and J. Łuczka

Institute of Physics, University of Silesia, 40-007 Katowice, Poland

(Received 28 December 2006; revised manuscript received 11 April 2007; published 31 July 2007)

Magnetic flux in mesoscopic rings in the quantum Smoluchowski regime is investigated. Quantum corrections to the dissipative current are shown to form multistable steady states and can result in statistical enhancement of the magnetic flux. The relevance of quantum correction effects is supported via the entropic criterion. A possible application for a qutrit architecture of quantum information is proposed.

DOI: [10.1103/PhysRevB.76.045337](https://doi.org/10.1103/PhysRevB.76.045337)

PACS number(s): 73.23.-b, 05.40.-a, 73.23.Ra

I. INTRODUCTION

Mesoscopic systems belong to one of the most intriguing parts of present-day investigations. They occupy territory between the physics of small quantum objects and the physics of macroscopic objects. Many aspects of that territory remains *terra incognita* to both experimentalists and theoreticians. For example, over one decade after the first experiments¹⁻³ proving the existence of theoretically predicted^{4,5} persistent currents in normal-metal multiply connected samples, there is an unsolved central question: which mechanism is responsible for the unexpectedly large amplitude of the measured current?⁶ There is a suggestion that the large current is due to *nonequilibrium* noise presented in the system.⁷ It is also theoretically predicted⁸ that currents in mesoscopic rings can flow even in the absence of any driving. Such *self-sustaining* currents have not been observed so far. Their existence is a desired property for quantum information retrieval and computing technologies based on non-superconducting devices.⁹

In our earlier work¹⁰ we proposed the two-fluid model of noisy dynamics of the magnetic flux in mesoscopic rings and cylinders. The dynamics of the magnetic flux is described by an evolution equation which is equivalent to a Langevin equation for an overdamped motion of a classical Brownian particle, and the steady state of the system is characterized by the asymptotic probability density being a stationary solution of the corresponding Fokker-Planck equation. In this approach, self-sustaining fluxes are long-living states of the system described by a multistable asymptotic probability density. This model is an example of a hybrid of quantum and classical parts and is a counterpart of the well-known model of a resistively shunted Josephson junction.¹¹ The classical part consists of “normal” electrons carrying dissipative current. The quantum part is formed by those electrons which maintain their phase coherence around the circumference of the cylinder or ring. The effective kinetics is determined by a classical Langevin equation with a Nyquist noise describing thermal equilibrium fluctuations. The coherent part of the system acts as an additional “force” driving normal electrons. It is natural to ask what the impact is of the quantum nature of dissipative kinetics on the properties of fluxes and currents flowing in such systems. To answer this question, we exploit the so-called quantum Smoluchowski equation introduced in Ref. 12 and, with Maxwell’s demon

successfully exorcised, in Refs. 13 and 14. First, we extend our model for overdamped kinetics¹⁰ to the domain where charging effects (corresponding to the inertial effects for particles) appear. This extension is necessary for a precise identification of the quantum Smoluchowski regime. The quantum corrections are of great importance for the existence and properties of self-sustaining currents or magnetic fluxes. It is shown below that in moderate, with respect to the gap at the Fermi level, temperatures these quantum corrections are destructive for their existence. It is not the case at lower temperature: one gets not only the multistability of the probability density but also significant enhancement of the probability of the occurrence of long-living states carrying magnetic flux of a certain amplitude.

It is shown that for the system under consideration the passage from the classical Smoluchowski regime into the quantum Smoluchowski regime is accompanied by a decrease of the Shannon entropy. It emphasizes the significance of the multistable ordered state. As the predicted multistability is formed by a set of an odd number of maxima in the asymptotic probability density it is natural to expect the ring or cylinder to be a candidate for a *qutrit* rather than a qubit.

The layout of the paper is as follows: In Sec. II, we construct an extended, capacitive model of dissipative magnetic flux dynamics in mesoscopic systems of a cylinder symmetry. Next, in Sec. III, we discuss the quantum Smoluchowski regime for the system. In Sec. IV, we study properties of the stationary magnetic flux in the quantum Smoluchowski domain. Section V contains a summary and conclusions.

II. CAPACITIVE MODEL OF DISSIPATIVE FLUX DYNAMICS

At zero temperature T , small metallic systems of cylinder symmetry (like rings, toroids, and cylinders) threaded by a magnetic flux ϕ display persistent and nondissipative currents I_{coh} run by coherent electrons. At nonzero temperature, a part of electrons becomes “normal” (noncoherent) and the amplitude of the persistent current decreases. Moreover, the resistance of the ring and thermal fluctuations start to play a role. Therefore, for temperatures $T > 0$, there are both coherent and dissipative parts of the total current: namely,

$$I_{tot} = I_{coh} + I_{dis}. \quad (1)$$

The persistent current I_{coh} as a function of the magnetic flux ϕ depends on the parity of the number of coherent electrons.

