Adiabatic domain wall motion and Landau-Lifshitz damping

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Recent theory and measurements of the velocity of current-driven domain walls in magnetic nanowires have reopened the unresolved question of whether Landau-Lifshitz damping or Gilbert damping provides the more natural description of dissipative magnetization dynamics. In this paper, we argue that (as in the past) experiment cannot distinguish the two, but that Landau-Lifshitz damping, nevertheless, provides the most physically sensible interpretation of the equation of motion. From this perspective, (i) adiabatic spin-transfer torque dominates the dynamics with small corrections from nonadiabatic effects, (ii) the damping always decreases the magnetic free energy, and (iii) microscopic calculations of damping become consistent with general statistical and thermodynamic considerations.

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I. BACKGROUND

Experiments designed to study the effect of electric current on domain wall motion in magnetic nanowires show that domain walls move over large distances with a velocity proportional to the applied current.[1–10] Most theories ascribe this behavior to the interplay between *spin-transfer* (the quantum mechanical transfer of spin angular momentum between conduction electrons and the sample magnetization) and magnetization damping of the Gilbert type.[11] Contrary to the second point, we argue in this paper that Landau-Lifshitz damping[12] provides the most natural description of the dynamics. This conclusion is based on the premises that damping should always reduce magnetic free energy and that microscopic calculations must be consistent with statistical and thermodynamic considerations.

Theoretical studies of current-induced domain wall motion typically focus on one-dimensional models, where current flows in the x direction through a magnetization $\mathbf{M}(x)=M\hat{\mathbf{M}}(x)$. When M is constant, the equation of motion is

$$\dot{\mathbf{M}}=-\gamma\mathbf{M}\times\mathbf{H}+\mathbf{N}_{\mathrm{ST}}+\mathbf{D}. \tag{1}$$

The precession torque $-\gamma\mathbf{M}\times\mathbf{H}$ depends on the gyromagnetic ratio γ and an effective field $\mu_0\mathbf{H}=-\delta F/\delta\mathbf{M}$, which accounts for external fields, anisotropies, and any other effects that can be modeled by a free energy $F[\mathbf{M}]$ (μ_0 is the magnetic constant). The spin-transfer torque $\mathbf{N}_{\mathrm{ST}}$ is not derivable from a potential, but its form is fixed by symmetry arguments and model calculations.[13–21] A local approximation[22] (for current in the x direction) is

$$\mathbf{N}_{\mathrm{ST}}=-\upsilon[\partial_x\mathbf{M}-\beta\hat{\mathbf{M}}\times\partial_x\mathbf{M}]. \tag{2}$$

The first term in Eq. (2) occurs when the spin current follows the domain wall magnetization adiabatically, i.e., when the electron spins remain largely aligned (or antialigned) with the magnetization as they propagate through the wall. The constant υ is a velocity. If P is the spin polarization of the current, j is the current density, and μ_B is the Bohr magneton,

$$\upsilon=\frac{-Pj\mu_B}{eM}. \tag{3}$$

The second term in Eq. (2) arises from nonadiabatic effects. The constant β is model dependent.

The damping torque $\mathbf{D}$ in Eq. (1) accounts for dissipative processes (see Ref. 23 for a review). Two phenomenological forms for $\mathbf{D}$ are employed commonly: the Landau-Lifshitz form[12] with damping constant λ,

$$\mathbf{D}_L=-\lambda\hat{\mathbf{M}}\times(\mathbf{M}\times\mathbf{H}), \tag{4}$$

and the Gilbert form[11] with damping constant α,

$$\mathbf{D}_G=\alpha\hat{\mathbf{M}}\times\dot{\mathbf{M}}. \tag{5}$$

The difference between the two is usually very small and almost all theoretical and simulation studies of current-induced domain wall motion solve Eq. (1) with the Gilbert form of damping.[18–20,24–28] This is significant because, as we now discuss, Gilbert damping and Landau-Lifshitz damping produce quite different results for this problem when the same spin-transfer torque is used.

