Unconventional conductance plateau transitions in quantum Hall wires with spatially correlated disorder

Tohru Kawarabayashi and Yoshiyuki Ono
Department of Physics, Toho University, Miyama 2-2-1, Funabashi 274-8510, Japan

Tomi Ohtsuki
Department of Physics, Sophia University, Kioi-cho 7-1, Chiyoda-ku, Tokyo 102-8554, Japan

Stefan Kettemann
I. Institut für Theoretische Physik, Universität Hamburg, 20355 Hamburg, Germany

Alexander Struck
Department of Physics, University of Kaiserslautern, D-67663 Kaiserslautern, Germany

Bernhard Kramer
Jacobs University Bremen, 28757 Bremen, Germany
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Quantum transport properties in quantum Hall wires in the presence of spatially correlated random potential are investigated numerically. It is found that the potential correlation reduces the localization length associated with the edge state, in contrast to the naive expectation that the potential correlation increases it. The effect appears as the sizable shift of quantized conductance plateaus in long wires, where the plateau transitions occur at energies much higher than the Landau band centers. The scale of the shift is of the order of the strength of the random potential and is insensitive to the strength of magnetic fields. Experimental implications are also discussed.

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I. INTRODUCTION

Quantum transport property of two-dimensional (2D) systems in strong magnetic fields has been one of the central issues of condensed matter physics. Much work has been performed in connection with the quantum Hall effect.[1–6] Recent discovery of the quantum Hall effect in graphene[7] also stimulated theoretical interest. When the system is in a strong magnetic field, the so-called edge states are formed along the boundaries of the system.[8,9] The edge state corresponds to the classical skipping orbit along the boundary and is known to be less influenced by impurities and defects of the system.[10] It thus plays an important role in the quantum transport of a two-dimensional system in a strong magnetic field.

Numerical studies of the electron states of two-dimensional systems in a strong magnetic field in the presence of boundaries have already been performed by many authors.[11–15] It has been shown that the edge state is well defined and is extended along the boundary as long as its energy lies away from those of the bulk Landau subbands and the magnetic fields are strong enough. On the other hand, when its energy lies in the middle of the bulk Landau subbands, the edge state mixes with the bulk states by impurity scattering, which leads to the localization of edge states.[14,15] Recently, it has been demonstrated for the case of long quantum Hall wires with uncorrelated disorder potential that the conductance, indeed, vanishes when the Fermi energy is close to the centers of the bulk Landau subbands.[16,17] The vanishing conductance can be understood as the consequence of the mixing between the bulk states and the edge states having opposite current directions at each end of the system, as has been confirmed by a quantitative comparison with analytical results. This transition between a quantized conductance and the insulator is called the chiral metal-insulator transition (CMIT).[16,17]

In the present paper, the effect of potential correlation in such systems is investigated numerically. The potential correlation is, in general, important for the transport in low dimensions, since in some cases it changes the localization behavior drastically. For a carbon nanotube, it has been shown that the potential correlation leads to the absence of backscattering.[18] Even for pure one-dimensional systems, dimer type correlation or long-range correlation of potential gives rise to delocalized states.[19,20] For the two-dimensional bulk system in a magnetic field, the effect of potential correlation has also been studied based on the continuum model[21–23] as well as on the tight-binding model.[24] It has been demonstrated that the mixing between edge states is suppressed by the potential correlation.[22] In the analysis on the critical states of the Landau bands, it is seen that the critical energies are insensitive to the potential correlation as long as the strength of the disorder is weak.[24] It may then be expected naively that CMIT observed for the uncorrelated potential is suppressed and the quantized conductance steps are recovered when the potential correlation is introduced. We find, indeed, the suppression of CMIT, which yields the recovery of the quantized conductance steps. Quite unexpectedly, however, it is found that the localization length associated with the edge states is suppressed by the potential correlation. This causes the shift of the quantized conductance steps toward higher energies. It is remarkable that the potential correlation suppresses the conduction of wires, since the potential correlation is usually expected to enhance the conduction of the system. We find that the scale of the shift is of the order of the strength of random potential and is insensitive to the strength of the magnetic field as long as the Landau levels are well defined. On the other hand, no shift appears in the square geometry. It is therefore necessary to analyze the effect of potential correlation on the edge states in long wires to understand these shifts of conductance plateaus.

