Conductance oscillations with magnetic field of a two-dimensional electron gas–superconductor junction

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We develop a theory for the current-voltage characteristics of a two-dimensional electron gas–superconductor interface in magnetic field at arbitrary temperatures and in the presence of surface roughness. Our theory predicts that, in the case of disordered interface, the higher harmonics of the conductance oscillations with the filling factor are strongly suppressed as compared with the first one; it should be contrasted with the case of the ideal interface for which amplitudes of all harmonics involved are of the same order. Our findings are in qualitative agreement with recent experimental data.

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I. INTRODUCTION

The study of hybrid systems consisting of superconductors (S) in contact with high mobility two-dimensional electron gas (2DEG) in magnetic field has attracted considerable interest in recent years.[1–3] The quantum transport in this type of structures can be investigated in the framework of Andreev reflection theory.[4] When an electron quasiparticle in a normal metal (N) reflects from the interface of the S into a hole, Cooper pair transfers into the superconductor. A number of very interesting phenomena based on Andreev reflection had been studied in the past.[5] For example, if the normal metal is surrounded by superconductors, so we have a SNS junction, a number of Andreev reflections appear at the NS interfaces. In equilibrium, it leads to Andreev quasiparticle levels in the normal metal that carry a considerable part of the Josephson current; out of equilibrium, when superconductors are voltage biased, quasiparticles Andreev reflect about $2\Delta/eV$ times, transferring large quanta of charge from one superconductor to the other. This effect is called multiple Andreev reflection (MAR).[5]

Effect similar to MAR appears at a long enough NS interface in the presence of magnetic field; it bends quasiparticle trajectories and forces quasiparticles to reflect many times from the superconductor. If phase coherence is maintained, interference between electrons and holes can result in periodic, Aharonov-Bohm-like oscillations in the magnetoresistance. The conductance g of a S-2DEG interface in the presence of the magnetic field has been measured in recent experiments.[6–10] At large filling factors, it demonstrated highly nonmonotonic dependence with the magnetic field B. The most interesting effect was the oscillations of g with the filling factor ν in a somewhat similar manner as in the Shubnikov–de Haas effect.

Recently, a phenomenological analytical theory of these phenomena based on an "analogy" with the Aharonov-Bohm effect was suggested in Ref. 11. Numerical simulation was performed in Ref. 12. It was theoretically shown that the transport along the infinitely long 2DEG-S interface can be described in the framework of electron and hole edge states.[13] The 2DEG-S interfaces investigated in the experiments were not infinitely long. Their length, L, was typically of the order of a few cyclotron orbits, R_c, of an electron in 2DEG at the Fermi energy E_F. Quasiclassical theory of the charge transport through 2DEG-S interface of arbitrary length was developed in Ref. 14 for the case of large filling factors and vanishing temperature. In most theoretical papers mentioned above, the 2DEG-S interface with no roughness (ideal interface) was considered. As was shown,[14] when $L\sim 2R_c$, $g(\nu)$ oscillates nearly harmonically with ν, as $\cos(2\pi\nu)$. For $L\sim 4R_c$, the next harmonics with $\cos(4\pi\nu)$ appears, and so on. In experiments $L\gtrsim 6R_c$, therefore, one would expect to observe $\cos(2\pi n\nu)$ harmonics in the conductance, with $n=1,2,\ldots$. However, if we try to compare theoretically predicted $g(\nu)$ with the experimentally measured one, then we find that (i) although $L\gtrsim 6R_c$, the lowest harmonic $\cos(2\pi\nu)$ in the conductance survives, whereas higher harmonics are absent; and (ii) the amplitude of the conductance oscillations is much smaller than is predicted by theories. The reason for this disagreement is probably the roughness of the 2DEG-S interface in experiments and the perfect flatness of this interface in theory.

The main objective of the present paper is to develop a theory for the current-voltage characteristics of the 2DEG-S interface in magnetic field at finite temperature, which takes into account the surface roughness. Our approach with the surface roughness possibly helps to make a step toward explanation of the experimental results.

In this paper, we find the current-voltage characteristics of a 2DEG-S interface in magnetic field in the presence of surface roughness. Theories based on the assumption of interface perfectness predict the conductance $g=g_0+g_1\cos(2\pi\nu+\delta_1)+g_2\cos(4\pi\nu+\delta_2)+\ldots$; the amplitudes of the harmonics are of the same order: $g_1\sim g_2\sim\ldots$. This result cannot qualitatively describe the visibility of the conductance oscillations in experiments. As we have mentioned above, experimentally, the conductance behaves as $g=g_0+g_1\cos(2\pi\nu+\delta_1)$; higher harmonics: $g_2\cos(4\pi\nu+\delta_2),\ldots$, are hardly observed. Our approach that takes into account the surface roughness allows one to eliminate discrepancy between experiments and the theory. Due to the presence of a disorder at a 2DEG-S interface, the higher harmonics in the conductance

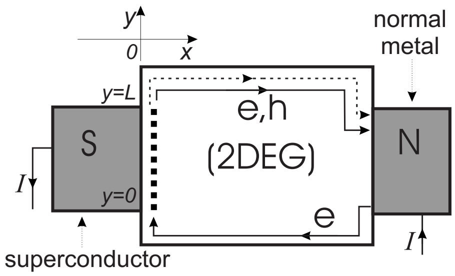

FIG. 1. The device which we investigate consists of a superconductor, 2DEG, and a normal conductor. An electron injected from the normal conductor to 2DEG in the integer quantum Hall regime transfers through an edge state to the superconductor. It reflects into a hole and an electron which returns to the normal contact through the other edge states.

oscillations with ν are suppressed, in qualitative agreement with experiments.

The paper is organized as follows. In Sec. II, we present the general framework in terms of the quasiparticle scattering states for the computation of the current-voltage characteristics for 2DEG-S interface. In Sec. III, we consider the case of an ideally flat 2DEG-S boundary. We generalize the result of Ref. 14 on finite temperature. In Sec. IV, we consider the effect of surface roughness (diffusive reflection case) on the current-voltage characteristics. We end the paper with conclusions in Sec. V.

II. FORMALISM

We consider a junction consisting of a superconductor, 2DEG, and a normal conductor segment (see Fig. 1). Magnetic field B is applied along the z direction, perpendicular to the plane of 2DEG. It is supposed that quasiparticle transport is ballistic; the mean free path of an electron $l_{tr} \gg L$. The current I flows between N and S terminals provided the voltage V is applied between them.

