Influence of Rashba coupling on the magnetoresistance of a smooth domain wall

A. Phirouznia,[1,2] M. M. Tehranchi,[1,2,*] and M. Ghanaatshoar[1]

[1]*Laser Research Institute, Shahid Beheshti University, Evin 1983963113, Tehran, Iran*

[2]*Department of Physics, Shahid Beheshti University, Evin 1983963113, Tehran, Iran*

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The effect of the Rashba-type spin-orbit interaction on the domain-wall (DW) magnetoresistance (MR) is investigated for a smooth linear DW within the semiclassical approach. Results of this study are indicative of a negative MR for current-in-wall geometry for some ranges of the impurity densities and the chirality dependence of the DW MR. The increase of the Rashba coupling strength can effectively increase the MR of the DW in the samples with low impurity concentrations.

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I. INTRODUCTION

Nowadays the spin-dependent transport and nanoscale physics have been very active and constitute a prolific field of inquiry in science. On the one hand, spin-dependent transport in the new generation of electronic devices in which the spin of electron plays the role of the active element has gained crucial importance. Spintronics has attracted noticeable interest and has turned into a fast-growing research field in condensed matter physics and semiconductor electronics. Such an interest is a direct result of extremely rapid success in giant magnetoresistance devices as magnetic-field sensors in the read heads of commercial high-performance hard disks. On the other hand, recent progress in nanotechnology has enabled researchers to fabricate low-dimensional devices. By embracing special quantum-mechanical effects, such low-dimensional structures produce a variety of exciting and novel features especially in spin-dependent transport phenomena. The spin-orbit Rashba interaction is a prime instance. It arises out of the presence of structure inversion asymmetry introduced by a heterojunction, surfaces, or external fields.[1] The spin precession associated with the Rashba coupling led Datta and Das to propose a spin field-effect transistor in which the spin of the electron passing through the device is controlled by the Rashba spin-orbit (SO) interaction.[2] Such a transistor generated great interest in mesoscopic spin-polarized transport in the presence of structure inversion asymmetry.

Subsequently, spin-dependent transport properties of ferromagnetic structures in nanometer scale and local magnetic structures have attracted much interest recently. For instance, contrary to bulk samples, it has been found out that the magnetoresistance associated with nanosize domain walls (DWs) can be significantly large, even up to 2000%.[3–8] Also, the recent experiments suggest that the DW magnetoresistance (MR) can be either positive or negative. Positive MR due to the DW has been reported by Gregg *et al.* in striped domain structures.[3] Positive MR has also been observed in the Ni wires,[9] in single-layer ferromagnetic wires of $Ni_{80}Fe_{20}$,[10] and in a junction of mesoscopic ferromagnetic NiFe wires.[11] However, a number of experiments conducted on very narrow wires and thin films show negative DW MR.[12–16]

Theoretically, in weakly localized regime, quantum decoherence caused by the DW has been mentioned as a source of reduction in the resistance (for example, see Ref. 17). Nevertheless, spin-dependent impurity scattering proposed by Levy and Zhang was found responsible for mixing the spin channels and positive DW MR.[18] van Gorkom *et al.*[19] described a semiclassical model in which the DW MR is either positive or negative, depending on the difference between the spin-dependent scattering lifetimes. There are also other reports in which some parameters such as thickness of the DW in very thin layers or value of the magnetic field applied to a quantum wire are responsible for a transition from positive to negative DW MR.[20,21]

Recently, the effect of the Rashba SO interaction on the MR has been calculated by Dugaev *et al.*,[22] indicating that spin-orbit interaction may result in an increase of the magnetoresistance of a semiconducting magnetic wire with a DW of width d. Such calculations were carried out in the limit of $k_{F\uparrow(\downarrow)}d \ll 1$, corresponding to the case of a sharp DW—which can be realized in semiconductors. It should be pointed out that inside a collinear ferromagnet, the wave vector $\mathbf{k}$ is a good quantum number for the Rashba Hamiltonian, which commutes with the current operator. In this case, the Rashba SO interaction couples two states with different spin bands and the same $\mathbf{k}$. Therefore, for a collinear ferromagnet, the Rashba SO interaction cannot be taken responsible for elastic scatterings at low temperatures, and regardless of thermal spin-dependent scatterings, it may not contribute to the resistivity of the sample. On the contrary, eigenstates of the Rashba Hamiltonian for noncollinear magnetization of incommensurate structures such as DW possess quite a different nature. In fact, the Rashba interaction can be considered as a scattering source with an effective order range of the DW width in the elastic regime.

In this paper, the effect of the Rashba interaction on the MR of a smooth DW identified by condition $k_{F\uparrow(\downarrow)}d \gg 1$ is investigated for a two-dimensional ferromagnetic metal. In this order, we consider a two-dimensional electron gas (2DEG) system which includes a linear magnetic DW between two ferromagnetic regions in opposite directions. Besides DW, the Rashba SO interaction corresponding to an electric field, perpendicular to the plane of the system, mixes different spin channels in the DW. Here, the problem of DW MR for an ideal 2DEG system in the presence of SO interaction of Rashba type is treated at the analytical level within the semiclassical approach at low temperatures. We study the

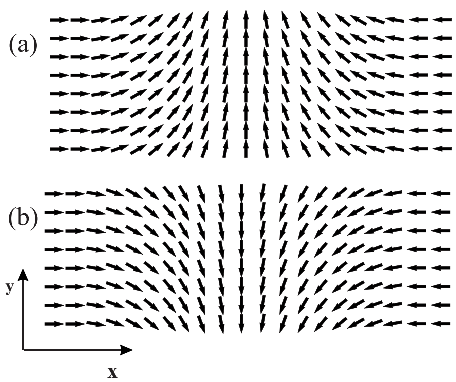

FIG. 1. (a) Positive chirality in which the DW rotation axis is parallel to the confinement electric field. (b) Negative chirality in which the DW rotation axis is antiparallel to the confinement electric field.

spin-dependent transport through the DW for current-in-wall (CIW) and current-perpendicular-to-wall (CPW) geometries. We also investigate the effect of the chirality on the MR in the presence of the Rashba interaction.

The paper is organized as follows: In Sec. II, we introduce the Hamiltonian including the Rashba term used to describe the DW MR. Section III describes the adiabaticity condition for spin transport through the DW. In Sec. IV we will calculate the scattering matrices. Section V will provide our descriptions for the results and, finally, a brief summary of the results concludes the paper.

II. MODEL

In the following investigation of a two-dimensional system, z axis is considered as the direction of confinement and the electric field is taken along such a direction. In accordance with Fig. 1, a two-dimensional linear Néel-type DW is assumed with two types of positive and negative chiralities depending on whether the domain-wall rotation axis is either parallel or antiparallel to the direction of the confining electric field, i.e., the z axis.

The Hamiltonian of the system can be written as

$$H = H_0 + H_{ex} + H_{im} + H_R. \tag{1}$$

H_0 and the exchange between the conduction electrons and the localized magnetic moments, H_{ex}, can be expressed as

$$H_0 = -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r}), \tag{2a}$$

$$H_{ex} = -\Delta\hat{\sigma}\cdot\hat{M}(\mathbf{r}), \tag{2b}$$

in which $V(\mathbf{r})$ is the lattice periodic potential, Δ is the exchange interaction strength, $\hat{\sigma}$ denotes the spin operator in terms of the Pauli spin matrices, $\hat{M}$ is the unit vector along the direction of local magnetization, and $\mathbf{r}=x\hat{x}+y\hat{y}$ denotes the position of the electron in the two-dimensional space. The interaction of the localized magnetic impurities with the electrons takes the form

$$H_{im} = \sum_j [v_{im} - \Delta_{im}\hat{\sigma}\cdot\hat{M}(\mathbf{r})]\delta(\mathbf{r}-\mathbf{r}_j), \tag{3}$$

where the summation is over all impurities. Δ_{im} and v_{im} are the exchange interaction strength and on-site electrical potential of the localized impurities, respectively. The last term in (1) for the Rashba interaction is

$$H_R = \frac{\alpha}{\hbar}\hat{\sigma}\times\mathbf{P}\cdot\hat{z} = i\alpha\left(\hat{\sigma}_y\frac{\partial}{\partial x} - \hat{\sigma}_x\frac{\partial}{\partial y}\right), \tag{4}$$

where $\mathbf{P}$ is the momentum operator and α is the Rashba parameter characterizing the strength of the SO coupling.