Let p denotes the probability of an even number of coherent electrons. Then the formula for coherent current reads¹⁵

$$I_{coh} = I_{coh}(\phi, T) = pI_{even}(\phi, T) + (1-p)I_{odd}(\phi, T), \quad (2)$$

where

$$I_{even}(\phi, T) = I_{odd}(\phi + \phi_0/2, T) = I_0 \sum_{n=1}^{\infty} A_n(T/T^*) \sin(2n\pi\phi/\phi_0). \quad (3)$$

The flux quantum $\phi_0 = h/e$ is the ratio of the Planck constant h and the charge of the electron and I_0 is the maximal current at zero temperature. The temperature-dependent amplitudes are determined by the relation¹⁵

$$A_n(T/T^*) = \frac{4T}{\pi T^*} \frac{\exp(-nT/T^*)}{1 - \exp(-2nT/T^*)} \cos(nk_F l), \quad (4)$$

where the characteristic temperature T^* is defined by the relation $k_B T^* = \Delta_F / 2\pi^2$, where Δ_F is the energy gap at the Fermi surface, k_B is the Boltzmann constant, k_F is the Fermi momentum, and l is the circumference of the ring.

The dissipative current I_{dis} is determined by Ohm's law and Lenz's rule,¹⁶

$$I_{dis} = I_{dis}(\phi, T) = -\frac{1}{R} \frac{d\phi}{dt} + \sqrt{\frac{2k_B T}{R}} \Gamma(t), \quad (5)$$

where R is the resistance of the ring and $\Gamma(t)$ models thermal Nyquist fluctuations of the Ohmic current. In the first approximation, this thermal noise is classical Gaussian white noise of zero average—i.e., $\langle \Gamma(t) \rangle = 0$ and δ -autocorrelated function $\langle \Gamma(t)\Gamma(s) \rangle = \delta(t-s)$. The noise intensity $D_0 = k_B T / R$ is chosen in accordance with the classical fluctuation-dissipation theorem.

Quantum corrections to classical thermal fluctuations will be considered below in the so-called Smoluchowski regime. To define precisely this regime, first we have to include charging effects.¹⁷ To this aim, we shall construct a formal Hamilton function (i.e., energy) of the system which consists of three parts. The first one corresponds to an effective potential related to the persistent current itself, the second is related to the energy of the magnetic flux, and the third is due to charging effects caused by capacitance C of the system (it corresponds to the kinetic energy of a particle).

We define a potential energy related to the persistent current by the relation

$$E_{coh}(\phi) = - \int I_{coh}(\phi, T) d\phi, \quad (6)$$

which reflects the well-known fact that the persistent current is an equilibrium and thermodynamic phenomenon. At zero temperature, it is an energy of the set of discrete energy levels carrying persistent current. For nonzero temperature, the persistent current is averaged over the thermal distribution function and the above relation holds for a thermodynamic potential.

We assume that the ring can be characterized by a capacitance C . To justify it we cite Kopietz¹⁸ who showed that in

the diffusive regime the energy associated with long-wavelength and low-energy charge fluctuations is determined by classical charging energies and therefore the ring behaves as it were a classical capacitor. The flux dependence of these energies yields the contribution to the persistent current. The speculation that the local charge fluctuations and charging energies could contribute to the persistent current has also been suggested by Imry and Altshuler.¹⁹

From the above it follows that the total energy takes the form¹⁷

$$E = \frac{C}{2} \left(\frac{d\phi}{dt} \right)^2 + \frac{1}{2L} (\phi - \phi_e)^2 + E_{coh}(\phi), \quad (7)$$

where ϕ_e is the magnetic flux induced by an external magnetic field B and L is a self-inductance of the system. The equation of motion, which corresponds to Eq. (7), has the form

$$C \frac{d^2\phi}{dt^2} = -\frac{1}{L} (\phi - \phi_e) + I_{coh}(\phi, T). \quad (8)$$

Now, we want to take into account dissipation effects. To this aim we generalize Eq. (8), replacing the coherent current I_{coh} by the total current I_{tot} given by Eq. (1). As a result we obtain the evolution equation

$$\begin{aligned} C \frac{d^2\phi}{dt^2} + \frac{1}{R} \frac{d\phi}{dt} &= -\frac{1}{L} (\phi - \phi_e) + I_{coh}(\phi, T) + \sqrt{\frac{2k_B T}{R}} \Gamma(t) \\ &= -\frac{dW(\phi)}{d\phi} + \sqrt{\frac{2k_B T}{R}} \Gamma(t), \end{aligned} \quad (9)$$

where the potential $W(\phi)$ reads

$$\begin{aligned} W(\phi) &= \frac{1}{2L} (\phi - \phi_e)^2 + \phi_0 I_0 \sum_{n=1}^{\infty} \frac{A_n(T/T^*)}{2n\pi} \left\{ p \cos\left(2n\pi \frac{\phi}{\phi_0}\right) \right. \\ &\quad \left. + (1-p) \cos\left[2n\pi \left(\frac{\phi}{\phi_0} + \frac{1}{2}\right)\right] \right\}. \end{aligned} \quad (10)$$

This equation is an extended one in comparison with the equation of motion studied in Ref. 10 by including the inertial, capacitive term. Its structure is similar to the model of capacitively and resistively shunted Josephson junctions.¹¹ Indeed, the dynamics of a trapped magnetic flux in a superconducting ring interrupted by the Josephson junction is described by Eq. (9) by changing $I_{coh}(\phi, T)$ into the Josephson supercurrent $I = I_c \sin(\phi)$.²⁰