Following Refs. 15–18, we shall describe the transport properties of the junction in terms of electron and hole quasiparticle scattering states, which satisfy Bogoliubov–de Gennes (BdG) equations. Then the current through the 2DEG-S surface is given as

$$I(V) = \frac{e}{h}\int_0^\infty dE\{f_e \,\mathrm{Tr}[\hat{1} - R_{ee} + R_{he}] - f_h \,\mathrm{Tr}[\hat{1} - R_{hh} + R_{eh}]\}, \tag{1}$$

where $f_{e(h)}$ denotes the Fermi-Dirac distribution function:

$$f_{e(h)} = \frac{1}{e^{(E \mp \mathrm{eV})/T} + 1}.$$

The energy E is measured from the chemical potential μ of the superconductor, and $R_{ee}(E, n_o, n_i)$ is the probability of the (normal) reflection of an electron with the energy E incident on the superconductor in the edge channel with quantum number n_i to an electron going from the superconductor in the channel n_o. The trace in Eq. (1) is taken over the channel space provided that spin degrees of freedom are included in the channel definition. According to Eq. (1), if we find the probabilities R_{ab}, $a, b = e, h$, then we shall be able to evaluate the current, the conductance, the current noise, and so on.[18] We shall focus on the case $R_c \lesssim L$. Then, quasiparticles reflected from the superconductor due to normal and Andreev reflections return to the superconductor again due to bending of their trajectories by the magnetic field.

At first sight, it seems that the reflection probabilities R_{ab} could be easily found using the approach of matching the incident and outgoing quasiparticle wave functions at $y=0$ and $y=L$ with the linear combinations of the quasiparticle wave functions at the 2DEG-S boundary, corresponding to Andreev edge states.[13] However, this procedure is not easily achieved, especially, in the presence of a disorder at the 2DEG boundaries. Even without a disorder, the matching approach is hardly accomplished because Andreev bound state wave functions[13] and the wave functions of 2DEG edge states are localized generally in different domains in the $\hat{x}$ direction. The Andreev bound state wave functions ψ penetrate inside the superconductor on the length scale of the order of the superconductor coherence length ξ,[13] but 2DEG edge state wave functions ϕ of the incident and outgoing quasiparticles penetrate inside the 2DEG edges at the length scale l_n, which, in general, is not equal to ξ (Ref. 19;) e.g., $l_n=0$ if we describe the edges of 2DEG by the model of solid impenetrable potential walls.

Let $\nu=1$, and we want to find the reflection probabilities by the matching procedure. Then, we should expand the wave function $\phi_e(x)$ of the incident quasiparticles with the energy E_F at $y=0$ over the basis of the two [because $\nu=1$] Andreev bound state wave functions: $\psi_e(x)$ and $\psi_h(x)$. If, e.g., $l_n=0$, we can hardly do it because then $\psi_{e(h)}(x)$ is finite but $\phi_e(x)=0$ for $-\xi \lesssim x < 0$; so it seems that the basis of Andreev bound state wave functions turns out to be incomplete in this case. When $\nu \gg 1$, the incompleteness problem of the Andreev bound state basis possibly will be asymptotically removed, but we do not know the accuracy of this statement.

Using the picture of quasiclassical trajectories, we develop here an alternative procedure of matching the quasiparticle wave functions. It allows us to take into account the disorder at the boundaries of the 2DEG.

At large filling factors, the quasiclassical approximation is applicable. An electron (hole) quasiparticle in 2DEG can be viewed in the quasiclassics as a beam of rays.[21,22] In a similar way, propagation of the light is described in optics within the eikonal approximation in terms of the ray beams.[25] Trajectories of the quasiparticle rays can be found from the equations of classical mechanics. In terms of the wave functions, this description means that we somehow make wave packets from wave functions of the edge states. Reflection of an electron from the superconductor is schematically shown in Figs. 1–5. In what follows, we demonstrate that, within the quasiclassical approximation, the matching problem can be solved even in the presence of an interface disorder and the probabilities R_{ab} can be explicitly evaluated.

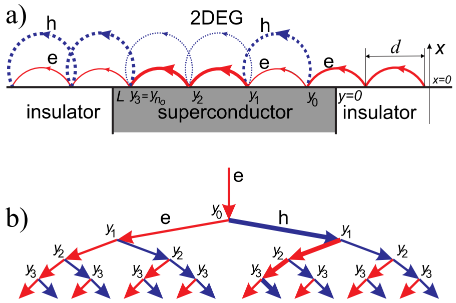

FIG. 2. (Color online) The propagation of quasiparticles in the quasiclassical approximation can be described in terms of rays. The state of a ray can be found from the equations of the classical mechanics. (a) and (b) show what happens if an electron ray from an edge state of 2DEG comes to the superconductor. The electron ray reflects at the point $y=y_0$ from the superconductor into electron and hole rays (normal and Andreev reflections). They reflect, in turn, at the position y_1 from the superconductor generating two other electron (red lines) and two other hole rays (blue lines), and so on. To find the probabilities, e.g., $R_{he}(E,n_o,n_i)$, it is necessary to know the sum of the amplitudes of the eight holes that appear after the last beam reflection at the point y_3. Drawing (a), we assume that Andreev approximation (Ref. 4) is applicable [see Eq. (19)] and the 2DEG-S interface is ideally flat.

III. IDEALLY FLAT 2DEG-S INTERFACE

We start from the transport properties of the ideally flat 2DEG-S interface which can be most simply described provided that the edge channels do not mix at such interface, $R_{he}(E;n_o,n_i)\propto\delta_{n_o,n_i}$. In the quasiclassical language, it means that electrons (holes) skip along the 2DEG edge along the same arc trajectories, like it is shown in Fig. 2.

The probability of Andreev reflection can be found as follows for the trajectories shown in Figs. 2 and 3:

$$R_{he}(y_0;n_o,n_i) = \delta_{n_o,n_i}\left|e^{i(S_e-\pi/2)}\{r_{he}r_{ee}r_{ee}r_{ee}e^{3iS_e-i3\pi/2-i\phi(y_3)} + r_{hh}r_{he}r_{ee}r_{ee}e^{iS_h+2iS_e-i\pi/2-i\phi(y_2)} + \dots\}\right|^2, \quad (2)$$

where r_{ba} is the amplitude of reflection of a quasiparticle (ray) a into a quasiparticle (ray) b from the superconductor. The quantity $S_{e(h)}$ is the quasiclassical action of an electron (hole) taken along the part of the trajectory connecting the adjacent points of reflection. The additional phase $\pm\pi/2$ is the Maslov index[20] of the electron trajectory. The phase $\phi(y)$ arises due to the screening supercurrents. We assume that if the superconductor satisfies the description within the London theory [it is usually the case in experiments], then $\phi(y)=\phi(0)+(2m/\hbar)\int_0^y d\tilde{y}v_s(\tilde{y})$,[23] where v_s is the superfluid velocity evaluated at $x=0$, and m is the electron mass in the superconductor. Here, we used the following property of London superconductors: the spatial dependence of the vector potential and v_s are small in the direction perpendicular to the superconductor edge on the length scale ξ.