Consider a Néel wall, where $\mathbf{M}$ lies entirely in the plane of a thin film when the current is zero. By definition, $\hat{\mathbf{M}}\times\mathbf{H}=0$ if we choose $\mathbf{M}(x)$ as the equilibrium structure, which minimizes the free energy $F[\mathbf{M}]$. The wall distorts if $\hat{\mathbf{M}}\times\mathbf{H}\neq 0$ for any reason. The theoretical literature cited above shows that, with damping omitted, the Néel wall moves undistorted at the speed υ [see Eq. (3)] when $\beta=0$ in Eq. (2). Gilbert damping brings this motion to a stop because $\mathbf{D}_G$ rotates $\mathbf{M}(x)$ out of plane until the torque from magnetostatic shape anisotropy cancels the spin-transfer torque. However, if the nonadiabatic term in Eq. (2) is nonzero, steady wall motion occurs at speed $\beta v/\alpha$.

Using this information, two recent experiments[9,10] used their observations of average domain wall velocities very near v to infer that $\beta\approx\alpha$ for Permalloy nanowires. This is consistent with microscopic calculations (which include disorder-induced spin-flip scattering) that report $\beta=\alpha$ (Ref. 29) or $\beta\approx\alpha$ (Ref. 30) for realistic band models of an itinerant ferromagnet. On the other hand, calculations for *s*-*d* models of ferromagnets with localized moments find little numerical relationship between β and α.[30,31]

A rather different interpretation of the data follows from a discussion of current-driven domain wall motion in the *s*-*d* model offered by Barnes and Maekawa.[32] These authors argue that there should be no damping of the magnetization when a wall which corresponds to a minimum of the free energy $F[\mathbf{M}]$ simply translates at constant speed. This is true of $\mathbf{D}_L$ in Eq. (4) because $\mathbf{M}\times\mathbf{H}=0$ but it is *not* true of $\mathbf{D}_G$ in Eq. (5) because $\dot{\mathbf{M}}\neq 0$ when $\mathbf{N}_{\mathrm{ST}}\neq 0$. From this point of view, the "correct" equation of motion is

$$\dot{\mathbf{M}} = -\gamma\mathbf{M}\times\mathbf{H} - v\partial_x\mathbf{M} - \lambda\hat{\mathbf{M}}\times(\mathbf{M}\times\mathbf{H}), \tag{6}$$

because it reduces (for energy-minimizing walls) to

$$\dot{\mathbf{M}} = -v\partial_x\mathbf{M}. \tag{7}$$

In the absence of extrinsic pinning, this argument identifies the experimental observation of long-distance wall motion with a uniformly translating solution $\mathbf{M}(x-vt)$ of Eq. (7) with minimum energy.

As we discuss below, it is possible to convert between descriptions with Landau-Lifshitz and Gilbert dampings by concurrently changing the value of the nonadiabatic spin-transfer torque. The Landau-Lifshitz description in Eq. (6) is equivalent to one with Gilbert damping with $\beta=\alpha$. The goal of this paper is to argue that there are conceptual reasons to prefer the description with Landau-Lifshitz damping even when $\beta\neq\alpha$.

Section II presents micromagnetic simulations that confirm the discussion above and describes further details. Then, the remainder of this paper provides three theoretical arguments which support the use of Landau-Lifshitz damping for current-driven domain wall motion (in particular) and for other magnetization dynamics problems (in general). First, we reconcile our preference for Landau-Lifshitz damping with the explicit microscopic calculations of Gilbert damping and nonadiabatic spin torque reported in Refs. 29–31. Second, we show that Gilbert damping can increase the magnetic free energy in the presence of spin-transfer torques. Finally, we show that Landau-Lifshitz damping is uniquely selected for magnetization dynamics when the assumptions of nonequilibrium thermodynamics are valid.