Using the approach of Ref. 18, one can find the eigenstates of the H_0+H_{ex}. According to this approach, for a linear DW and up to any order of approximation, the exchange interaction cannot produce any mixing between different $\mathbf{k}$ states. This is due to the position-independence perturbation potential which is introduced by the exchange interaction for linear DW. As mentioned earlier, the complexity of the magnetic structures has a significant effect on the transport characteristics. However, in this work we assume a linear functionality for the DW such as $\theta(x)=\pm(\pi/d)x$, where the positive and negative signs correspond to the positive and negative chiralities, respectively.

If we choose the DW width d as a unit of length and π/d as a unit of wave number, it will be convenient to express position $\mathbf{r}$ and wave number $\mathbf{k}$ in terms of dimensionless quantities $\mathbf{r}\to\mathbf{r}/d$ and $\mathbf{k}\to\mathbf{k}/(\pi/d)$. Then, the eigenstates of H_0+H_{ex} for the two-dimensional system may be formulated as follows:[18]

$$|\Psi_{\mathbf{k}}^{\uparrow}\rangle = \tilde{\alpha}(k_x)\frac{e^{i\pi(k_x x+k_y y)}}{\sqrt{L_xL_y}}R_{\theta(x)}\begin{pmatrix}1+ik_x\xi\\ 1-ik_x\xi\end{pmatrix}, \tag{5a}$$

$$|\Psi_{\mathbf{k}}^{\downarrow}\rangle = \tilde{\alpha}(k_x)\frac{e^{i\pi(k_x x+k_y y)}}{\sqrt{L_xL_y}}R_{\theta(x)}\begin{pmatrix}1+ik_x\xi\\ -1+ik_x\xi\end{pmatrix}, \tag{5b}$$

where $L_x=d$ and L_y are DW dimensions along the x and y axes; $\tilde{\alpha}(k_x)=[2(1+k_x^2\xi^2)]^{-1/2}$ is the normalization coefficient; $R_\theta=\exp[-i(\theta/2)\hat{\sigma}\cdot\hat{n}]$ is the rotation operator for spin of electrons; $\hat{n}$ is the direction of DW rotation axis, which in this case, because of the shape anisotropy, is assumed to be perpendicular to the plane of the system, i.e., along the z axis; and finally, $\xi=\pm\hbar^2\pi^2/(8m\Delta d^2)$ is the perturbation parameter for each chirality. This parameter characterizes the deviation of the eigenstates from their local minimum-energy states, i.e., the ξ-independent terms of Eq. (5), which are the eigenstates of the local direction of magnetization. The eigenstates of Eq. (5) are not pure spin states and correspond to energy eigenvalues $\epsilon_k^\sigma=\frac{\pi^2\hbar^2k^2}{2md^2}-\sigma\Delta$ $(\sigma=\pm1)$.

III. NONADIABATIC TRANSPORT INSIDE THE DISORDERED DOMAIN WALL

Berger[23] describes wall crossing by electrons as "purely adiabatic" for their spin and notes that the DW represents a very smooth and gradual disturbance which cannot be responsible for an appreciable electron scattering. This view is based on size considerations of the DW width, which is much greater than the Fermi wavelength. However, the ξ-dependent parts of Eq. (5) go beyond such an adiabatic approximation.[18]

Although the size of the DW width is introduced as the main factor for adiabatic transport, nonadiabatic transport inside the DW can be attributed to the weak magnitude of the exchange interaction, which reduces the effect of the DW width on the adiabatic transport. This interaction in the absence of the magnetic field and other magnetic interactions is a unique interaction in the imposing magnetic order, and the size argumentation itself cannot describe the whole physical problem. Nevertheless, for very wide DWs, the spin follows adiabatically the local magnetization orientation and also low-energy electrons are transmitted adiabatically.[24]

If DW configuration is assumed to be a nonhomogeneous effective magnetic field experienced by electrons as a result of exchange interaction, only in the limit of a high exchange coupling strength, i.e., when $\Delta\to\infty$, can the electron spin adiabatically follow the spatially varying magnetization direction, and the spin wave function acquires a geometrical or Berry phase.[25] The limit of this high magnetic interaction for adiabatic transport in one-dimensional ballistic (disorder-free) systems for CPW geometry can be characterized by the condition of $\omega_P/\omega_L \gg 1$, in which $\omega_P=2\Delta/\hbar$ is the frequency of spin precession around a local direction of magnetization and $\omega_L=v_F/d=\hbar k_F/md$ the frequency of orbital motion,[26–28] where the DW width is considered as length scale over which the effective field changes. On the other hand, since the mean free paths of the electrons are small compared with the DW width, the simple picture of ballistic transport inside the DW needs some modifications.[29] Some corrections are required for adiabatic spin transport through the disordered systems.[30] The condition for adiabaticity in the two-dimensional disordered systems reads $\omega_P/\omega_L \gg (1/1.4)\times(d/l)^{0.95}$, in which l is the elastic mean free path.[30] This condition cannot be satisfied even for a metallic ferromagnet in the semiclassical regime, which is the framework of the present paper.

The problem of adiabaticity will become more complicated if a realistic band-dependent effective mass of electrons is taken into account. However, when some of the geometrical distortions of the DW such as DW bulging for an ideal DW are ignored, the CIW geometry [$k_x=0$ in Eqs. (5)] shows spin transport in a purely homogeneous effective magnetic field. This is because there is no variation in magnetization along the DW (y axis) and a charge moving in this direction may not experience any nonhomogeneous effective magnetic field and thus flows in its local minimum-energy state with a spin along the local magnetization until it experiences a scattering.

IV. SCATTERING MATRICES AND DOMAIN-WALL MAGNETORESISTANCE

The scattering matrices of the Rashba interaction read

$$[H_R]^{\sigma,\sigma'}_{\mathbf{k},\mathbf{k}'} = \frac{\alpha}{\hbar}\langle\Psi^{\sigma}_{\mathbf{k}}|(\hat{\sigma}_y P_x - \hat{\sigma}_x P_y)|\Psi^{\sigma'}_{\mathbf{k}'}\rangle = \frac{\alpha}{\hbar}\tilde{\alpha}(k_x)\tilde{\alpha}(k'_x) \times\langle\mathbf{k}\sigma|R^{-1}_{\theta(x)}(\hat{\sigma}_y P_x - \hat{\sigma}_x P_y)R_{\theta(x)}|\mathbf{k}'\sigma'\rangle, \tag{6}$$

in which we have defined

$$|\Psi^{\sigma}_{\mathbf{k}}\rangle = \tilde{\alpha}(k_x)R_{\theta(x)}|\mathbf{k}\sigma\rangle, \quad \sigma=\uparrow(\downarrow), \tag{7a}$$

$$|\uparrow\rangle = \begin{pmatrix} 1+ik_x\xi \\ 1-ik_x\xi \end{pmatrix}, \tag{7b}$$

$$|\downarrow\rangle = \begin{pmatrix} 1+ik_x\xi \\ -1+ik_x\xi \end{pmatrix}, \tag{7c}$$

by using the following relations:

$$R_{\theta(x)} = I\cos[\theta(x)/2] - i\hat{\sigma}_z\sin[\theta(x)/2], \tag{8a}$$

$$R^{-1}_{\theta(x)}H_R R_{\theta(x)} = R^{-1}_{\theta(x)}[H_R,R_{\theta(x)}] + H_R, \tag{8b}$$

where I is the identity operator. As a result, the scattering matrices can be obtained as follows:

$$[H_R]^{\sigma,\sigma'}_{\mathbf{k},\mathbf{k}'} = -\frac{\alpha\pi}{4d}\tilde{\alpha}(k_x)\tilde{\alpha}(k'_x)\{\tilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'}\beta^{\sigma,\sigma'}_1(\mathbf{k}') + \tilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'}\beta^{\sigma,\sigma'}_2(\mathbf{k}')\} + \frac{\alpha\pi}{d}\tilde{\alpha}(k_x)\tilde{\alpha}(k'_x)\delta_{\mathbf{k},\mathbf{k}'}\beta^{\sigma,\sigma'}_3(\mathbf{k}'), \tag{9}$$

in which $\tilde{\delta}^{(\pm)}_{\mathbf{k},\mathbf{k}'}$ and $\beta^{\sigma,\sigma'}_i(\mathbf{k}')$ are defined as

$$\tilde{\delta}^{(\pm)}_{\mathbf{k},\mathbf{k}'} = \delta_{k_y,k'_y}(\delta_{k_x-1,k'_x} \pm \delta_{k_x+1,k'_x}), \tag{10a}$$

$$\beta^{\sigma,\sigma'}_1(\mathbf{k}') = [i\,\langle\sigma|\hat{\sigma}_x\hat{\sigma}_z|\sigma'\rangle - 2ik'_x\langle\sigma|\hat{\sigma}_x|\sigma'\rangle) + 2ik'_y\langle\sigma|\hat{\sigma}_y|\sigma'\rangle], \tag{10b}$$

$$\beta^{\sigma,\sigma'}_2(\mathbf{k}') = [\langle\sigma|\hat{\sigma}_y\hat{\sigma}_z|\sigma'\rangle + 2ik'_x\langle\sigma|\hat{\sigma}_x\hat{\sigma}_z|\sigma'\rangle + 2ik'_y\langle\sigma|\hat{\sigma}_y\hat{\sigma}_z|\sigma'\rangle], \tag{10c}$$

$$\beta^{\sigma,\sigma'}_3(\mathbf{k}') = i[k'_x\langle\sigma|\hat{\sigma}_z\hat{\sigma}_x|\sigma'\rangle - k'_y\langle\sigma|\hat{\sigma}_z\hat{\sigma}_y|\sigma'\rangle] + k'_x\langle\sigma|\hat{\sigma}_y|\sigma'\rangle - k'_y\langle\sigma|\hat{\sigma}_x|\sigma'\rangle. \tag{10d}$$