II. MODEL AND METHOD

We adopt the tight-binding model described by the following Hamiltonian on the square lattice:

$$H = \sum_{\langle i,j\rangle} V \exp(i\theta_{i,j}) c_i^\dagger c_j + \sum_i \varepsilon_i c_i^\dagger c_i, \tag{1}$$

where $c_i^\dagger$ (c_i) denotes the creation (annihilation) operator of an electron on the site i. The summation of the phases $\{\theta_{i,j}\}$ around a plaquette is equal to $-2\pi\phi/\phi_0$, where ϕ is the magnetic flux through the plaquette and $\phi_0 = h/e$ stands for the flux quantum. The elementary charge and the Planck constant are denoted by e and h, respectively. All length scales are measured in units of the lattice spacing. The site energies $\{\varepsilon_i\}$ are assumed to be distributed with the Gaussian probability density

$$P(\varepsilon) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-\varepsilon^2}{2\sigma^2}\right), \tag{2}$$

and to have the spatial correlation as

$$\langle \varepsilon_i \varepsilon_j \rangle = \langle \varepsilon^2 \rangle \exp(-|\boldsymbol{R}_i - \boldsymbol{R}_j|^2/4\eta^2). \tag{3}$$

Here, σ and η are the parameters which determine the strength of the random potential and the strength of its spatial correlation, respectively. The position vector for the site i is denoted by $\boldsymbol{R}_i$. The spatially correlated potential is made from the uncorrelated potential v_j's as[24]

$$\varepsilon_i = \frac{\sum_j v_j \exp(-|\boldsymbol{R}_j - \boldsymbol{R}_i|^2/2\eta^2)}{\sqrt{\sum_j \exp(-|\boldsymbol{R}_j - \boldsymbol{R}_i|^2/\eta^2)}}. \tag{4}$$

When the uncorrelated potential v_j's obey the Gaussian distribution with variance σ, it is easy to verify that ε_i satisfies the relations in Eqs. (2) and (3). The parameter η, therefore, represents the range of the impurity potential. In the following, we specify the disorder strength by a parameter $w = \sigma\sqrt{12}$, since it can be regarded as the effective width of the potential distribution.

We consider a system with length L and width M. Two leads are attached to both ends of the system, and the fixed boundary condition is assumed in the transverse direction. For the realization of the isotropic correlation for ε_i in the sample region $L \times M$, we consider the additional regions of width 5η outside the sample region in performing the summation over v_j in Eq. (4).

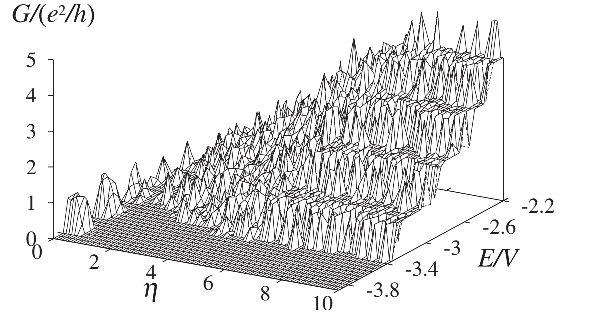

FIG. 1. Conductance for $w/V=0.8$, $\phi/\phi_0=1/40$, and $L/M=250$ as a function of the range of the impurity potential η and the Fermi energy E/V. In the present system, ratio of the width M to the magnetic length l is $M/l \approx 8$.