It is convenient to introduce the matrix

$$M(y) = \begin{pmatrix} r_{ee}e^{i(S_e-\pi/2)} & r_{eh}e^{i(S_h+\pi/2)+i\phi(y)} \\ r_{he}e^{i(S_e-\pi/2)-i\phi(y)} & r_{hh}e^{i(S_h+\pi/2)} \end{pmatrix} \quad (3)$$

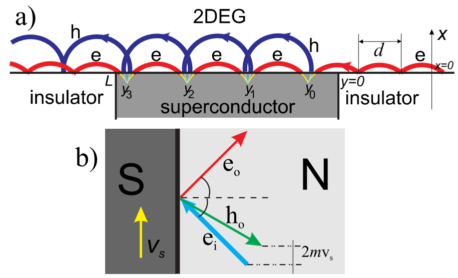

FIG. 3. (Color online) When the conditions of the Andreev approximation are violated, we cannot use the assumption that an Andreev-reflected hole velocity is exactly opposite to the velocity of the incident quasiparticle at the superconductor electron. Then, the quasiparticle rays propagate along the 2DEG-S interface as sketched in the figure. The orbits are organized as if the scattering occurs not at the 2DEG-S interface but from the interface in the superconductor lying at some distance from the 2DEG-S interface. According to Ref. 13, Andreev reflection couples electron and hole orbits with the guiding center x coordinates $-\delta\pm X$, where $\delta \simeq l_B^2 m v_s/\hbar$, $l_B=\sqrt{\hbar c/eB}$, and the value of v_s should be taken at the 2DEG-S interface. For superconductors wider than the London penetration length λ_M, $\delta=\lambda_M$. (b) illustrates schematically how Andreev and normal quasiparticle reflections occur at the superconducting interface along which a supercurrent flows. Indices i and o label the incident and reflected quasiparticles, respectively.

that contains the amplitudes of Andreev (r_{he},r_{eh}) and normal (r_{ee},r_{hh}) quasiparticle reflections from the superconductor at the point y. Then, the matrix product

$$S^{(3)} = M(y_3)M(y_2)M(y_1)M(y_0) \quad (4)$$

describes the Andreev and normal scattering amplitudes for the case shown in Figs. 2 and 3. In the case of $n+1$ reflections of a quasiparticle from the 2DEG-S interface, we obtain the result

$$S^{(n)} = M(y_n)M(y_{n-1})\cdots M(y_1)M(y_0). \quad (5)$$

Hence, the probabilities R_{ab} can be found as

$$R_{ee}(E;y_0;n_i,n_i) = |S_{11}^{(n)}|^2, \quad R_{eh}(E,y_0;n_i,n_i) = |S_{12}^{(n)}|^2, \quad (6)$$

$$R_{he}(E;y_0;n_i,n_i) = |S_{21}^{(n)}|^2, \quad R_{hh}(E;y_0;n_i,n_i) = |S_{22}^{(n)}|^2. \quad (7)$$

The matrix $S^{(n)}$ can be calculated analytically for any integer n when the superfluid velocity $v_s(x=0,y)$ is constant and the phase $\phi(y)$ is a linear function of y. Then, $\phi(y_n)-\phi(y_0)=n\delta\phi$, with $\delta\phi=2mv_sd/\hbar$. The matrix $M(y_n)$ can be conveniently written as

$$M(y_n) = \Phi^{\dagger}(n)M(y_0)\Phi(n), \quad (8)$$

where

$$\Phi(n) \equiv \begin{pmatrix} \exp\left\{-\frac{i}{2}n\delta\phi\right\} & 0 \\ 0 & \exp\left\{\frac{i}{2}n\delta\phi\right\} \end{pmatrix}. \quad (9)$$

Next, we obtain

$$S^{(n)} = \Phi^\dagger(n)(M\Phi)^{n+1}\Phi^\dagger, \quad (10)$$

where $M=M(y_0)$ and $\Phi=\Phi(1)$. Given Eq. (A1) [see Appendix A], it is easy to find

$$(M\Phi)^{n+1} = e^{i[(n+1)/2](S_e+S_h)}(r_{ee}r_{hh}-r_{eh}r_{he})^{(n+1)/2}\begin{pmatrix} \frac{r_{ee}e^{i\Omega_0}U_n(a)}{\sqrt{r_{ee}r_{hh}-r_{eh}r_{he}}} - U_{n-1}(a) & \frac{r_{eh}e^{-i\Omega_0}}{\sqrt{r_{ee}r_{hh}-r_{eh}r_{he}}}U_n(a) \\ \frac{r_{he}e^{i\Omega_0}}{\sqrt{r_{ee}r_{hh}-r_{eh}r_{he}}}U_n(a) & \frac{r_{hh}e^{-i\Omega_0}U_n(a)}{\sqrt{r_{ee}r_{hh}-r_{eh}r_{he}}} - U_{n-1}(a) \end{pmatrix}. \quad (11)$$

Here, $U_n(a)=\sin[(n-1)\arccos a]/\sqrt{1-a^2}$ denotes the Chebyshev polynomial of the second kind and

$$a = \frac{r_{ee}e^{i\Omega_0-i\delta\phi/2} + r_{hh}e^{-i\Omega_0+i\delta\phi/2}}{2\sqrt{r_{ee}r_{hh}-r_{eh}r_{he}}}, \quad (12)$$

with $\Omega_0=(S_e-S_h-\pi)/2$.

If the amplitudes of local reflection r_{ab} are given, then we can find the probabilities R_{ab} from Eqs. (10)–(12). We obtain for $n+1$ reflections:

$$R_{ee} = |r_{ee}r_{hh}-r_{eh}r_{he}|^{n+1}\left|\frac{r_{ee}e^{i\Omega_0}U_n(a)}{\sqrt{r_{ee}r_{hh}-r_{eh}r_{he}}} - U_{n-1}(a)\right|^2,$$

$$R_{hh} = |r_{ee}r_{hh}-r_{eh}r_{he}|^{n+1}\left|\frac{r_{hh}e^{-i\Omega_0}U_n(a)}{\sqrt{r_{ee}r_{hh}-r_{eh}r_{he}}} - U_{n-1}(a)\right|^2,$$

$$R_{eh} = |r_{ee}r_{hh}-r_{eh}r_{he}|^{n}|r_{eh}|^2|U_n(a)|^2,$$

$$R_{he} = |r_{ee}r_{hh}-r_{eh}r_{he}|^{n}|r_{he}|^2|U_n(a)|^2. \quad (13)$$

The probabilities R_{ab} depend on the position of the first reflection from the 2DEG-S interface, y_0, that varies in the range $(0,d)$ [see Fig. 2(a)]. The number of reflections n depends on the choice of y_0 as $n=1+[(L-y_0)/d]$, where $[x]$ denotes the integer part. So, the solution strategy is to calculate the current using Eq. (1) with the probabilities R_{ab} defined in Eqs. (13) and, then, average the result over y_0. The natural choice for the distribution of y_0 is the uniform distribution: $P_{n_i}[y_0]=\theta[d(n_i)-y_0]/d(n_i)$. Hence, we find

$$I(V) = \frac{e}{h}\int_0^\infty dE\sum_{n_i}\int dy_0 P_{n_i}(y_0)\{[1-R_{ee}+R_{he}]f_e - [1-R_{hh}+R_{eh}]\}f_h. \quad (14)$$

Equations (13) and (14) constitute the main result of the paper. They allow one to find the current through the 2DEG-S surface at arbitrary temperature provided the amplitudes of local reflection r_{ab} are given.

