Three-particle collisions in quantum wires: Corrections to thermopower and conductance

Anders Mathias Lunde,[1,2] Karsten Flensberg,[1] and Leonid I. Glazman[2]
[1]*Nano-Science Center, Niels Bohr Institute, University of Copenhagen, DK-2100 Copenhagen, Denmark*
[2]*William I. Fine Theoretical Physics Institute, University of Minnesota, Minneapolis, Minnesota 55455, USA*
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We consider the effect of electron-electron interaction on the electron transport through a finite length single-mode quantum wire with reflectionless contacts. The two-particle scattering events cannot alter the electric current and therefore we study the effect of three-particle collisions. Within the Boltzmann equation framework, we calculate corrections to the thermopower and conductance to the leading order in the interaction and in the length of wire L. We check explicitly that the three-particle collision rate is identically zero in the case of several integrable interaction potentials. In the general (nonintegrable) case, we find a positive contribution to the thermopower to leading order in L. The processes giving rise to the correction involve electron states deep in the Fermi sea. Therefore, the correction follows an activation law with the characteristic energy of the order of the Fermi energy for the electrons in the wire.

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I. INTRODUCTION

Short clean one-dimensional (1D) mesoscopic wires, often referred to as quantum point contacts, show conductance quantization[1,2] as a function of the channel width. The quantization is well described by the theory of adiabatic propagation of free electrons.[3] For noninteracting particles, conductance quantization should occur in longer channels too, as long as there is no backscattering off inhomogeneities within the channel.

A lot is known about the role of electron-electron interaction of 1D channels. Electron-electron repulsion in a wire enhances dramatically the reflection coefficient, making it energy dependent.[4] However, interaction between electrons does not alter the quantization (in units of $2e^2/h$) of an ideal channel conductance in the limit of zero temperature.[5,6] What is still an open question is whether there are other manifestations of interactions due to inelastic processes, which influence the transport properties.

In the absence of interactions, left- and right-moving particles in a wire are at equilibrium with the reservoirs they originate from. If a bias is applied between the reservoirs, then these equilibria differ from each other, giving rise to a particular form of the nonequilibrium distribution inside the channel. On the other hand, in a long ideal channel and in the presence of interactions, one may expect equilibration to occur between the left and right movers into a single distribution characterized by an equilibrium with respect to a reference frame moving with some drift velocity. Interestingly, in a model with momentum-independent electron velocity for left and right movers (as it is the case in the Tomonaga-Luttinger model), there is no difference between the two distributions. Effects originating from the particle-hole asymmetry, however, may discriminate between the two. Thermopower and Coulomb drag[7–9] are examples of such effects.

At present, little is known about equilibration in a 1D electron system. In higher dimensions, the electron-electron interaction provides the most effective relaxation mechanism at low temperatures and therefore we include this relaxation mechanism as the first approach. However, in 1D pair collisions cannot change the distribution function for quadratic dispersion, since the momentum and energy conservation[10] laws result in either zero-momentum exchange or an interchange of the two momenta.[11] In either case, the distribution function remains the same. Thus, the leading equilibration mechanism is due to three-particle collisions, which we study in this paper.[12]

We investigate here the effects of three-particle collisions in reasonably short wires (see Fig. 1), where electron-electron scattering can be considered perturbatively. As measurable quantities, we evaluate the temperature dependence of the thermopower and conductance. Note that for more than one mode, pair collisions become important for certain fillings.[13]

The paper is organized as follows. First we review the noninteracting limit of thermopower and give a qualitative explanation of the effects due to three-particle collisions. Then, we describe how to include the electron interactions using the Boltzmann equation. Next, we calculate the main ingredient for our perturbation theory, namely, the three-particle matrix element and scattering rate using a T-matrix expansion. We note several interesting properties of this scattering rate. Finally, we derive the conductance and thermopower corrections and discuss the deviation from the so-called Mott formula. Furthermore, some technical details are

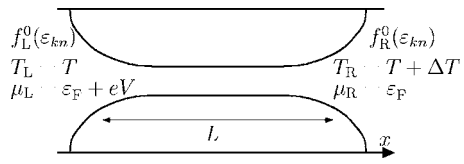

FIG. 1. A schematic picture of two metallic gates depleting the underlying two-dimensional electron gas and thereby forming a short 1D quantum wire of length L. This fabrication method has the advantage of producing reflectionless contacts to the leads (Ref. 3), so that the boundary conditions of the distribution function are given by the Fermi function of the reservoirs. We define the thermopower as $S=V/\Delta T|_{I=0}$, i.e., the voltage V required to counteract a current due to the temperature difference ΔT.

put in two appendices, and in Appendix A we show that the number of left and right movers have to change in a scattering event for the current to change.

A. Thermopower in the noninteracting limit

For a wire without interactions, the distribution function $f^{(0)}$ is determined solely by the electron reservoirs,

$$f_k^{(0)} = \begin{cases} f^0(\varepsilon_k - \mu_L, T_L) \equiv f_L^0(\varepsilon_k) & \text{for } k > 0, \\ f^0(\varepsilon_k - \mu_R, T_R) \equiv f_R^0(\varepsilon_k) & \text{for } k < 0, \end{cases} \tag{1}$$

where ε_k is the dispersion relation for momentum k and spin σ (suppressed in the notation), and $f^0(\varepsilon-\mu,T)=(1+\exp[(\varepsilon-\mu)/k_BT])^{-1}$ is the Fermi function with $\mu_{L/R}$ and $T_{L/R}$ denoting the chemical potential and temperature of the left/right contact, respectively (see Fig. 1). The electric current for low temperature $T \ll T_F$ and in linear response to the applied bias V and temperature difference $\Delta T \ll T$ then follows as ($e>0$) (Ref. 14)

$$I^{(0)} = \frac{(-e)}{L} \sum_{\sigma k>0} v_k [f_L^0(\varepsilon_k) - f_R^0(\varepsilon_k)], \tag{2}$$

$$\simeq -\frac{2e^2}{h} V(1 - e^{-T_F/T}) + \frac{2e}{h} k_B \Delta T \frac{T_F}{T} e^{-T_F/T}. \tag{3}$$

From this, the well-known leading-order results for conductance,

$$G^{(0)} = \frac{2e^2}{h}(1 - e^{-T_F/T}), \tag{4}$$

and for thermopower,

$$S^{(0)} = \frac{k_B}{e}\frac{T_F}{T} e^{-T_F/T}, \tag{5}$$

for a fully open channel are obtained. Here $T_F \equiv \varepsilon_F/k_B$ is the Fermi temperature.

B. Main results and a simple picture of the effect of the three-particle scattering

One of the main results of this paper is that the three-particle collisions give a *positive* contribution to the thermopower, i.e., the current due to a temperature difference is *increased* by the three-particle scattering. This can be explained in simple terms. Firstly, to change the current the number of left- and right-moving electrons need to change, since it is the number of electrons going through a mesoscopic structure that determines the current and not their velocity (see Appendix A). Secondly, we find the dominant scattering process at low temperature to only involve a single electron changing direction. This occurs near the bottom of the band, as pictured on Fig. 2(a). For the initial electronic distribution, the left-moving electrons have a higher temperature than the right-moving ones, which favors scattering into the warmer distribution, as seen on Fig. 2(b). This thus creates more left-moving electrons and thereby increases the particle current toward the colder reservoir, i.e., increasing the thermopower.

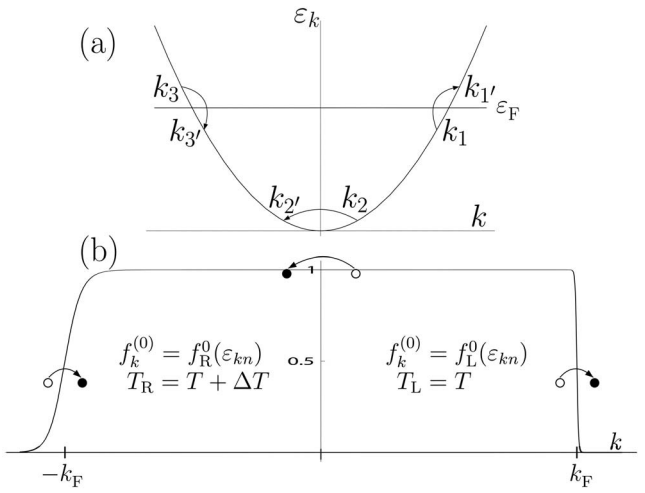

FIG. 2. (a) The dominant three-particle scattering process at low temperature in a single energy band. (b) The three-particle scattering process perturbing the initial distributions shown with warm left-moving electrons ($k<0$) and cold right-moving electrons ($k>0$). Due to the temperature difference of the initial distributions, the scattering process creating left movers dominates compared to the opposite scattering and therefore it gives a positive correction to the thermopower.