From the transport point of view, it can be easily shown that the last term of Eq. (9), which includes $\delta_{\mathbf{k},\mathbf{k}'}$, cannot contribute to the scatterings. The Rashba SO coupling in homogeneous ferromagnets cannot make couplings of different $\mathbf{k}$ states, but inside the DW this interaction results in momentum transfer of the order of $\Delta k\sim 1$ (in the unit of π/d).

The other type of SO interaction is the so-called Elliott-Yafet SO coupling,[31,32] which is given by

$$V_{so} = \frac{\hbar}{4m^2c^2}\nabla V_{sc}\times\mathbf{P}\cdot\hat{\sigma}, \tag{11}$$

where V_{sc} is the periodic lattice potential. This type of SO interaction cannot couple spin bands of a DW obtained while considering the periodic nature of the lattice. The order of transitions which can be made by this interaction is about $\Delta k\sim d/a$ (in the unit of π/d), in which a is the lattice constant. This is because of the translational symmetry of V_{sc}. This value cannot satisfy the conditions of elastic scatterings for a simple two-band model and typical acceptable exchange interaction strength, Δ, which defines the gap between the two spin bands. The magnitude of the Elliott-Yafet SO interaction is very small (about $V_{sc}[\hbar^2\pi^2/(16ma^2)]/mc^2 \sim 10^{-5}$ or 10^{-4} eV). However, in some magnetic modulated structures such as rare-earth manganese oxides, this type of SO interaction can contribute to the elastic scattering if the length of the translational magnetic symmetry of the system is comparable with the lattice constant a.

Scattering matrices of impurities are calculated by

$$[H_{im}]^{\sigma,\sigma'}_{\mathbf{k},\mathbf{k}'} = 2C_{\mathbf{k},\mathbf{k}'}\tilde{\alpha}(k_x)\tilde{\alpha}(k'_x)M_{\sigma,\sigma'}, \quad (12)$$

where we have defined

$$M = \begin{pmatrix} v_{im} - \Delta_{im} + k_x k'_x \xi^2(v_{im}+\Delta_{im}) & \xi[-ik_x(v_{im}+\Delta_{im}) + ik'_x(v_{im}-\Delta_{im})] \\ \xi[ik_x(v_{im}+\Delta_{im}) - ik'_x(v_{im}-\Delta_{im})] & v_{im} + \Delta_{im} + k_x k'_x \xi^2(v_{im}-\Delta_{im}) \end{pmatrix}, \quad (13)$$

and $C_{\mathbf{k},\mathbf{k}'}$ has the following form:

$$C_{\mathbf{k},\mathbf{k}'} = \sum_j e^{i\pi(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_j}, \quad (14)$$

in which the summation is over the impurities inside the DW.

If we define $|V^{\sigma,\sigma'}_{\mathbf{k},\mathbf{k}'}|^2 = |H^{\sigma,\sigma'}_R(\mathbf{k},\mathbf{k}') + H^{\sigma,\sigma'}_{im}(\mathbf{k},\mathbf{k}')|^2$ as the total scattering matrix, then in the dimensionless k space the transport relaxation times are given by

$$[\tau^\sigma(\mathbf{k})]^{-1} = \frac{L_y}{d}\frac{\pi}{2\hbar}\sum_{\sigma'} I_{\sigma\sigma'}, \quad (15)$$

in which $I_{\sigma\sigma'}$ is as follows:

$$I_{\sigma\sigma'} = \int d^2k' |V^{\sigma,\sigma'}_{\mathbf{k},\mathbf{k}'}|^2[1-\cos(\mathbf{k},\mathbf{k}')]\delta(\epsilon^\sigma_k - \epsilon^{\sigma'}_{k'}). \quad (16)$$

To drive Eq. (16), the electron velocity is taken to be parallel to its wave vector for spherical Fermi surfaces of up- and down-spin bands. Therefore, one can write $[1-\mathbf{v}_\mathbf{k}\cdot\hat{e}/\mathbf{v}_{\mathbf{k}'}\cdot\hat{e}] = [1-\cos(\mathbf{k},\mathbf{k}')]$, where $\hat{e}$ is a unit vector along the electric field of a given current.[33]

Random nature of the impurity distribution results in decoupling of impurity and the Rashba interactions as $|V^{\sigma,\sigma'}_{\mathbf{k},\mathbf{k}'}|^2 = |H^{\sigma,\sigma'}_{im}(\mathbf{k},\mathbf{k}')|^2 + |H^{\sigma,\sigma'}_R(\mathbf{k},\mathbf{k}')|^2$ (Appendix A). The following relations:

$$\delta(\epsilon^\sigma_k - \epsilon^\sigma_{k'}) = \frac{1}{8\Delta|\xi|k}\{\delta(k'-k) + \delta(k'+k)\}, \quad (17a)$$

$$\delta(\epsilon^\uparrow_k - \epsilon^\downarrow_{k'}) = \frac{1}{8\Delta|\xi|k_-}\{\delta(k'-k_-) + \delta(k'+k_-)\}, \quad (17b)$$

$$\delta(\epsilon^\downarrow_k - \epsilon^\uparrow_{k'}) = \frac{1}{8\Delta|\xi|k_+}\{\delta(k'-k_+) + \delta(k'+k_+)\}, \quad (17c)$$

in which $k_\pm = \sqrt{k^2 \pm 1/(2|\xi|)}$, $k=|\mathbf{k}|$, and $k'=|\mathbf{k}'|$ indicate that the Rashba interaction cannot contribute to scatterings not corresponding to a spin band change in the elastic regime. This is because $\tilde{\delta}^{(\pm)}_{\mathbf{k},\mathbf{k}'}\delta(\epsilon^\sigma_k - \epsilon^\sigma_{k'}) \equiv 0$. Therefore, the Rashba interaction contributes only in the $I_{\sigma,-\sigma}$. Neverthless, using the definition of the $\tilde{\delta}^{(\pm)}_{\mathbf{k},\mathbf{k}'}$, one can write

$$(\tilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'})^2 = \tilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'}, \quad (18a)$$

$$(\tilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'})^2 = \tilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'}, \quad (18b)$$

$$\tilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'}\tilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'} = \tilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'}, \quad (18c)$$

$$|a_1\tilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'} + a_2\tilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'}|^2 = (|a_1|^2 + |a_2|^2)\tilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'} + (a_1^* a_2 + a_1 a_2^*)\tilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'}. \quad (18d)$$

The relations in Eq. (18) result in the following expressions for $I_{\sigma,\sigma'}$:

$$I_{\uparrow,\uparrow} = \frac{\tilde{\alpha}^2(k_x)}{16\Delta|\xi|k}\Lambda_1(k_x,k_y), \quad (19a)$$

$$I_{\downarrow,\downarrow} = \frac{\tilde{\alpha}^2(k_x)}{16\Delta|\xi|k}\Lambda_2(k_x,k_y), \quad (19b)$$

$$I_{\uparrow,\downarrow} = \frac{1}{16\Delta|\xi|}\left\{\frac{\tilde{\alpha}^2(k_x)}{k}\Lambda'_1(k_x,k_y) + \tilde{\alpha}^2(-\mu_1)\Gamma_1(k_x,k_y)\delta_{k_x,-\mu_1} + \tilde{\alpha}^2(\mu_1)\Gamma_2(k_x,k_y)\delta_{k_x,\mu_1}\right\}, \quad (19c)$$

$$I_{\downarrow,\uparrow} = \frac{1}{16\Delta|\xi|}\left\{\frac{\tilde{\alpha}^2(k_x)}{k}\Lambda'_2(k_x,k_y) + \tilde{\alpha}^2(-\mu_2)\Gamma_3(k_x,k_y)\delta_{k_x,-\mu_2} + \tilde{\alpha}^2(\mu_2)\Gamma_4(k_x,k_y)\delta_{k_x,\mu_2}\right\}, \quad (19d)$$