The action $S_e=\int\tilde{\mathbf{k}}_e\cdot d\mathbf{l}$, where $\tilde{k}_e$ is the generalized momentum and the integral is over the quasiparticle trajectory that connects the adjacent points of reflection from the 2DEG-S interface:

$$S_e = k_e l_e + \frac{e}{\hbar c}\int \mathbf{A}\cdot d\mathbf{l} = k_e l_e - \frac{|e|}{\hbar c}\Phi_e, \quad (15)$$

$$S_h = -k_h l_h + \frac{|e|}{\hbar c}\Phi_h. \quad (16)$$

Here, $k_{e(h)}=\sqrt{2m[\mu_{2DEG}\pm(E+g_L\mu_B\sigma B)]/\hbar^2}$ and $\sigma=\pm 1$, with μ_{2DEG} and g_L being the chemical potential and the g factor of 2DEG, respectively; $l_{e(h)}$ is the trajectory length; and $\Phi_{e(h)}$ is the absolute value of the magnetic field flux through the area bounded by the quasiparticle trajectory arc and the 2DEG-S interface. The actions can be explicitly written in terms of the filling factor ν and the y component of the quasiparticle velocity, $v_y^{e(h)}$, at the 2DEG-S interface (when $E, g_L\mu_B\sigma B \ll \mu_{2DEG}/\nu$, so $k_{e(h)}\approx k_F$):

$$S_{e(h)} = s_{e(h)} \pm \pi(\nu+1/2), \quad (17)$$

where

$$s_a = 2\left(\nu+\frac{1}{2}\right)\left[\arcsin\frac{v_y^a}{v^a} - \frac{v_y^a}{v^a}\sqrt{1-\left(\frac{v_y^a}{v^a}\right)^2}\right]. \quad (18)$$

Often the Andreev approximation can be used.[4] Then, the Andreev-reflected hole velocity direction may be considered opposite to the velocity direction of the incident quasiparticle at the superconductor electron (see Fig. 2). The conditions are as follows:

$$v_s \ll v_F^{(2DEG)}, \quad \max(|eV|, T, g_L \mu_B B) < \Delta < E_F^{(2DEG)}. \tag{19}$$

Then, $s_e = s_h$ and

$$S_e - S_h = \frac{|e|}{2\hbar c}\Phi = 2\pi\left(\nu + \frac{1}{2}\right). \tag{20}$$

Here, Φ is the flux through the Larmor ring trajectory of an electron in magnetic field B at the Fermi shell.

The problem of how to evaluate the r_{ab} amplitudes also simplifies within the parameter range (19). The conditions mean that the magnetic field could be neglected in the BdG equations; it is already taken into account by the phase ϕ. Then, r_{ab} can be evaluated according to the Blonder-Tinkham-Klapwijk (BTK) theory.[15]

In general, when the conditions (19) are violated, our quasiclassical transport picture in terms of quasiparticle rays can still be applied (see Fig. 3). The amplitudes r_{ab} are the solutions of the scattering problem for Bogoliubov–de Gennes equations:

$$(E - g_L \mu_B B)u = \left(\frac{[\mathbf{p} + m\mathbf{v}_s]^2}{2m} - \mu\right)u + \Delta v, \tag{21}$$

$$(E - g_L \mu_B B)v = -\left(\frac{[\mathbf{p} - m\mathbf{v}_s]^2}{2m} - \mu\right)v + \Delta u. \tag{22}$$

Here, m, g_L, and μ should be considered different in S and 2DEG. The spatial distribution of the superfluid velocity is fixed by the London equation, rot $m\mathbf{v}_s = -\mathbf{B}e/c$. We do not write the contribution to the BdG equations from a barrier which exists usually at the 2DEG-S interface.

At $T=0$, the amplitudes of local reflection r_{ab} are related to each other as $r_{hh} = r_{ee}^\star$, $r_{eh} = -r_{he}^\star$, and $|r_{ee}|^2 + |r_{he}|^2 = 1$. Then, using Eq. (14), we find the conductance at zero temperature and voltage:

$$g = \frac{2e^2}{h}\sum_{n_i}\sum_{s} P_s \frac{|r_{eh}|^2 \sin^2\{s \arccos[\sqrt{|r_{ee}|^2}\cos(\Omega)]\}}{1 - |r_{ee}|^2 \cos^2(\Omega)}, \tag{23}$$

where spin is combined with the channel index, $\Omega = (S_e - S_h - \pi)/2 + \theta - \delta\phi/2$, and $\theta = \arg(r_{ee})$ is the phase of the amplitude of electron-electron reflection from the superconductor. If the superconductor characteristic dimensions in the x direction are larger than the Meissner penetration length λ_M, then $\delta\phi/2 = 2\lambda_M k_\perp$, where $k_\perp = k_\perp(n_i)$ is the perpendicular component of the quasiparticle momentum when it reflects from the superconductor. The function P_s is the probability that the orbit experiences s reflections from the surface of the superconductor. The function P_s originates from the integration over y_0 in Eq. (14). It can be expressed through the maximum number of jumps, $[L/d]$, over the S-2DEG surface as

$$P_s = \begin{cases} \frac{L}{d} - \left[\frac{L}{d}\right] & \text{if } s = 1 + \left[\frac{L}{d}\right], \\ 1 - \frac{L}{d} + \left[\frac{L}{d}\right] & \text{if } s = \left[\frac{L}{d}\right], \\ 0 & \text{otherwise}. \end{cases} \tag{24}$$

The conductance [Eq. (23)] as well as the current [Eq. (14)] are an oscillating function of ν:

$$g(\nu) = \sum_{n=0}^{\infty} g_n \cos(2\pi\nu n + \delta_n), \tag{25}$$

where g_n are the Fourier coefficients and δ_n the "phase shifts." For the length of the interface $L \lesssim 2R_c$, the leading contribution to the conductance (current) is given by the zero harmonic g_0. In the case $2R_c \lesssim L \lesssim 4R_c$, the conductance is determined by the zero and first harmonics, $g \approx g_0 + g_1 \cos(2\pi\nu + \delta_1)$. If $4R_c \lesssim L \lesssim 6R_c$, the second harmonics $g_2 \cos(4\pi\nu + \delta_2)$ becomes relevant, and so on.