Another important point is that the thermopower and conductance corrections are exponential in temperature, i.e., proportional to $\exp(-T_F/T)$. This is a direct consequence of the dominant three-particle scattering process requiring an empty state near the bottom of the band. We find the form of the thermopower correction at low temperatures due to the three-particle scattering to be given by

$$S^{\text{int}} \propto L|V|^4 \left(\frac{T}{T_F}\right)^6 \exp(-T_F/T) > 0, \tag{6}$$

where V is the electron-electron interaction strength and T_F the Fermi temperature. This is found perturbatively in the short-wire limit. The long-wire limit remains an open question, and we expect that the length dependence of thermopower saturates once L exceeds some relaxation length (which increases for decreasing temperature).

In contrast, the conductance correction is negative. To understand this, note that the chemical potential of the initial distribution is higher for the right-moving electrons than the left-moving ones. This favors scattering into the left-moving branch [still with the process shown in Fig. 2(a)] for nonzero temperature and thereby decreasing the current. The form of the conductance correction is similar to the thermopower correction,

$$G^{\text{int}} \propto -L|V|^4 \left(\frac{T}{T_F}\right)^7 \exp(-T_F/T) < 0. \tag{7}$$

II. CURRENT CALCULATION IN THE BOLTZMANN EQUATION FORMALISM

A. Effect of interactions on the current

To model the current through a short 1D quantum wire including perturbatively the three particle interactions, we use the Boltzmann equation

$$v_k \partial_x f_k(x) = \mathcal{I}_{kx}[f], \tag{8}$$

where $f_k(x)$ is the distribution function at a space point x between zero and L (see Fig. 1), $v_k = \frac{1}{\hbar}\partial_k \varepsilon_k$ is the velocity, and $\mathcal{I}_{kx}[f]$ is the three-body electronic collision integral, i.e., no impurity or interface roughness effects are included here. We include the voltage and temperature difference in the boundary conditions of the reflectionless contacts,[3] i.e.,

$$f_k(x=0) = f_L^0(\varepsilon_k) \quad \text{for } k>0, \tag{9a}$$

$$f_k(x=L) = f_R^0(\varepsilon_k) \quad \text{for } k<0, \tag{9b}$$

and therefore omit the term $\dot{k}\partial_k f_k(x)$ in the Boltzmann equation allowed in the linear-response regime.[15] A similar method has been used to investigate electron-phonon interactions in short quantum wires,[16] quantum Hall effect in quantum wires,[17] and ballistic Coulomb drag.[18]

The three-particle collision integral is assumed to be local in space and is given by

$$\mathcal{I}_{k_1 x}[f] = -\sum_{\substack{\sigma_2\sigma_3 \\ \sigma_1'\sigma_2'\sigma_3'}} \sum_{\substack{k_2 k_3 \\ k_1' k_2' k_3'}} W_{123;1'2'3'}[f_1 f_2 f_3 (1-f_{1'})(1-f_{2'}) \times (1-f_{3'}) - f_{1'} f_{2'} f_{3'} (1-f_1)(1-f_2)(1-f_3)], \tag{10}$$

where the quantum numbers are primed (unprimed) after (before) the scattering event, $f_i \equiv f_{k_i}(x)$, and the scattering rate $W_{123;1'2'3'}$ is found in the next section. Without interactions ($W_{123;1'2'3'}=0$), the solution of the Boltzmann equation is simply given by $f^{(0)}$ in Eq. (1). When interactions are included, it becomes a very difficult task to solve the Boltzmann equation to all orders in the interaction. However, for a short wire the interactions only have a short time to change the distribution function away from the initial distribution $f^{(0)}$ and therefore we expand the distribution function in orders of $W_{123;1'2'3'}$ as

$$f = f^{(0)} + f^{(1)} + \cdots. \tag{11}$$

To find $f^{(1)}$ to the first order in W, we insert the expansion of f in the Boltzmann equation and realize that only $f^{(0)}$ is necessary in the collision integral. Since $\mathcal{I}_{kx}[f^{(0)}] = \mathcal{I}_k[f^{(0)}]$ is independent of x, we find that

$$f_k^{(1)}(x) = \frac{x}{v_k}\mathcal{I}_k[f^{(0)}] \quad \text{for } k>0, \tag{12a}$$

$$f_k^{(1)}(x) = \frac{x-L}{v_k}\mathcal{I}_k[f^{(0)}] \quad \text{for } k<0, \tag{12b}$$

using the boundary conditions [Eq. (9)]. Therefore, the current to the first order in W is

$$I = I^{(0)} + e\sum_{\sigma k<0} \mathcal{I}_k[f^{(0)}] \equiv I^{(0)} + I^{\text{int}}, \tag{13}$$

where $I^{(0)}$ is the noninteracting (Landauer) part of the current from Eq. (2) and I^{int} is the part due to interactions.

B. The linear-response limit

The form of the interacting part of the current is now known and the next step is therefore to evaluate it to linear response to V and ΔT to obtain the thermopower and conductance corrections. To this end, we define $\psi_k^{(0)}$ via

$$f_k^{(0)} \equiv f^0(\varepsilon_k) + f^0(\varepsilon_k)[1-f^0(\varepsilon_k)]\psi_k^{(0)}, \tag{14}$$

where $f^0(\varepsilon_k)$ is the Fermi function with temperature T and Fermi level ε_F. It turns out that $\psi_k^{(0)}$ is proportional to either V or ΔT. This is seen by using the identity

$$-k_B T \partial_\varepsilon f^0(\varepsilon_k) = f^0(\varepsilon_k)[1-f^0(\varepsilon_k)], \tag{15}$$

so we can identify $\psi_k^{(0)}$ by expanding the noninteracting distribution function $f_k^{(0)}$ [see Eq. (1) and Fig. 1],

$$f_L^0(\varepsilon_k) \simeq f^0(\varepsilon_k) + [-\partial_\varepsilon f^0(\varepsilon_k)]eV, \tag{16a}$$

$$f_R^0(\varepsilon_k) \simeq f^0(\varepsilon_k) + [-\partial_\varepsilon f^0(\varepsilon_k)](\varepsilon - \varepsilon_F)\frac{\Delta T}{T}, \tag{16b}$$

i.e.,

$$\psi_k^{(0)} = \begin{cases} \dfrac{eV}{k_B T} & \text{for } k>0 \\ \dfrac{\varepsilon_k - \varepsilon_F}{k_B T}\dfrac{\Delta T}{T} & \text{for } k<0. \end{cases} \tag{17}$$

Therefore, to get I^{int} in linear response to V and ΔT, we linearized the collision integral $\mathcal{I}_k[f^{(0)}]$ [Eq. (10)] with respect to $\psi_k^{(0)}$ and insert it into I^{int} [Eq. (13)] to obtain

$$I^{\text{int}} = (-e)\sum_{\substack{\sigma_1\sigma_2\sigma_3 \\ \sigma_1'\sigma_2'\sigma_3'}} \sum_{\substack{k_1<0, k_2 k_3 \\ k_1' k_2' k_3'}} \Delta_{123;1'2'3'} \times [\psi_1^{(0)} + \psi_2^{(0)} + \psi_3^{(0)} - \psi_{1'}^{(0)} - \psi_{2'}^{(0)} - \psi_{3'}^{(0)}], \tag{18}$$

where we defined

$$\Delta_{123;1'2'3'} = W_{123;1'2'3'} f_1^0 f_2^0 f_3^0 (1-f_{1'}^0)(1-f_{2'}^0)(1-f_{3'}^0), \tag{19}$$

using the shorthand notation $\psi_i^{(0)} \equiv \psi_{k_i}^{(0)}$ and $f_i^0 \equiv f^0(\varepsilon_{k_i})$. To linearize the collision integral and thereby the correction to the current due to interactions I^{int}, we have used the relation

$$f_1^0 f_2^0 f_3^0 (1-f_{1'}^0)(1-f_{2'}^0)(1-f_{3'}^0) = f_{1'}^0 f_{2'}^0 f_{3'}^0 (1-f_1^0)(1-f_2^0)(1-f_3^0), \tag{20}$$

valid at $\varepsilon_1+\varepsilon_2+\varepsilon_3 = \varepsilon_{1'}+\varepsilon_{2'}+\varepsilon_{3'}$.