The two-terminal conductance G is obtained by means of the Landauer formula

$$G = (e^2/h) \mathrm{Tr}\, T^\dagger T, \tag{5}$$

where T is the transmission matrix. We adopt the transfer matrix method[25] to evaluate the transmission matrix. Throughout the present analysis, we consider an independent impurity configuration for each value of energy E and η. The width M is set to be 20. The smallest magnetic flux ϕ/ϕ_0 per plaquette is $1/40$, and the corresponding magnetic length $l = \sqrt{\hbar/eB}$ is about 2.5, much smaller than the system width M. This leads to the existence of edge states along the boundaries.

III. NUMERICAL RESULTS

We first show the results of conductance for $w=0.8V$ and $\phi/\phi_0=1/40$ in Fig. 1. The length of the system is $L/M=250$. In the absence of potential correlation ($\eta=0$), we see the chiral metal-insulator transition as a function of energies. The conductance peaks at energies $E/V \approx -3.7$ and -3.4 are the contributions from the lowest and second lowest Landau bands, respectively. It is clearly seen that these peaks disappear for $\eta > 1$ and that the quantized conductance steps are recovered for $\eta > 6$. It is to be noted here that the recovery starts at $E/V \approx -3.4$. Since the contribution from the lowest Landau band starts at $E/V \approx -3.8$ for the case of $\eta=0$, the conductance steps can be understood as being shifted toward higher energies with an amount of $\delta E/V \approx 0.4$ in the presence of long potential correlation.

In order to see the shift more clearly, we examine the conductance in the case of stronger magnetic fields $\phi/\phi_0 = 1/16$ and $1/8$. In Fig. 2, the results for $\phi/\phi_0=1/16$ are shown. The length of systems is assumed to be 1000 ($L/M=50$). Here, it is clearly seen that the conductance plateaus shift toward higher energies as the correlation length η is increased. In both cases, the scale of the shifts in the presence of the long correlation turns out to be approximately $0.3V$, insensitive to the magnetic fields. We also evaluate the conductance for $L/M=1$, namely, for the square system. It is

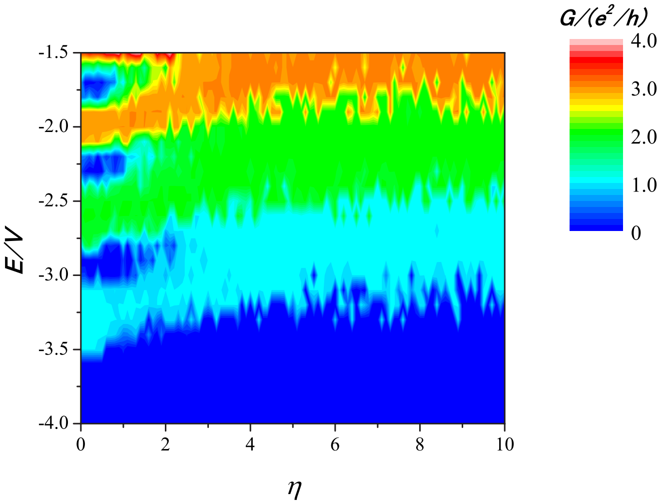

FIG. 2. (Color online) Conductance $G/(e^2/h)$ for $w/V=0.8$ and $L/M=50$ as a function of the potential range η and the Fermi energy E/V. The magnetic flux ϕ/ϕ_0 is assumed to be 1/16. The magnetic length $l\approx 1.6$ and, accordingly, $M/l\approx 12.5$.

then found that no shift appears in that case. This is consistent with the result for the 2D bulk system that the levitation of critical states due to the potential correlation for the present range of disorder strength ($w/V\approx 1$) is much smaller than the present scale of the shifts of conductance plateaus.[24] The present shift of conductance plateaus is, thus, a specific feature to long wires, suggesting that the shift arises from the one-dimensional character of the system rather than the 2D bulk properties. Apart from the shift of plateaus, it is also confirmed that CMIT fades away for larger values of η ($\gtrsim 2.5$). Detailed energy dependencies of the conductance in the presence of potential correlation ($\eta=5$) and that in its absence ($\eta=0$) are shown in Fig. 3. It is clearly seen that the shifts are common to these three plateaus and CMIT observed for $\eta=0$ (Refs. 16 and 17) disappears for $\eta=5$. The random jumps between the quantized plateaus for $\eta=5$ are fluctuations due to different realizations of disorder potential.