How the conductance changes at $T=0$ with ν is illustrated in Fig. 4(a). The thin black curve in Fig. 4 corresponds to $L/2R_c \gtrsim 3$, which is a typical value in experiments. Applying BTK model for extracting $r_{ee}, \ldots$, one can plot figures of the zero-bias conductance $g(\nu)$ at finite temperatures using such parameters that many harmonics are presented, as in the case $T=0$ shown in Fig. 4. It is worthwhile mentioning that the amplitudes $g_1, g_2, g_3, \ldots$, of all visible harmonics of the conductance oscillations decrease more or less equally when temperature grows from zero to T_c; finally, they vanish at $T=T_c$. However, contrary to these predictions where a large

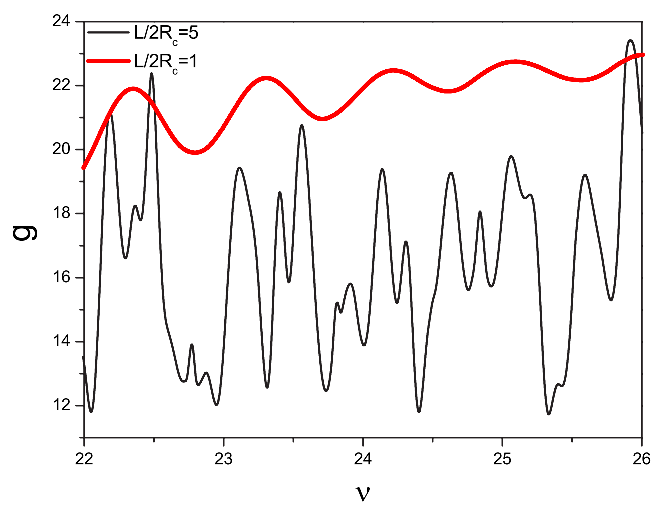

FIG. 4. (Color online) The oscillations with ν of the dimensionless zero-bias conductance at $T=0$. The parameters are as follows. Fermi wave vector $k_F^{(2DEG)} = 2\times10^6$ cm^{-1} and $\delta\phi/4 = 20$ that correspond to NbN film with a width of the order of 100 nm. The $L=3$ μm ($L/2R_c \simeq 5$ at $\nu=25$) for the lower (black thin) curve and $L=0.6$ μm ($L/2R_c \simeq 1$ at $\nu=25$) for the upper (red thick) curve. We neglect the Zeeman splitting, which is typically small. The 2DEG-S interface scattering amplitudes r_{ab} were taken according to the BTK model (Ref. 15), with $Z=0.6$. The curves were produced using Eq. (23).

number of harmonics are seen well, experimentally, only the zeroth and first harmonics, g_0 and g_1, survive. The reason for the discrepancy between our theory and the experiment is the assumption that the 2DEG-S interface is ideally flat. Indeed, as we shall demonstrate in the next section, disorder at the 2DEG-S interface makes $g_0 > g_1 > g_2 \ldots$

IV. DISORDERED 2DEG-S INTERFACE

Usually, 2DEG-S interface is not ideally flat. The disorder at the interface can be divided into two classes: long range and short range with the respect to the characteristic wavelength, λ_F, in 2DEG. The presence of the long-range disorder implies that the 2DEG-S interface position fluctuates around the line $x=0$ at length scales much larger than $\lambda_F^{(2DEG)} \sim 10^{-6}$ μm. Photographs of the experimental setups do not allow one to think that 2DEG-S interface bends strongly from the line $x=0$. Therefore, in the experiments of Refs. 6–10, this kind of disorder is likely not very important.

The short-range disorder includes the fluctuations of the surface at length scales smaller than $\lambda_F^{(2DEG)}$. Usually, this type of disorder is provided by impurities, clusters of atoms at the surface due to defects of the lithography, etc. When, for example, an electron ray falls on the disordered 2DEG-S surface, the reflected electron rays go off the surface not at a fixed angle but they may go at any angle with certain disorder-induced probability distribution (diffusive reflection). The phases that carry the reflected electron rays going off the surface at different angles may be considered random, so the reflected electron rays can be considered as incoherent.[27] However, to any reflected electron ray, an Andreev-reflected hole ray is attached that is coherent with the electron. So the interference of rays that produces the conductance oscillations may not be suppressed completely by the short-range disorder.

"Weak" short-range disorder at 2DEG-S interface does not destroy the Andreev edge states, but it induces transitions between the edge states [see Fig. 5(b)]. Andreev edge states in quasiclassics fix electron-hole orbit arcs with the same beginning and end. The quasiclassical picture of the disorder-induced transitions is shown in Fig. 5(a).

In what follows, we assume that if the Andreev approximation conditions [Eq. (19)] are fulfilled, then $s_e = s_h$ and the only fluctuating quantity is $\delta\phi$. In the general case, qualitatively similar results for the current can be obtained after cumbersome calculations.

In close analogy with the ideal case, we introduce the matrix

$$M(y_n) = \Phi^\dagger(n) M \Phi(n) \exp\left(i \sum_{i=0}^{n} s_i \right), \qquad (26)$$

where s is defined in Eq. (17), the matrix

$$\Phi(n) \equiv \begin{pmatrix} \exp\left\{ -\frac{i}{2} \sum_{i=0}^{n} \delta\phi_i \right\} & 0 \\ 0 & \exp\left\{ \frac{i}{2} \sum_{i=0}^{n} \delta\phi_i \right\} \end{pmatrix}, \qquad (27)$$

and

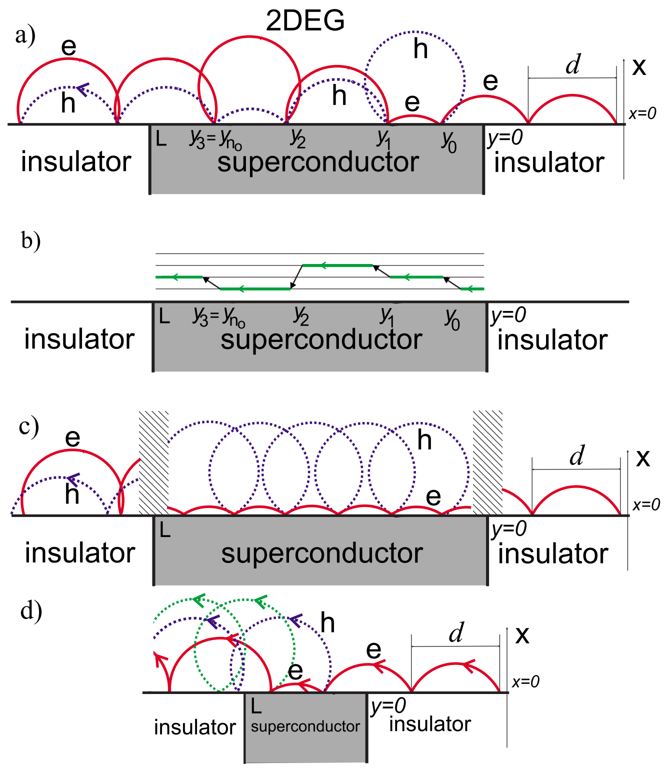

FIG. 5. (Color online) The propagation of quasiparticles in the quasiclassical approximation along the 2DEG-S interface in the presence of disorder [see (a)]. It corresponds to the transitions between the Andreev edge states, as shown in (b), which are induced by relatively weak short-range disorder. Disorder at the edges of the 2DEG-S interface leads to the orbits depicted in (c). Strong disorder destroys Andreev edge states. There are no oscillations in the conductance because quasiparticles reflected from the strongly disordered 2DEG-S surface are incoherent. Their orbits are shown in (d).

$$M = \begin{pmatrix} r_{ee} e^{i\pi\nu} & r_{eh} e^{-i\pi\nu} \\ r_{he} e^{i\pi\nu} & r_{hh} e^{-i\pi\nu} \end{pmatrix}. \qquad (28)$$

As before, for the situation sketched in Fig. 5(a), we find $S^{(3)} = M(y_3) M(y_2) M(y_1) M(y_0) \exp(i\Sigma_{i=0}^{3} s_i)$ and, e.g., $R_{he} = |S_{21}^{(3)}|^2$. It is worthwhile mentioning that the factor $\exp(i\Sigma_{i=0}^{3} s_i)$ does not influence the probabilities R_{ab}; therefore, we shall omit this term below. In the general case of $n+1$ reflections, we obtain

$$S^{(n)} = \Phi^\dagger(n) [M\Phi_n \cdots M\Phi_1 M\Phi_0] \Phi_0^\dagger. \qquad (29)$$

Here,

$$\Phi_n \equiv \begin{pmatrix} \exp\left\{ -\frac{i}{2} \delta\phi_n \right\} & 0 \\ 0 & \exp\left\{ \frac{i}{2} \delta\phi_n \right\} \end{pmatrix}, \qquad (30)$$

with $\delta\phi_n = \phi_n - \phi_{n-1}$ and $\delta\phi_0 \equiv 0$.