Since $\psi_i^{(0)}$ is different for positive and negative k_i, we need to divide the summation in I^{int} [Eq. (18)] into positive

and negative k sums, which gives $2^5=32$ terms. For this purpose, we introduce the notation

$$\sum_{\substack{k_1<0,k_2>0,k_3<0\\k_{1'}>0,k_{2'}>0,k_{3'}<0}}(\cdot)\equiv\sum_{\substack{-+-\\++-}}(\cdot),\qquad\sum_{\substack{\sigma_1\sigma_2\sigma_3\\\sigma_{1'}\sigma_{2'}\sigma_{3'}}}(\cdot)\equiv\sum_{\text{spin}}(\cdot),\tag{21}$$

and similarly for other combinations of the summation intervals. The 32 terms can be simplified to only three terms using energy conservation and symmetry properties of $\Delta_{123;1'2'3'}$ in Eq. (19) under interchange of indices. There are pairwise exchanges of indices $\Delta_{123;1'2'3'}=\Delta_{213;1'2'3'}=\Delta_{123;1'3'2'}$, etc., and interchanges between primed and unprimed indices, $\Delta_{123;1'2'3'}=\Delta_{1'2'3';123}$, using Eq. (20) and the fact that $W_{123;1'2'3'}$ contains a matrix element squared. This leads to six terms. Furthermore, $\Delta_{123;1'2'3'}$ is invariant under $k_i\to-k_i$ for all $i=1,2,3,1',2',3'$ simultaneously due to time-reversal symmetry, also seen explicitly from the form of $W_{123;1'2'3'}$ (derived below). An example of how the simplifications occurs can be seen in Eq. (A6). Thus, we obtain the result

$$\begin{aligned}I^{\text{int}}=2(-e)\sum_{\substack{\text{spin}\\+++}}\sum_{-++}\Delta_{123;1'2'3'}&\left[\frac{\Delta T}{k_BT^2}(\varepsilon_1-\varepsilon_F)-\frac{eV}{k_BT}\right]\\+4(-e)\sum_{\substack{\text{spin}\\+++}}\sum_{--+}\Delta_{123;1'2'3'}&\left\{\frac{\Delta T}{k_BT^2}[(\varepsilon_1-\varepsilon_F)+(\varepsilon_2-\varepsilon_F)]-\frac{2eV}{k_BT}\right\}\\+3(-e)\sum_{\substack{\text{spin}\\+-+}}\sum_{+--}\Delta_{123;1'2'3'}&\left\{\frac{\Delta T}{k_BT^2}[-(\varepsilon_3-\varepsilon_F)+(\varepsilon_{2'}-\varepsilon_F)+(\varepsilon_{3'}-\varepsilon_F)]-\frac{eV}{k_BT}\right\},\end{aligned}\tag{22}$$

where the definition of $\psi_i^{(0)}$ in Eq. (17) was inserted. An important point is that the number of positive and/or negative wave-vector intervals is not the same before and after the scattering. Therefore, we note that only scattering events that change the number of left- and right-moving electrons contribute to the interaction correction to the current. The origin of this is the cancellation of the velocity in the definition of the current and in the distribution functions [Eq. (12)].

This cancellation thus leads to an expression for the interaction correction to the current in Eq. (22) where all the in-going and out-going momenta enter on equal footing. In Appendix A, we show that this is valid to all orders in perturbation theory. Due to this property and momentum conservation, there are no processes that alter the current possible near the Fermi level. Consequently, states far away from the Fermi level have to be involved in the scattering, which, as we will see, leads to a suppression of I^{int} by a factor $\exp(-T_F/T)$. The distribution function, on the other hand, *can* be changed by scattering processes near the Fermi level.

To identify the important processes, we find in the next section the scattering rate $W_{123;1'2'3'}$.

III. THREE-PARTICLE SCATTERING RATE

The three-particle scattering rate $W_{123;1'2'3'}$ is calculated using the generalized Fermi golden rule inserting the T matrix, $T\equiv V+VG_0T$, iterated to second order in the interaction V to get the three-particle interaction amplitude, i.e.,

$$W_{123;1'2'3'}=\frac{2\pi}{\hbar}|\langle 1'2'3'|VG_0V|123\rangle_{\text{c}}|^2\delta(E_i-E_f),\tag{23}$$

where $E_i=\varepsilon_1+\varepsilon_2+\varepsilon_3$ is the initial energy, $E_f=\varepsilon_{1'}+\varepsilon_{2'}+\varepsilon_{3'}$ the final energy, G_0 is the resolvent operator (or free Green's function), j is shorthand for k_j, and the subscript "c" means connected in the sense that the scattering process cannot be effectively a two-particle process, where one of the incoming particles does not participate in the scattering. Explicitly G_0 and V are given by

$$G_0=\frac{1}{E_i-H_0+i\eta},\quad(\eta\to0^+),\tag{24}$$

$$V=\frac{1}{2L}\sum_{k_1k_2q}\sum_{\sigma_1\sigma_2}V_qc^\dagger_{k_1+q\sigma_1}c^\dagger_{k_2-q\sigma_2}c_{k_2\sigma_2}c_{k_1\sigma_1}.\tag{25}$$

Here, H_0 is the unperturbed Hamiltonian (i.e., kinetic energy with some dispersion), V_q the Fourier-transformed interaction potential, and $c_{k\sigma}$ ($c^\dagger_{k\sigma}$) is the annihilation (creation) operator. To calculate the matrix element $\langle 1'2'3'|VG_0V|123\rangle_{\text{c}}$, we write the initial and final states as

$$|123\rangle=c^\dagger_{k_1\sigma_1}c^\dagger_{k_2\sigma_2}c^\dagger_{k_3\sigma_3}|0\rangle,\tag{26}$$

$$|1'2'3'\rangle=c^\dagger_{k_{1'}\sigma_1,}c^\dagger_{k_{2'}\sigma_2,}c^\dagger_{k_{3'}\sigma_{3'}}|0\rangle,\tag{27}$$

where $|0\rangle$ is the empty state. Using the anticommutator algebra $\{c_i,c_j^\dagger\}=\delta_{i,j}$, we obtain

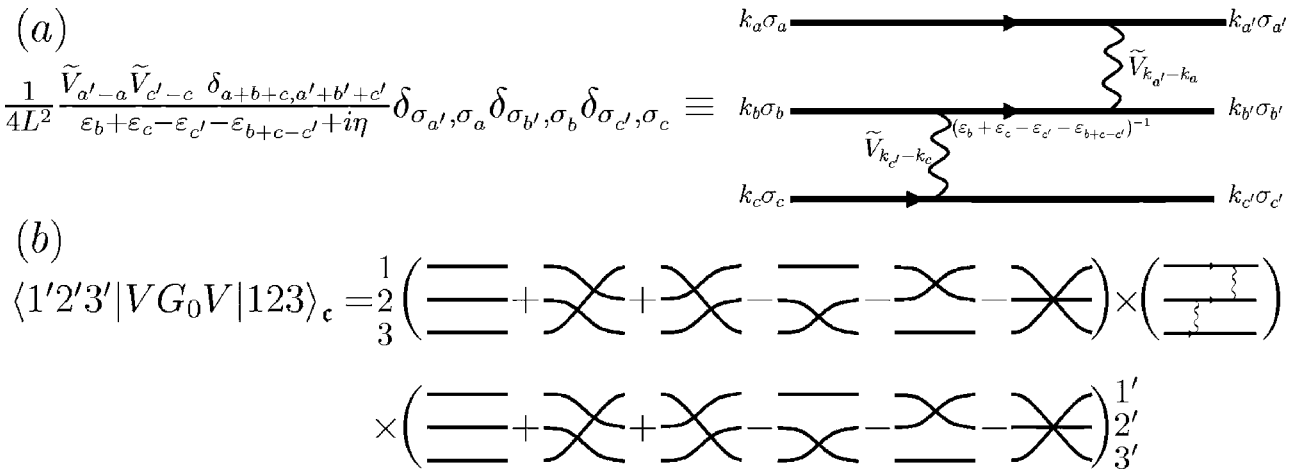

FIG. 3. A visualization of the connected three-particle scattering matrix element [Eq. (30)], where three particles interchange their momenta and energy. This matrix element enters the scattering rate via the generalized Fermi golden rule [Eq. (23)]. (a) The basic three-particle interaction consisting of two interaction lines and a free propagation [see Eq. (30)]. (b) Picture of the exchange processes times the basic interaction needed to form the matrix element [Eq. (30)].

$$G_0V|123\rangle = \frac{1}{2L}\sum_q V_q \sum_{(abc)\in P(123)} \frac{\mathrm{sgn}(abc)}{\varepsilon_{ba}(q)} \times c^\dagger_{k_a+q\sigma_a} c^\dagger_{k_b-q\sigma_b} c^\dagger_{k_c\sigma_c}|0\rangle, \tag{28}$$

where we introduced

$$\varepsilon_{ba}(q) = \varepsilon_b + \varepsilon_a - \varepsilon_{b-q} - \varepsilon_{a+q} + i\eta = \frac{\hbar^2}{m} q(k_b - k_a - q) + i\eta \tag{29}$$

(the last equality is only valid for a quadratic dispersion), and where the set of permutations is given by $P(123) = [(123)^+, (231)^+, (312)^+, (132)^-, (321)^-, (213)^-]$. Here, the signs of the permutation, $\mathrm{sgn}(abc)$, are shown as superscripts.