in which $\mu_1=\frac{1}{2}(1+\frac{1}{2|\xi|})$ and $\mu_2=\frac{1}{2}(1-\frac{1}{2|\xi|})$. By direct integration over dimensionless k space in polar coordinates, the impurity dependent parts are given by

$$\Lambda_1(k_x,k_y)=\frac{8k\pi c_i(v_{im}-\Delta_{im})^2}{\sqrt{1+k^2\xi^2}}+\frac{8\pi}{k}\left(1-\frac{1}{\sqrt{1+k^2\xi^2}}\right)\times c_ik_x{}^2(v_{im}+\Delta_{im})[(-2+k^2\xi^2)v_{im}+(2+k^2\xi^2)\Delta_{im}], \tag{20a}$$

$$\Lambda_2(k_x,k_y)=\frac{8k\pi c_i(v_{im}+\Delta_{im})^2}{\sqrt{1+k^2\xi^2}}+\frac{8\pi}{k}\left(1-\frac{1}{\sqrt{1+k^2\xi^2}}\right)\times c_ik_x{}^2(v_{im}-\Delta_{im})[(-2+k^2\xi^2)v_{im}-(2+k^2\xi^2)\Delta_{im}], \tag{20b}$$

$$\Lambda_1'(k_x,k_y)=\frac{8k\pi\xi^2c_ik_x{}^2(v_{im}+\Delta_{im})^2}{\sqrt{1+\xi^2k_-{}^2}}+\frac{8\pi}{k_-}\left(1-\frac{1}{\sqrt{1+\xi^2k_-{}^2}}\right)c_i(v_{im}-\Delta_{im})\times[kk_-(v_{im}-\Delta_{im})+2k_x{}^2(v_{im}+\Delta_{im})], \tag{20c}$$

$$\Lambda_2'(k_x,k_y)=\frac{8k\pi\xi^2c_ik_x{}^2(v_{im}+\Delta_{im})^2}{\sqrt{1+\xi^2k_+{}^2}}+\frac{8\pi}{k_+}\left(1-\frac{1}{\sqrt{1+\xi^2k_+{}^2}}\right)c_i(v_{im}-\Delta_{im})\times[kk_+(v_{im}-\Delta_{im})+2k_x{}^2(v_{im}+\Delta_{im})], \tag{20d}$$

where Λ_1 and Λ_2 represent the contribution of impurity interactions in the relaxation times during non-spin-flip processes, while Λ_1' and Λ_2' represent the same contribution during spin-flip processes in which c_i is the impurity concentration (see Appendix A). In addition, the Rashba-dependent terms are given by (see Appendix B)

$$\Gamma_1(k_x,k_y)=\frac{|\beta_1^{\uparrow\downarrow}(\mathbf{k}')-\beta_2^{\uparrow\downarrow}(\mathbf{k}')|^2}{(1+\xi^2k_x'^2)\sqrt{\frac{-1}{2|\xi|}+k_x{}^2}}\eta_{\mathbf{k},\mathbf{k}'}\delta_{k_x',-\mu_1+1}\delta_{k_y',k_y}, \tag{21a}$$

$$\Gamma_2(k_x,k_y)=\frac{|\beta_1^{\uparrow\downarrow}(\mathbf{k}')+\beta_2^{\uparrow\downarrow}(\mathbf{k}')|^2}{(1+\xi^2k_x'^2)\sqrt{\frac{-1}{2|\xi|}+k_x{}^2}}\eta_{\mathbf{k},\mathbf{k}'}\delta_{k_x',\mu_1-1}\delta_{k_y',k_y}, \tag{21b}$$

$$\Gamma_3(k_x,k_y)=\frac{|\beta_1^{\downarrow\uparrow}(\mathbf{k}')-\beta_2^{\downarrow\uparrow}(\mathbf{k}')|^2}{(1+\xi^2k_x'^2)\sqrt{\frac{+1}{2|\xi|}+k_x{}^2}}\eta_{\mathbf{k},\mathbf{k}'}\delta_{k_x',-\mu_2+1}\delta_{k_y',k_y}, \tag{21c}$$

$$\Gamma_4(k_x,k_y)=\frac{|\beta_1^{\downarrow\uparrow}(\mathbf{k}')+\beta_2^{\downarrow\uparrow}(\mathbf{k}')|^2}{(1+\xi^2k_x'^2)\sqrt{\frac{+1}{2|\xi|}+k_x{}^2}}\eta_{\mathbf{k},\mathbf{k}'}\delta_{k_x',\mu_2-1}\delta_{k_y',k_y}, \tag{21d}$$

where

$$\eta_{\mathbf{k},\mathbf{k}'}=\left(\frac{\alpha\pi}{4d}\right)^2\left[1-\frac{k_x'k_x+k_y'k_y}{\sqrt{(k_x'^2+k_y'^2)(k_x{}^2+k_y{}^2)}}\right]. \tag{22}$$

Γ_i functions correspond to the contribution of the Rashba coupling in the relaxation times through the spin-flip scatterings at a given momentum transfer identified by Eqs. (19c) and (19d).

Because of the small magnitude of the Rashba and impurity interaction strengths, the condition for applicability of Born approximation for fast electrons given by $|V_{\mathbf{k},\mathbf{k}'}^{\sigma,\sigma'}|\ll\hbar^2k_F/md$ can be satisfied.[34] Therefore, within the two-band model, semiclassical approach is applicable and we can find a solution for the distribution function for each eigenstate of the H_0+H_{ex} within the relaxation time approximation as

$$f^\sigma(\mathbf{k})=f_0(\epsilon_k)-\frac{\hbar}{m}\frac{\pi}{d}e\mathbf{k}\cdot\mathbf{E}\tau^\sigma(\mathbf{k})\frac{\partial f_0(\epsilon_k)}{\partial\epsilon}, \tag{23}$$

where f_0 is the equilibrium distribution function. The last term in Eq. (23) represents the deviation from the equilibrium distribution function due to the external electric field $\mathbf{E}$, which is responsible for the current flow and relaxation mechanisms H_{im} and H_R, which are included in τ^σ. Then, the conductivity of the sample is given as

$$\sigma_{\hat{n}_e}=\left(\frac{\hbar\pi e}{2md^2}\right)^2\sum_\sigma\tilde{I}_{\hat{n}_e}^\sigma, \tag{24}$$

$$\tilde{I}_{\hat{n}_e}^\sigma=\int(\mathbf{k}\cdot\hat{n}_e)^2\tau^\sigma(\mathbf{k})\delta(\epsilon_k^\sigma-\epsilon_F)d^2k, \tag{25}$$

in which $\hat{n}_e=\mathbf{E}/E$ is the unit vector along the electric field, which, according to Fig. 1, identifies two transport geometries CPW ($\hat{n}_e=\hat{x}$) and CIW ($\hat{n}_e=\hat{y}$).

Using the discrete form of integration and the relation

$$\lim_{L_i\to\infty}\delta(k_i-k_i')\Delta k=\lim_{L_i\to\infty}\delta(k_i-k_i')\frac{2\pi}{L_i}=\delta_{k_i,k_i'}, \tag{26}$$

Eqs. (15), (20), and (21) result in

$$\tilde{I}_x^{\uparrow} = \frac{2d\hbar}{\pi L_y}\left\{ \int \frac{16\Delta|\xi|k}{\tilde{\alpha}^2(k_x)} k_x^2 \frac{\delta(\epsilon_k^{\uparrow} - \epsilon_F)}{\Lambda_1(k_x,k_y) + \Lambda_1'(k_x,k_y)} d^2k + \frac{2\mu_1^2}{\tilde{\alpha}^2(\mu_1)k_F^{\uparrow}}[\Gamma^{\uparrow}(\mu_1,k_{\mu_1}^{\uparrow}) + \Gamma^{\uparrow}(\mu_1,-k_{\mu_1}^{\uparrow})] \right\}, \quad (27)$$

where

$$\Gamma^{\uparrow}(k_x,k_y) = \left\{ -\frac{2}{(k_F^{\uparrow})^{-1}(\Lambda_1 + \Lambda_1')} + \frac{1}{(k_F^{\uparrow})^{-1}(\Lambda_1 + \Lambda_1') + \Gamma_1} + \frac{1}{(k_F^{\uparrow})^{-1}(\Lambda_1 + \Lambda_1') + \Gamma_2} \right\}. \quad (28)$$

