Evolution and analysis of the vortex solid to vortex liquid melting line in $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ and $YBa_2Cu_3O_{6.5}$ to 45 tesla

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By extending magnetotransport measurements to fields up to 45 T, we have been able to examine the melting line of $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ (x=0–0.4) thin film samples and that of an oxygen deficient $YBa_2Cu_3O_{6.5}$ single crystal over an extended temperature range of $0.03T_c \leqslant T \leqslant T_c$, larger than heretofore reported. These measurements provide an opportunity to examine the evolution of the behavior of vortices along nearly the entire melting line of this important high-T_c superconducting system. Within the context of the quantum-thermal fluctuation model of Blatter and Ivlev [Phys. Rev. Lett. **70**, 2621 (1993)], we empirically arrive at a modified form of their expression for the vortex-lattice melting line by considering that the mechanism responsible for the vortex flux line relaxation time is not simply the scattering time of quasiparticles within the vortex core. This form of the melting line is demonstrated to fit over the *entire* melting line of each sample, implying no crossover in dynamics from three dimensions to two dimensions. This result implies that the physical mechanism responsible for the manner and conditions under which the melting of the vortex-solid ensemble takes place can be described smoothly over the entire temperature-field range. It is proposed that the relaxation time of a single vortex flux line, displaced by quantum-thermal fluctuations, is related to the critical behavior associated with the scaling properties of the fluctuation conductivity.

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I. INTRODUCTION

Since the discovery of the high-temperature layered cuprate superconductors, the physics of the mixed state of strongly type-II superconductors has received considerable attention. A significant amount of effort has been devoted to the development of a consistent theory of the melting transition of the vortex lattice. As there are many processes to be taken into consideration—thermal and quantum fluctuations, pinning mechanisms, anisotropy, coupling of the vortex lattice to the underlying electronic structure, and the critical dynamics of vortex motion as the melting transition is approached—arriving at an expression that is relevant over the entire vortex-lattice melting line has proven to be elusive.

In this paper, we present the results of an experimental investigation of the vortex-glass melting lines $H_g(T)$ of $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ thin film samples (x=0–0.4) and an ultrahigh purity oxygen deficient $YBa_2Cu_3O_{6.5}$ single crystal in magnetic fields up to 45 T, allowing a larger part of the $H_g(T)$ phase diagram to be accessed than previously possible. We analyze the $H_g(T)$ data within the context of the theory of Blatter-Ivlev (BI),[1,2] by introducing certain modifications. The vortex-lattice melting theory of BI takes into account the contribution of both quantum and thermal fluctuations to the melting of the vortex lattice, resulting in a universal form of the vortex-lattice melting line based on a Lindemann criterion approach. We find that the *entire* vortex-glass melting line $H_g(T)$ of each sample investigated here over reduced temperature ranges as large as $0.03 \leqslant T/T_c \leqslant 1$, as well as that of a $Sm_{1.85}Ce_{0.15}CuO_{4-y}$ film[3] ($0.1 \leqslant T/T_c \leqslant 1$) and a $Bi_2Sr_2CaCu_2O_8$ single crystal[4] ($0.2 \leqslant T/T_c \leqslant 1$), can be described by the expression of BI if we account for the temperature dependences of parameters held constant in their expression *and* if we assume that the relaxation time of a single vortex line takes the form

$$\tau_r^v = \tau_0 \left(\frac{T}{T_c}\right)^s \left(1 - \frac{T}{T_c}\right)^{-s}, \tag{1}$$

where T_c is the superconducting critical temperature in zero field. The $H_g(T)$ data from this study are shown in Fig. 1 in linear and semilogarithmic plots to emphasize the quality of the fits to the data of the phenomenological melting line equation given below.

The value of the exponent s, found by the fitting of our modified melting line expression of BI to the data, is seen to be in very good agreement with that found by a scaling of the resistivity data in the context of the vortex-glass melting model of Fisher-Fisher-Huse[5] (FFH) for each sample examined. This suggests that the two different approaches of BI

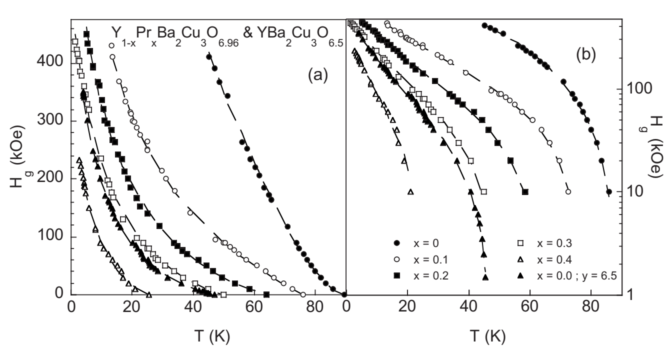

FIG. 1. Vortex melting line H_g vs T data for $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ films (1 kOe$<H<$450 kOe) and a $YBa_2Cu_3O_{6.5}$ single crystal (100 Oe$<H<$450 kOe) with fits of the modified melting line [Eq. (7)] shown in (a) linear and (b) semilogarithmic plots to emphasize the quality of the fits over the entire H-T range.