II. MICROMAGNETICS

Our analysis begins with a check on the robustness of the foregoing model predictions using full three-dimensional micromagnetic simulations of current-driven domain wall motion.[33] We studied nanowires 12 nm thick and 100 nm wide, with material parameters chosen to simulate $Ni_{80}Fe_{20}$. At zero current, this geometry and material system support in-plane magnetization with stable domain walls of transverse type.[34] Figure 1 shows the wall position as a function of time for a transverse domain wall for several values of applied current density j. The curves labeled Gilbert ($\alpha=0.02$) show that wall motion comes quickly to a halt. Examination of the magnetization patterns confirms the torque cancellation mechanism outlined above. The curves labeled Landau-Lifshitz show that the wall moves uniformly with the velocity given by Eq. (3), which is independent of the damping parameter λ.[35]

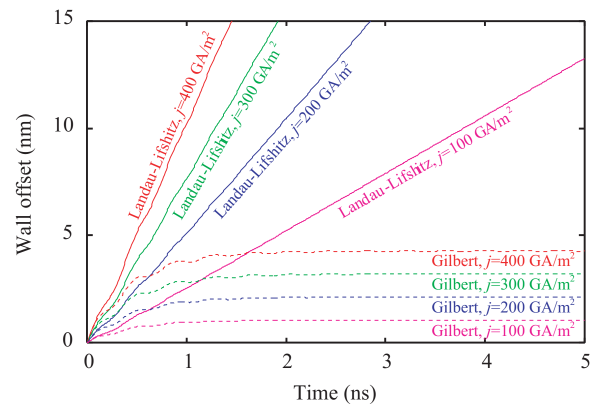

FIG. 1. (Color online) Position versus time for a transverse domain wall, and several values of the applied current density computed with adiabatic spin torques ($\beta=0$) and the two forms of damping in Eqs. (4) and (5).

The sudden turn-on of the current and hence Oersted magnetic field at $t=0$ generates the small amplitude undulations of the curves in Fig. 1 but, otherwise, has little effect on the dynamics. An initial state of a stable vortex wall in a 300 nm wide wire produces similar results, except that under the Gilbert formulation, the vortex wall moves about 20 times farther before stopping as compared to the transverse wall in the 100 nm wire. We conclude from these simulations that the basic picture of domain wall dynamics gleaned from one-dimensional models is correct.

The magnetic free energy behaves differently in simulations depending on whether Landau-Lifshitz or Gilbert damping is used. Before the current is turned on, the domain wall is in a configuration that is a local minimum in the energy. For Landau-Lifshitz damping, the energy remains largely constant near this minimum and is exactly constant if the Oersted fields are ignored. For Gilbert damping, the energy increases when the current is turned on and the walls distort. For a transverse wall, the distortion is largely an out of plane tilting. Initially, the energy increases at a rate proportional to the damping parameter (ignoring higher order corrections discussed in the next section). The details of this behavior are somewhat obscured by the oscillations due to the Oersted magnetic field, but are quite apparent in simulations in which this field is omitted. As the wall tilts out of plane, the torque due to the magnetostatic field opposes the wall motion and the wall slows down. Eventually, the torque balances the adiabatic spin-transfer torque and the wall stops.

In simulations using Gilbert damping, the change in magnetic free energy between the initial and final configurations is independent of the damping parameter, and is determined by the balance between the magnetostatic torque and the adiabatic spin-transfer torque. However, the amount of time before the wall stops and the distance the wall moves are inversely proportional to the damping parameter. The Gilbert damping torque is responsible for this increase in energy, as can be seen from analyzing the directions of the other torques. Precessional torques, like those due to the exchange and magnetostatic interactions that are important in these simulations, by their nature are directed in constant energy directions and do not change the magnetic free energy. The adiabatic spin-transfer torque is in a direction that translates the domain wall and does not change the energy in systems where the energy does not depend on the position of the wall. Thus, in simulations of ideal domain wall motion without Oersted fields, the Gilbert damping torque is the only torque that changes the energy. Throughout these simulations, the Gilbert damping torque is in a direction that increases rather than decreases the magnetic free energy.