For the sake of simplicity, we have dropped the (k_x,k_y) dependence of Λ_1, Λ_1', Γ_1, and Γ_2 and have defined $k_F^{\sigma} = \sqrt{k_F^2 + \sigma/(4|\xi|)}$ ($\sigma = +1,-1$ for up- and down-spin bands, respectively), $k_F = \epsilon_F/(4\Delta|\xi|)$, and $k_{\mu}^{\sigma} = \sqrt{(k_F^{\sigma})^2 - \mu^2}$. To evaluate the first term of Eq. (27), one can divide $\Lambda_1(k_x,k_y) + \Lambda_1'(k_x,k_y)$ as

$$\Lambda_1(k_x,k_y) + \Lambda_1'(k_x,k_y) = \gamma_{11}(k) + \gamma_{12}(k)k_x^2, \quad (29)$$

where $\gamma_{11}(k)$ and $\gamma_{12}(k)$ have been defined as

$$\gamma_{11}(k) = \frac{8k\pi c_i(v_{im} - \Delta_{im})^2(-\lambda_k + \lambda_{k_-} + \lambda_k\lambda_{k_-})}{\lambda_k\lambda_{k_-}} \quad (30)$$

and

$$\gamma_{12}(k) = \frac{8\pi c_i(v_{im} + \Delta_{im})}{kk_-\lambda_k\lambda_{k_-}}(v_{im}\{2k\lambda_k(-1 + \lambda_{k_-}) + k_-[-2(-1 + \lambda_k)\lambda_{k_-} + k^2\xi^2(\lambda_k - \lambda_{k_-} + \lambda_k\lambda_{k_-})]\} + \Delta_{im}\{-2k\lambda_k(-1 + \lambda_{k_-}) + k_-[2(-1 + \lambda_k)\lambda_{k_-} + k^2\xi^2(\lambda_k - \lambda_{k_-} + \lambda_k\lambda_{k_-})]\}), \quad (31)$$

in which $\lambda_k = \sqrt{1 + k^2\xi^2}$ and $\lambda_{k_\pm} = \sqrt{1 + k_\pm^2\xi^2}$. Then, using the relation

$$\delta(\epsilon_F - \epsilon_k^{\sigma}) = \frac{1}{8\Delta|\xi|k_F^{\sigma}}\{\delta(k - k_F^{\sigma}) + \delta(k + k_F^{\sigma})\}, \quad (32)$$

we can evaluate the first term of Eq. (27) in the polar coordinate as

$$\int \frac{16\Delta|\xi|k}{\tilde{\alpha}^2(k_x)} k_x^2 \frac{\delta(\epsilon_k^{\uparrow} - \epsilon_F)}{\Lambda_1(k_x,k_y) + \Lambda_1'(k_x,k_y)} d^2k = 4(k_F^{\uparrow})^3 \left\{ 2\pi \left[1 - \frac{\sqrt{\gamma_{11}}}{\sqrt{\gamma_{11} + (k_F^{\uparrow})^2\gamma_{12}}} \right] \times \frac{1}{(k_F^{\uparrow})^2\gamma_{12}} + \pi\xi^2 \frac{1}{(k_F^{\uparrow})^2{\gamma_{12}}^2} \times \left[-2\gamma_{11} + (k_F^{\uparrow})^2\gamma_{12} + \frac{2{\gamma_{11}}^{3/2}}{\sqrt{\gamma_{11} + (k_F^{\uparrow})^2\gamma_{12}}} \right] \right\}, \quad (33)$$

where γ_{11} and γ_{12} are evaluated at $k = k_F^{\uparrow}$, i.e., $\gamma_{11} = \gamma_{11}(k_F^{\uparrow})$ and $\gamma_{12} = \gamma_{12}(k_F^{\uparrow})$.

Similarly, if we define

$$\Gamma^{\downarrow}(k_x,k_y) = \left\{ -\frac{2}{(k_F^{\downarrow})^{-1}(\Lambda_2 + \Lambda_2')} + \frac{1}{(k_F^{\downarrow})^{-1}(\Lambda_2 + \Lambda_2') + \Gamma_3} + \frac{1}{(k_F^{\downarrow})^{-1}(\Lambda_2 + \Lambda_2') + \Gamma_4} \right\} \quad (34)$$

and

$$\Lambda_2(k_x,k_y) + \Lambda_2'(k_x,k_y) = \gamma_{21}(k) + \gamma_{22}(k)k_x^2, \quad (35)$$

in which

$$\gamma_{21}(k) = 8k\pi c_i\left\{ {v_{im}}^2\left(1 + \frac{1}{\lambda_k} - \frac{1}{\lambda_{k_+}}\right) + {\Delta_{im}}^2\left(1 + \frac{1}{\lambda_k} - \frac{1}{\lambda_{k_+}}\right) + 2v_{im}\Delta_{im}\left(-1 + \frac{1}{\lambda_k} + \frac{1}{\lambda_{k_+}}\right) \right\} \quad (36)$$

and

$$\gamma_{22}(k) = \frac{8\pi c_i}{kk_+\lambda_k\lambda_{k_+}}[2k^2\xi^2k_+v_{im}\Delta_{im}(\lambda_k + \lambda_{k_+} - \lambda_k\lambda_{k_+}) + {v_{im}}^2\{2k\lambda_k(-1 + \lambda_{k_+}) + k_+[-2(-1 + \lambda_k)\lambda_{k_+} + k^2\xi^2(\lambda_k - \lambda_{k_+} + \lambda_k\lambda_{k_+})]\} + {\Delta_{im}}^2\{-2k\lambda_k(-1 + \lambda_{k_+}) + k_+[2(-1 + \lambda_k)\lambda_{k_+} + k^2\xi^2(\lambda_k - \lambda_{k_+} + \lambda_k\lambda_{k_+})]\}]. \quad (37)$$

Then, by the same approach adopted above, we can find all components of the $\tilde{I}_{\hat{n}_e}^{\sigma}$, summarized as

$$\tilde{I}_x^{\uparrow} = \frac{2d\hbar}{\pi L_y}\left(4(k_F^{\uparrow})^3\left\{ 2\pi\left(1 - \frac{\sqrt{\gamma_{11}}}{\sqrt{\gamma_{11} + (k_F^{\uparrow})^2\gamma_{12}}}\right)\frac{1}{(k_F^{\uparrow})^2\gamma_{12}} + \pi\xi^2\frac{1}{(k_F^{\uparrow})^2{\gamma_{12}}^2}\left[-2\gamma_{11} + (k_F^{\uparrow})^2\gamma_{12} + \frac{2{\gamma_{11}}^{3/2}}{\sqrt{\gamma_{11} + (k_F^{\uparrow})^2\gamma_{12}}} \right] \right\} + \frac{2\mu_1^2}{\tilde{\alpha}^2(\mu_1)k_F^{\uparrow}}[\Gamma^{\uparrow}(\mu_1,k_{\mu_1}^{\uparrow}) + \Gamma^{\uparrow}(\mu_1,-k_{\mu_1}^{\uparrow})] \right), \quad (38)$$

$$\tilde{I}_x^{\downarrow} = \frac{2d\hbar}{\pi L_y}\left(4(k_F^{\downarrow})^3\left\{ 2\pi\left(1 - \frac{\sqrt{\gamma_{21}}}{\sqrt{\gamma_{21} + (k_F^{\downarrow})^2\gamma_{22}}}\right)\frac{1}{(k_F^{\downarrow})^2\gamma_{22}} + \pi\xi^2\frac{1}{(k_F^{\downarrow})^2{\gamma_{22}}^2}\left[-2\gamma_{21} + (k_F^{\downarrow})^2\gamma_{22} + \frac{2{\gamma_{21}}^{3/2}}{\sqrt{\gamma_{21} + (k_F^{\downarrow})^2\gamma_{22}}} \right] \right\} + \frac{2\mu_2^2}{\tilde{\alpha}^2(\mu_2)k_F^{\downarrow}}[\Gamma^{\downarrow}(\mu_2,k_{\mu_2}^{\downarrow}) + \Gamma^{\downarrow}(\mu_2,-k_{\mu_2}^{\downarrow})] \right) \quad (39)$$