and FFH to the problem of the melting of the solid vortex ensemble are both valid and compatible perspectives on the physics involved, a point we return to in Sec. IV. We propose below that the above expression for τ_r^v can be obtained naturally from the scaling expression for the fluctuation conductivity,[5,6] and that the exponent s is the critical scaling exponent associated with the vanishing of resistivity $\rho \sim (T-T_g)^s$ at the critical temperature T_g, where s is either that found from the vortex-glass model of FFH[5] or the Bose glass model of Nelson and Vinokur.[7] Thus, any changes in the critical behavior of the vortex lattice along the melting transition, i.e., a change of the value of the critical exponent, should be reflected in the expression for the melting line describing each section with the different exponents. A vortex lattice in a highly anisotropic material, such as the high-T_c cuprates, may undergo a transition from three-dimensional (3D) to a two-dimensional (2D)-like (or quasi-2D) structure at a characteristic temperature or field when the interaction between vortex lines within a Cu-O plane becomes comparable to the elastic strength of a single vortex line, resulting in a decoupling of the pancake vortices.[8] We find that our phenomenological expression for the melting transition line describes the entire vortex-glass melting lines of $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ ($x=0-0.4$) thin film and $YBa_2Cu_3O_{6.5}$ single crystal samples and the electron-doped cuprate $Sm_{1.85}Ce_{0.15}CuO_{4-y}$,[3] with constant values of the critical exponent indicating no change in the dimensionality of the vortex lattice. However, the melting line of the highly anisotropic superconductor $Bi_2Sr_2CaCu_2O_8$, along which a well known 3D to 2D transition occurs at $H \approx 1$ kOe,[4] exhibits a change in the critical exponent from that of a 3D-*XY*-like (~ 3) to a 2D-*XY*-like ($\sim 6-8$) value at this field.

We use the terms vortex lattice and vortex glass interchangeably, mainly due to the different terminology used in Refs. 1, 2, and 5; however, it is also well known that in systems with sufficient random disorder, the first order vortex-lattice melting transition is replaced by a second order vortex-glass melting transition,[5] similar to what occurs when the elasticity of the vortex lattice becomes relevant.[2,9] It should be noted though that the expression for the melting line derived below is limited neither to a weak pinning regime nor to the anisotropic high-T_c cuprate materials.

Before continuing, we would like to state precisely the approach we have followed, of which there are three key points:

(1) The vortex-glass melting lines $H_g(T)$ [or $T_g(H)$] of the various samples in this study were determined experimentally via magnetoresistivity data within the context of the theory of FFH. It is important to recognize that the position of melting lines could have been determined without employing the formalism of the FFH model and, instead, by a criterion wherein the resistivity vanishes, with differences between the two methods well within the experimental error. Thus, we stress that the use of the FFH model to determine the position of the melting lines in no way biases the data and subsequent theoretical considerations or analysis toward a result that must then invoke the FFH model.

(2) An initial analysis of the vortex-glass melting line data was carried out in the context of the quantum-thermal fluctuation based vortex-lattice melting model of BI. Subsequently, two modifications to the model were introduced: (a) we allowed for the temperature dependence of the various physical properties which enter into the expression of BI, ξ, η, μ, etc., and (b) we considered the possibility that the relaxation time τ_r^v of a single vortex flux line would have a temperature dependence different than that assumed by BI. The striking result is that the modified expression is found to describe the entire vortex-glass melting line $H_g(T)$, with exceptional accuracy, over a temperature range well beyond that heretofore reported. We emphasize that only these changes were made and that all of the theoretical development presented with respect to the model of BI involves only straightforward algebraic procedure. The resulting melting line expression describes the melting field H_g as a function of temperature T with six physical quantities uniquely characterizing the shape of the melting line in the *H*-*T* phase diagram, $H_{c2}(0)$, T_c, G_i, c_L, s, and Q_0. Values for the first three quantities are determined from independent measurements or taken from the literature. In the analysis performed here, the latter three parameters are left as fitting parameters when fitting the expression for $H_g(T)$ to the experimental data set $[H_g, T_g]$. We also point out here that the values obtained from the fits for c_L, s, and Q_0 all agree well with those found independently.[1,3]

(3) Finally, we consider an ansatz wherein the single vortex relaxation time τ_r^v is related to the critical behavior of the fluctuation conductivity and, subsequently, to the scaling properties of the superfluid density. From this ansatz, an expression for τ_r^v is found, which is in agreement with the empirically determined expression. Support for this ansatz is found by comparing the theoretical expressions for the time scales of the quantum tunneling of a vortex core[10] t_{qt} and the Ginzburg-Landau Cooper pair fluctuation lifetime[8] τ_{GL}, from which it can be seen that the region surrounding the 'classical' vortex core likely contains a significant density of Cooper pairs in the fluctuation regime. We believe that the ansatz provides a reasonable and consistent explanation for the empirical result and, furthermore, stress that the ansatz in no way affects the steps involved in the modification of the melting line expression of BI and the subsequent analysis, with the sole exception being that the exponent s is then found to be related to the well known critical exponents ν and z of the model of FFH. The meaning of the values of the remaining parameters in the melting line expression, c_L, and Q_0 are independent of whether or not the exponent s is, in fact, a critical exponent.