III. QUANTUM SMOLUCHOWSKI REGIME

The dissipative part of the current, given by Eq. (5), is a classical one in which the quantum character of thermal fluctuations is ignored. At lower temperatures, it can be insufficient and leading quantum corrections might be important. We do not know how to incorporate quantum corrections in the general case described by Eq. (9). However, in the regimes where the charging effects can be neglected, the system can be described by the so-called quantum Smoluchowski equation.^{12,13} It has the same structure as a classical

Smoluchowski equation, in which the potential $W(\phi)$ and diffusion coefficient $D_0 = k_B T / R$ are modified due to quantum effects like tunneling, quantum reflections, and fluctuations. In terms of the Langevin equation (9), it assumes the form

$$\frac{1}{R} \frac{d\phi}{dt} = - \frac{dW_m(\phi)}{d\phi} + \sqrt{2D_m(\phi)} \Gamma(t). \quad (11)$$

This equation has to be interpreted in the Ito sense.²¹ The modified potential $W_m(\phi)$ and the modified diffusion coefficient $D_m(\phi)$ take the form¹³

$$W_m(\phi) = W(\phi) + \frac{1}{2} \Lambda W''(\phi), \quad (12)$$

$$D_m(\phi) = \frac{D_0}{1 - \Lambda W''(\phi) / k_B T}, \quad (13)$$

where the prime denotes differentiation with respect to the argument of the function. The quantum corrections are characterized by the parameter Λ . It measures the deviation of the quantal flux fluctuations from their classical counterparts: namely,

$$\Lambda = \langle \phi^2 \rangle_Q - \langle \phi^2 \rangle_C, \quad (14)$$

where $\langle \cdots \rangle$ denotes equilibrium average and the subscripts Q and C refer to quantal and classical cases, respectively. The explicit form of Λ reads²²

$$\Lambda = \frac{\hbar R}{\pi} [\Psi(1 + \lambda_1 / \nu) - \Psi(1 + \lambda_2 / \nu)], \quad (15)$$

where the psi function $\Psi(z)$ is the logarithmic derivative of the gamma function and

$$\lambda_{1/2} = \omega_0 [k \pm \sqrt{k^2 - 1}], \quad (16)$$

$$k = (2\omega_0 CR)^{-1}, \quad \nu = 2\pi k_B T / \hbar.$$

The frequency ω_0 is a typical frequency of the bare system, and its inverse corresponds to a characteristic time of the system.

Now, let us determine the range of applicability of the quantum Smoluchowski regime. The classical Smoluchowski limit corresponds to the neglect of charging effects. Formally, we should put $C=0$ in the inertial term of Eq. (9), which is related to the strong damping limit of the Brownian particle. In the case studied here it means that

$$k \gg 1 \text{ or } \omega_0 CR \ll 1, \quad (17)$$

and then Eq. (15) takes the form

$$\Lambda = \frac{\hbar R}{\pi} \left[\gamma + \Psi \left(1 + \frac{\hbar}{2\pi CR k_B T} \right) \right], \quad (18)$$

where $\gamma = 0.5772$ is the Euler constant.

The separation of time scales, on which the flux relaxes and the conjugate observable (a charge)²³ is already equilibrated, requires the second condition: namely,

$$\omega_0 CR \ll k_B T / \hbar \omega_0. \quad (19)$$

In the deep quantum regime, i.e., when

$$k_B T \ll \frac{\hbar}{2\pi CR}, \quad (20)$$

the correction (18) assumes the form

$$\Lambda = \frac{\hbar R}{\pi} \left[\gamma + \ln \left(\frac{\hbar}{2\pi CR k_B T} \right) \right]. \quad (21)$$

In order to identify precisely the quantum Smoluchowski regime, we have to determine a typical frequency ω_0 or the corresponding characteristic time $\tau_0 \propto 1/\omega_0$. There are many characteristic times in the system, which can be explicitly extracted from the evolution equation (9)—e.g., CR , $\hbar/k_B T$, and $\phi_0/(RI_0)$. The characteristic time $\tau_0 = L/R$ is the relaxation time of the flux in the classical (noncoherent) systems, and below we scale time with respect to τ_0 . As to why time is scaled in this way, we refer the readers to our previous paper.¹⁷ Therefore, in the quantum Smoluchowski regime, all the above inequalities, (17), (19), and (20) should be fulfilled for $\omega_0 \propto 1/\tau_0$. Because the diffusion coefficient cannot be negative, the parameter Λ should be chosen small enough to satisfy the condition $D_m(\phi) \geq 0$ for all values of ϕ . We note that the passage from the classical Smoluchowski domain to the quantum Smoluchowski domain allows for the identification of the physical regime because of the formal similarities of the inertial and capacitive terms in the equations of motion for the Brownian particle and the magnetic flux, respectively.