In order to exclude the effectively two-particle processes when multiplying Eq. (28) by $\langle 1'2'3'|V$ from the left, k_c $(c=1,2,3)$ needs to be different from $k_{j'}$ $(j=1,2,3)$. The result is

$$\langle 1'2'3'|VG_0V|123\rangle_c = \frac{1}{(2L)^2} \sum_{(abc)\in P(123)} \sum_{(a'b'c')\in P(1'2'3')} \mathrm{sgn}(abc)\mathrm{sgn}(a'b'c') \times \frac{\tilde{V}_{a'-a}\tilde{V}_{c'-c}\delta_{a+b+c,a'+b'+c'}}{\varepsilon_b+\varepsilon_c-\varepsilon_{c'}-\varepsilon_{b+c-c'}+i\eta} \delta_{\sigma_{a'},\sigma_a}\delta_{\sigma_{b'},\sigma_b}\delta_{\sigma_{c'},\sigma_c}, \tag{30}$$

where $\tilde{V}_q = V_q + V_{-q}$ is the symmetrized interaction. The matrix element consists of 36 terms and the scattering rate thus has $36^2 = 1296$ terms. To obtain this result, we did not use energy conservation. For a quadratic dispersion, the denominator is only zero if we have an effective pair collision or if the momentum transfer is zero, as seen from the expression $\varepsilon_{ba}(q) = \frac{\hbar^2}{m} q(k_b - k_a - q) + i\eta$. A picture of the matrix element is found in Fig. 3(b), where the exchange processes (including the sign) are visualized as different ways to connect two interaction lines and an intermediate propagation (G_0) seen on Fig. 3(a). The inclusion of the Fermi statistics makes a substantial difference for the properties of the scattering rate as compared to the case described in Ref. 19, which is obtained by setting all $\mathrm{sgn}(\cdots) = +1$.

We can rewrite the matrix element [Eq. (30)] in a more transparent way in terms of quantum-mechanical exchange symmetry. First, we introduce the following combination of three-particle scattering amplitudes:

$$\mathbb{V}(11',22',33') = \frac{\delta_{\sigma_{1'},\sigma_1}\delta_{\sigma_{2'},\sigma_2}\delta_{\sigma_{3'},\sigma_3}}{4L^2} \left(\frac{\tilde{V}_{1'-1}\tilde{V}_{3'-3}}{\varepsilon_3+\varepsilon_2-\varepsilon_{3'}-\varepsilon_{2+3-3'}} + \frac{\tilde{V}_{2'-2}\tilde{V}_{1'-1}}{\varepsilon_1+\varepsilon_3-\varepsilon_{1'}-\varepsilon_{3+1-1'}} + \frac{\tilde{V}_{3'-3}\tilde{V}_{2'-2}}{\varepsilon_2+\varepsilon_1-\varepsilon_{2'}-\varepsilon_{1+2-2'}} + \frac{\tilde{V}_{1'-1}\tilde{V}_{2'-2}}{\varepsilon_2+\varepsilon_3-\varepsilon_{2'}-\varepsilon_{3+2-2'}} + \frac{\tilde{V}_{3'-3}\tilde{V}_{1'-1}}{\varepsilon_1+\varepsilon_2-\varepsilon_{1'}-\varepsilon_{2+1-1'}} + \frac{\tilde{V}_{2'-2}\tilde{V}_{3'-3}}{\varepsilon_3+\varepsilon_1-\varepsilon_{3'}-\varepsilon_{1+3-3'}} \right), \tag{31}$$

and after some rewriting, we then obtain

$$\langle 1'2'3'|VG_0V|123\rangle_c = \delta_{k_1+k_2+k_3,k_{1'}+k_{2'}+k_{3'}}[\mathbb{V}(11',22',33') + \mathbb{V}(12',23',31') + \mathbb{V}(13',21',32') - \mathbb{V}(11',23',32') - \mathbb{V}(13',22',31') - \mathbb{V}(12',21',33')]. \quad (32)$$

We interpret this result in a way similar to a two-particle matrix element,

$$\langle 1'2'|V|12\rangle = \frac{\delta_{k_1+k_2,k_{1'}+k_{2'}}}{L}(V_{k_{1'}-k_1}\delta_{\sigma_1,\sigma_{1'}}\delta_{\sigma_2,\sigma_{2'}} - V_{k_{2'}-k_1}\delta_{\sigma_1,\sigma_{2'}}\delta_{\sigma_2,\sigma_{1'}}), \quad (33)$$

which contains a direct (first term) and an exchange term (where $1' \leftrightarrow 2'$).

In the three-particle case, $\mathbb{V}(11',22',33')$ is the direct term and one can make five exchange processes (instead of one) by exchanging the three final states $1'$, $2'$, and $3'$. This gives Eq. (32). The sign in front of each $\mathbb{V}(\cdots)$ is determined by the number of exchanges made, e.g., in $\mathbb{V}(11',23',32')$ a single exchange, $2' \leftrightarrow 3'$, gives a minus $(-1)^1$ whereas for $\mathbb{V}(12',23',31')$ two exchanges ($1' \leftrightarrow 3'$ followed by $3' \leftrightarrow 2'$) give a positive sign $(-1)^2$. Furthermore, the arguments in $\mathbb{V}(11',22',33')$ are ordered in three pairs such that the differences between the elements in each pair are the only arguments of the interaction potential [see Eq. (31)]. This is useful when constructing approximations having a specific scattering process in mind.

How the matrix element was rewritten into the form of Eq. (32) can also be described in terms of the drawings of Fig. 3. The direct term $\mathbb{V}(11',22',33')$ is the sum of the six terms having mirror-symmetric exchanges before and after the scattering. The other terms in Eq. (32) then can be obtained by suitable changes of out-going lines.

A. Zero three-particle scattering rate for integrable models

The expressions we obtain for the three-particle scattering rates [Eq. (23)] are quite cumbersome. Nevertheless, the obtained results allow for some consistency checks. Remarkably, for some two-body potentials, scattering of the particles of an N-body system is exactly equivalent to a sequence of two-body collisions. Such "special" potentials were studied in the context of integrable quantum many-body problems.[11] We recall now that for a quadratic band, a pair collision does not change the momenta of the incoming particles or simply permutes the two momenta. Therefore, three-particle scattering for the integrable potentials may result *only* in permutations within the group of three momenta of the colliding particles; all other three-particle scattering amplitudes must be zero for such potentials. In the context of this work, it means that even three-particle (or higher-order) collisions would not bring electron equilibration for such types of electron-electron interaction.

In this section, we check that the three-particle scattering amplitudes are indeed zero for two special potentials.

1. Pointlike interaction

In the case of contact interaction, $\tilde{V}_q = \text{const} \equiv \tilde{V}_0$, and for any kind of electron dispersion relation (i.e., not necessarily quadratic), we find by using the energy conservation law that

$$\sum_{\text{spin}} |\langle 1'2'3'|VG_0V|123\rangle_c|^2 = \frac{2\tilde{V}_0^4}{(2L)^4}\delta_{k_1+k_2+k_3,k_{1'}+k_{2'}+k_{3'}}(|A_{121'} - A_{122'} - A_{131'} + A_{132'}|^2 + |A_{121'} - A_{123'} - A_{131'} + A_{133'}|^2 + |A_{122'} - A_{123'} - A_{132'} + A_{133'}|^2 + |A_{121'} - A_{122'} - A_{231'} + A_{232'}|^2 + |A_{131'} - A_{132'} - A_{231'} + A_{232'}|^2 + |A_{121'} - A_{123'} - A_{231'} + A_{233'}|^2 + |A_{131'} - A_{133'} - A_{231'} + A_{233'}|^2 + |A_{122'} - A_{123'} - A_{232'} + A_{233'}|^2 + |A_{132'} - A_{133'} - A_{232'} + A_{233'}|^2), \quad (34)$$

where $A_{abc} = (\varepsilon_a + \varepsilon_b - \varepsilon_c - \varepsilon_{a+b-c} + i\eta)^{-1}$. This is a major simplification from $36^2 = 1296$ to $9 \times 4^2 = 144$ terms by performing the spin summation. If, furthermore, the dispersion is quadratic, $\varepsilon_k \propto k^2$, then we find the (at first sight) surprising cancellation

$$\sum_{\text{spin}} |\langle 1'2'3'|VG_0V|123\rangle_c|^2 \delta(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 - \varepsilon_{1'} - \varepsilon_{2'} - \varepsilon_{3'}) = 0. \quad (35)$$

This can be seen directly from Eq. (34) or by noting that

$$\mathbb{V}(1a',2b',3c')\delta(\varepsilon_1 + \varepsilon_2 + \varepsilon_3 - \varepsilon_{a'} - \varepsilon_{b'} - \varepsilon_{c'}) = 0, \quad (36)$$

for a quadratic dispersion and constant interaction for $(a'b'c') \in P(1'2'3')$, i.e., each term of Eq. (32) is zero. $\mathbb{V}(11',22',33')$ cancels in such a way that the three first terms of Eq. (31) cancel each other [the even permutations of (123) combined with the same primed permutation] and the three last terms cancel each other [the odd permutations of

(123) combined with the same primed permutation].