II. EXPERIMENTAL DETAILS

The $H_g(T)$ line was determined from electrical resistivity $\rho(H,T)$ measurements with $H \parallel c$. The value of $T_g(H_g)$ was determined from the temperature (field) at which the resistance disappears, in accordance with the FFH vortex-glass scaling expression of the resistivity as the melting transition is approached by decreasing T at constant H,[5] $\rho \sim (T-T_g)^{\nu(z+2-d)}$ [or decreasing H at constant T, $\rho \sim (H-H_g)^{\nu(z+2-d)}$],[11] where ν and z are the correlation length and dynamical exponents and d is the dimensionality of the vortex lattice. The value of T_g or H_g is the temperature or field at which the linear relation between $[d\log(\rho)/d(T,H)]^{-1}$ and (T,H) that results from the FFH expression vanishes. Low field measurements, $H\leqslant 9$ T, were made at UCSD, whereas high field measurements up to 45 T were carried out at the National High Magnetic Field Laboratory (Tallahassee, FL). Further experimental details will be included in a separate publication.[11]

III. MODIFICATION OF THE BLATTER-IVLEV MODEL

In their seminal work, BI[1,2] included the contribution of quantum fluctuations to the statistical mechanics of the vortex ensemble of a type-II superconductor. The scope of the classical formalism based on the continuum elastic theory for the vortex lattice was extended to a dynamic formalism based on the Euclidean action. Combining this theoretical framework with the Lindemann criterion, they establish a universal form of the melting line $H_m(T)$ $[\equiv H_g(T)]$.

It should be noted that two slightly different expressions are derived in Refs. 1 and 2. In their initial work,[1] when calculating the mean squared displacement amplitude $\langle u^2\rangle$, a term involving compressional modes is dropped. In their later work,[2] this term is retained.

For the first case, BI obtain

$$H_m(t) = \frac{4H_{c2}(0)\theta^2}{(1+\sqrt{1+4Q\theta})^2}, \tag{2}$$

where θ is a reduced temperature given by $\theta=(\pi c_L^2/\sqrt{G_i})(1-t)$, $Q=[\tilde{Q}_u/(\pi^2\sqrt{G_i})]\Omega\tau_r$ is a parameter measuring the relative strength of quantum to thermal fluctuations, $t\equiv T/T_c$, $\tilde{Q}_u=\frac{e^2}{\hbar}\frac{\rho_N}{d}$ is the dimensionless quantum of resistance, c_L is the Lindemann number, $G_i=[T_c/H_c^2(0)\epsilon\xi^3(0)]^2/2$ is the Ginzburg number, Ω is a cutoff frequency, τ_r is the scattering relaxation time of the quasiparticles in the vortex core given by the Drude formula $\rho_N^{-1}=\sigma_N=e^2 n\tau_r/m$ (σ_N is the normal state conductivity, n is the free-charge-carrier density, and m the electron mass), and here, d is the distance between the superconducting planes. Ω is given by $\Omega=\min[\Omega_\rho,\Omega_i]$, where $\Omega_\rho\approx\sqrt{\eta_\ell/\mu_\ell\tau_r}$ is the kinetic cutoff frequency associated with the electromagnetic contribution to the vortex mass, and $\Omega_i\approx\frac{2}{\hbar}\Delta$ is the intrinsic cutoff given by the energy gap due to the creation of quasiparticles by vortex motion.

For the latter case, they find the same form as Eq. (2) with Q replaced by S/t, and now $\theta=c_L^2\sqrt{\frac{\beta_{th}}{G_i}}\frac{T_c}{T}(1-t)$, $S=q+c_L^2\sqrt{\frac{\beta_{th}}{G_i}}$, and $q=\frac{2\sqrt{\beta_{th}}}{\pi^3}\frac{Q_u}{\sqrt{G_i}}\Omega\tau_r$. Either expression can be approximated by the power-law form $H_m\sim(1-t)^\alpha$ over temperatures T ranging from T_c down to $0.6T_c$. By estimating values for $\tilde{Q}_u$ and $\sqrt{G_i}$ and leaving c_L and $\Omega\tau_r$ as fitting parameters in Ref. 2, they find $\alpha\approx 1.45$, in close agreement with experimental values. As pointed out by BI, the value of the approximate exponent α depends on the quantum parameter Q and the reduced temperature θ. This then explains the experimentally observed increase of α as the temperature drops below $T\sim 0.6T_c$.[12]