IV. STEADY-STATE ANALYSIS

From the mathematical point of view, the Langevin equation (11) describes a classical Markov stochastic process. Therefore its all statistical properties can be obtained from the corresponding Fokker-Planck equation for the probability density. To analyze its stationary solution, let us introduce dimensionless variables in Eq. (11): the rescaled flux $x = \phi/\phi_0$ and rescaled time $s = t/\tau_0$, where the characteristic time $\tau_0 = L/R$. Then Eq. (11) can be rewritten in the dimensionless form

$$\frac{dx}{ds} = - \frac{dV_{eff}(x)}{dx} + \sqrt{2D(x)} \xi(s). \quad (22)$$

The rescaled modified potential $V_{eff}(x)$ and the modified diffusion coefficient $D(x)$ take the form

$$V_{eff}(x) = V(x) + \frac{1}{2} \lambda B''(x), \quad (23)$$

$$V(x) = \frac{1}{2} (x - x_e)^2 + B(x), \quad (24)$$

$$D(x) = \beta^{-1} \{1 - \lambda \beta [1 + B''(x)]\}^{-1}, \quad (25)$$

where

$$B(x) = \alpha \sum_{n=1}^{\infty} \frac{A_n(T_0)}{2n\pi} \{p \cos(2n\pi x) + (1-p) \cos[2n\pi(x + 1/2)]\}, \quad (26)$$

with the rescaled temperature $T_0 = T/T^*$. The remaining dimensionless parameters are $x_e = \phi_e l \phi_0$, $\alpha = LL_0 l \phi_0$, $\lambda = \Lambda / \phi_0^2$, and $1/\beta = k_B T / 2E_m = k_0 T_0$, where the elementary magnetic flux energy $E_m = \phi_0^2 / 2L$ and $k_0 = k_B T^* / 2E_m$ is the ratio of two characteristic energies. The rescaled zero-mean Gaussian white noise $\xi(s)$ has the same statistical properties as the thermal noise $\Gamma(t)$. The dimensionless quantum correction parameter

$$\lambda = \lambda_0 \left[\gamma + \Psi \left(1 + \frac{\epsilon}{T_0} \right) \right], \quad \lambda_0 = \frac{\hbar R}{\pi \phi_0^2}, \quad \epsilon = \frac{\hbar / 2 \pi C R}{k_B T^*}. \quad (27)$$

The probability density $p(x, s)$ of the process (22) evolves according to the corresponding Fokker-Planck equation with natural boundary conditions. The stationary probability density $P(x)$ can be obtained from the steady-state Fokker-Planck equation and reads

$$P(x) = \lim_{s \rightarrow \infty} p(x, s) \propto D^{-1}(x) \exp[-\Phi(x)], \quad (28)$$

where the generalized thermodynamic potential

$$\Phi(x) = \int \frac{V'_{eff}(x)}{D(x)} dx. \quad (29)$$

Due to both the x dependence of the modified diffusion coefficient $D(x)$ and the temperature dependence of the modified potential $V_{eff}(x)$, the stationary state (28) is a thermal equilibrium state; however, it is not a Gibbs state: $P_G(x) \propto \exp[-\beta V(x)]$.

A. Quantum renormalization of potential and diffusion coefficient

In Figs. 1 and 2, we present the influence of quantum corrections on the shape of the potential and diffusion coefficient. We compare the potential $V(x)$ and the modified quantum potential $V_{eff}(x)$ with each other, as well as by analyzing the modified diffusion function $D(x)$ (which is constant in the classical Smoluchowski domain). In the regime presented in Fig. 1, the potential $V(x)$ (dashed line) is bistable and possesses the barrier in contrary to $V_{eff}(x)$ (solid line) and the generalized thermodynamic potential $\Phi(x)$ (not shown in the figure) which are monostable and barrier less. The state-dependent modified diffusion function $D(x)$ possesses maxima and minima. The maxima and minima can be interpreted as higher and lower effective local temperatures. It means that quantum fluctuations mimic a state-dependent periodic effective temperature. For the escape dynamics the generalized thermodynamic potential $\Phi(x)$ is decisive: It contains the combined influences of the modified potential and the modified diffusion. In the regime presented in Fig. 1, $\Phi(x)$ has the same properties as the modified quantum potential $V_{eff}(x)$.