III. MAGNETIC DAMPING WITH SPIN-TRANSFER TORQUE

When $\mathbf{N}_{\mathrm{ST}}=0$, it is well known that a few lines of algebra convert the equation of motion (1) with Gilbert damping into Eq. (1) with Landau-Lifshitz damping (and vice versa) with suitable redefinitions of the precession constant γ and the damping constants λ and α.[36] The same algebraic manipulations[29] show that Eq. (6) is mathematically equivalent to a Gilbert-type equation with $\alpha=\lambda/\gamma$:

$$\dot{\mathbf{M}} = -\gamma(1+\alpha^2)\mathbf{M}\times\mathbf{H} + \alpha\hat{\mathbf{M}}\times\dot{\mathbf{M}} - v[\partial_x\mathbf{M} - \alpha\hat{\mathbf{M}}\times\partial_x\mathbf{M}]. \tag{8}$$

To analyze Eq. (8), we first ignore spin transfer (put $v=0$) and note that this rewritten Landau-Lifshitz equation differs from the conventional Gilbert equation only by an $O(\alpha^2)$ renormalization of the gyromagnetic ratio. Consequently, first-principles derivations of any equation of motion for the magnetization must be carried out to second order in the putative damping parameter if one hopes to distinguish Landau-Lifshitz damping from Gilbert damping. This observation shows that papers that derive Gilbert damping[29–31,37–39] or Landau-Lifshitz damping[40,41] from microscopic calculations carried out only to first order in α cannot be used to justify one form of damping over the other.

Now restore the spin-transfer terms in Eq. (8) and note that the transformation to this equation from Eq. (6) automatically generates a nonadiabatic torque with $\beta=\alpha$. This transformation means that, to lowest order in α and β, an equation of motion with Gilbert damping and a nonadiabatic coefficient β_G is equivalent to an equation of motion with Landau-Lifshitz damping with nonadiabatic coefficient $\beta_L=\beta_G-\alpha$. This shows that equivalent equations of motion can be made using either form of damping, albeit with rather different descriptions of current-induced domain wall motion. Nevertheless, as we argue below, there are conceptual advantages to the Landau-Lifshitz form.

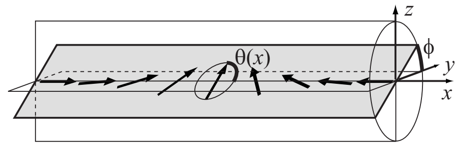

FIG. 2. A one-dimensional Néel domain wall with magnetization $\mathbf{M}(x)$.

IV. LANDAU-LIFSHITZ DAMPING UNIQUELY REDUCES MAGNETIC FREE ENERGY

Landau-Lifshitz damping irreversibly reduces magnetic free energy when spin-transfer torque is present. The same statement is not true for Gilbert damping. This can be seen from the situation described in Sec. II, where Gilbert damping causes a minimum-energy domain wall configuration to distort and tilt out of plane. Nothing prevents an increase in magnetic free energy for this open system, but it is clearly preferable if changes in magnetic configurations that increase $F[\mathbf{M}]$ come from the effects of spin-transfer torque rather than from the effects of a torque intended to model dissipative processes. This is an important reason to prefer $\mathbf{D}_L$ in Eq. (4) to $\mathbf{D}_G$ in Eq. (5). This argument depends crucially on the fact that the adiabatic spin-transfer torque is *not* derivable from a free energy. This we discuss next.