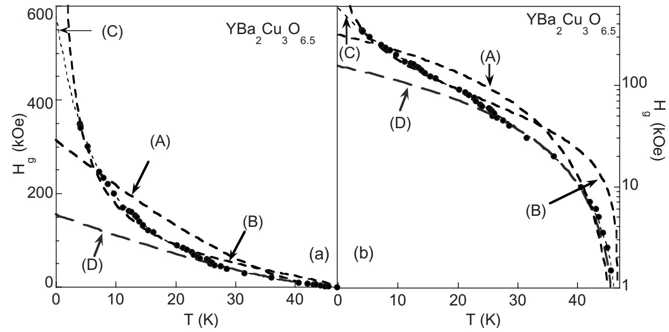

FIG. 2. Comparison of the quality of the fits to the entire vortex melting line data of the $YBa_2Cu_3O_{6.5}$ single crystal by the expressions of BI (Ref. 1) (A), Lundqvist *et al.* (Ref. 13) (B), and that obtained here (C) [Eq. (7)] shown in (a) linear and (b) semilogarithmic plots, demonstrating that only Eq. (7) adequately describes the data over the entire H-T range. Also shown is the empirically observed expression (Ref. 12) $H_m\sim(1-t)^\alpha$, for $T\gtrsim 0.6T_c$ (D).

For simplicity, we focus here on the original expression in Ref. 1. The analytical procedure below is valid for the latter expression in Ref. 2 as well, and further details will be given in a future publication.[11] Some results are cited here for comparison. In particular, in the large Q (or q) limit, the modified form of either expression, which is found here and in Ref. 10, reduces to the form of the expression found empirically by Lundqvist *et al.*,[13] $H_g=H_0[(1-T/T_c)/(T/T_c)]^\alpha$, with $\alpha=s/2$. This expression was found to fit smoothly to the entire range of data examined there ($H\leqslant 12T$) but, as seen in Fig. 2, failed to fit the larger range of data shown here.

Instead of putting in constant values or approximations for the various factors of the quantum parameter Q, we leave them in their exact temperature and field dependent forms. Additionally, we allow for two separate intrinsic relaxation times; one given by the quantized level splittings of the quasiparticle states within the vortex core,[14,15] τ_r^{core}, and the second given by the motion of the single vortex flux line through the surrounding superfluid environment, τ_r^v, with $\tau_r^v\gg\tau_r^{core}$ in the region of the melting transition. These time scales, which arise from different physical mechanisms—one involving the electronic states within the vortex core and the other the critical behavior associated with single vortex flux line fluctuations—together determine the final melting condition.

With $\sigma_N\propto\tau_r^{core}$, we then identify τ_r^v as the relevant relaxation time to be associated with the cutoff frequency $\Omega_\rho\approx\sqrt{\eta_\ell/\mu_\ell\tau_r^v}$ and the expression for the quantum parameter $Q=\frac{\tilde{Q}_u}{\pi\sqrt{G_i}}\Omega\tau_r^v$. If the condition $\tau_r^v\gg\tau_r^{core}$ holds, then $\Omega_\rho<\Omega_i$. (For $YBa_2Cu_3O_{7-\delta}$, BI[2] find $\Omega_\rho\sim 10\Omega_i$ using the assumption $\tau_r^v=\tau_r^{core}$.) We include the temperature dependence of $\xi=\xi_0/(1-t)^{1/2}$ in the kinetic cutoff frequency $\Omega_\rho\approx\sqrt{\eta_\ell/\mu_\ell\tau_r^v}\sim 1/(\xi\sqrt{\tau_r^v})$ and in the expression for the dimensionless quantum of resistance given in Ref. 15,

$$\tilde{Q}_u=\frac{e^2}{\hbar}\frac{\rho_N}{\epsilon\xi}. \tag{3}$$

Then, using the field dependent expression of the Ginzburg number,[8]

$$G_i(H) \approx (G_i)^{1/3}\left(\frac{H}{H_{c2}(0)}\right)^{2/3}, \tag{4}$$

where H is evaluated along the melting line at $H=H_g$, the vortex relaxation time τ_r^v, given in Eq. (1), the temperature dependence of ξ, and with all expressions to be evaluated at the melting temperature, $T=T_g\equiv T_m$, we arrive at expressions for $\Omega[H,T]$, $Q[H,T]$, and, lastly, the modified form of vortex-glass melting line:

$$\Omega = \Omega_0(t)^{-s/2}(1-t)^{(1+s)/2}, \tag{5}$$

$$Q(H_g,T_g) = \frac{\tilde{Q}_0\Omega_0\tau_0}{\pi^2\sqrt{G_i(H_g)}}t^{s/2}(1-t)^{1-s/2}, \tag{6}$$