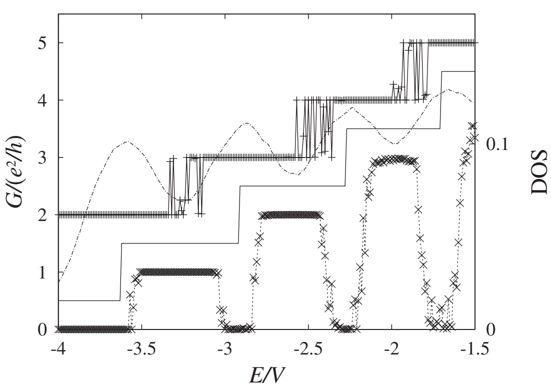

FIG. 3. Conductance $G/(e^2/h)$ for $\eta=0$ ($\times$) and $\eta=5\approx 3.13l$ (+) in the case of $w/V=0.8$, $\phi/\phi_0=1/16$, and $L/M=50$. For $\eta=5$, $G/(e^2/h)+2$ is plotted instead of $G/(e^2/h)$. Conductance G_0 for $w=0$ is also calculated and $G_0/(e^2/h)+0.5$ is plotted as a solid line. The density of states (DOS) for $\eta=5$ obtained by the Green's function method (Ref. 11) is plotted as a dotted curve.

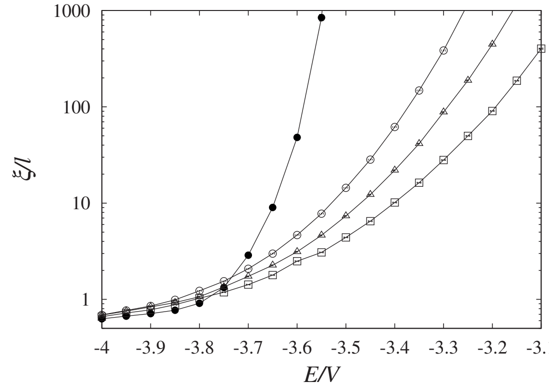

FIG. 4. Localization length along the wire of width 20 in the cases of $\eta=0$ g3 (○), 5 (△), and 10 (□). The disorder and the magnetic field are assumed to be $w/V=0.8$ and $\phi/\phi_0=1/16$, respectively. The magnetic length is $l\approx 1.6$.

It should be emphasized that the conductance plateau transitions occur away from the peaks of the density of states in the case of $\eta=5$.

In order to clarify how the scale of the shift depends on w, we calculate the conductance also for $w/V=0.4$ and 1.6 in the case of $L/M=50$ and $\phi/\phi_0=1/8$. We obtain the shifts to be $(0.1-0.2)V$ and $(0.6-0.7)V$, respectively. These results suggest that the scale of the shift is proportional to w.

The shift of plateaus means that the edge states which are transmitted in the case of $\eta=0$ are reflected for $\eta\neq 0$. The localization length in the case of $\eta\neq 0$ must therefore be much smaller than that in the case of $\eta=0$. This is surprising since the potential correlation normally reduces the electron localization. We, thus, examine the effect of potential correlation on the localization length along the wire in the presence of edge states. The localization length ξ estimated by the transfer matrix method[26] is shown in Fig. 4 for $\phi/\phi_0=1/16$ and various values of η. Here, we clearly see that the localization length ξ is much shorter than the system length $L/l=625$ for the energies from -3.6V to -3.3V in the presence of potential correlation. This is consistent with the vanishing conductivity in the presence of long potential correlation for energies lower than $E/V\sim -3.3$ in Figs. 2 and 3.

