Mesoscopic Hall effect driven by chiral spin order

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A Hall effect due to spin chirality in mesoscopic systems is investigated. We consider a four-terminal Hall system including local spins with geometry of a vortex domain wall, where strong spin chirality appears near the center of the vortex. The Fermi energy of the conduction electrons is assumed to be comparable to the exchange coupling energy where the adiabatic approximation ceases to be valid. Our results show a Hall effect where a voltage drop and a spin current arise in the transverse direction, which is shown to survive in the presence of weak disorder. The similarity between this Hall effect and the conventional spin Hall effect in systems with spin-orbit interaction is pointed out.

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I. INTRODUCTION

Recent research on the anomalous Hall effect has shown that the spin chirality of a local spin system induces a Hall conductance via exchange coupling.[1–5] The anomalous Hall effect can be seen in ferromagnetic metallic systems, where the time reversal symmetry (TRS) is broken. When TRS is preserved, the spin current in a transverse direction is driven by a longitudinal voltage drop. Such a spin current, the so-called spin Hall current,[6–8] can be seen in semiconductor systems with a spin-orbit interaction. Both anomalous and spin Hall effects were originally expected in bulk systems, where the gauge field related to monopoles in momentum space plays a crucial role. The spin Hall effect can also be seen in mesoscopic two-dimensional samples with Rashba spin-orbit interaction.[7] Numerical calculations using both the Kubo and Landauer-Büttiker formulas predict the spin Hall effect. Note that the Landauer-Büttiker formula[9,10] does not explicitly assume a local electric field inside the sample.[11]

Recently, it has been shown that a Hall conductance is expected in mesoscopic systems, such as dilute magnetic semiconductors with artificial magnetic structures.[12–14] The spin of the conduction electron couples to the local magnetic moment via an exchange interaction. The Hall conductance is determined in such a well-ordered magnetic system by using a local gauge transformation and the adiabatic approximation, in which only a majority spin component is considered. This approximation changes the symmetry of the system from SU(2) to U(1). However, the minority spin of conduction electrons cannot be neglected when the exchange coupling energy $J_{\rm ex}$ is comparable to the Fermi energy E_F as in magnetic semiconductors.[15] Furthermore, the characteristic length of the local spin modulation ξ can be comparable to the Fermi wavelength λ_F due to the small Fermi energy ($E_F \sim 10$ meV). For such conditions, one cannot apply the adiabatic approximation.[12]

In this paper, we show that mesoscopic systems with internal chiral magnetic order exhibit Hall effects in such a way that both the charge and the spin Hall effects occur simultaneously. We consider a two-dimensional electron system that interacts with local spins via exchange coupling. The local spins have a vortex structure with a finite out-of-plane component that determines the spin chirality. We assume that the Fermi energy of the conduction electron is comparable to the exchange coupling energy as in some magnetic semiconductors. In such energy region, the adiabatic approximation and the U(1) mean field theory[4] that explains the anomalous Hall effect cannot be applied. We calculate the spin-resolved Hall conductance numerically by using the recursive Green's function method.[16,17] Our numerical results show that a Hall voltage is induced when the system has spin chirality. Furthermore, a spin Hall current can be observed even if the system does not have a spin chirality. This spin Hall current does not require a uniform electric field inside the system unlike conventional spin Hall effects in bulk systems with spin-orbit interaction.[6] We also study the effect of randomness and find that the Hall effect survives in the presence of the impurities. We mention that the system considered here is related to a two-dimensional spin-orbit system for which the spin Hall effect has been reported. We show that the previously reported spin Hall effects[18,19] obtained from the Landauer-Büttiker formula are similar to the Hall effect presented here. We investigate the coupling constant dependence of the spin Hall conductances for these systems. Both of the spin Hall conductances oscillate when increasing the exchange or spin-orbit coupling strength and show linear dependences in the weak coupling regime.