for CPW geometry and

$$\tilde{I}_y^{\uparrow} = \frac{2d\hbar}{\pi L_y}\Bigg\{ 4(k_F^{\uparrow})^3 \Bigg(\frac{2\pi[\gamma_{11} + (k_F^{\uparrow})^2\gamma_{12} - \sqrt{\gamma_{11}}\sqrt{\gamma_{11} + (k_F^{\uparrow})^2\gamma_{12}}]}{(k_F^{\uparrow})^2\sqrt{\gamma_{11}}\gamma_{12}\sqrt{\gamma_{11} + (k_F^{\uparrow})^2\gamma_{12}}} + \frac{\pi\xi^2\{-2\sqrt{\gamma_{11}}[\gamma_{11} + (k_F^{\uparrow})^2\gamma_{12}] + \sqrt{\gamma_{11} + (k_F^{\uparrow})^2\gamma_{12}}[2\gamma_{11} + (k_F^{\uparrow})^2\gamma_{12}]\}}{(k_F^{\uparrow})^2{\gamma_{12}}^2\sqrt{\gamma_{11} + (k_F^{\uparrow})^2\gamma_{12}}} \Bigg) + \frac{2(k_{\mu_1}^{\uparrow})^2}{\tilde{\alpha}^2(\mu_1)k_F^{\uparrow}}[\Gamma^{\uparrow}(\mu_1, k_{\mu_1}^{\uparrow}) + \Gamma^{\uparrow}(\mu_1, -k_{\mu_1}^{\uparrow})] \Bigg\}, \quad (40)$$

$$\tilde{I}_y^{\downarrow} = \frac{2d\hbar}{\pi L_y}\Bigg\{ 4(k_F^{\downarrow})^3 \Bigg(\frac{2\pi[\gamma_{21} + (k_F^{\downarrow})^2\gamma_{22} - \sqrt{\gamma_{21}}\sqrt{\gamma_{21} + (k_F^{\downarrow})^2\gamma_{22}}]}{(k_F^{\downarrow})^2\sqrt{\gamma_{21}}\gamma_{22}\sqrt{\gamma_{21} + (k_F^{\downarrow})^2\gamma_{22}}} + \frac{\pi\xi^2\{-2\sqrt{\gamma_{21}}[\gamma_{21} + (k_F^{\downarrow})^2\gamma_{22}] + \sqrt{\gamma_{21} + (k_F^{\downarrow})^2\gamma_{22}}[2\gamma_{21} + (k_F^{\downarrow})^2\gamma_{22}]\}}{(k_F^{\downarrow})^2{\gamma_{22}}^2\sqrt{\gamma_{21} + (k_F^{\downarrow})^2\gamma_{22}}} \Bigg) + \frac{2(k_{\mu_2}^{\downarrow})^2}{\tilde{\alpha}^2(\mu_2)k_F^{\downarrow}}[\Gamma^{\downarrow}(\mu_2, k_{\mu_2}^{\downarrow}) + \Gamma^{\downarrow}(\mu_2, -k_{\mu_2}^{\downarrow})] \Bigg\} \quad (41)$$

for CIW geometry, where γ_{ij} parameters are given by $\gamma_{11}=\gamma_{11}(k_F^{\uparrow})$, $\gamma_{12}=\gamma_{12}(k_F^{\uparrow})$, $\gamma_{21}=\gamma_{21}(k_F^{\downarrow})$, and $\gamma_{22}=\gamma_{22}(k_F^{\downarrow})$. Subsequently, the CPW and CIW resistivities can be determined using relations $R_{CPW}=(\sigma_x)^{-1}$ and $R_{CIW}=(\sigma_y)^{-1}$.

If we replace the DW with a ferromagnet, the eigenstates of the H_0+H_{ex} will be pure spin states given by

$$|\Phi_{\mathbf{k}}^{\uparrow}\rangle = \frac{1}{\sqrt{2}}\frac{e^{i\pi(k_x x + k_y y)}}{\sqrt{L_x L_y}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad (42a)$$

$$|\Phi_{\mathbf{k}}^{\downarrow}\rangle = \frac{1}{\sqrt{2}}\frac{e^{i\pi(k_x x + k_y y)}}{\sqrt{L_x L_y}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}. \quad (42b)$$

As mentioned before, the Rashba interaction cannot contribute to the elastic scatterings inside a ferromagnet (neglecting very small change of Fermi energy due to this interaction). Therefore, in the face of the present problem, the only relaxation mechanism which should be taken into account for the ferromagnetic reign is the impurity scattering. Since this relaxation cannot produce any spin-flip transition between the pure spin eigenstates introduced in Eqs. (42) [which is contrary to the case of the presence of DW (Ref. 18)], the scattering matrix of impurities inside a ferromagnet and in the spin space can be expressed as

$$|V_{im}^{ferr}(\mathbf{k},\mathbf{k}')|^2 = c_i\begin{pmatrix} (v_{im}-\Delta_{im})^2 & 0 \\ 0 & (v_{im}+\Delta_{im})^2 \end{pmatrix}. \quad (43)$$

Similarly, one can show that, inside a ferromagnet, the relaxation times associated with impurity scatterings are

$$\tau_{ferr}^{\uparrow} = \frac{16d\hbar}{L_y\pi^2}\frac{\Delta|\xi|}{c_i(v_{im}-\Delta_{im})^2}, \quad (44a)$$

$$\tau_{ferr}^{\downarrow} = \frac{16d\hbar}{L_y\pi^2}\frac{\Delta|\xi|}{c_i(v_{im}+\Delta_{im})^2}. \quad (44b)$$

As a result, the resistivity of the ferromagnet is found to be

$$R_{ferr} = c_i\left\{\left(\frac{\pi\hbar e}{2md^2}\right)^2\left(\frac{2d\hbar}{\pi L_y}\right)\right\}^{-1}\left[\frac{(k_F^{\uparrow})^2}{(v_{im}-\Delta_{im})^2} + \frac{(k_F^{\downarrow})^2}{(v_{im}+\Delta_{im})^2}\right]^{-1}. \quad (45)$$

Finally, the magnetoresistance amounts of CIW and CPW geometries are given by

$$\delta\rho_{CPW} = \left(\frac{R_{CPW}}{R_{ferr}} - 1\right), \quad (46a)$$

$$\delta\rho_{CIW} = \left(\frac{R_{CIW}}{R_{ferr}} - 1\right). \quad (46b)$$

V. RESULTS AND DISCUSSION

The DW MR as a function of the Rashba coupling strength is shown in Fig. 2 for CPW and CIW geometries and different chiralities. The typical acceptable parameters have been chosen to be $\epsilon_F=10$ eV, $d=10$ nm, $m=m_e$, $\Delta=0.1$ eV, $\Delta_{im}/\Delta=0.8$, and $v_{im}=1$ eV. As can be seen, the DW MR for the mentioned smooth DW is found to be relatively small, which is in agreement with the results of some other papers. This is in contrast to Dugaev *et al.*, who showed that the DW MR in the case of sharp DW can be considerably large especially at low Fermi energies.[22]

According to Fig. 2, CIW MR is very sensitive to the impurity density. For low impurity concentrations, the Rashba interaction is more effective and, according to Fig. 2(a), one can find both positive and negative CIW magnetoresistance types depending on the Rashba coupling strength. However, since for the CPW geometry, $R_{CPW}=(\pi\hbar e/2md^2)^{-2}(\tilde{I}_x^{\uparrow}+\tilde{I}_x^{\downarrow})^{-1} > R_{ferr}$, CPW MR is always positive for all values of the Rashba coupling strengths and impurity concentrations.

High impurity densities suppress the effect of the Rashba interaction contribution to CPW and CIW resistivities. The reason is that the Rashba coupling-dependent part of the relaxations, Γ^{σ}, vanishes at high impurity concentrations. It can be shown that in the limit of $c_i \to \infty$ and at the Fermi

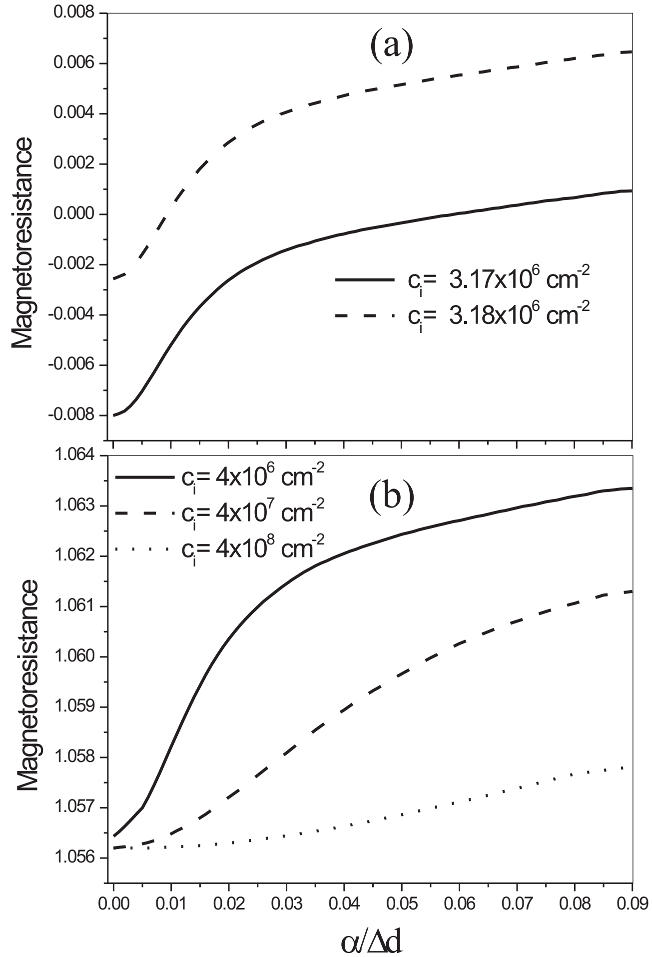

FIG. 2. Domain-wall magnetoresistance as a function of the Rashba coupling strength for (a) CIW geometry and (b) CPW geometry.