In fact, the cancellation described above is in agreement with the general factorization results for the S matrix of one-dimensional N-body problem with δ-function interaction in real space.[20] In this context, it is crucial that the particles have a quadratic dispersion relation; if we use, e.g., $\varepsilon_k \propto k^4$, then the cancellation does not occur. Notice also that the cancellation we demonstrate is not a trivial zero. Indeed, the underlying two-particle amplitudes [Eq. (33)] are finite for a q-independent potential if one includes spins. (For spinless fermions and contact interaction, the matrix element would be zero because the direct and the exchange terms cancel in accordance with the Pauli principle.)

2. $\tilde{V}_q = V_0(1 - q^2/q_0^2)$ interaction

We also observed that the energy conserving part of the matrix element $\langle 1'2'3'|VG_0V|123\rangle_c$ in the case of *spinless* fermions, quadratic dispersion, and the Fourier transformed interaction potential of the form

$$\tilde{V}_q = V_0\left(1 - \frac{q^2}{q_0^2}\right) \tag{37}$$

becomes equal to zero. This is also possible to expect because of the relation of the potential [Eq. (37)] to the integrable 1D bosonic Lieb-Liniger model.[21] Indeed, the bosonic model with contact interaction potential $\propto g_B\delta(x_1 - x_2)$ may be exactly mapped[22] onto the spinless fermionic model with interaction $V_F(x_1 - x_2) \propto -(1/g_B)\delta''(x_1 - x_2)$. The integrability of the bosonic model guarantees the integrability of the corresponding fermionic one. Adding a contact interaction to V_F does no harm, as we are considering spinless fermions. Finally, Fourier transformation takes us to Eq. (37).

We observed that including the spin degree of freedom spoils the remarkable cancellation for a three-particle amplitude.

In the following sections, we assume a general case interaction potential for which the three-particle scattering amplitudes lead to a nontrivial redistribution of the momenta between the particles.

IV. THERMOPOWER AND CONDUCTANCE CORRECTIONS DUE TO THREE-PARTICLE INTERACTION

In this section, we go through the main ideas and approximations in evaluating the current correction due to interactions I^{int} [Eq. (22)] to lowest order in the temperature, $T \ll T_F$. We give a more detailed calculation in Appendix B.

As noted previously, all three terms in I^{int} [Eq. (22)] are exponentially suppressed, since momentum conservation

$$k_1 + k_2 + k_3 = k_{1'} + k_{2'} + k_{3'}$$

forbids scattering processes near the Fermi level for the given combinations of positive and negative k intervals. To be more specific, it is the phase-space restrictions of the Fermi functions that give the exponential suppression, i.e.,

$$f_1^0 f_2^0 f_3^0 (1 - f_{1'}^0)(1 - f_{2'}^0)(1 - f_{3'}^0) \propto e^{-T_F/T}. \tag{38}$$

We begin by identifying the most important three-particle scattering process. The three terms in I^{int} [Eq. (22)] are the following: (i) two right movers backscattering a left mover while remaining right movers, (ii) one right mover keeping its direction while backscattering two left movers, and (iii) a left and a right mover keeping their directions while backscattering the third particle. From now on, we will concentrate on the case of Coulomb interaction $\tilde{V}_q$, which is the largest for small q; therefore we want to identify processes where the initial and final states are close in momentum space.[23] Further, the process(es) should not require more than one electron in states suppressed exponentially by the Fermi functions. One can see that due to the constraints stemming from momentum and energy conservation, in fact, only process (iii) allows both initial and final states to be close to each other in momentum space and at the same time having only a single exponentially suppressed factor. The corresponding scattering process is of the type shown in Fig. 2(a). Therefore, to the first order in $\exp(-T_F/T)$, we include only the third one in Eq. (22). This leads to

$$I^{\text{int}} \simeq 3(-e)\sum_{\text{spin}} \sum_{\substack{++-\\+-}} \Delta_{123;1'2'3'} \times \left[\frac{\Delta T}{k_B T^2}(-\varepsilon_3 + \varepsilon_{2'} + \varepsilon_{3'} - \varepsilon_F) - \frac{eV}{k_B T}\right]. \tag{39}$$

Here, $\Delta_{123;1'2'3'}$ expresses the available phase space in form of the Fermi functions and the three-particle scattering rate [see Eq. (19)].

One essential approximation is that for the scattering process depicted in Fig. 2(a), we may replace the full Fermi distribution functions by the exponential tales or the low-temperature limit expressions, i.e.,

$$f_1^0 \simeq \theta(k_F - k_1)\theta(k_1), \quad 1 - f_{1'}^0 \simeq \theta(k_{1'} - k_F), \tag{40a}$$

$$f_2^0 \simeq \theta(k_F - k_2)\theta(k_2), \quad 1 - f_{2'}^0 \simeq e^{(\varepsilon_{2'} - \varepsilon_F)/k_B T}, \tag{40b}$$

$$f_3^0 \simeq e^{-(\varepsilon_3 - \varepsilon_F)/k_B T}, \quad 1 - f_{3'}^0 \simeq e^{(\varepsilon_{3'} - \varepsilon_F)/k_B T}. \tag{40c}$$

Note that k_1, $k_{1'}$, and k_2 are all positive. We see that the product of the Fermi functions is indeed exponentially suppressed, i.e., $\propto \exp(-T_F/T)$.

The second essential approximation is that for the scattering process seen in Fig. 2(a), the initial and final states differ by a small momentum. Therefore, the matrix element in the transition rate $W_{123;1'2'3'}$ is dominated by the direct term $\mathbb{V}(11',22',33')$ in Eq. (32), since the five exchange terms are suppressed by the Coulomb interaction $|\tilde{V}_{|q|\sim k_F}| \ll |\tilde{V}_{|q|\ll k_F}|$, i.e.,

$$\langle 1'2'3'|VG_0V|123\rangle_c \simeq \delta_{k_1+k_2+k_3,k_{1'}+k_{2'}+k_{3'}} \mathbb{V}(11',22',33'). \tag{41}$$

The direct term [Eq. (41)] would be zero for $\tilde{V}_q$=const. In the case of quadratic dispersion relation and general $\tilde{V}_q$, it vanishes in the limit $(k_{i'}-k_i)\to 0$ and $(k_{j'}-k_j)\to 0$ for $i,j \in \{1,2,3\}$ due to the Pauli principle. For a quadratic dispersion and for a general symmetrized interaction $\tilde{V}_q=V_q+V_{-q}$, the direct term $\mathbb{V}(11',22',33')$ simplifies to the following expression:

$$\mathbb{V}(11',22',33') = \frac{\delta_{\sigma_{1'},\sigma_1}\delta_{\sigma_{2'},\sigma_2}\delta_{\sigma_{3'},\sigma_3}}{4L^2\hbar^2/m}(q_1+q_3) \times \frac{[-(q_1+q_3)\tilde{V}_{q_1}\tilde{V}_{q_3}+\tilde{V}_{q_1+q_3}(q_3\tilde{V}_{q_1}+q_1\tilde{V}_{q_3})]}{(k_1-k_3+q_1)q_1q_3(k_1-k_3-q_3)}, \quad (42)$$

where we used energy conservation and introduced $q_1=k_{1'}-k_1$ and $q_3=k_{3'}-k_3$.