II. MODEL AND METHOD

We consider a two-dimensional electron system with exchange interaction,[20]

$$H = \sum_{i,\sigma} W_i c_{i\sigma}^{\dagger} c_{i\sigma} - t \sum_{\langle i,j\rangle,\sigma} c_{i\sigma}^{+} c_{j\sigma} - J \sum_{i,\sigma,\sigma'} c_{i\sigma}^{\dagger} \boldsymbol{\sigma}_{\sigma,\sigma'} c_{i\sigma'} \cdot \mathbf{S}(x,y), \tag{1}$$

with the nearest-neighbor hopping parameter $t=\hbar^2/2m^* a^2$ (m^* the effective mass and a the lattice parameter). Fermi energy is defined as $E_F=4t+E$ from the band edge. The operator $c_{i\sigma}^{+}$ ($c_{i\sigma}$) creates (annihilates) an electron of spin σ at

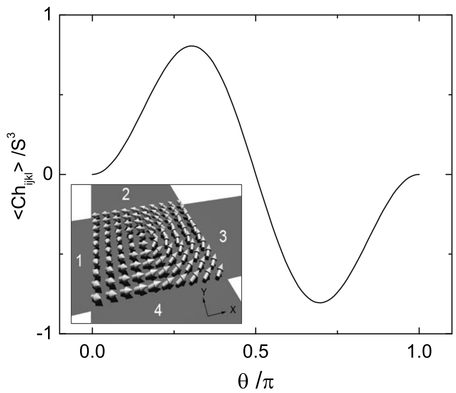

FIG. 1. Spatial average of the chirality Ch_{ijkl} in Eq. (3) as a function of θ. Inset: schematic of proposed four-terminal system, including chiral magnetic structure. Labels 1–4 indicate leads that are free from randomness, local magnets, or spin-orbit interaction.

lattice site i, $\boldsymbol{\sigma}$'s are the Pauli matrices, and $J(>0)$ is the exchange coupling constant. W_i is the random potential distributed uniformly in the range of $-W/2$ and $W/2$. The local spin $\mathbf{S}(x,y)$ has the geometry of a vortex in the x-y plane, in addition to the uniform S_z component,

$$\mathbf{S}(x,y) = S[\cos\phi(x,y)\sin\theta, \sin\phi(x,y)\sin\theta, \cos\theta]. \quad (2)$$

Here, S is the modulus of the local spin, $\phi(x,y) = -\tan^{-1}(y/x)$, such that the center of the vortex is located at the origin. We assume that the dynamics of the local spins is much slower than the dynamics of the conduction electrons, and treat the local spins as static. A schematic view of the system is shown in the inset of Fig. 1, in which we assume four-terminal geometry. The leads are labeled as 1–4 and the $+x(y)$ direction is set to the direction from lead 1(4) to 3(2). We define the chirality of the local spin system as

$$\mathrm{Ch}_{ijkl} \equiv E_{ijk} + E_{ikl}, \quad (3)$$

where $E_{ijk}=\mathbf{S}_i\cdot(\mathbf{S}_j\times\mathbf{S}_k)$ for a plaquette of a square lattice labeled as (i,j,k,l) counterclockwise. The system shows large spin chirality near the center of the vortex. Figure 1 shows the spatial average of the spin chirality as a function of θ. The value of the spin chirality changes its sign when the sign of the S_z component changes. When the conduction electron spin is parallel to the local spin, the conduction electrons feel the effective magnetic flux by propagating around the square lattice.[21] The effective magnetic flux is proportional to Ch_{ijkl}, and the sign of the Hall conductance will change at $\theta=\pi/2$. Obviously, the local spin system does not have a chirality, $\langle \mathrm{Ch}_{ijkl}\rangle=0$, for $\theta=0,\pi/2,\pi$. As in the case of the bulk anomalous Hall effect due to spin chirality,[4] we expect that the Hall effect can be obtained except for these values of θ.

We calculate the spin-resolved transmission amplitudes by using the recursive Green's function method.[16,17] By employing the Landauer-Büttiker formula, we assume that the net current of leads 2 and 4 is zero. The charge current of lead $I_l=\Sigma_{\sigma\sigma'}(N_l-R_{l\nu\sigma,l\nu\sigma'})\mu_l-\Sigma_{l\neq l'\sigma\sigma'}T_{l\nu\sigma,l'\nu\sigma'}\mu_{l'}$, where N_l is the number of propagating channels per spin for the lead l, $T_{l\nu\sigma,l'\nu\sigma'}(R_{l\nu\sigma,l'\nu\sigma'})$ is the transmission (reflection) amplitude from the σ'-spin channel (polarized in the ν direction) of lead l' to the σ-spin channel of the lead l, and μ_l is the chemical potential of the reservoir attached to the lead l. We assume that the chemical potential of lead 3 is zero. The Hall conductance is defined as $G_H=-r_{yx}/(r_{xx}^2+r_{yx}^2)$, where $r_{yx}=(\mu_2-\mu_4)/I_1$ and $r_{xx}=\mu_1/I_1$ are the Hall resistance and the resistance, respectively. On the other hand, the spin current (polarized in the ν direction) of lead l is given by $I_l^{\nu\sigma}=\Sigma_{\sigma'}(N_l-R_{l\nu\sigma,l\nu\sigma'})\mu_l-\Sigma_{l'\neq l\sigma'}T_{l\nu\sigma,l'\nu\sigma'}\mu_{l'}$. The spin Hall conductance is defined as