Provided that $R_{ee} - R_{he} = R_{hh} - R_{eh}$, we find the probabilities

$$R_{ee}-R_{he}=\frac{1}{2}\mathrm{Tr}\{\sigma_z M\Phi_n M\Phi_{n-1}\cdots\Phi_2 M\Phi_1 M\sigma_z M^\dagger\Phi_1^\dagger \times M^\dagger\Phi_2^\dagger\cdots\Phi_{n-1}^\dagger M^\dagger\Phi_n^\dagger M^\dagger\}. \tag{31}$$

For reasons to be explained shortly, we neglect the term $\Phi^\dagger(n)$. If one wants to find R_{ee}, then σ_z should be substituted by $(\sigma_0+\sigma_z)/2$ in the last equation. In order to obtain R_{eh}, one should substitute the first σ_z by $(\sigma_0+\sigma_z)/2$ and the second one by $(\sigma_0-\sigma_z)/2$.

As in the previous section, we should average the current over position y_0. This operation is closely related to the disorder averaging, because by shifting y_0 we make the trajectories go through different disorder realizations. The phase jumps $\delta\phi_n$ fluctuate due to the disorder.

We assume the following disorder average:

$$\langle[\Phi_a]_{ij}[\Phi_b]_{pq}^\dagger\rangle=\delta_{ab}\delta_{ij}\delta_{pq}(\delta_{ip}+\Lambda_{ip}), \tag{32}$$

$$\Lambda_{ip}=\begin{pmatrix}0 & \langle e^{-i\delta\phi_a}\rangle\\ \langle e^{i\delta\phi_a}\rangle & 0\end{pmatrix}. \tag{33}$$

Then, for example, the averages of the type $\langle(1-\delta_{ab})e^{-i(\delta\phi_a\pm\delta\phi_b)/2}\rangle=0$, because quasiparticles acquire a random phase reflecting from the disordered interface, as was mentioned above. The quantity $R_{ee}-R_{he}$ can be treated perturbatively over small Λ. For the Gaussian distribution of $\delta\phi_n$:

$$\langle\delta\phi_a\rangle=\overline{\delta\phi}, \tag{34}$$

$$\langle\langle\delta\phi_a\delta\phi_b\rangle\rangle=2\eta\delta_{ab}, \tag{35}$$

where $\langle\langle\cdots\rangle\rangle$ denotes the irreducible average, one finds

$$\langle e^{i\delta\phi_a}\rangle=e^{i\overline{\delta\phi}}e^{-\eta}. \tag{36}$$

For weak fluctuations: $\eta\ll 1$, the results of the previous section are valid. The opposite case, $\eta\gg 1$, we shall discuss below.

To the lowest order in Λ, the average probabilities for $n=3$ are given as

$$R_{ee}-R_{he}=\mathrm{Tr}\{(\sigma_z)_{i_2i_2}|M_{i_2i_3}|^2|M_{i_3i_4}|^2|M_{i_4i_5}|^2|M_{i_5i_6}|^2(\sigma_z)_{i_6i_6}\}. \tag{37}$$

We see, therefore, that in the "completely incoherent case," $\Lambda=0$, the average probabilities can be found as the corresponding elements of the matrix $\tilde{S}^n$

$$\tilde{S}^n=\begin{pmatrix}|r_{ee}|^2 & |r_{eh}|^2\\ |r_{he}|^2 & |r_{hh}|^2\end{pmatrix}^n. \tag{38}$$

For example, $R_{ee}=[\tilde{S}^n]_{11}$. Given the theorem presented in Appendix A, it is easy to find the explicit form of $\tilde{S}^n$.

More interesting, however, is the first order term in the expansion in powers of Λ for the probabilities R_{ab}. After calculations diagrammatically illustrated in Figs. 6 and 7 see Appendix B), we find for the average probabilities at $T=0$:

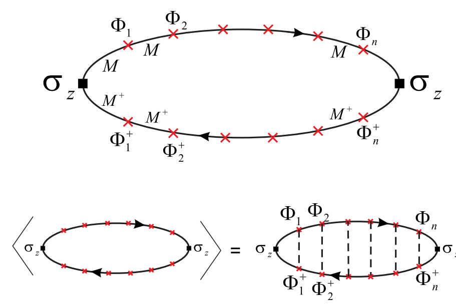

FIG. 6. (Color online) Disorder averaging for the transmission probabilities. The loop corresponds to the trace in Eq. (31). The $\times$ vertices are $\Phi,\Phi^\dagger$; the solid lines represent $M,M^\dagger$; and the dashed lines are the $\langle\Phi\Phi^\dagger\rangle$ correlators.

$$R_{ee}-R_{he}=(|r_{ee}|^2-|r_{eh}|^2)^s \times\left[1+e^{-\eta}\frac{4(s-1)|r_{ee}|^2|r_{eh}|^2}{(|r_{ee}|^2-|r_{eh}|^2)^2}\cos(2\Omega)\right]. \tag{39}$$

Here, $\Omega=\pi\nu+\theta_{ee}-\overline{\delta\phi}/2$, and s is the number of reflections from the 2DEG-S interface. All trajectory-dependent quantities that enter Eq. (39) should be evaluated for the trajectory with equal electron and hole arcs.

Finally, we obtain the desired result for the zero-bias conductance at $T=0$:

$$g=\frac{4e^2}{h}\nu\sum_s P_s\left\{1-(|r_{ee}|^2-|r_{eh}|^2)^s \times\left[1+e^{-\eta}\frac{4(s-1)|r_{ee}|^2|r_{eh}|^2}{(|r_{ee}|^2-|r_{eh}|^2)^2}\cos(2\Omega)\right]\right\}, \tag{40}$$

where P_s is defined in Eq. (24), but with d being substituted by $2R_c$. As one can see, in the limit $e^{-\eta}\ll 1$, the conductance oscillations are described by the first harmonics $g_1\cos(2\pi\nu$

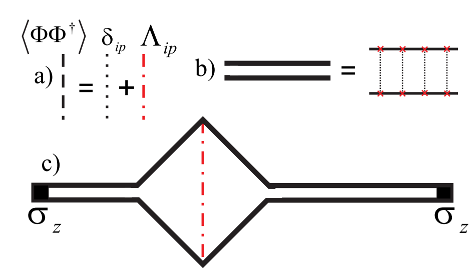

FIG. 7. (Color online) It is convenient to split the $\langle\Phi\Phi^\dagger\rangle$ line into the sum of two. The first one corresponds to the δ_{ip} term in Eq. (32) and the second one to the Λ term [(a)]. The parallel lines show the part of the bubble [Eq. (31)] where the zero order in Λ is taken ("diffusion"). It does not carry interference information, (b). (c) Typical contribution to the average of the bubble of the first order in Λ is schematically shown. The diamond denotes the place where the Λ line is inserted. It is this diamond that provides oscillations of the current with ν. In order to find the contribution to the current of the first order in Λ, one should sum all n diagrams like those shown in (c) that differ by the position of the diamond.