Next, we give a qualitative explanation for the power law in T for the interacting current correction [Eq. (39)] using the quadratic dispersion. First, we consider the phase-space constraint. To do the sum over all k in Eq. (39), we use the momentum and energy conservation and introduce new variables $q_1=k_{1'}-k_1$ and $q_3=k_{3'}-k_3$, i.e., change the summation variables,

$$k_1,k_2,k_3,k_{1'},k_{2'},k_{3'} \to k_1,k_3,q_1,q_3. \quad (43)$$

The energy conservation for a quadratic dispersion gives a factor of $1/|q_1+q_3|$ [see, e.g., Eq. (B6)]. For the process at hand, k_1 and k_3 are close to the Fermi level and each of their sums contributes with a factor of q_1 and q_3, respectively. The Fermi functions give the exponential suppression and a contribution to the phase space in form of an exponential tail, i.e.,

$$f_1^0 f_2^0 f_3^0 (1-f_{1'}^0)(1-f_{2'}^0)(1-f_{3'}^0) \propto e^{-T_F/T} e^{(\varepsilon_{2'}-\varepsilon_3+\varepsilon_{3'})/k_BT} \quad (44)$$

[see Eqs. (40a)–(40c)]. To get the low-temperature result for I^{int} [Eq. (39)], we use the method of steepest decent to calculate the integral. To this end, we note that the exponent $\varepsilon_{2'}-\varepsilon_3+\varepsilon_{3'}$ is a function of q_1 and q_3 and in the limit $T/T_F\to 0$ the most important part is around the origin $q_1=q_3=0$. Here, $\varepsilon_{2'}-\varepsilon_3+\varepsilon_{3'}$ vanishes as $-\frac{1}{2}\hbar v_F(q_1+q_2)$ (see Appendix B for details). Therefore, collecting the phase-space factors, the current correction due to three-particle interactions [Eq. (39)] becomes

$$I^{\text{int}} \propto \varepsilon^{-T_F/T} \int dq_1 \int dq_3 \frac{q_1q_3}{|q_1+q_3|} e^{-(T_F/T)(q_1+q_3)/k_F} \times |\mathbb{V}(11',22',33')|^2 \left[\frac{\Delta T}{T}\frac{T_F}{T}\left(-\frac{q_1+q_3}{k_F}-1\right)-\frac{eV}{k_BT}\right], \quad (45)$$

in the limit $T \ll T_F$. Furthermore, it turns out that the constraints $k_2>0$ and $k_{2'}<0$ in the sum [Eq. (39)] only leaves phase space close to $q_1=q_3$ for $T/T_F\to 0$, so we can set $q_3=q_1$ in the integrand and do the integral over q_3, which is $\propto q_1^2$ due to the phase-space limits. To lowest order in temperature, this yields

$$I^{\text{int}} \propto \varepsilon^{-T_F/T} \int_0^\infty dq q^3 e^{-(T_F/T)2q/k_F} |\mathbb{V}(11',22',33')|^2 \times \left(\frac{\Delta T}{T}\frac{T_F}{T}+\frac{eV}{k_BT}\right). \quad (46)$$

From this, we conclude that phase space alone (i.e., assuming $|\mathbb{V}(11',22',33')|^2$ to be a constant) gives a temperature dependence of the form

$$I^{\text{int}} \propto \varepsilon^{-T_F/T} T^4 \left(\frac{\Delta T}{T}\frac{T_F}{T}+\frac{eV}{k_BT}\right) \quad \text{(phase space only)}. \quad (47)$$

However, as we have seen the three-particle interaction rate has a delicate momentum dependence that needs to be taken into account. Therefore, to calculate the direct interaction term $\mathbb{V}(11',22',33')$, we expand the symmetrized potential $\tilde{V}_q$ for small q as

$$\tilde{V}_q = V_0\left[1-\left(\frac{q}{q_0}\right)^2+\mathcal{O}(q^4)\right], \quad (48)$$

where the parameter $q_0 \ll k_F$ describes the screening due to the metallic gates near the quantum wire and V_0 is (twice) the $q=0$ Fourier transform of the Coulomb potential cut off by the screening. Setting $q_3=q_1\equiv q$ into the three-particle scattering rate [Eq. (42)], we obtain

$$\mathbb{V}(11',22',33') \propto V_0^2\left(\frac{k_F}{q_0}\right)^2 q^2 \quad (49)$$

to lowest order in q. Inserting this into Eq. (46), the final result for the current correction, including both phase-space factors and the momentum dependent scattering rate, becomes

$$I^{\text{int}} \propto e^{-T_F/T} T^8 V_0^4 \left(\frac{k_F}{q_0}\right)^4 \left(\frac{\Delta T}{T}\frac{T_F}{T}+\frac{eV}{k_BT}\right). \quad (50)$$

(Here we noticed that the nonconstant three-particle scattering rate gave rise to four extra powers in temperature.) The detailed calculation given in Appendix B yields a prefactor, and the end result is

$$I^{\text{int}} = \frac{8505}{2048\pi^4} e^{-T_F/T} \frac{e}{\hbar}\frac{(V_0k_F)^4}{\varepsilon_F^3}(Lk_F)\left(\frac{k_F}{q_0}\right)^4\left(\frac{T}{T_F}\right)^7\left(\frac{\Delta T}{T}+\frac{eV}{\varepsilon_F}\right) + \mathcal{O}\left[\left(\frac{T}{T_F}\right)^8\right]. \quad (51)$$

Combining this result with the zero order in the interaction terms see [Eqs. (4) and (5)], we find for the thermopower and conductance in the low-temperature limit,

$$S = \frac{k_B}{e}\frac{T_F}{T}e^{-T_F/T}\left(1+\frac{L}{\ell_{eee}}\right), \quad (52)$$

$$G = \frac{2e^2}{h} - \frac{2e^2}{h} e^{-T_F/T}\left(1 + \frac{L}{\ell_{eee}}\right). \tag{53}$$

Here, we introduced the effective length ℓ_{eee} by the relation[24]

$$\ell_{eee}^{-1} = \frac{8505}{2048\pi^3}\frac{(V_0 k_F)^4}{\varepsilon_F^4}\left(\frac{k_F}{q_0}\right)^4\left(\frac{T}{T_F}\right)^7 k_F, \tag{54}$$

which may be viewed as a mean free path with respect to backscattering for a hole near the bottom of the band.

To recapitulate, the temperature dependence $T \propto T^7$ in Eq. (54) can be understood in the following way: the three-particle scattering of a single particle leaves five free momenta, and since two are taken by energy and momentum conservation this gives T^3. In addition, the interaction, V_q, is proportional to q^2, and when squared it gives rise to four more powers, which results in the T^7 dependence.

In the limit of a pointlike interaction, $q_0 \to \infty$, the corrections are zero in agreement with the result of Sec. III A.

It is known from the Luttinger liquid theory that in the limit of linear spectrum, which corresponds to $T_F \to \infty$, the conductance remains finite even if the wire is infinitely long ($L \to \infty$). Therefore, it is tempting to speculate that the two terms in the square brackets of Eq. (53) are the first terms of an expansion in $\lambda = L/\ell_{eee}$ of some function $F_G(\lambda)$ which saturates at a constant value in the limit $\lambda \to \infty$. One may also have a similar speculation generalizing Eq. (52) for the thermopower, $[\cdots] \to F_S(L/\ell_{eee})$.

As a final remark, we note that the so-called Mott formula[25] relating the thermopower to the low-temperature conductance,

$$S = \frac{\pi^2}{3}\frac{k_B}{e} k_B T \frac{1}{G}\frac{dG}{d\varepsilon_F}, \tag{55}$$

is clearly violated by Eqs. (53) and (52). This violation could be expected because the conventional derivation of the Mott formula (for the noninteracting case) assumes that the main contribution to the conductance *and* thermopower comes from the states around the Fermi level in an energy interval of the order of temperature.[26] However, in the considered case the main contribution to S comes from the "deep" states, even in the zeroth order with respect to the interaction potential. Correspondingly, there is no surprise that Eqs. (52) and (53) being substituted, respectively, in the left- and right-hand sides of Eq. (55) produce a parametrically large mismatch $\sim T_F/T$.

V. SUMMARY AND DISCUSSION

We have calculated the leading interaction correction to the transport properties of a clean mesoscopic wire adiabatically connected to the leads, using perturbation theory in the length of the wire.

For a single-mode wire, the leading interaction corrections turns out to be given by three-particle scattering processes. This is because two-particle processes cannot change the current due to momentum and energy conservation. To calculate the effect of the three-body processes, we have utilized the Boltzmann equation formalism, with three-particle scattering events defining the collision integral. We have identified the leading-order scattering processes and found that they involve at least one state near the bottom of the band, i.e., far from the Fermi level. The involvement of such "deep" states results in an exponentially small, $\propto e^{-T_F/T}$, interaction-induced correction to thermopower and conductance at low temperatures.

The account for interaction in this paper is performed for relatively short wires, where perturbation theory in the interaction or equivalently in the wire length is valid. For longer wires, one needs to find the distribution function by treating the collision integral in the Boltzmann equation nonperturbatively. It is not clear whether the relaxation of the distribution function would instead yield nonexponential corrections to the transport coefficients for longer wires. However, since the scattering processes that contribute to the current must involve a particle that changes direction (which is proven in Appendix A), one might speculate that the exponential suppression is valid for all lengths, as long as electron-electron scattering is the only active relaxation mechanism.

The question of what the relaxed distribution function looks like for a mesoscopic wire is an interesting and unsolved problem. Here, we have only given a partial answer for the leading contributions for a short wire, i.e., to lowest order in the interaction. Further studies should involve a self-consistent determination of the distribution function.

Since thermopower is sensitive to the electron distribution function, it might be a good experimental tool for answering the fundamental questions regarding the effect of electron-electron collisions. Indeed, refined measurements of thermopower of short 1D quantum wires have been performed, yielding reasonably good agreement with the free-electron theory.[27–29] It remains an open question whether the accuracy of thermopower measurements is high enough to see the interaction effects in longer wires.