level, $(k_F^\sigma)^{-1}[\Lambda_i(k_x,k_y)+\Lambda_i'(k_x,k_y)] \gg \Gamma_j(k_x,k_y)$ (i.e., when the impurity-dependent scattering contribution is much higher than the Rashba contribution); therefore, $\Gamma^\uparrow(k_x,k_y) \to 0$ and $\Gamma^\downarrow(k_x,k_y) \to 0$. It can also be demonstrated that in the case of CIW geometry, $R_{CIW}=(\pi\hbar e/2md^2)^{-2}(\tilde{I}_y^\uparrow+\tilde{I}_y^\downarrow)^{-1} \ll R_{ferr}$; therefore, CIW MR is nearly -1. Consequently, it can easily be deduced that the DW MR is not a monotonic function of impurity concentration since the increasing rates of the DW resistivity and R_{ferr} as a function of impurity concentration are not the same.

The negative MR for CIW geometry is obtained when $R_{ferr} > R_{CIW}$. To understand the negative CIW MR in the case of a 2DEG, the key feature is the concept of electron motion in its local minimum of the magnetic energy. For two-dimensional systems, in addition to the two-dimensional confinement, electron experiences a weak confinement due to the DW in CIW geometry as seen schematically in Fig. 3(a). In this case, spin-flip scatterings can only take place when the electron undergoes a nonadiabatic transition [Fig. 3(a)]. Because of the nonadiabatic nature of spin-flip scatterings, the magnetic moment of electron cannot immediately adapt itself to the new local direction of magnetization; as a result, the electron experiences a higher effective magnetic

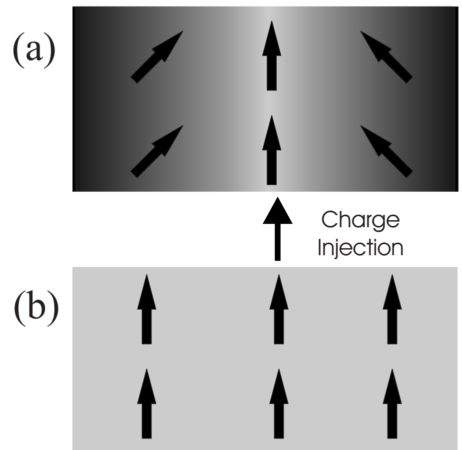

FIG. 3. (a) Effective magnetic potential for an electron in CIW geometry for possible directions of scatterings due to the nonadiabatic nature of the transport in states with $k_x \neq 0$. Darker regions correspond to higher magnetic potentials. (b) Effective magnetic potential for an electron in a homogeneous ferromagnetic conductor.

potential during a scattering to the states with nonzero k_x. From the physical point of view, these confinements reduce the final states of a single scattering relative to the possible final states of a homogeneous two-dimensional ferromagnetic system [Fig. 3(b)], especially at low temperatures where thermal scatterings can be ignored. However, scattering potentials can effectively increase the possible number of scatterings. For example, as the Rashba coupling strength increases, the negative CIW MR increases and gradually becomes positive [Fig. 2(a)]. This is due to the enhancement of spin-flip scattering rate by the Rashba interaction in the DW. Meanwhile, it cannot contribute to the elastic scatterings in the ferromagnetic region. As a result, as the Rashba coupling strength increases, the DW MR grows monotonically.

According to some theoretical and experimental reports on thin layers, it seems that there is a critical thickness where the total rate of scatterings due to the DW becomes less than the total rate in a ferromagnet, which results in negative DW MR.[13,15,20] It should be pointed out that, to be realistic, we have to consider the dependence of the DW width on the thickness of the sample.[35,36] Moreover, because of some alternative effects such as DW bulging or the presence of zigzag DWs,[20] the current experiences a mixture of these two types of geometries and the prediction of the sign of MR becomes more complicated.

Results of the present work qualitatively confirm the idea of some experiments in which CPW resistivity has been reported to be greater than that of CIW.[23,35] This may be explained by considering electron transport across DW as electron tunneling through a spin-dependent potential barrier in CPW geometry, while in the case of CIW geometry, the electron adapts its magnetic moment to the local magnetization which is homogeneous along the y axis.[35] Other reasons for $R_{CPW} > R_{CIW}$ not included in the present approach are the Hall effect of the effective magnetic field of DW (Ref. 23) and spin accumulation, which can effectively increase CPW resistivity.[5,37]

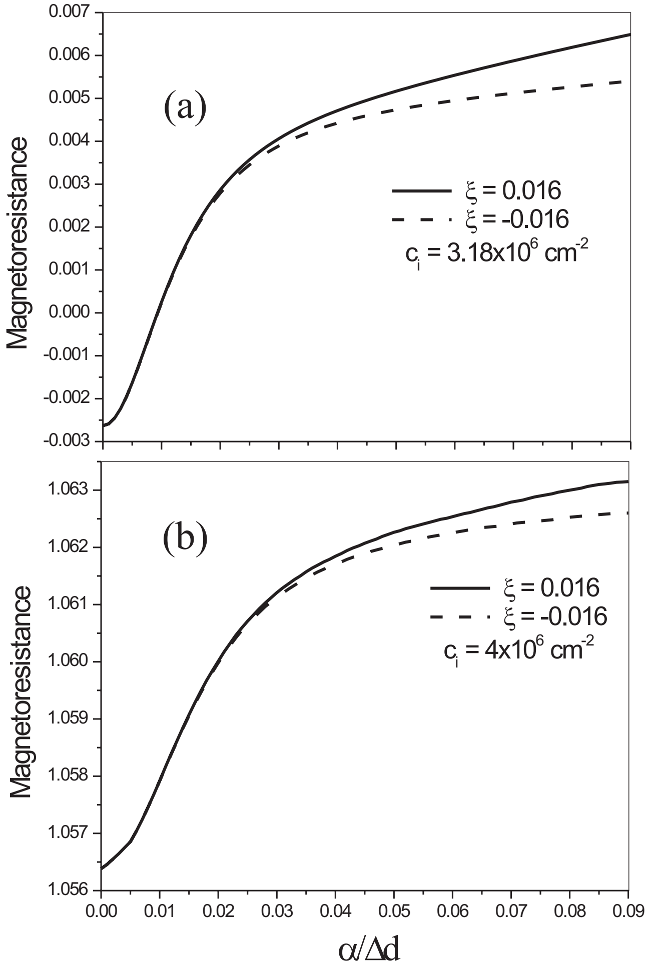

FIG. 4. Chirality dependence of the domain-wall magnetoresistance for (a) CIW geometry and (b) CPW geometry, where $\xi>0$ and $\xi<0$ correspond to positive and negative chiralities, respectively.

We have also determined the chirality dependence of the DW MR, which is induced by the Rashba coupling. As can be seen in Fig. 4, in the presence of the Rashba interaction, the anisotropy induced by the confining electric field results in chirality dependence of the DW MR. The deference of MRs for the two types of chiralities vanishes at zero Rashba coupling strength. The chirality-dependent parts of MR are those terms of the relaxation times which are not even functions of ξ. Therefore, it can be seen that the Rashba-dependent terms of $\widetilde{I}^{\sigma}_{\hat{n}_e}$, i.e., $\Gamma^{\uparrow(\downarrow)}(k_x,k_y)$, are responsible for the chirality dependence of MR. However, because the impurity-dependent parts of the scattering matrices are even functions of ξ [Eqs. (20)], there is no chirality dependence owing to this interaction.

VI. CONCLUSION

Using the semiclassical approach, we have studied the effect of Rashba SO interaction on the MR of a linear smooth DW. Although the magnitude of this interaction is small, it mixes different spin channels and plays a significant role in spin-dependent scatterings. MR calculations for two types of CPW and CIW geometries show that the MR of the sample increases as the strength of the Rashba coupling increases. Especially in the case of the low concentrations of impurities, its contribution to the MR of DW becomes more evident, while the increment of impurity density suppresses the effect of the Rashba interaction contribution to MR. The results also show that CPW resistivity is generally greater than CIW resistivity. This approach also results in negative MR for CIW geometry in some ranges of impurity concentrations for ideal two-dimensional systems and at low temperatures, which might be due to the confinement introduced by the DW for this special geometry. There is also some evidence indicating that the DW MR is chirality dependent owing to the induced anisotropy introduced by the confining electric field.