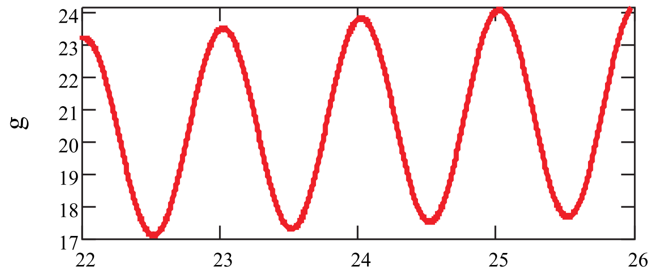

FIG. 8. (Color online) The figure shows the dependence of the conductance on ν at $T=0$ for the case of the disordered 2DEG-S interface. The parameters used for this graph: length L of the S-2DEG interface and the normal conductance of the interface, are the same as for the black thin curve in Fig. 4. The comparison with the data of Refs. 6 and 8 suggests $e^{-\eta}\simeq 0.1$ that we choose for this figure. The harmonics $g_2, g_3, \ldots$, do not contribute to the conductance in the case of the disordered interface in contrast to the clean interface case of Fig. 4. Remarkably, features of the conductance oscillations, such as suppression of the harmonics higher than g_1, the order of the oscillation amplitude, etc., agree qualitatively with the experimental data.

$+\delta_1)$ in Eq. (25); higher harmonics are exponentially suppressed by disorder as $g_{n+1}/g_n \propto e^{-\eta} \ll 1$.

The conductance oscillations [Eq. (40)] are illustrated in Fig. 8. The conductance behavior agrees qualitatively with the experimental data (for detailed discussion, see Ref. 8).

An important question is the temperature dependence of the conductance. Generally, the visibility of the conductance oscillations is maximal at $T=0$ and is strongly suppressed as T tends to T_c.[11–13] Given Eqs. (B13), one can find the conductance at arbitrary temperature: $g=g_0+g_1\cos(2\pi\nu+\delta_1)$. Here, the amplitude $g_1 \propto e^{-\eta}$. In accordance with general expectations, finite temperature suppresses the amplitude g_1 of the conductance oscillations. In Fig. 9, we draw the temperature dependence of the conductance maximum using Eq. (1) for the case of strong disorder at the interface; parameters correspond to the conductance maximum at $\nu_{\max}\approx 23$ for the curve on Fig. 8. The curve in Fig. 9 qualitatively reproduces the shape of the experimental data, see detailed comparison in Ref. 8. Surprisingly, the T behavior of the conductance at different maxima (different $\nu_{\max}$) are qualitatively the same and resembles the dependence of conductance on temperature of the 2DEG-S junction in zero magnetic field.

Finally, we mention that the result similar to Eqs. (39) and (40) could be obtained by calculation of the influence of the disorder at the edges of 2DEG-S interface on the magnetoconductance [see Fig. 5(c)].

V. CONCLUSIONS

To summarize, in this paper, we developed a theory for the current-voltage characteristics of a 2DEG-S interface in a magnetic field at finite temperature and in the presence of surface roughness (diffusive reflection). We predict that the surface roughness at the 2DEG-S interface suppresses higher harmonics in the conductance oscillations with the filling factor of 2DEG. We believe that it removes the contradiction between the theory and the experiment that existed so far.

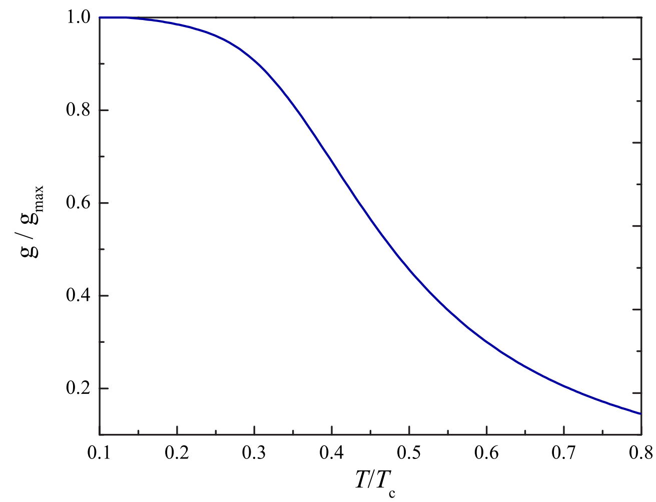

FIG. 9. (Color online) The temperature dependence of the conductance. The visibility of the conductance oscillations is maximal at $T=0$ and is strongly suppressed as T tends to T_c. We plot the temperature dependence of the conductance at the maximum using Eq. (1) for the case of strong disorder at the interface; parameters correspond to the conductance maximum of the curve at $\nu\approx 23$ (Fig. 8).

The magnetoconductance of the 2DEG-S boundary seems to be sensitive to the degree of the boundary roughness, which then offers an independent way of probing the interface quality.

ACKNOWLEDGMENTS

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APPENDIX A: ABELÉS THEOREM

For the sake of the reader's convenience, we present in this appendix the following theorem.[24,25] Let Q be a 2×2 (complex) matrix with the determinant equal to unity, then

$$Q^{n+1} = \begin{pmatrix} Q_{11}U_n(a) - U_{n-1}(a) & Q_{12}U_n(a) \\ Q_{21}U_n(a) & Q_{22}U_n(a) - U_{n-1}(a) \end{pmatrix}, \tag{A1}$$

where $a=\mathrm{Tr}\,Q/2$, and $U_n(a)$ is the Chebyshev polynomial of the second kind[26]

$$U_n(a) = \sin[(n+1)\arccos(a)]/\sqrt{1-a^2}.$$

APPENDIX B: DISORDER AVERAGING

In this appendix, we present the details of derivation for Eq. (39). Let us consider the matrix

$$\begin{pmatrix} A_{n+1} & B_{n+1} \\ C_{n+1} & D_{n+1} \end{pmatrix} = \left\langle M\Phi_{n+1}\begin{pmatrix} A_n & B_n \\ C_n & D_n \end{pmatrix}\Phi_{n+1}^{\dagger}M^{\dagger}\right\rangle \quad \text{(B1)}$$

with

$$\begin{pmatrix} A_0 & B_0 \\ C_0 & D_0 \end{pmatrix} = M\sigma_z M^{\dagger}. \quad \text{(B2)}$$