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APPENDIX A: SCATTERING PROCESSES CONTRIBUTING TO THE CURRENT

In this appendix, we show that the particle current changes due to electronic scattering *if and only* if the scattering changes the number of left- and right-moving electrons. In the main text [see Eq. (22)], this was shown to first order in the transition rate, but here we show it to *all orders in the interaction*.

We show it explicitly in the Boltzmann equation framework; however, we suspect it to be a general feature of mesoscopic systems. Intuitively, the statement means that it is the *number* of particles that passes through the mesoscopic system that matters and not their velocity. In contrast to this

is, e.g., a long 1D wire or a bulk metal, where a velocity change of the particles is enough to change the current.

To show the above statement explicitly, we formally rewrite the Boltzmann equation [Eq. (8)] including the boundary conditions [Eq. (9)] as

$$f_k(x) = f_L^0(\varepsilon_k) + \int_0^x dx' \frac{\mathcal{I}_{kx'}[f]}{v_k} \quad \text{for } k > 0, \tag{A1}$$

$$f_k(x) = f_R^0(\varepsilon_k) + \int_L^x dx' \frac{\mathcal{I}_{kx'}[f]}{v_k} \quad \text{for } k < 0. \tag{A2}$$

Note that this is not a closed solution of the Boltzmann equation, since the distribution function is still contained inside the collision integral. However, this rewriting enables us to find the current without finding the distribution function first, i.e., by inserting Eqs. (A1) and (A2) into the current definition

$$I = \frac{(-e)}{L} \sum_{\sigma k} v_k f_k(x) \tag{A3}$$

and obtain (after a few manipulations)

$$I = \frac{(-e)}{L} \sum_{\sigma k>0} v_k [f_L^0(\varepsilon_k) - f_R^0(\varepsilon_k)] - \frac{(-e)}{L} \int_0^L dx \sum_{\sigma k<0} \mathcal{I}_{kx}[f] + \frac{(-e)}{L} \int_0^x dx' \overbrace{\sum_{\sigma k} \mathcal{I}_{kx'}[f]}^{=0} \equiv I^{(0)} + I^{(\text{int})}, \tag{A4}$$

where the x-dependent part can be seen to be zero by changing variables. We note the cancellation of the velocity in the distribution function [Eqs. (A1) and (A2)] and the current definition [Eq. (A3)], which is the origin of the statement we are showing (as in the first order calculation). A similar cancellation occurs in the Landauer formula, thus relating the transmission to the conductance. By using the explicit form of the collision integral Eq. [(10)], the current from the interactions is

$$I^{(\text{int})} = \frac{(-e)}{L} \int_0^L dx \sum_{\substack{\sigma_1\sigma_2\sigma_3 \\ \sigma_1'\sigma_2'\sigma_3'}} \sum_{\substack{k_1<0,k_2,k_3 \\ k_1'k_2'k_3'}} W_{123;1'2'3'} [f_1 f_2 f_3 (1-f_{1'}) \times (1-f_{2'})(1-f_{3'}) - f_{1'} f_{2'} f_{3'} (1-f_1)(1-f_2)(1-f_3)]. \tag{A5}$$

We can divide the summation over k quantum number into positive and negative intervals as in the main text (see Sec. II B). The essential point is now that all terms that have the same number of positive (and negative) intervals for the primed and unprimed wave numbers k are zero. In other words, if the number of left- and right-moving electrons does not change, then the contribution is zero by symmetry of the transition rate. We show this cancellation in practice by an example [using the notation of Eq. (21)],

$$\begin{aligned} &\sum_{\text{spin}} \sum_{\substack{-+- \\ +--}} W_{123;1'2'3'} [f_1 f_2 f_3 (1-f_{1'})(1-f_{2'})(1-f_{3'}) - f_{1'} f_{2'} f_{3'} (1-f_1)(1-f_2)(1-f_3)] \\ &= \sum_{\text{spin}} \sum_{\substack{-+- \\ -+-}} W_{123;1'2'3'} [f_1 f_2 f_3 (1-f_{1'})(1-f_{2'})(1-f_{3'}) - f_{1'} f_{2'} f_{3'} (1-f_1)(1-f_2)(1-f_3)] \\ &= \sum_{\text{spin}} \sum_{\substack{-+- \\ -+-}} W_{123;1'2'3'} f_1 f_2 f_3 (1-f_{1'})(1-f_{2'})(1-f_{3'}) - \sum_{\text{spin}} \sum_{\substack{-+- \\ -+-}} \underbrace{W_{123;1'2'3'} f_{1'} f_{2'} f_{3'} (1-f_1)(1-f_2)(1-f_3)}_{\text{interchange } (123) \leftrightarrows (1'2'3')} = 0, \end{aligned} \tag{A6}$$

interchanging $1'$ and $2'$ at the first equality using $W_{123;1'2'3'} = W_{123;2'1'3'}$ and interchanging $(123) \leftrightarrow (1'2'3')$ in the second term as indicated. Thereby, we have shown to all orders that to change the current by electronic interactions, the number of left and right movers have to change.

The statement is *not limited* to only three-particle scattering and can be shown equivalently for pair interaction including several bands, electron-phonon coupling, or any other interaction with the same kind of symmetry under particle interchange. Furthermore, the statement is still true if the collision is nonlocal in space, since that only introduce some spatial integrals in the collision integral that can be

handled similarly. Note, however, that the distribution function can be changed by processes that do not change the number of left and right movers.

APPENDIX B: DETAILED CALCULATION OF THE THERMOPOWER AND CONDUCTANCE CORRECTION DUE TO THE THREE-PARTICLE SCATTERING

The purpose of this appendix is to calculate I^{int} in Eq. (39),

$$I^{\text{int}} \simeq 3(-e)\sum_{\text{spin}}\sum_{\substack{++-\\+--}}\Delta_{123;1'2'3'} \times\left[\frac{\Delta T}{k_B T^2}(-\varepsilon_3+\varepsilon_{2'}+\varepsilon_{3'}-\varepsilon_F)-\frac{eV}{k_B T}\right], \quad \text{(B1)}$$

in the low-temperature limit, $T \ll T_F$, step by step to find the prefactor given in Eq. (51). As already mentioned, we perform the calculation with the scattering process seen in Fig. 2(a) in mind. Therefore, we use the Fermi functions as given in Eq. (40) and the matrix element entering in the scattering rate from Eqs. (41) and (42), i.e. using a quadratic dispersion.

We perform the summation over all the k in Eq. (B1) in the following way. First of all, we note that due to the momentum and energy conservation in the interaction process described, the scattering of k_3 to $k_{3'}$ has to be from above to below the Fermi level, i.e.,

$$k_3 < -k_F < k_{3'} \Rightarrow \theta(-k_F-k_3)\theta(k_F+k_{3'}). \quad \text{(B2)}$$

This is due to the signs of k_2 and $k_{2'}$ and can be understood as a sign of the difference between the curvature of the dispersion near the bottom of the band and near the Fermi level. Next, we introduce the momentum transfer around the Fermi level $q_i \equiv k_{i'} - k_i$ for $i=1,3$ and using the momentum conservation to do the $k_{2'}$ summation, we obtain

$$\sum_{\substack{++-\\+--}}(\cdots) \to \sum_{k_1>0,k_2>0,k_3<0}\sum_{q_1,q_3}(\cdots), \quad \text{(B3)}$$

remembering the constraint $k_{1'}=k_1+q_1>0$, $k_{2'}=k_2+q_1+q_3<0$, and $k_{3'}=k_3+q_3<0$. The Fermi factors $f_1^0(1-f_{1'}^0)$ and $f_3^0(1-f_{3'}^0)$ restrict the momentum transfer q_1 and q_3 to be much smaller then k_F and the k_1 and k_3 to be near the Fermi level for the process in mind. Therefore, we can use the Fermi functions $f_1^0(1-f_{1'}^0)$ to do the summation over k_1. Assuming slow variation of the scattering rate over a range of $q_1 \ll k_F$ at the Fermi level, the k_1 summation becomes

$$\sum_{k_1>0}\theta(k_F-k_1)\theta(k_1+q_1-k_F)=\frac{L}{2\pi}q_1\theta(q_1). \quad \text{(B4)}$$

Similarly, the k_3 summation is done using the phase-space constraint in Eq. (B2),