$$H_g(t) = \frac{4H_{c2}(0)\dfrac{(\pi c_L^2)^2}{G_i(H_g)}(1-t)^2}{\left(1+\sqrt{1+4(\tilde{Q}_0\Omega_0\tau_0)\dfrac{c_L^2}{\pi G_i(H_g)}t^{s/2}(1-t)^{2-s/2}}\right)^2}, \tag{7}$$

with $\Omega_0\equiv\sqrt{\frac{\eta_\ell(0)}{\mu_\ell^{em}(0)}}$, $\tilde{Q}_0\equiv\frac{e^2}{\hbar}\frac{\rho_N}{\epsilon\xi_0}$, and $t\equiv T/T_c$.

IV. DISCUSSION

The melting line data $H_g(T)$ obtained in this study of $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ films and a $YBa_2Cu_3O_{6.5}$ single crystal were fitted by Eq. (7) [Figs. 1(a) and 1(b)] and that obtained in Ref. 2 from the latter equation of BI[2] (not given). With the experimental value $Q_0=0.34\pm0.15$ obtained from Eq. (7), we solve for the value of $\Omega_0\tau_{r0}^v$ for the $YBa_2Cu_3O_{7-\delta}$ film. Using $\xi_0=12$ Å, $\epsilon=1/8$,[8] and $\rho_N\approx 2\times10^{-5}$ Ω cm, we obtain $\Omega_0\tau_{r0}^v\approx 1.1$. If we use the second modified melting line equation found in Ref. 2 derived from the form given in Ref. 4, then we get an equally good fit, indistinguishable from Eq. (7) with similar values of $\nu(z-1)$ and c_L, with $q_0 \sim(1/2-1/10)\times Q_0$. The result $q<Q$ can be understood by recalling that Q (and q) is a measure of the ratio of the quantum to thermal contributions to vortex line displacements. The inclusion of the additional thermal contribution of the compressional modes in the modified form in Ref. 2 of the second melting line equation of BI[2] leads to a relative increase of the contribution of thermal displacements over that of quantum displacements; so naturally, $q<Q$. The nonmonotonic variation of Q_0 with respect to Pr doping and reduced oxygen content (see Table I) results from the dependence on many physical properties of these systems, including the magnetic field penetration depth λ, the superconducting coherence length ξ, the critical temperature T_c, anisotropy of the system, and the strength and type of disorder. The value of the quantum of resistance $\tilde{Q}_u(0)$ of each system is given in Table I and is seen to exhibit a similar trend, particularly with respect to the values for the $YBa_2Cu_3O_{7-\delta}$ and $YBa_2Cu_3O_{6.5}$ samples. The values of s found here are notably different for the clean $YBa_2Cu_3O_{7-\delta}$ and the disordered $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ and $YBa_2Cu_3O_{6.5}$ systems and are presumably due the dynamical response of the vortices being dependent on more than just the extent of disorder. A possible explanation is that the dominant type of disorder expected in the pure $YBa_2Cu_3O_{7-\delta}$ sample consists of screw dislocations and that in the $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ and $YBa_2Cu_3O_{6.5}$ samples is primarily composed of clusters of Pr atoms and oxygen vacancies, respectively.

TABLE I. Values of the Lindemann number c_L, critical exponent $s\equiv\nu(z+2-d)$, the quantum parameter $Q_0\equiv\tilde{Q}_0\Omega_0\tau_0$ using Eq. (7) and the quantum of resistance $\tilde{Q}_u(0)\equiv\tilde{Q}_0$ for the data in Fig. 1.

x	c_L	s	Q_0	$\tilde{Q}_0$
0	0.31	3.33	0.34	0.32
0.1	0.28	1.90	11.5	10.1
0.2	0.29	2.07	12.8	16.2
0.3	0.30	2.10	9.8	15.1
0.4	0.27	2.22	9.6	18.0
		$y=6.5$		
0	0.28	2.21	16.9	37.6

Additionally, Eq. (7) was used to analyze melting line data from a $Sm_{1.85}Ce_{0.15}CuO_{4-y}$ film[3] [Fig. 3(a)] and a $Bi_2Sr_2CaCu_2O_8$ single crystal[4] [Fig. 3(b)]. The data for all of the above samples are well described by Eq. (7) over the entire range of field and temperatures examined, with the exception of the $Bi_2Sr_2CaCu_2O_8$ data, which can be fitted by Eq. (7) in two segments, with a corresponding change in the critical exponent s. This latter case is interpreted readily as evidence for a 3D-2D vortex-glass transition at $H_{2D}\approx 1$ kOe, in agreement with the conclusions of many different studies in this highly anisotropic compound.[4,16–19] Since the data in the other cases considered here can be fitted over the entire (available) range of the $H_g(T)$ line with one set of critical exponents corresponding to three dimensions, this implies that the vortex glass remains in a 3D state along the entire line.