Then, the quantity $R_{ee}-R_{he}$ in Eq. (31) is equal to A_{n-1}. It is convenient to introduce a vector $\Psi_n=(A_n,D_n,B_n,C_n)^T$:

$$\Psi_{n+1} = \mathbb{T}(e^{-\eta})\Psi_n, \quad \mathbb{T}(x) = \begin{pmatrix} \widetilde{S} & -xU \\ W & -xV \end{pmatrix}, \quad \text{(B3)}$$

where the elements U, V, and W of the transfermatrix $\mathbb{T}$ are the following 2×2 matrices:

$$W = \begin{pmatrix} r_{ee}r_{he}^{\star} & r_{eh}r_{hh}^{\star} \\ r_{he}r_{ee}^{\star} & r_{hh}r_{eh}^{\star} \end{pmatrix}, \quad \text{(B4)}$$

$$U = \begin{pmatrix} r_{ee}r_{eh}^{\star}e^{i2\pi\nu-i\overline{\delta\phi}} & r_{eh}r_{ee}^{\star}e^{-i2\pi\nu+i\overline{\delta\phi}} \\ r_{he}r_{hh}^{\star}e^{i2\pi\nu-i\overline{\delta\phi}} & r_{hh}r_{he}^{\star}e^{-i2\pi\nu+i\overline{\delta\phi}} \end{pmatrix}, \quad \text{(B5)}$$

$$V = \begin{pmatrix} r_{ee}r_{hh}^{\star}e^{i2\pi\nu-i\overline{\delta\phi}} & r_{eh}r_{he}^{\star}e^{-i2\pi\nu+i\overline{\delta\phi}} \\ r_{he}r_{eh}^{\star}e^{i2\pi\nu-i\overline{\delta\phi}} & r_{hh}r_{ee}^{\star}e^{-i2\pi\nu+i\overline{\delta\phi}} \end{pmatrix}. \quad \text{(B6)}$$

By using $\Psi_0=\mathbb{T}(0)\Psi_{-1}$ with $\Psi_{-1}=(1,-1,0,0)^T$, we find

$$\Psi_n = \mathbb{T}^n(x)\mathbb{T}(0)\Psi_{-1}. \quad \text{(B7)}$$

To the lowest order in $\exp(-\eta)$, we obtain from Eq. (B7) the following:

$$A_n = \lim_{x\to 0}\mathrm{Tr}\begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}\left[\widetilde{S}^{n+1} - 2e^{-\eta}\,\mathrm{Re}\, e^{i(2\pi\nu-\phi)}\frac{\partial}{\partial x}(\widetilde{S}+xZ)^n\right], \quad \text{(B8)}$$

where

$$Z = \begin{pmatrix} r_{ee}^2 r_{eh}^{\star}r_{he}^{\star} & |r_{eh}|^2 r_{ee}^{\star}r_{hh}^{\star} \\ |r_{he}|^2 r_{hh}^{\star}r_{ee} & r_{hh}^{\star 2}r_{eh}r_{he} \end{pmatrix}. \quad \text{(B9)}$$

1. Zero temperature (T=0)

At $T=0$, the matrices $\widetilde{S}$ and Z can be simplified drastically:

$$\widetilde{S} = \begin{pmatrix} |r_{ee}|^2 & |r_{eh}|^2 \\ |r_{eh}|^2 & |r_{ee}|^2 \end{pmatrix}, \quad Z = |r_{ee}|^2|r_{eh}|^2 e^{2i\theta}\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}. \quad \text{(B10)}$$

With the help of the following identity:

$$\begin{pmatrix} a & b \\ b & a \end{pmatrix} = \frac{\sigma_x-\sigma_z}{\sqrt{2}}[a-b\sigma_z]\frac{\sigma_x-\sigma_z}{\sqrt{2}}, \quad \text{(B11)}$$

we obtain Eq. (39) as

$$A_{n-1} = (|r_{ee}|^2-|r_{eh}|^2)^n\left[1+\frac{4(n-1)e^{-\eta}|r_{ee}|^2|r_{eh}|^2}{(|r_{ee}|^2-|r_{eh}|^2)^2}\cos 2\Omega\right]. \quad \text{(B12)}$$

2. Arbitrary temperature

At arbitrary temperature (energy) there is no special relation between r_{ab}. Then, with the help of Eq. (A1), we find

$$A_n = A_n^{(0)} - 2e^{-\eta}\,\mathrm{Re}\, e^{i(2\pi\nu-\overline{\delta\phi})}A_n^{(1)}, \quad \text{(B13)}$$

where

$$A_n^{(0)} = \det_0^{n/2}\left[\frac{|r_{ee}|^2-|r_{he}|^2}{\sqrt{\det_0}}U_{n-1}(a_0) - U_{n-2}(a_0)\right] \quad \text{(B14)}$$

and

$$\begin{aligned} A_n^{(1)} = {} & \det_0^{(n-1)/2}\,U_{n-1}(a_0)(r_{ee}^2 r_{he}^{\star}r_{eh}^{\star} - |r_{eh}|^2 r_{ee}r_{hh}^{\star}) \\ & - \det_0^{n/2}\left[\alpha a_0 U_{n-2}'(a_0) + \frac{n}{2}\beta U_{n-2}(a_0)\right] \\ & + \det_0^{(n-1)/2}\left[\frac{n-1}{2}U_{n-1}(a_0) - \alpha a_0 U_{n-2}'(a_0)\right](|r_{ee}|^2 \\ & - |r_{eh}|^2). \end{aligned} \quad \text{(B15)}$$

Here, we have introduced the following notations:

$$\det_0 = |r_{ee}|^2|r_{hh}|^2 - |r_{eh}|^2||r_{he}|^2, \quad \text{(B16)}$$

$$a_0 = \frac{|r_{ee}|^2+|r_{hh}|^2}{2\sqrt{\det_0}}, \quad \text{(B17)}$$

and

$$\beta = \det_0^{-1}(|r_{hh}|^2 r_{ee}^2 r_{eh}^{\star}r_{he}^{\star} + |r_{ee}|^2 r_{hh}^{\star 2}r_{eh}r_{he} - 2|r_{eh}|^2|r_{he}|^2 r_{ee}r_{hh}^{\star}), \quad \text{(B18)}$$

$$\alpha = \frac{r_{ee}^2 r_{eh}^{\star}r_{he}^{\star} + r_{hh}^{\star 2}r_{eh}r_{he}}{|r_{ee}|^2+|r_{hh}|^2} - \frac{\beta}{2}. \quad \text{(B19)}$$

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[27] This is the usual assumption that the reflected electron (hole) rays can be considered as incoherent with the incident ray on a (slightly) disordered interface. This assumption is widely used in nanophysics. Well-known Zaitsev boundary conditions for quasiclassical Green functions were derived with this assumption. With the help of this assumption, the so-called "nondiagonal," quickly oscillating components of the Green functions that carry information about the coherence were regarded as quickly fading (if we go off the surface) (Refs. 28 and 29).
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