Nonlinear spin waves in cylindrical ferromagnetic nanowires

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We study the nonlinear evolution of bulk spin waves in a charge-free, isotropic ferromagnetic nanowire with negligible surface anisotropy, restricted to the modes with no azimuthal dependence. Using a multiple scale analysis, we find that the magnetization oscillations are always restricted to one particular plane for the Fourier component $p=1$, while the Fourier component $p=2$ comes out of the plane. Moreover, the magnetization excitations are governed by the cubic nonlinear Schrödinger equation. We also find that the ferromagnetic nanowire facilitates the propagation of dark solitons with the stable continuous wave.

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I. INTRODUCTION

In recent years, magnetic nanostructures have attracted much attention as they have potential applications in ultrahigh-density memory devices and sensors.[1,2] The magnetic excitations or spin waves of these nanostructures are best investigated using Brillouin light scattering (BLS) techniques.[3] In particular, the linear and nonlinear evolutions of magnetostatic spin waves (SWs) in charge-free and isotropic ferromagnetic nanotubes[4] have been recently studied. As a result, the ferromagnetic nanotubes facilitate the propagation of elliptically polarized waves, which induces soliton excitations in the medium governed by the cubic nonlinear Schrödinger (NLS) equation. The stability of soliton excitations is determined by the external field related to the magnetized nanotube.

Apart from nanotubes, other forms of nanostructures include nanowires. Two-dimensional arrays of nickel nanowires were fabricated using a two-step electrochemical anodization process and pulsed electrodeposition.[5,6] These nanowires are uniform in cross section with lengths of about 1 μm and diameters in the range of 30–55 nm and, hence, can be realized as infinite in length to excellent approximation. The diameter range considered for the isolated nanowire prompts one to include exchange and dipolar contributions to the excitation energy.

Recently, the linear theory of spin excitations in ferromagnetic nanowires with exchange and dipolar contributions has been studied.[7] The bulk standing modes of the nanowires were obtained by using the general form of magnetic scalar potential calculated from the Bloch equation of motion with dipole and exchange fields within the nanowire. It showed the existence of discrete spin modes due to the quantization of bulk spin waves accordingly with the usual dipole-exchange theory, which has been successfully validated experimentally.[8] The boundary conditions generate the mixing of bulk SW with the surface SW. In Ref. 7, this complex mechanism was taken into account numerically, while Ref. 8 gives approximate analytic expressions. The theory considered a cylindrical cross section with the magnetization parallel to the axis of the wire. A generalization to a magnetization not parallel to the axis has been proposed.[9] The continuous description allows one to determine the number of guided modes.[10] Other theoretical approaches have also been considered; using discrete spin lattices,[10–12] the validity of which has been confirmed experimentally.[13] From the nonlinear viewpoint, the existence of solitons in thin magnetic films has been demonstrated long time ago,[14] and spin-wave bullets have been recently observed.[15]

In the present paper, we study the nonlinear theory of spin excitations in a long charge-free isotropic cylindrical ferromagnetic nanowires by including contributions due to the dipolar and exchange fields. In Sec. II, we formulate the model, discuss the various length scales involved by the problem, and derive the linear propagation modes. Section III is devoted to the derivation of a nonlinear Schrödinger equation by means of a multiple scale expansion method, to the discussion of the existence of solitons, and to the description of the corresponding wave profiles. The results are commented in Sec. IV, while the Appendix contains the details of the derivation.

II. SETTING THE PROBLEM

A. Model and dynamical equations

We consider long charge-free isotropic ferromagnetic nanowire with circular cross section and the magnetization parallel to the symmetry axis of the nanowire, the z axis. We consider that both the exchange and dipolar couplings between the spins are comparable in magnitude, and we use the same macroscopic approach as in Ref. 7. Although a quantum theory approach could be thought to be more relevant at the nano scale, the linear theory of spin waves in nanowires of Ref. 7 has been found to be in good agreement with experiment.[8] Consequently, we consider that, in the nonlinear regime, the spin excitations in the nanowire are governed by the Landau-Lifshitz equation,[16]

$$\partial_t \mathbf{M} = -\delta\mu_0 \mathbf{M} \wedge (\mathbf{H} + \beta\Delta\mathbf{M}), \quad (1)$$

where $\mathbf{M}$ is the magnetization of the medium, $\mathbf{H}$ the dipolar field, Δ the Laplacian operator, δ the gyromagnetic ratio, μ_0 the magnetic permeability in vacuum, $\beta = D/M_s$ with D being the exchange stiffness, and M_s the saturation magnetization. The quantities $\mathbf{M}$ and $\mathbf{H}$ are rescaled to $\mathbf{M}/(\delta\mu_0)$, $\mathbf{H}/(\delta\mu_0)$, so that the coefficient $\delta\mu_0$ in Eq. (1) is replaced by 1.