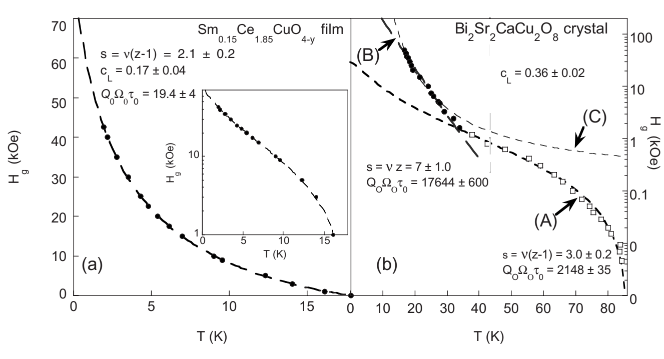

FIG. 3. (a) Fit of Eq. (7) to the melting line $H_g(T)$ of a thin film of $Sm_{1.85}Ce_{0.15}CuO_{4-y}$ (Ref. 3). The inset shows the same data on a semilogarithmic plot. (b) Fit of Eq. (7) to $H_g(T)$ of a $Bi_2Sr_2CaCu_2O_8$ single crystal (Ref. 4). A 3D-2D transition at $H_{2D}\approx 1$ kOe is inferred by a corresponding change of the exponent $\nu(z+2-d)$ from a 3D-*XY*-like (A) to 2D-*XY*-like (B) value. The upper portion of the curve is fitted to the 2D melting line expression of Schilling *et al.* (Ref. 4) (C) for comparison.

While the expression we arrive at in Eq. (7) is empirical in the sense that we have assumed a temperature form of the single vortex line relaxation time τ_r^v, we find a natural explanation for this form by considering the following ansatz.

The relaxation time of a vortex line associated with a displacement from an equilibrium position, in the region of the melting transition, has its origins in the critical behavior of the fluctuation conductivity associated with the zero field superconducting transition near T_c, not just that of the quasiparticle scattering times in the normal core. The basic idea is that since the quantum fluctuation induced motion of the vortex (over the length scale of the core diameter) involves a continuous process of breaking and reforming Cooper pairs in the physical region encompassed by the fluctuating position of the normal state core, the fluid through which the individual vortex line is moving (on the length scale of the quantum fluctuation) has the same properties as the zero field fluctuation conductivity. The relevant temperature scale for this process remains that of the zero field case, i.e., T_c.

The vortex-glass model of FFH is a generalization of the zero field transition where the ac fluctuation conductivity in the critical region as $T \rightarrow T_c$ scales as

$$\sigma(\omega) \sim \tau\rho_s \sim \xi^{z+2-d}\mathcal{S}(\omega\xi^z),$$

where ρ_s is the superfluid density, τ is the time scale associated with the coherence of fluctuating Cooper pairs, and $\xi \sim |1-T/T_c|^{-\nu}$. If the single vortex line relaxation time τ_r^v does, in fact, scale with the fluctuation conductivity, then it can be seen that τ_r^v is also related to the critical dynamics of the vortex-glass melting transition. While the Bose glass model of Nelson and Vinokur describes the physics of the melting transition of a correlated vortex-glass in terms of a wandering length scale and wandering time, in contrast to the correlation length scale and correlation time of FFH, Lidmar and Wallin[20] have shown that the scaling properties of a Bose glass also arise from the scaling properties of the superfluid density; thus, the two vortex-glass melting theories are quite related. It is the expression for the single vortex relaxation time associated with vortex fluctuations that we seek to use in the model of Blatter and Ivlev.[1,2] It should be emphasized that the vortex relaxation time we are referring to *is not* the "relaxation time" directly associated with the correlation time of the vortex-glass correlation length fluctuations, $\tau_{VG} \sim \xi_{VG}^z \sim (T-T_g)^{-\nu z}$.

Blatter and Ivlev assume that the relaxation time τ_r for the vortex displacements comes from the normal state conductivity where $\tau_r = \frac{m}{ne^2}\sigma$. Since we are concerned with the dynamics of vortex fluctuations, following the reasoning in the ansatz above, we use the expression for the zero field ac fluctuation conductivity to arrive at an expression for the single vortex relaxation time in the region of $T=T_g$,

$$\tau_r^v \sim \sigma_f \sim \xi^{z+2-d} \sim (T_c/T)^s(1-T/T_c)^{-s},$$

where $s \equiv \nu(z+2-d)$, and τ_r^v is to be evaluated along the melting line at $T=T_g$.