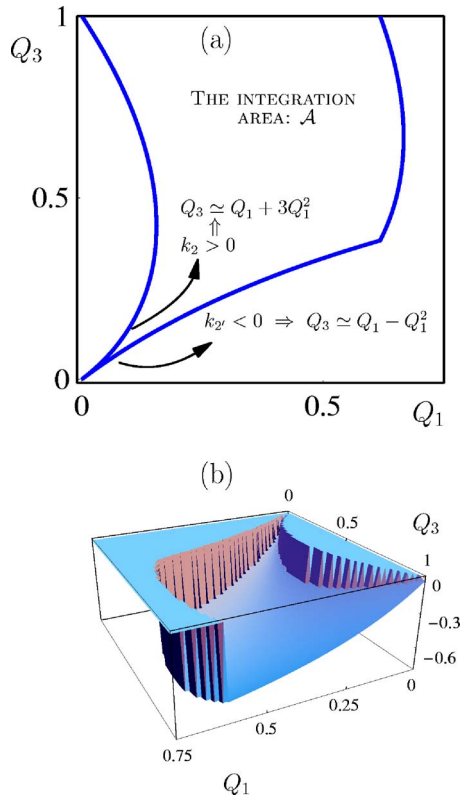

FIG. 4. (Color online) (a) The integration region $\mathcal{A}$ for the integral [Eq. (B13)] to calculate the current due to interactions. The two boundaries for the integration area *close to the origin* stemming from the signs of k_2 and $k_{2'}$ are indicated. (b) The $\Xi(Q_1,Q_3)=\xi(k_F Q_1, k_F Q_3)/\varepsilon_F$ function, a dimensionless version of $\xi(q_1,q_3)$ [Eq. (B9)], important in the calculation using the method of steepest decent.

$$\sum_{k_3<0}\theta(-k_F-k_3)\theta(k_F+k_{3'})=\frac{L}{2\pi}q_3\theta(q_3). \quad \text{(B5)}$$

We see that since k_1 and k_3 are restricted to the Fermi level, we can insert $k_1 \simeq k_F$ and $k_3 \simeq -k_F$ in the rest of the integrand. To do the $k_2>0$ summation, we use the energy conservation contained in the scattering rate. It is rewritten as (inserting $k_1=k_F$ and $k_3=-k_F$)

$$\delta(\varepsilon_{1'}+\varepsilon_{2'}+\varepsilon_{3'}-\varepsilon_1-\varepsilon_2-\varepsilon_3) \simeq \frac{m}{\hbar^2}\frac{1}{|q_1+q_3|} \times\delta\left(k_2-k_F\frac{q_1-q_3}{q_1+q_3}-\frac{1}{2}(q_1+q_3)-\frac{1}{2}\frac{q_1^2+q_3^2}{q_1+q_3}\right). \quad \text{(B6)}$$

We have now done the summation over k_1, k_2, and k_3 and are left with the summation over q_1 and q_3 of the scattering rate, some Fermi functions, and the phase factors described above. To this end, we introduce $u(q_1,q_3)$ by inserting $k_1=k_F$ and $k_3=-k_F$ in Eq. (42),

$$\mathbb{V}(11',22',33')=\frac{\delta_{\sigma_{1'},\sigma_1}\delta_{\sigma_{2'},\sigma_2}\delta_{\sigma_{3'},\sigma_3}}{4L^2\hbar^2/m}u(q_1,q_3) \quad \text{(B7)}$$

for a general symmetrized interaction $\tilde{V}_q=V_q+V_{-q}$.

Furthermore, we collect the exponential tales of the Fermi functions [Eqs. (40b) and (40c)],

$$(1-f^0_{2'})f^0_3(1-f^0_{3'}) = e^{(\varepsilon_{2'}-\varepsilon_3+\varepsilon_{3'}-\varepsilon_F)/k_BT}, \tag{B8}$$

defining

$$\xi(q_1,q_3) \equiv \varepsilon_{2'} - \varepsilon_3 + \varepsilon_{3'} = \varepsilon_F\left(\frac{q_1-q_3}{q_3+q_1}\right)^2 - \frac{1}{2}\hbar v_F(q_1+q_3) + \frac{1}{2}\hbar v_F\frac{(q_1-q_3)(q_1^2+q_3^2)}{(q_3+q_1)^2} + \frac{\hbar^2}{2m}\frac{q_3^2(2q_1^2+2q_1q_3+q_3^2)}{(q_1+q_3)^2}, \tag{B9}$$

inserting $k_{2'}=k_2-q_1-q_3$, k_2 from the energy conservation Eq. (B6) and $k_1=k_F$ and $k_3=-k_F$. Therefore, we finally get the interacting contribution to the current in Eq. (B1) as

$$I^{\text{int}} = \frac{3(-e)Lm^3}{32\pi^4\hbar^7}\int_0^\infty dq_1\int_0^\infty dq_3\frac{q_1q_3}{q_1+q_3}|u(q_1,q_3)|^2 \times\theta(k_F-k_2)\,\theta(k_2)\,\theta(-k_2+q_1+q_3) \times\theta(k_F-q_3)e^{(\xi(q_1,q_3)-\varepsilon_F)/k_BT} \times\left\{\frac{\Delta T}{k_BT^2}[\xi(q_1,q_3)-\varepsilon_F]-\frac{eV}{k_BT}\right\}. \tag{B10}$$

Here, only the step functions that restricts the integral are included. Next, we introduce the dimensionless integration variables $Q_i=q_i/k_F$ for $i=1,3$ and the dimensionless functions

$$U(Q_1,Q_3) = k_F^2u(k_FQ_1,k_FQ_3), \tag{B11}$$

$$\Xi(Q_1,Q_3) = \frac{\xi(k_FQ_1,k_FQ_3)}{\varepsilon_F} \tag{B12}$$

in the integral

$$I^{\text{int}} = \frac{3(-e)Lm^3}{32\pi^4\hbar^7k_F}e^{-T_F/T}\int_{\mathcal{A}}dQ_1dQ_3\frac{Q_1Q_3}{Q_1+Q_3}|U(Q_1,Q_3)|^2 \times e^{\Xi(Q_1,Q_3)T_F/T}\left\{\frac{T_F\Delta T}{T^2}[\Xi(Q_1,Q_3)-1]-\frac{eV}{k_BT}\right\}, \tag{B13}$$

where $\mathcal{A}$ is the integration area shown in Fig. 4(a). Note that this expression is valid for a general interaction $\tilde{V}_q$ and that it is not possible to extract a power law in temperature times some integral by defining new integration variables.

To proceed, we consider the low-temperature limit $T/T_F \ll 1$ by using the method of steepest decent. Due to the exponential function $e^{\Xi(Q_1,Q_3)T_F/T}$, the maximum of $\Xi(Q_1,Q_3)$ will dominate the integral for $T/T_F\to 0$, since $\Xi(Q_1,Q_3) \leqslant 0$. The maxima are $\Xi(0,0)=0$ and $\Xi(0,1)=0$, and $\Xi(Q_1,Q_3)$ is shown in Fig. 4(b). For a decreasing interaction, the area of $Q_1\ll 1$ and $Q_3\ll 1$ dominates even though the integrand is zero for $Q_1=Q_3\to 0$. Therefore, we expand around the maximum $(Q_1,Q_3)=(0,0)$ to get the lowest-order result in T/T_F. In view of the integration region [Fig. 4(a)], we use $Q_3=Q_1\equiv Q$ in the integral [Eq. (B13)] and thereby do the Q_3 integral using the approximate limits seen in Fig. 4(a), i.e.,

$$\int_{Q_1-Q_1^2}^{Q_1+3Q_1^2} 1dQ_3 = 4Q_1^2. \tag{B14}$$

To model the symmetrized potential $\tilde{V}_q$ for small q, we include the deviation from a constant, as described in Eq. (48). This gives

$$\frac{Q_1Q_3}{Q_1+Q_3}|U(Q_1,Q_3)|^2\Big|_{Q_3=Q_1\equiv Q} \to V_0^4\left(\frac{k_F}{q_0}\right)^4\frac{9}{2}Q^5 \tag{B15}$$

to lowest order in Q. In the exponential, we keep Ξ to lowest order in Q, i.e.,

$$e^{\Xi(Q_1,Q_3)T_F/T} \to e^{-2QT_F/T}. \tag{B16}$$

So using the lowest order in Q in the integrand (leading to lowest order in T), the interacting contribution to the current is

$$I^{\text{int}} = \frac{3(-e)Lm^3}{32\pi^4\hbar^7k_F}e^{-T_F/T}\int_0^\infty dQV_0^4\left(\frac{k_F}{q_0}\right)^4\frac{9}{2}Q^5 4Q^2e^{-2QT_F/T} \times\left[\frac{T_F\Delta T}{T^2}(0-1)-\frac{eV}{k_BT}\right] \tag{B17}$$

$$\simeq \frac{8505}{2048\pi^4}e^{-T_F/T}\frac{e}{\hbar}\frac{(V_0k_F)^4}{\varepsilon_F^3}(Lk_F)\left(\frac{k_F}{q_0}\right)^4\left(\frac{T}{T_F}\right)^7\left(\frac{\Delta T}{T}+\frac{eV}{\varepsilon_F}\right) \tag{B18}$$

to lowest order in temperature. This is the result stated in the text in Eq. (51).

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