The field $\mathbf{H}$ in Eq. (1) is the (negative) gradient of the magnetic free energy. The component of this gradient in the direction that does not change the size of the magnetization is $-\hat{\mathbf{M}}\times[\mathbf{M}\times\mathbf{H}]$. Since this direction is exactly that of the Landau-Lifshitz form of the damping [Eq. (4)], it follows that this form of the damping always reduces this magnetic free energy. When the Gilbert form of the damping [Eq. (5)] is used in Eq. (1), it is possible to rewrite the damping term as $D_G=-\alpha\gamma\hat{\mathbf{M}}\times[\mathbf{M}\times\mathbf{H}-(1/\gamma)\mathbf{N}_{\mathrm{ST}}]+\mathcal{O}(\alpha^2)$. Further, one can always write $\mathbf{N}_{\mathrm{ST}}=-\gamma\mathbf{M}\times\mathbf{H}_{\mathrm{ST}}$, where $\mathbf{H}_{\mathrm{ST}}$ is an effective "spin-transfer magnetic field." However, unlike the field $\mu_0\mathbf{H}=-\delta\mathbf{F}/\delta\mathbf{M}$ in Eq. (1), there is no "spin-transfer free energy" F_{ST}, which gives $\mathbf{H}_{\mathrm{ST}}$ as its gradient:

$$\mu_0\mathbf{H}_{\mathrm{ST}} = -\frac{\delta F_{\mathrm{ST}}}{\delta\mathbf{M}} \quad \text{(not correct)}. \tag{9}$$

If Eq. (9) were true, the lowest order (in α) Gilbert damping term $-\alpha\gamma\hat{\mathbf{M}}\times[\mathbf{M}\times(\mathbf{H}+\mathbf{H}_{\mathrm{ST}})]$ would, indeed, always decrease the sum $F+F_{\mathrm{ST}}$. Unfortunately, a clear and convincing demonstration of the nonconservative nature of the spin-transfer torque is not easy to find. Therefore, in what follows, we focus on the adiabatic contribution to Eq. (2) and show that a contradiction arises if Eq. (9) and its equivalent,

$$dF_{\mathrm{ST}} = -\mu_0\mathbf{H}_{\mathrm{ST}}\cdot d\mathbf{M}, \tag{10}$$

are true.

For this argument, we consider a simpler model than that discussed in Sec. II. Figure 2 shows the magnetization $\mathbf{M}(x)$ for a one-dimensional Néel wall in a system with uniaxial anisotropy along the x direction. The domain wall of width w is centered at $x=0$ and the plane of the magnetization is tilted out of the x-y plane by an angle ϕ. A convenient parametrization of the in-plane rotation angle $\theta(x)$ is

$$\theta(x) = \pi/2 + \sin^{-1}[\tanh(x/w)]. \tag{11}$$

Therefore,

$$\mathbf{M} = M[\cos\theta(x), \sin\theta(x)\cos\phi, \sin\theta(x)\sin\phi], \tag{12}$$

where $\cos\theta(x) = -\tanh(x/w)$ and

$$\sin\theta(x) = \mathrm{sech}(x/w). \tag{13}$$

The magnetic free energy of this domain wall is independent of both its position and its orientation (angle ϕ).

For electron flow in the x direction, Eq. (2) shows that the adiabatic piece of the spin-transfer torque lies entirely in the plane of the magnetization:

$$\mathbf{N}_{\mathrm{ST}}^{\mathrm{ad}} \propto \theta'(x)(-\sin\theta, \cos\theta\cos\phi, \cos\theta\sin\phi). \tag{14}$$

This torque rotates the magnetization in a manner which produces uniform translation of the wall in the x direction with no change in ϕ. Since

$$\theta'(x) = (1/w)\mathrm{sech}(x/w), \tag{15}$$

comparison with Eq. (13) shows that $\mathbf{N}_{\mathrm{ST}}^{\mathrm{ad}}=0$ outside the wall as expected. The magnetic free energy of the domain wall does not change as the wall is translated.