APPENDIX A

Using the relations (9) and (12), one can write

$$
\begin{aligned}
|V^{\sigma,\sigma'}_{\mathbf{k},\mathbf{k}'}|^2 = \widetilde{\alpha}^2(k_x)\widetilde{\alpha}^2(k'_x)\Bigg(& 4c_i|M^{\sigma,\sigma'}_{\mathbf{k},\mathbf{k}'}|^2 \\
& - \frac{\alpha\pi}{2d}\{C^*_{\mathbf{k},\mathbf{k}'}M^{*,\sigma,\sigma'}_{\mathbf{k},\mathbf{k}'}[\widetilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'}\beta_1^{\sigma,\sigma'}(\mathbf{k}') + \widetilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'}\beta_2^{\sigma,\sigma'}(\mathbf{k}')] \\
& + C_{\mathbf{k},\mathbf{k}'}M^{\sigma,\sigma'}_{\mathbf{k},\mathbf{k}'}[\widetilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'}\beta_1^{*,\sigma,\sigma'}(\mathbf{k}') + \widetilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'}\beta_2^{*,\sigma,\sigma'}(\mathbf{k}')]\} \\
& + \left(\frac{\alpha\pi}{4d}\right)^2|[\widetilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'}\beta_1^{\sigma,\sigma'}(\mathbf{k}') + \widetilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'}\beta_2^{\sigma,\sigma'}(\mathbf{k}')]|^2\Bigg),
\end{aligned}
\tag{A1}
$$

where we have used the following approximation for a homogeneous impurity distribution:

$$
\begin{aligned}
c_i = C_{\mathbf{k},\mathbf{k}'}C^*_{\mathbf{k},\mathbf{k}'} &= \sum_{j,j'} e^{i\pi(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_j}e^{-i\pi(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}'_j} \\
&= \sum_j 1 + \sum_{j\neq j'} e^{i\pi(\mathbf{k}-\mathbf{k}')\cdot(\mathbf{r}_j-\mathbf{r}'_j)} \simeq \sum_j 1.
\end{aligned}
\tag{A2}
$$

c_i is the number of impurities inside the DW, i.e., the concentration of impurities in the unit surface defined by DW dimensions. Since the nondiagonal terms of $C_{\mathbf{k},\mathbf{k}'}$ are oscillating and very small, these terms can be canceled out in averaging for different impurity configurations. $\widetilde{\delta}^{(\pm)}_{\mathbf{k},\mathbf{k}'}$ selects the nondiagonal elements of $C_{\mathbf{k},\mathbf{k}'}$ in second and third terms of Eq. (A1); therefore, keeping the nonoscillating terms immediately results in reduction of Eq. (A1) to

$$
\begin{aligned}
|V^{\sigma,\sigma'}_{\mathbf{k},\mathbf{k}'}|^2 = 4c_i\widetilde{\alpha}^2(k_x)\widetilde{\alpha}^2(k'_x)|M^{\sigma,\sigma'}_{\mathbf{k},\mathbf{k}'}|^2 + \widetilde{\alpha}^2(k_x)\widetilde{\alpha}^2(k'_x) \\
\times\left(\frac{\alpha\pi}{4d}\right)^2|[\widetilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'}\beta_1^{\sigma,\sigma'}(\mathbf{k}') + \widetilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'}\beta_2^{\sigma,\sigma'}(\mathbf{k}')]|^2,
\end{aligned}
\tag{A3}
$$

which is identical to $|V^{\sigma,\sigma'}_{\mathbf{k},\mathbf{k}'}|^2 = |H^{\sigma,\sigma'}_{im}(\mathbf{k},\mathbf{k}')|^2 + |H^{\sigma,\sigma'}_R(\mathbf{k},\mathbf{k}')|^2$.

APPENDIX B

The Rashba coupling-dependent terms of the scattering matrix are

$$|H_R^{\sigma,\sigma'}(\mathbf{k},\mathbf{k}')|^2 = \tilde{\alpha}^2(k_x)\tilde{\alpha}^2(k_x')\left(\frac{\alpha\pi}{4d}\right)^2 |[\tilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'}\beta_1^{\sigma,\sigma'}(\mathbf{k}') + \tilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'}\beta_2^{\sigma,\sigma'}(\mathbf{k}')]|^2. \tag{B1}$$

Then Eq. (18d) results in

$$\begin{aligned}|[\tilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'}\beta_1^{\sigma,\sigma'}(\mathbf{k}') + \tilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'}\beta_2^{\sigma,\sigma'}(\mathbf{k}')]|^2 &= [|\beta_1^{\sigma,\sigma'}(\mathbf{k}')|^2 + |\beta_2^{\sigma,\sigma'}(\mathbf{k}')|^2]\tilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'} \\ &\quad + \{[\beta_1^{\sigma,\sigma'}(\mathbf{k}')]^*\beta_2^{\sigma,\sigma'}(\mathbf{k}) + \beta_1^{\sigma,\sigma'}(\mathbf{k}')[\beta_2^{\sigma,\sigma'}(\mathbf{k})]^*\}\tilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'}.\end{aligned} \tag{B2}$$

Using Eq. (17) and assuming $\epsilon_k^{\sigma}=4|\xi|k^2\Delta-\sigma\Delta$, we can then easily show that

$$\begin{aligned}\tilde{\delta}^{(\pm)}_{\mathbf{k},\mathbf{k}'}\delta(\epsilon_k^{\downarrow}-\epsilon_{k'}^{\uparrow})d^2k' &= \frac{\tilde{\delta}^{(\pm)}_{\mathbf{k},\mathbf{k}'}}{8\Delta|\xi|k_+}\{\delta(k_x'-k_{x+}) + \delta(k_x'+k_{x+})\}d^2k' \\ &= \frac{\delta_{k_y,k_y'}}{8\Delta|\xi|k_+}\{\delta_{k_x,\mu_2}\delta_{k_x',\mu_2-1} \pm \delta_{k_x,-\mu_2}\delta_{k_x',-\mu_2+1}\},\end{aligned} \tag{B3}$$

and similarly,

$$\tilde{\delta}^{(\pm)}_{\mathbf{k},\mathbf{k}'}\delta(\epsilon_k^{\uparrow}-\epsilon_{k'}^{\downarrow})d^2k' = \frac{\delta_{k_y,k_y'}}{8\Delta|\xi|k_-}\{\delta_{k_x,\mu_1}\delta_{k_x',\mu_1-1} \pm \delta_{k_x,-\mu_1}\delta_{k_x',-\mu_1+1}\}, \tag{B4}$$

in which $k_{x\pm}=\sqrt{k_x^2\pm\frac{1}{2|\xi|}}$. Therefore, it can be demonstrated that for Rashba induced up-down transitions, the following relations can be satisfied:

$$\begin{aligned}&|[\tilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'}\beta_1^{\uparrow\downarrow}(\mathbf{k}') + \tilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'}\beta_2^{\uparrow\downarrow}(\mathbf{k}')]|^2\delta(\epsilon_k^{\uparrow}-\epsilon_{k'}^{\downarrow})[1-\cos(\mathbf{k},\mathbf{k}')] \\ &= \eta_{\mathbf{k},\mathbf{k}'}\frac{\delta_{k_y,k_y'}}{8\Delta|\xi|k_-}\{[|\beta_1^{\uparrow\downarrow}(\mathbf{k}') - \beta_2^{\uparrow\downarrow}(\mathbf{k}')|^2]\delta_{k_x,-\mu_1}\delta_{k_x',-\mu_1+1} \\ &\quad + [|\beta_1^{\uparrow\downarrow}(\mathbf{k}') - \beta_2^{\uparrow\downarrow}(\mathbf{k}')|^2]\delta_{k_x,\mu_1}\delta_{k_x',\mu_1-1}\}.\end{aligned} \tag{B5}$$

Repeating the same procedure for $|[\tilde{\delta}^{(-)}_{\mathbf{k},\mathbf{k}'}\beta_1^{\downarrow\uparrow}(\mathbf{k}') + \tilde{\delta}^{(+)}_{\mathbf{k},\mathbf{k}'}\beta_2^{\downarrow\uparrow}(\mathbf{k}')]|^2\delta(\epsilon_k^{\downarrow}-\epsilon_{k'}^{\uparrow})[1-\cos(\mathbf{k},\mathbf{k}')]$ results in Eqs. (21).

*Electronic address: teranchi@cc.sbu.ac.ir

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