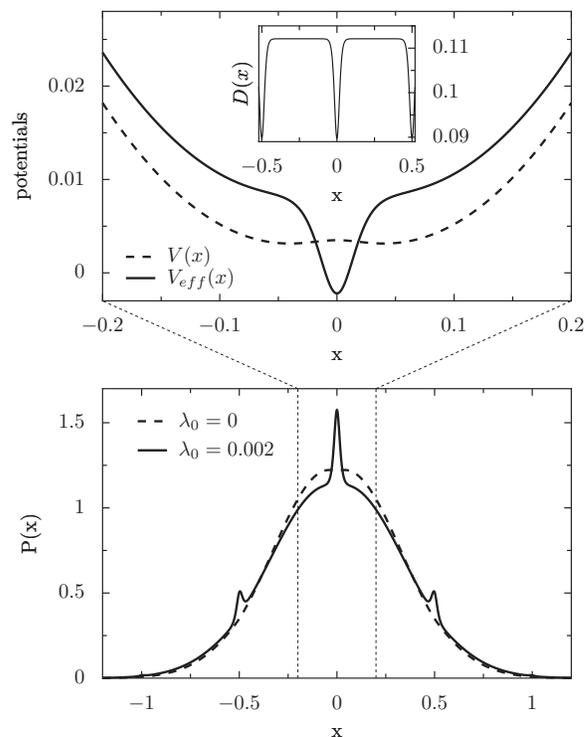


FIG. 1. The quantum Smoluchowski regime is compared to its classical counterpart. The potential $V(x)$ given by Eq. (24) and the modified potential $V_{eff}(x)$ in Eq. (23) are shown in the upper panel. In the inset, the modified diffusion function $D(x)$ defined in Eq. (25) is depicted. The lower panel shows the stationary probability density $P(x)$ in the classical Smoluchowski ($\lambda_0=0$) and quantum Smoluchowski ($\lambda_0=0.002$) regimes. Other parameters are set as follows: $x_e=0$, $T_0=0.2$, $\epsilon=10$, $\alpha=0.1$, $p=0.5$, $k_0=0.5$, and $k_F l = 0.001$.

The regime shown in Fig. 2 is much more interesting. The potential $V(x)$ (dashed line) is also bistable and possesses the barrier. However, the modified potential $V_{eff}(x)$ (solid line) and $\Phi(x)$ (not shown) are now multistable and possess many barriers. In fact, they possess infinitely many barriers and their heights are smaller and smaller as absolute value of the flux increases. As in the previous case, the state-dependent modified diffusion function $D(x)$ possesses maxima and minima which now are more distinct.

The values of the parameters in Figs. 1 and 2 seem to be feasible. A part of the values of the parameters have been evaluated from experimental data. E.g., following Mohanty,³ $T^* \approx 170$ mK and $T > 5$ mK. Therefore the rescaled temperature $T_0 > 0.03$. From Ref. 2, we have estimated the quantum correction parameter λ_0 . The parameters α and k_0 can be related to each other. The value of the parameter ϵ is unconfirmed. Fortunately, it enters only into the quantum correction parameter λ_0 which depends weakly (logarithmically) on it; cf. Eq. (21).

B. Multistability

In the following discussion we focus on *self-sustaining fluxes*. Such fluxes, contrary to the superconducting quantum

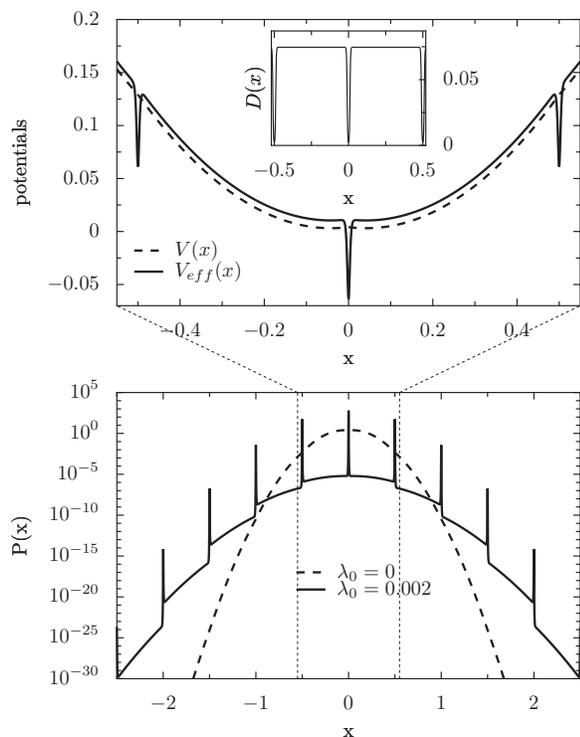


FIG. 2. The quantum Smoluchowski regime is compared to its classical counterpart. The potential $V(x)$ given by Eq. (24) and the modified potential $V_{eff}(x)$ in Eq. (23) are shown in the upper panel. In the inset, the modified diffusion function $D(x)$ defined in Eq. (25) is depicted. The lower panel shows the stationary probability density $P(x)$ in the classical Smoluchowski ($\lambda_0=0$) and quantum Smoluchowski ($\lambda_0=0.002$) regimes. Other parameters are set as follows: $x_e=0$, $T_0=0.04$, $\epsilon=10$, $\alpha=0.1$, $p=0.5$, $k_0=0.5$, and $k_{Fl}=0.001$.

interference device (SQUIDs), has not been observed in mesoscopic rings so far. Therefore, there is a question if it may be due to additional (quantum) noise in the system. In the noiseless system, they are related to minima of the multistable generalized potential.¹⁰

In the regime where quantum corrections are negligible ($\lambda \rightarrow 0$), it is a one-to-one correspondence between the minima of the potential $V(x)$ and the maxima of the stationary probability density $P(x)$.¹⁰ It is clearly not the case in the quantum Smoluchowski regime as the modified diffusion coefficient is flux dependent. Nevertheless, we relate the formation of the self-sustaining currents to the appearance of the multi peaked probability density at sufficiently low temperatures. As the steady state is always reflection invariant, self-sustaining fluxes are in fact *finitely long living* and appear if the peaks of the steady-state probability distribution are sufficiently high.