$$G_{\mathrm{sH}}^{\nu} = \frac{I_2^{\nu\uparrow}-I_2^{\nu\downarrow}}{\mu_1} = 2\frac{I_2^{\nu\uparrow}}{\mu_1} = 2\left[-T_{2\nu\uparrow,1\nu\uparrow}-T_{2\nu\uparrow,1\nu\downarrow}+\frac{(N_2-R_{2\nu\uparrow,2\nu\uparrow}-R_{2\nu\downarrow,2\nu\uparrow})\mu_2-(T_{2\nu\uparrow,4\nu\uparrow}+T_{2\nu\uparrow,4\nu\downarrow})\mu_4}{\mu_1}\right], \quad (4)$$

where the subscripts $\nu\uparrow$ and $\nu\downarrow$ indicate the eigenstates of σ_ν. Second equality of Eq. (4) comes from the condition $I_2^{\nu\uparrow}+I_2^{\nu\downarrow}=0$. Chemical potentials are calculated by $\mu_1=I_1\mathbf{R}_{11}$, $\mu_2=I_1\mathbf{R}_{21}$, and $\mu_4=I_1\mathbf{R}_{31}$. $\mathbf{R}$ is the inverse matrix of $\mathbf{G}$ given by

$$\mathbf{G} = \begin{pmatrix} 2N_1-\Sigma_{\sigma\sigma'}R_{1\nu\sigma,1\nu\sigma'} & -\Sigma_{\sigma,\sigma'}T_{1\nu\sigma,2\nu\sigma'} & -\Sigma_{\sigma,\sigma'}T_{1\nu\sigma,4\nu\sigma'} \\ -\Sigma_{\sigma,\sigma'}T_{2\nu\sigma,1\nu\sigma'} & 2N_2-\Sigma_{\sigma\sigma'}R_{2\nu\sigma,2\nu\sigma'} & -\Sigma_{\sigma,\sigma'}T_{2\nu\sigma,4\nu\sigma'} \\ -\Sigma_{\sigma,\sigma'}T_{4\nu\sigma,1\nu\sigma'} & -\Sigma_{\sigma,\sigma'}T_{4\nu\sigma,2\nu\sigma'} & 2N_4-\Sigma_{\sigma\sigma'}R_{4\nu\sigma,4\nu\sigma'} \end{pmatrix}. \quad (5)$$

III. RESULTS

The uppermost panel of Fig. 2 shows the Hall conductance as a function of the energy of the conduction electrons and the angle of the local spin θ for $JS=1.0t$ in a system size of $30a\times 30a$. The Hall conductance is nonzero for $\theta\neq 0$, $\pi/2,\pi$. Its sign changes when the sign of S_z changes. The amplitude of the Hall conductance oscillates with the energy. This is because the Hall effect is proportional to the momentum of the x direction; hence, it decreases when the Fermi energy is close to the energy where new propagating channels open. We note that the (charge) Hall effect also induces spin current density to be polarized parallel to the local spins near the interface between lead 2 (or 4) and the sample.