IV. SUMMARY AND DISCUSSION

Now we show that the present shift of conductance plateaus can be understood as a semiclassical effect. It is useful to recall here the fact that the edge state is reflected by the potential barrier when its energy measured from the corresponding Landau level $\Delta E_{\text{edge}}=E-(n+1/2)\hbar\omega_c$ is smaller than the energy of the potential barrier,[27,28] where ω_c denotes the cyclotron frequency. This suggests that the shift of conductance plateaus also occurs in the presence of a potential barrier across the wire instead of a disorder potential. We have confirmed in the present lattice model, that the shifts expected in the continuum model indeed, occur when the thickness of the barrier is larger than the magnetic length.

With this property of the edge states, it is natural to expect that the edge states having lower energies ($\Delta E_{\text{edge}}<w/2$) are

deformed considerably by the correlated potential varying with scales larger than the magnetic length, because such a potential would act as a local potential barrier. Due to the deformation of the edge state trajectory, in a certain region of a long sample, the effective width of the sample becomes narrow enough to induce the mixing of edge states with opposite directions, leading to the reflection. As the potential correlation is increased, the deformation would become larger and the probability for the reflection of edge states is likely to increase, accordingly. The same order of the shift ($\sim w/2$) in each plateau transition is also naturally understood, since ΔE_{edge} determines the probability of reflection.

The above semiclassical argument is consistent with our results that the shift becomes significant when the potential range η/l is large enough, and that in the long correlation limit, it saturates around $w/2$, which is effectively the maximum of the potential energy (Fig. 2). It would also account for the fact that the contributions from the lowest and second lowest Landau levels vanish when the potential range η approaches the order of the magnetic length in Fig. 1. In the presence of long potential correlation, it is thus expected that the conductance steps occur at critical energies $E_c \approx (n+1/2)\hbar\omega_c + w/2$ in long wires. For certain fixed impurity configurations, we have observed a complex structure around the critical energy, which is expected to be the resonant tunneling[29,30] between edge states.

It is important to note that the mixing of the edge states at one place of the wire would be enough to reflect the whole current associated with them. The probability to have such a place in a particular sample apparently depends on the length of the wire. It is natural that there is no shift in the case of the short system ($L/M=1$), since such probability is very small. The probability P to have high potential regions across the wire in a sample is estimated as $P \approx p^{M/\eta}(L/\eta)$ for a small p, which is the probability that the potential energy in the box of size η is larger than ΔE_{edge}. Requiring P to be of the order of unity, we get

$$\log(L/\eta) \cong (-\ln p)(M/\eta), \tag{6}$$

which suggests that M must be practically several times η or less to observe these phenomena. In reality, when the magnetic field is 10 T, the magnetic length is $l \sim 8$ nm, and the present systems correspond to wires whose width M is in the range 70–140 nm. The present numerical results suggest that the shift of conductance plateaus can be observed when $\eta > 3l \sim 25$ nm and $L = 3.5$–7 μm.

In summary, we have investigated the effect of potential correlation in quantum Hall wires. It has been shown clearly that the potential correlation shifts the quantized conductance plateaus toward higher energies. The scale of the shifts is of the order of the strength of the random potential. This shift is specific to systems with the wire geometry and is insensitive to the strength of the magnetic fields. This phenomenon is related to the transport property of edge states in the presence of long potential correlation. We have argued that the potential correlation enhances the mixing of the edge states at the opposite edges, which yields the reflection of edge channels. For correlated potential, the positions of the conductance plateau transitions in quantum Hall wires do not necessarily coincide with the positions of the bulk Landau levels. The chiral metal-insulator transition is absent when the potential correlation is much larger than the magnetic length of the system.

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