Let us consider two qualitatively different regimes. The first is the moderate-temperature regime where the noiseless system is bistable. It is shown in Fig. 1. For this case, the system, if it can be described in terms of the “quantum Smoluchowski” equation, is not able to accommodate self-sustaining flux due to the destructive role of quantum fluctuations since the steady state is effectively monostable.

The second regime is the regime presented in Fig. 2, where the onset to the multistable state of a noiseless system occurs. This regime is accessible either by lowering the temperature or using systems with a larger amplitude of persistent current—i.e., accommodating more coherent electrons. Here, the quantum corrections change significantly the properties of the system. Both $V_{eff}(x)$ and $\Phi(x)$ become multistable, which results in multistability of the steady state. The peaks are new since they do not appear at the “classically predicted” position but rather are shifted by approximately a quarter of flux quantum ϕ_0 . There is a natural interpretation of such peaks: if they occur at $x \neq 0$, they are related to self-sustaining fluxes in the system. Their lifetimes can be estimated using the well-established first-passage-time method.²¹

C. Lifetimes of self-sustaining flux states

The lifetimes of the zero- and non-zero-flux stationary states depend strongly on relation between the depth of the potential well of $V_{eff}(x)$ and temperature. Therefore they can be controlled by the system parameters. It is desirable to obtain these lifetimes much longer than the characteristic time τ_0 , according to which time is scaled; cf. the beginning of Sec. IV. Let us consider the regime presented in Fig. 2. The lifetime of any stationary state $x=x_s$ can be calculated as the mean first passage time $\tau(x_s; a, b)$ to leave the interval $[a, b]$ assuming that $x_s \in [a, b]$. It depends on the interval $[a, b]$ as well as on the boundary conditions (BCs). We can define the lifetime of the state $x_s=0$ as $\tau(0; -a, a)$ with two absorbing BCs at $x=\pm a$, where a is a little bit larger than the local maximum stucked around $x=0.0172$. Such a calculated time $\tau(0; -a, a) \approx 11 \times 10^3$. The lifetimes of the remainder states $|x_s| > 0$ can be defined as $\tau(x_s; a, b)$ with one absorbing and one reflecting BC. E.g., for $x_s=1/2$, one can take $a=0.488$ (which is on the left of the local maximum stucked around $x=0.4885$) as an absorbing BC and $b=0.52$ as a reflecting BC. Then $\tau(1/2; a, b) \approx 4.9 \times 10^3$. Analogously, $\tau(1; a, b) \approx 1.9 \times 10^3$. For comparison, the mean passage time from $x_s=1/2$ into $x_s=0$ is $\tau(1/2 \rightarrow 0) \approx 15 \times 10^4$ and from $x_s=1$ into $x_s=1/2$ is $\tau(1 \rightarrow 1/2) \approx 3 \times 10^4$. Moreover, $\tau(0 \rightarrow 1/2) \approx 8.6 \times 10^5$ and $\tau(1/2 \rightarrow 1) \approx 4 \times 10^7$. As a result, the system in this regime can effectively be treated as *tristable* with the reasonable level of accuracy.

D. Statistical enhancement of the magnetic flux

The problem of the flux amplitude is more subtle. The modified diffusion coefficient, depicted in the insets of Figs. 1 and 2, is periodic with respect to the magnetic flux x . If the magnetic flux is close to half-integer, the modified diffusion coefficient is smaller than the “classical” Einstein one. As a result of the interplay between this phenomenon and the shape of the modified potential, one observes *statistical enhancement* of the magnetic flux due to quantum noise. This enhancement is statistical since it allows one to *expect* an occurrence of the flux of some amplitudes with higher *probability* due to quantum features of thermal equilibrium fluctuations in the quantum Smoluchowski regime. This en-

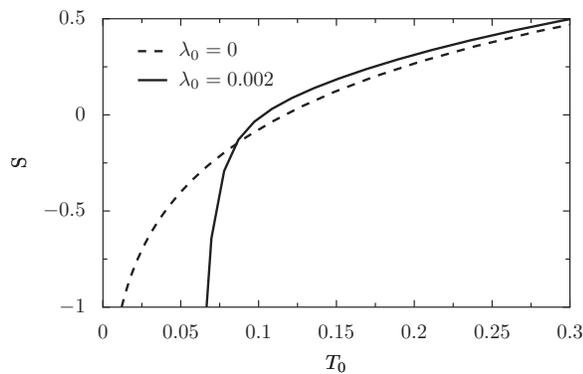


FIG. 3. “Quantum-corrected” entropy (solid line) and its fully classical counterpart versus temperature T_0 . The parameters are set as follows: $x_e=0$, $\epsilon=10$, $\alpha=0.1$, $p=0.5$, $k_0=0.5$, and $k_{Fl}=0.001$.

hancement is quantitative and, contrary to different approaches,^{6,7} this is a purely equilibrium effect.