Now, as indicated above Eq. (9), we are free to interpret the foregoing wall translation as resulting from local precession of $\mathbf{M}(x)$ around an effective field $\mathbf{H}_{\mathrm{ST}}(x)$ directed perpendicular to the plane of the domain wall. Specifically,

$$\mathbf{H}_{\mathrm{ST}}(x) \propto \theta'(x)(0, -\sin\phi, \cos\phi). \tag{16}$$

However, if Eq. (9) and thus Eq. (10) are assumed to be correct, the magnitude and direction of $\mathbf{H}_{\mathrm{ST}}$ imply that the putative free energy F_{ST} decreases when $\mathbf{M}(x)$ rotates rigidly around the x axis in the direction of increasing ϕ.[42] On the other hand, the free energy must return to its original value when ϕ rotates through 2π. Since the gradient (9) can never increase the free energy, we are forced to conclude that our assumption that F_{ST} exists is incorrect.

V. LANGEVIN EQUATION FOR THE MAGNETIZATION

Neglected work by Iwata[43] treats magnetization dynamics from the point of view of the thermodynamics of irreversible processes.[44] His nonperturbative calculations uniquely generates the Landau-Lifshitz form of damping. In this section, we make equivalent assumptions but go farther and derive an expression for the damping constant. Yamada *et al.* did this using a projection operator method.[45] Our more accessible discussion follows Reif's derivation of a Langevin equation for Brownian motion.[46]

We begin by taking the energy change in a unit volume

$$dE = -\mu_0 H_\alpha dM_\alpha, \tag{17}$$

where the repeated index α implies a sum over Cartesian coordinates. It is crucial to note that the magnitude $|\mathbf{M}|=M$ is fixed, so only rotations of $\mathbf{M}$ toward the effective field $\mathbf{H}$ change the energy of the system. The interaction with the environment enters the equation of motion for the magnetization through a fluctuating torque N'_α:

$$\frac{dM_\alpha}{dt} = -\gamma(\mathbf{M}\times\mathbf{H})_\alpha + N'_\alpha. \tag{18}$$

The torque $\mathbf{N}'$ is perpendicular to $\mathbf{M}$ since $|\mathbf{M}|=M$.

We consider the evolution of the magnetization over a time interval Δt, which is much less than the precession period, but much greater than the characteristic time scale for the fluctuations τ^*. After this time interval, the statistical average of the change in magnetization $\Delta M_\alpha = M_\alpha(t+\Delta t) - M_\alpha(t)$ is

$$\Delta M_\alpha = -\gamma(\mathbf{M}\times\mathbf{H})_\alpha(\Delta t) + \int_t^{t+\Delta t} dt' \langle N'_\alpha(t')\rangle. \tag{19}$$

The equilibrium Boltzmann weighting factor W_0 gives $\langle N'_\alpha(t')\rangle_0=0$. However, $\langle N'_\alpha(t')\rangle \neq 0$ when the magnetization is out of equilibrium. Indeed, this method derives the damping term precisely from the bias built into the fluctuations due to the changes $\Delta E=-\mu_0 H_\nu \Delta M_\nu$ in the energy of the magnetic system.

The Boltzmann weight used to calculate $\langle N'_\alpha(t')\rangle$ is $W = W_0 \exp[-\Delta E/(k_B T)]$, where (assuming that $\mathbf{H}$ does not change much over the integration interval)

$$\Delta E(t') = -\mu_0 H_\nu(t') \int_t^{t'} \frac{dM_\nu(t'')}{dt''} dt'' \approx -\mu_0 H_\nu(t) \int_t^{t'} N'_\nu(t'') dt''. \tag{20}$$