Alternatively, since the FFH vortex-glass theory is a generalization of the zero field dynamic scaling of a second order transition where the ac fluctuation conductivity in the critical region as $T \rightarrow T_c$ scales as

$$\sigma(\omega) \sim \xi^{z+2-d}\mathcal{S}(\omega\xi^z),$$

with $\xi \sim |1-T/T_c|^{-\nu}$, it can then be seen that each $T_g(\omega,H)$ line is shifted from $T_g(H)$ according to the generalized scaling properties of the zero field critical point T_c.[5,21] The vortex-glass melting line is understood as a line of critical points, where $T_c \Rightarrow T_g$, so that $\xi_{VG} \sim |1-T/T_g|^{-\nu}$. From this, we would expect the behavior of the intrinsic vortex relaxation frequency ω_r to also have its origin in the properties of the critical point T_c, so that the fluctuation conductivity $\sigma \sim |1-T/T_c|^{-\nu(z+2-d)}$ will determine the behavior of the time scale of vortex line fluctuations along the melting line. From $\omega_m(T_g) \sim \sigma_{VG}^{-1}(T_g)$, we would expect

$$\omega_r(T=T_g) \sim \sigma^{-1}(T_g) \sim (T_c/T_g)^s(1-T_g/T_c)^{-s}.$$

The idea that the intrinsic single vortex relaxation time τ_r changes as a function of temperature along the melting line is akin to the idea that the pinning potential changes along the melting line, an idea which Rydh *et al.*[22] used to modify the vortex-glass theory of FFH. This modification of the FFH theory was motivated by a prior empirical scaling form of the vortex melting line by Lundqvist *et al.*,[13] who referred to the model of Blatter and Ivlev[2] as possible theoretical support for their melting line expression. In the large Q limit, our expression for the melting line reduces to that found by Lundqvist *et al.*, so, from the prior work of Lundqvist *et al.* and Rydh *et al.*, it can be seen that there does indeed appear to be a relation between the thermal- and quantum-fluctuation Lindemann criterion based model of BI and the critical dynamic vortex-glass model of FFH, and, by extension, the Bose glass model.

Additional support for the ansatz can be found by considering two important time scales: the lifetime of fluctuating Cooper pairs,[8] τ_{GL}, and the quantum tunneling time of the vortex core,[10] $t_{qt}=(\eta_\ell L_c^2)/(\epsilon_o)$. With $\epsilon_o=[\Phi_0^2/(4\pi\lambda)]^2$ and $L_c \simeq 10\epsilon\xi$ (Ref. 15) ($\epsilon=1/\gamma$ is the anisotropy parameter), we can compare the ratio t/τ_{GL} (in mks units) using

$$\tau_{GL} = \frac{\pi\hbar}{16k_BT_c}\frac{1}{(1-T/T_c)}, \tag{8}$$

$$t_{qt} \approx 2\mu_0(10\epsilon)^2\frac{\lambda_0^2}{\rho_N}\frac{1}{(1-T/T_c)}, \tag{9}$$

giving

$$\frac{t_{qt}}{\tau_{GL}} \approx \left[\frac{32\mu_0k_B}{\pi\hbar}\right]\frac{\lambda_0^2}{\rho_N}(10\epsilon)^2T_c. \tag{10}$$

In the absence of a magnetic field, the momentum dependence of the Cooper pairs has been accounted for leading to an expression for the fluctuation Cooper pair lifetime $\tau_{GL}(\mathbf{p},t)$ such that[8]

TABLE II. Values of the ratio of the quantum tunneling time of the vortex core with respect to the momentum independent and momentum dependent Cooper pair lifetimes, t/τ_{GL} and $t/\tau_{GL}(\mathbf{p})$, calculated from Eq. (10) using the values of T_c and λ_0 from Ref. 11 and the values of the anisotropy parameter $\gamma \equiv 1/\epsilon$ for $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ from Ref. 26 and for $YBa_2Cu_3O_{6.5}$ given in Ref. 27. The values of ξ_0 are derived from the value of $H_{c2}(0)$ obtained from the fits to the data.

x	T_c (K)	ξ_0 (Å)	λ_0 (nm)	ρ_N ($\mu\Omega$ m)	t/τ_{GL}	$t/\tau_{GL}(\mathbf{p})$
0	89.4	13	143	0.2	28	150
0.1	76.0	17	172	8.4	0.6	3.4
0.2	64.0	20	193	9.3	0.4	2.2
0.3	50.0	26	219	9.8	0.4	2.1
0.4	25.5	29	293	10.3	0.4	0.9
			$y=6.5$			
0	47.0	22	149	2.2	0.8	1.6

$$\tau_{GL}(\mathbf{p},t) = \frac{\tau_{GL}}{[1+(\xi\mathbf{p}/\hbar)^2]}. \tag{11}$$

Using the relations $\mathbf{p}^2=(m^* v_F)^2$ and $v_F=\pi\Delta\xi_0/\hbar$, Eq. (11) becomes

$$\tau_{GL}(\mathbf{p},t) = \frac{\tau_{GL}}{[1+(\pi m^* \Delta_0 \xi_0^2/\hbar^2)^2]}. \tag{12}$$