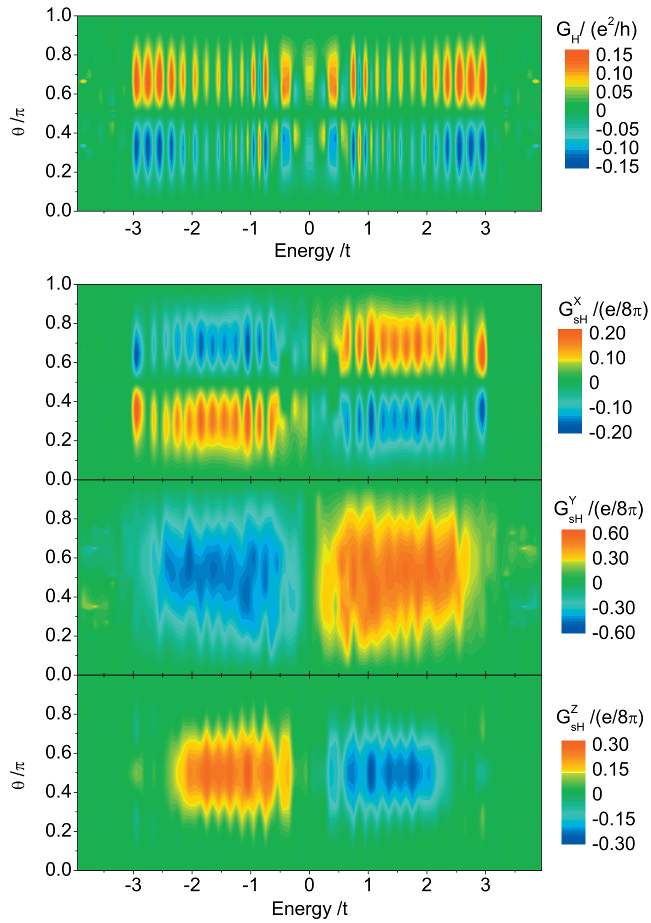

FIG. 2. (Color) Hall conductance G_H and spin Hall conductance $G^{\nu}_{\rm sH}(\nu=X,Y,Z)$ for each polarization: Exchange coupling constant is $JS=t$ and the system size of $30a\times 30a$. Hall conductance and x component of the spin Hall conductance disappear at $\theta=0,\pi/2,\pi$, where the spin chirality vanishes.

If the Fermi energy is comparable to the exchange coupling energy, the adiabatic approximation that neglects the minority spin components is no longer valid, and we expect a spin current with a polarization that is not parallel to the local spins. To confirm this, we plot the spin Hall conductance for each polarization direction in the lower three panels of Fig. 2. $G^{Y,Z}_{\rm sH}$ vanish at $\theta=0,\pi$, while $G^X_{\rm sH}$ vanishes at $\theta=0,\pi/2,\pi$. At $\theta=\pi/2$, the local spins near the interface between the sample and lead 2 are almost direct in the x direction, and the suppression of the Hall conductance results in a suppression of $G^X_{\rm sH}$. The nonvanishing Y and Z components of the spin Hall conductance at $\theta=\pi/2$ do not induce a voltage drop in the transverse direction, such as the spin Hall effect predicted in the spin-orbit system.[18,19] The direction of polarization rotates while electrons propagate in the sample due to the precession induced by the exchange coupling. This precession is an important feature of the mesoscopic spin Hall effect that is also obtained in a two-dimensional electron system with spin-orbit interaction. In contrast, only the Z component of the spin current is expected in bulk spin-orbit systems.

Figure 3 shows the averaged Hall conductances in the presence of impurities. We plot the conductance as a function of the ratio between λ_F and the mean free path $L_m=(6\lambda_F^3E_F^2)/(W^2\pi^3a^2)$.[16] The Hall conductances survive when the strength of the impurities is weak. We also observe that the sign of each Hall conductance is not sensitive to weak disorder. The spin Hall conductances, especially $\langle G^X_{\rm sH}\rangle$, survive when the strength of the impurities is not so strong.

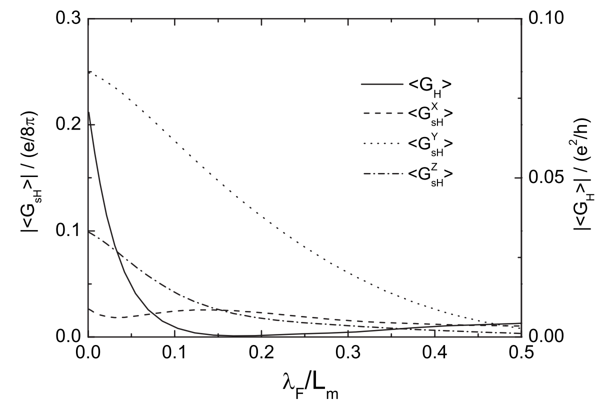

FIG. 3. Averaged Hall conductance $\langle G_H\rangle$ and spin Hall conductances $\langle G^{\nu}_{\rm sH}\rangle(\nu=X,Y,Z)$ as a ratio of the Fermi wavelength λ_F and the mean free path L_m. Parameters are $E_F=1.45t$, $\theta=0.3\pi$, and $JS=t$. Average over 10 000 random configurations has been performed.