The components of the dipolar field satisfy the magnetostatic Maxwell equations,

$$\nabla \cdot (\mathbf{H} + \mathbf{M}) = 0, \quad (2)$$

$$\nabla \wedge \mathbf{H} = 0. \quad (3)$$

Within the magnetostatic approximation, the dipolar field can be expressed as the gradient of the magnetic potential Φ according to

$$\mathbf{H} = \mathbf{H}_0 - \nabla\Phi. \quad (4)$$

Here, $\mathbf{H}_0$ is a uniform applied magnetic field, directed along the z axis. The magnetization of the nanowire in the absence of wave is uniform and directed along the symmetry axis, it is, thus, $\mathbf{M}_0 \equiv \mathbf{m} = (0, 0, M_s)$. Hence, $\mathbf{H}_0 = \alpha\mathbf{m}$, and α measures the strength of the applied field in units of M_s.

The boundary conditions are the magnetostatic ones (see detail in Appendix). However, since the exchange interaction is taken into account, we have to consider the pinning conditions describing the behavior of the magnetization at the surfaces. Using cylindrical coordinates (r, θ, z), they read as[7,17]

$$(\partial_r M^\theta)_{(r=R)} = 0, \quad (5)$$

$$(\partial_r M^r)_{(r=R)} = \gamma (M^r)_{(r=R)}, \quad (6)$$

where R is the radius of the nanowire, and γ is the surface anisotropy parameter.

B. Length scales

The characteristic lengths of the nanowire are its length L, its radius R, and the exchange length $l = \sqrt{\beta}$ of the medium. Typical values are $L = 10-20$ μm, $R = 15-250$ nm,[18] and $l = 5.8$ nm for nickel. Hence $l \lesssim R$ and $R \ll L$. For spin waves, typical wavelengths λ can range from about 1 nm to 100 μm. Let us consider a typical value about $\lambda = 0.5$ μm; we have clearly $l \ll \lambda \ll L$. This is typical for a long-wave approximation, according to the reductive perturbation method.[19] We introduce a perturbation parameter ε such that

$$\varepsilon \sim \frac{l}{\lambda} \sim \frac{\lambda}{L} \ll 1, \quad (7)$$

and take the exchange length l as a reference length. Hence, l is of order ε^0, while the wavelength $\lambda \sim l/\varepsilon$ is very large, corresponding to a long wave. The wave profile is accounted for by introducing the slow space variable,

$$\zeta = \varepsilon(z - Vt), \quad (8)$$

which involves a characteristic length l/ε, and a velocity V to be determined, the velocity of the wave pulse.

Then we consider nonlinear and dispersive propagation of the wave profile on long distances $L \sim \lambda/\varepsilon \sim l/\varepsilon^2$, or equivalently its evolution on large times $t \sim t_0/\varepsilon^2$, where t_0 is some zero-order reference time (typically, $1/t_0$ can be the ferromagnetic resonance frequency). The evolution at such times is accounted for using a slow time variable,

$$\tau = \varepsilon^2 t. \quad (9)$$

Then the derivation operators with respect to x and y remain unchanged, while the derivation operators with respect to z and t become

$$\frac{\partial}{\partial z} = \varepsilon \frac{\partial}{\partial \zeta}, \quad \frac{\partial}{\partial t} = -V\varepsilon \frac{\partial}{\partial \zeta} + \varepsilon^2 \frac{\partial}{\partial \tau}. \quad (10)$$

With the above numerical values, we get $\lambda/l \simeq 86$ and $L/\lambda \simeq 40$. According to relation (7), it allows us to take the value $\varepsilon = 1/100$ for the perturbation parameter. In many situations, comparison between asymptotic models and numerical resolution of the full initial set of equations have shown such a value of the perturbation parameter to be small enough to ensure the validity of the long-wave approximation. The latter will obviously be improved if longer nanowires are considered.

However, there is some important discrepancy between the present situation and the usual long-wave approximation. Consider, indeed, the dispersion relation for the spin waves in nanowires, as can be found in Ref. 7. As the wave number k tends to zero, the frequency ω tends to a finite limit Ω, with Ω/M_s a few less than 1. For spin waves in nanowires, long waves does not imply slow oscillations in time. Hence, a standing carrier $\exp(i\Omega t)$ must be introduced, and the long-wave approximation concerns the slow space-time evolution of the envelope of the standing carrier.