E. Relevance of peaks—entropic criterion

There is a question if the peaks in the multistable state are meaningful, i.e., if they occur in a typical experiment performed on the system. The problem can be quantified in the following equivalent way: one can ask if the equilibrium statistics of the system is governed by ordered or quasiordered “phases.” As a measure of such the quasiorder, we exploit the celebrated Shannon entropy^{24,25}

$$S[P] = - \int_{-\infty}^{\infty} P(x) \ln P(x) dx. \quad (30)$$

A nonzero value of the quantum correction parameter $\lambda > 0$ results in a decrease of entropy at low temperature; i.e., the system becomes more ordered.²⁴ It is obvious that an effective order is due to increasing significance of the “events” occurring with high probability which are either vanishing or self-sustaining fluxes. We would like to stress that the entropic criterion does not characterize the stability of maxima or their lifetimes but rather a relative frequency of their occurrence. The Shannon entropy plotted for two systems—with and without quantum Smoluchowski corrections—is given, as a function of temperature T_0 , in Fig. 3. Working in the classical Smoluchowski regime—i.e., neglecting quantum fluctuations—results in a lowering of the overall order in the system at low temperature. We would like to clarify that this effect should not be interpreted as a *noise-induced* order. The lower entropy means simply that, contrary to the quantum Smoluchowski domain, the “classical” regime corresponds to the disorder which is *overestimated*.

F. Qutrit?

Bistable systems are natural candidates for qubits. The celebrated examples are Josephson-junction based devices which can be generally divided into two classes: charge and flux qubits.²⁶ It seems that a qubit can also be based on

nonsuperconducting materials.⁹ Because within tailored parameter regimes in the quantum Smoluchowski domain there are symmetric peaks in the multistable state, such a system is a good candidate for a *qutrit*. The problem of the qutrit implementation is of a central importance for quantum cryptography.²⁷

The following discussion is purely qualitative. We assume for simplicity that there are only three significant (in the statistical sense) peaks in probability distribution, as, e.g., in Fig. 2. Following Feynman’s discussion of the ammonia molecule²⁸ one can propose the “Hamiltonian” of the system as a 3×3 real symmetric matrix with diagonal elements proportional to the energy of the system calculated at magnetic flux extremal value via Eq. (7). The off-diagonal elements are proportional to the inverse of interpeak transition times. Let us notice that in the quantum Smoluchowski regime this transitions include tunneling effects. The phenomenological modeling of quantum dynamics of the classically dissipative system may cause certain difficulties: one arrives directly at a quantum dissipative system the “conservative” component of which may be chosen, to some extent, arbitrarily. The system under consideration can be effectively truncated to the “qutrit,” and it is a *mesoscopic example* of the generic V system.²⁹ Such a system controlled by external coherent driving—i.e., equipped with an auxiliary bosonic field(s)—can be naturally studied via quantum jump approach.²⁹

V. CONCLUSIONS

A steady state of the magnetic flux in mesoscopic rings is both qualitatively and quantitatively different in the classical and quantum Smoluchowski regimes. Quantum effects are responsible, in dependence of parameters values for both the destruction of bistability at moderate temperatures and the formation of n -stability, with n odd, at low temperatures. The nontrivial flux dependence of the steady state results in statistical enhancement of fluxes of certain amplitudes. This qualitative effect is caused by equilibrium quantum noise. Validity of the multistability has been verified via the entropic criterion. We showed that the quantum Smoluchowski regime is more ordered compared to its classical counterpart. As the mesoscopic ring is formally identical to the zero-capacitance SQUID, it seems that the quantum Smoluchowski regime is a valid regime for a wide range of parameters of the system and hence the effects described in the paper are of importance in experiments performed on mesoscopic rings which are multistable systems.

According to the common wisdom of *today*, solid-state devices seem promising for implementation of quantum computers. Both theoretical and experimental efforts have been mainly directed to superconducting qubits. They are relatively stable with respect to decoherence and are relatively accessible. The formation of flux qubits in a superconducting ring with a junction requires an external bias which shifts the system into the bistable state. It is not the case for the rings considered in the paper, and our results can be of importance for possible qutrit architecture based on nonsuperconducting devices. Such devices, due to their small diameters, can effectively become decoupled from the mag-

netic environment.⁹ This may equilibrate the absence of the superconducting phase with its collective properties. It is clear that capacitance, resistance, and coherent currents are the properties of the *whole* nonsuperconducting mesoscopic rings which are thus candidates for highly integrated quantum or semiclassical circuits.³⁰

ACKNOWLEDGMENTS

The work supported by the ESF Program “Stochastic Dynamics: fundamentals and applications” and the Polish Ministry of Science and Higher Education under Grant No. 202 131 32/3786.