Note that precession does not contribute to $\Delta E(t')$. Only motions of the magnetization that change the energy of the magnetic subsystem produce bias in the torque fluctuations. Therefore, since $W = W_0[1-\Delta E/(k_B T)]$ for small $\Delta E/(k_B T)$, the last term in Eq. (19) now involves only an average over the equilibrium ensemble:

$$\Delta M_\alpha \approx -\gamma(\mathbf{M}\times\mathbf{H})_\alpha(\Delta t) + \frac{\mu_0 H_\nu(t)}{k_B T} \int_t^{t+\Delta t} dt' \int_t^{t'} dt'' \langle N'_\alpha(t') N'_\nu(t'')\rangle_0. \tag{21}$$

We recall now that the torque fluctuations are correlated over a microscopic time τ^* that is much shorter than the small but macroscopic time interval over which we integrate. Therefore, to the extent that memory effects are negligible, we define the damping constant λ (a type of fluctuation-dissipation result) from

$$\int_t^{t'} dt'' \langle N'_\alpha(t') N'_\nu(t'')\rangle \approx \lambda(k_B T M/\mu_0)\delta^\perp_{\alpha\nu}, \tag{22}$$

for $|t'-t| \gg \tau^*$ and with $\delta^\perp_{\alpha\nu} = \delta_{\alpha\nu} - \hat{M}_\alpha \hat{M}_\nu$, which restricts the fluctuations to be transverse to the magnetization, but otherwise uncorrelated. This approximation reduces the last term in Eq. (21) to $\lambda M H_{\perp\alpha}\Delta t$, where $\mathbf{H}_\perp = -\hat{\mathbf{M}}\times(\hat{\mathbf{M}}\times\mathbf{H})$ is the piece of $\mathbf{H}$ which is perpendicular to $\mathbf{M}$. Substituting Eq. (22) into Eq. (21) gives the final result in the form

$$\frac{d\mathbf{M}}{dt} \approx -\gamma(\mathbf{M}\times\mathbf{H}) - \lambda\hat{\mathbf{M}}\times(\mathbf{M}\times\mathbf{H}). \qquad (23)$$

Equation (23) is the Landau-Lifshitz equation for the statistically averaged magnetization. It becomes a Langevin equation when we add an (now) unbiased random torque to the right hand side.

The procedure outlined above generates higher order terms in λ from the expansion of the thermal weighting to higher order in ΔE. The second order terms involve an equilibrium average of three powers of N'. These are zero for Gaussian fluctuations. The third order terms involve an average of four powers of N', and are nonzero. They lead to a term proportional to $\lambda^2 H_\perp^2 \mathbf{H}_\perp$, which we expect to be small and to modify only large-angle motions of the magnetization.

VI. SUMMARY

In this paper, we analyzed current-driven domain wall motion using both Gilbert-type and Landau-Lifshitz-type damping of the magnetization motion. Equivalent equations of motion can be written with either type of damping, but the implied description of the dynamics (and the relative importance of adiabatic and nonadiabatic effects) is very different in the two cases.

With Landau-Lifshitz damping assumed, adiabatic spin-transfer torque dominates and produces uniform translation of the wall. Nonadiabatic contributions to the spin-transfer torque distort the wall, raise its magnetic energy, and thus produce a magnetostatic torque which perturbs the wall velocity. Damping always acts to reduce the distortion back toward the original minimum-energy wall configuration. With Gilbert damping assumed, the damping torque itself distorts and thereby raises the magnetic energy of the moving wall. The distortion-induced magnetostatic torque stops domain wall motion altogether. Additional wall distortions produced by nonadiabatic spin-transfer torque are needed to produce wall motion.

In our view, Landau-Lifshitz damping is always preferable to Gilbert damping. When spin-transfer torque is present, this form of damping inexorably moves the magnetic free energy toward a local minimum; Gilbert damping does not. Even in the absence of spin-transfer torque, arguments based on irreversible thermodynamics show that the Landau-Lifshitz form of damping is uniquely selected for a macroscopic description.[43] Here, we proceeded equivalently and derived the Landau-Lifshitz equation of motion as the unique Langevin equation for the statistical average of a fluctuating magnetization with fixed spin length.

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