To make contact with the two-dimensional system with spin-orbit interaction, we consider the Rashba spin-orbit interaction represented as

$$H=-t\sum_{\langle i,j\rangle,\sigma,\sigma'} V_{i\sigma,j\sigma'}c^{\dagger}_{i\sigma}c_{j\sigma'}, \tag{6}$$

with

$$V_{i+\hat{\mathbf{x}},i}=\begin{pmatrix}\cos\gamma & \sin\gamma\\ -\sin\gamma & \cos\gamma\end{pmatrix} \tag{7}$$

and

$$V_{i+\hat{\mathbf{y}},i}=\begin{pmatrix}\cos\gamma & -i\sin\gamma\\ -i\sin\gamma & \cos\gamma\end{pmatrix}, \tag{8}$$

where γ is the coupling strength of the Rashba spin-orbit interaction.[22–24] The parameters JS and γ can be regarded as a gauge field strength.[25,26] Figure 4 shows the dependence on the coupling strength of the spin Hall conductances both for the chiral spin system and for the spin-orbit system. The spin Hall conductances oscillate with the coupling constant in both cases. Indeed, the component of the spin Hall conductance in the spin-orbit system shows spin precession by changing the length of the lead where the spin-obit interaction is present.[27] The precession of the spin current is the feature of ballistic systems, such as a Datta-Das spin transistor.[28] In order to observe the spin precession of the spin current, one needs the long spin relaxation time. In this sense, the spin Hall current obtained in the present paper should be distinguished from the bulk spin Hall effect calculated from the spin current-charge current correlation of Kubo formula. We also show the absolute value of the spin

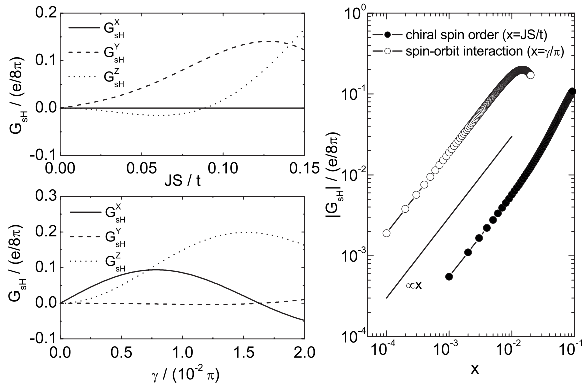

FIG. 4. Coupling parameter dependences of the spin Hall conductances of the chiral spin order system with $\theta=\pi/2$ (left upper panel) and of the Rashba spin-orbit interaction system (left lower panel). The right panel shows the log-log plot of absolute values of spin Hall conductances $|G_{\mathrm{sH}}|=\sqrt{(G_{\mathrm{sH}}^X)^2+(G_{\mathrm{sH}}^Y)^2+(G_{\mathrm{sH}}^Z)^2}$ in a weak coupling regime. Both spin Hall conductances show a linear dependence on the coupling parameters JS and γ.

Hall conductance $|G_{\mathrm{sH}}|=\sqrt{(G_{\mathrm{sH}}^X)^2+(G_{\mathrm{sH}}^Y)^2+(G_{\mathrm{sH}}^Z)^2}$ in the weak coupling regime. Here, both spin Hall conductances show linear dependence on the coupling constant in weak coupling regime.

IV. CONCLUSIONS

We have investigated a mesoscopic Hall effect driven by a local spin system with spin chirality, which might be experimentally detected in two-dimensional electron system embedded in ferromagnetic semiconductors. The local spin system is assumed to have the geometry of a vortex with a chirality at the center. We have predicted a Hall effect, which induces both a charge and a spin Hall conductance. Our numerical results based on the Landauer-Buttiker formula and the recursive Green's function technique show that a voltage drop is obtained in the presence of spin chirality. The sign and the magnitude of the Hall conductances are rather insensitive to weak disorder, and the Hall effect is expected to be observable experimentally. We have pointed out that the present Hall effect is related to the spin Hall effect obtained for a two-dimensional spin-orbit system, but should be distinguished from the usual bulk spin Hall effect driven by monopoles in momentum space described.

For measuring the present Hall effect in actual systems, an experimental setup using ferromagnetic semiconductors, such as (Ga, Mn)As, can be used. The proposed vortex spin configuration[29] can be obtained in dilute magnetic semiconductors with low Curie temperatures.[30,31] Because of the small saturation magnetization (≈0.01 T) of magnetic semiconductors, the coupling energy between the conduction spin and local magnetic moment should be comparable to the Fermi energy.[15] For our calculations, by setting the tight binding parameter $a=10$ nm and $m^*=0.05m_e$, the corresponding exchange energy becomes $J\sim6.9$ meV. The Fermi energy is $E_F\sim10$ meV for $E_F=1.45t$. θ should be adjusted approximately to $\pi/2$ to minimize the heating effect that destroys the spin order. Indirectly, the effect has been reported by spin torque effect in a vortex domain wall.[32,33]

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