-
- ¹L. P. Levy, G. Dolan, J. Dunsmuir, and H. Bouchiat, *Phys. Rev. Lett.* **64**, 2074 (1990); V. Chandrasekhar, R. A. Webb, M. J. Brady, M. B. Ketchen, W. J. Gallagher, and A. Kleinsasser, *ibid.* **67**, 3578 (1991); B. Reulet, M. Ramin, H. Bonchiat, and D. Mailly, *ibid.* **75**, 124 (1995); W. Rabaud, L. Saminadayar, D. Mailly, K. Hasselbach, A. Benoit, and B. Etienne, *ibid.* **86**, 3124 (2001); R. Deblock, R. Bel, B. Reulet, H. Bouchiat, and D. Mailly, *ibid.* **89**, 206803 (2002).
- ²D. Mailly, C. Chapelier, and A. Benoit, *Phys. Rev. Lett.* **70**, 2020 (1993).
- ³E. M. Q. Jariwala, P. Mohanty, M. B. Ketchen, and R. A. Webb, *Phys. Rev. Lett.* **86**, 1594 (2001); P. Mohanty, *Ann. Phys.* **8**, 549 (1999).
- ⁴F. Hund, *Ann. Phys.* **32**, 102 (1938).
- ⁵I. O. Kulik, *JETP Lett.* **11**, 275 (1970); M. Büttiker, Y. Imry, and R. Landauer, *Phys. Lett.* **96A**, 365 (1993).
- ⁶U. Eckern and P. Schwab, *J. Low Temp. Phys.* **126**, 1291 (2002).
- ⁷V. E. Kravtsov and B. L. Altshuler, *Phys. Rev. Lett.* **84**, 3394 (2000).
- ⁸D. Wohlleben, M. Esser, P. Freche, E. Zipper, and M. Szopa, *Phys. Rev. Lett.* **66**, 3191 (1991); M. Lisowski, E. Zipper, and M. Stebelski, *Phys. Rev. B* **59**, 8305 (1999).
- ⁹E. Zipper, M. Kurpas, M. Szelag, J. Dajka, and M. Szopa, *Phys. Rev. B* **74**, 125426 (2006).
- ¹⁰J. Dajka, J. Łuczka, M. Szopa, and E. Zipper, *Phys. Rev. B* **67**, 073305 (2003); J. Dajka, M. Kostur, J. Łuczka, M. Szopa, and E. Zipper, *Acta Phys. Pol. B* **34**, 3793 (2003).
- ¹¹A. Barone and G. Paterno, *Physics and Applications of the Josephson Effect* (Wiley, New York, 1982).
- ¹²J. Ankerhold, P. Pechukas, and H. Grabert, *Phys. Rev. Lett.* **87**, 086802 (2001).
- ¹³Ł. Machura, M. Kostur, P. Hänggi, P. Talkner, and J. Łuczka, *Phys. Rev. E* **70**, 031107 (2004); J. Łuczka, R. Rudnicki, and P. Hänggi, *Physica A* **351**, 60 (2005); Ł. Machura, J. Łuczka, P. Talkner, and P. Hänggi, *Acta Phys. Polon. B* **38**, 1855 (2007).
- ¹⁴J. Ankerhold, *Europhys. Lett.* **67**, 280 (2004); J. Ankerhold, P. Pechukas, and H. Grabert, *Chaos* **15**, 026106 (2005).
- ¹⁵H. F. Cheung, Y. Gefen, E. K. Riedel, and W. H. Shih, *Phys. Rev. B* **37**, 6050 (1989).
- ¹⁶J. Dajka, J. Łuczka, and P. Hänggi, *Phys. Status Solidi B* **242**, 196 (2005).
- ¹⁷J. Dajka, S. Rogoziński, Ł. Machura, and J. Łuczka, *Acta Phys. Polon. B* **38**, 1737 (2007).
- ¹⁸P. Kopietz, *Phys. Rev. Lett.* **70**, 3123 (1993).
- ¹⁹Y. Imry and B. L. Altshuler, in *Nanostructures and Mesoscopic Phenomena*, edited by W. P. Kirk and M. A. Reed (Academic, San Diego, 1992).
- ²⁰U. Weiss, *Quantum Dissipative Systems*, 2nd ed. (World Scientific, Singapore, 1999).
- ²¹C. W. Gardiner, *Handbook of Stochastic Methods* (Springer, Berlin, 1983).
- ²²H. Grabert, U. Weiss, and P. Talkner, *Z. Phys. B: Condens. Matter* **55**, 87 (1984).
- ²³W. H. Luisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973).
- ²⁴C. Beck and F. Schlögl, *Thermodynamics of Chaotic Systems: An Introduction* (Cambridge University Press, Cambridge, England, 1993).
- ²⁵A. Lasota and M. Mackey, *Probabilistic Properties of Chaotic Systems* (Cambridge University Press, Cambridge, England, 1985).
- ²⁶Y. Makhlin, G. Schön, and A. Schnirman, *Rev. Mod. Phys.* **73**, 357 (2001).
- ²⁷G. Molina-Terriza, A. Vaziri, R. Ursin, and A. Zeilinger, *Phys. Rev. Lett.* **94**, 040501 (2005).
- ²⁸R. P. Feynman, *Lectures on Physics* (Addison-Wesley, Reading, MA, 1964), Vol. 3.
- ²⁹M. B. Plenio and P. L. Knight, *Rev. Mod. Phys.* **70**, 101 (1998).
- ³⁰M. H. Devoret in *Quantum Fluctuations*, edited by S. Reynaud, E. Giacobino, and J. Zinn-Justin, Proceedings of the Les Houches Summer School of Theoretical Physics (North-Holland, Amsterdam, 1997), Vol. LXIII, p. 351; M. H. Devoret, A. Wallraff, and J. M. Martinis, arXiv:cond-mat/0411174 (unpublished).