Using the values of ξ_0, ρ_N, and $\epsilon=1/\gamma$ given in Table II and λ_0 for the $x=[0,0.4]$ samples from Ref. 11 and estimated for the $YBa_2Cu_3O_{6.5}$ sample from Ref. 23 (given in the table above) and of Δ in Ref. 24 and estimating Δ_0 for the $x=0.1-0.4$ samples from the $x=0$ value, using $\Delta_0 \propto \sqrt{T_c}/(\lambda_0\xi_0)$ [see Eq. (5) of Ref. 25], we give the results in Table II above. We have tabulated values of t/τ_{GL} and $t/\tau_{GL}(\mathbf{p})$ for the condition $L_c \approx 10\epsilon\xi$, and with the requirement that $L_c=\max[10\epsilon\xi,\xi]$. This latter condition is considered to be more likely since for $L_c \approx \xi$, the distortion of the vortex flux line is large, and simple elastic theory breaks down.[10] We must emphasize that the values presented are order of magnitude estimates, recognizing that the relations used above are not necessarily exact. In particular, the expression for $\tau_{GL}(\mathbf{p},t)$ will most likely change in field, and the relation $L_c \simeq 10\epsilon\xi$ was estimated by Blatter *et al.*[10] using physical quantities characteristic of pure YBCO. Nevertheless, the calculated values demonstrate that these two key time scales are near to or are of the same order of magnitude *over the entire temperature range* of the vortex-lattice and/or -glass melting line. This result then strongly suggests that, since quantum fluctuations of the vortex core must create and destroy Cooper pairs within and adjacent to the classical core position, the region over which the vortex core is smeared retains a large density of Cooper pairs in a fluctuation regime. It follows then that conductivity of the immediate environment of the classical vortex core will have a non-negligible fluctuation contribution, and hence the dynamics of the core will reflect this contribution.

The inclusion of the relaxation time found above [Eq. (8)] in the model of BI may seem contradictory, i.e., the use of a single vortex relaxation time τ_r^v involving a critical exponent, which implies a second order phase transition, with a Lindemann criterion based model of the first order melting transition of a vortex lattice. However, as pointed out in Refs. 28–30, while the Lindemann criterion is useful for locating important physical phenomena in the phase diagram, it cannot by itself predict whether a given transition is first order, second order, or simply a crossover.[31] It can be applied in various forms to both ordered and disordered systems. Recently, three variations of the Lindemann criterion were developed by Kierfeld and Vinokur to describe transitions in the phase diagram of type-II superconductors, one of which is the vortex-glass to vortex-liquid transition.[32] Further examples include the work of Morello *et al.*[33] and of Okuma *et al.*,[28] where the experimentally determined irreversibility and vortex-glass melting lines of $Bi_{2-x}Sr_{2-(x+y)}Cu_{1+y}O_{6\pm\delta}$ single crystals and α-Mo_xSi_{1-x} amorphous films, respectively, were analyzed using the same Lindemann criterion based theory of BI[1,2] on which our expression for the melting line is based.

V. CONCLUSION

The H_g-T phase diagrams of the high-temperature superconducting cuprate systems $Y_{1-x}Pr_xBa_2Cu_3O_{6.97}$ ($x=0-0.4$) and $YBa_2Cu_3O_{6.5}$ have been extended to high magnetic fields not previously explored for this system. These measurements have made it possible to examine almost the entire melting line. Within the context of the quantum-thermal fluctuation model of Blatter and Ivlev,[1,2] we have empirically arrived at a modified form of their universal expression for the vortex-lattice melting line by considering that the vortex flux line relaxation time is not solely dependent on the normal state scattering time of quasiparticles within the vortex core. This form of the melting line is demonstrated to fit over the entire melting line of each sample, implying no crossover in dynamics from three dimensions to two dimensions. This result also implies that the physical mechanism responsible for the manner and conditions under which the melting of the vortex-solid ensemble takes place can be described continuously over the entire temperature-field range. It is proposed that the single vortex relaxation time is related to the critical behavior associated with the scaling properties of the fluctuation conductivity.

The exceptionally good agreement between the experimentally determined vortex-lattice melting lines and the fits to the data by the melting line expression proposed here, and the agreement of the exponent s found from the above analysis with the exponent $\nu(z+2-d)$, found from scaling of the resistivity in the critical region of the vortex-glass melting transition, supports the idea that dynamical properties of a vortex-glass and of a single vortex line at the melting transition have the same physical origin, arising from the scaling properties of the fluctuation conductivity. Incorporating this notion into the work of BI[1,2] apparently results in an expression for the melting line that unifies the quantum-thermal nature of vortex fluctuations with the critical dynamic properties of vortices in the region of the melting line transition, providing a perspective from which to investigate the microscopic origin of superconducting properties.

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