We expand, thus, the magnetization about the uniform magnetization $\mathbf{M}_0$ and the scalar potential representing the dipolar field in a series of harmonics given by

$$\mathbf{M} = \mathbf{M}_0 + \varepsilon\mathbf{M}_1(x, y, \zeta, \tau)e^{i\Omega t} + \text{c.c.} + \sum_{j \geq 2, p} \varepsilon^j \mathbf{M}_j^p(x, y, \zeta, \tau)e^{ip\Omega t}, \quad (11)$$

$$\Phi = \varepsilon\Phi_1(x, y, \zeta, \tau)e^{i\Omega t} + \text{c.c.} + \sum_{j \geq 2, p} \varepsilon^j \Phi_j^p(x, y, \zeta, \tau)e^{ip\Omega t}. \quad (12)$$

Here, c.c. stands for complex conjugate, ε is the small parameter, and Ω is the frequency of the discrete standing modes which determine the transverse profile of the the guided spin waves.

C. Linear modes

The Landau equation (1) at order ε yields the following equations.

$$i\Omega M_1^r = -m\left(\alpha - \beta\left(\Delta_\perp - \frac{1}{r^2}\right)\right)M_1^\theta, \tag{13}$$

$$i\Omega M_1^\theta = m\left(1+\alpha - \beta\left(\Delta_\perp - \frac{1}{r^2}\right)\right)M_1^r. \tag{14}$$

Hence, M_1^θ and M_1^r are eigenfunctions of the operator $(\Delta_\perp - \frac{1}{r^2})$. Contrarily to Ref. 7 which considered the modes with nonzero azimuthal dependence only, we restrict the study to the purely radial modes, i.e., we assume that M_1^θ and M_1^r do not depend on θ. With this condition, the eigenfunctions of $(\Delta_\perp - \frac{1}{r^2})$ are well known, precisely

$$\left(\Delta_\perp - \frac{1}{r^2}\right)J_1(\kappa r) = -\kappa^2 J_1(\kappa r), \tag{15}$$

for any κ, J_1 being the Bessel function. We assume κ real, so that $J_1(\kappa r)$ is regular at $r=0$ [i.e., $J_1(0)=0$]. The transverse modes are, thus, given by

$$M_1^r = -m(\alpha+\beta\kappa^2)f(\zeta,\tau)J_1(\kappa r), \tag{16}$$

$$M_1^\theta = i\Omega f(\zeta,\tau)J_1(\kappa r). \tag{17}$$

κ can be interpreted as a transverse wave vector. Together with the frequency Ω, it satisfies the modal dispersion relation,

$$\Omega^2 = m^2(\alpha+\beta\kappa^2)(1+\alpha+\beta\kappa^2). \tag{18}$$

It is worth comparing the dispersion relation (18) to the equivalent relation obtained in Ref. 7 [Eq. (16) in the cited reference], which reads, with the notations of the present paper, as

$$\beta^2 m^2(\kappa^2+k^2)^3 + \beta m^2(2+\alpha)(\kappa^2+k^2)^2 + (\alpha(1+\alpha)m^2 - \Omega^2 - \beta m^2 k^2)(\kappa^2+k^2) = \alpha m^2 k^2. \tag{19}$$

It is straightforwardly seen that relation (19) reduces to Eq. (18) as $k=0$, which corresponds to the long-wave approximation we use here. In the case of zero applied magnetic field, Eq. (17) is similar to Eq. (4) in Ref. 8. It coincides with the dispersion relation of spin waves in a bulk medium (p. 19 of Ref. 20), which confirms the interpretation of κ as the transverse wave vector. Assuming azimuthal invariance, we show from Eq. (A4) that $M_1^z=0$. At this point, the first order ε^1 of the perturbative scheme, involving the harmonics $p=\pm 1$, has been completed. The evolution law of the function $f(\zeta,\tau)$ will be determined at next order.

Using expressions (16) and (17) of M_1^r and M_1^θ in the pinning conditions (5) and (6), we obtain the two conditions

$$J_1'(\kappa R) = 0, \tag{20}$$

where the prime denotes the derivative, and

$$\gamma J_1(\kappa R) = 0. \tag{21}$$

Since a Bessel function J_1 and its derivative J_1' have no zero in common, we get the condition $\gamma=0$ as a solvability condition. In other words, the above ansatz cannot account for nonlinear spin-wave propagation if surface anisotropy is not neglected. In real materials, surface anisotropy is not always negligible. Further, it has been shown that an effective nonzero pinning parameter may arise in thin stripes, even if the material itself does not present any exchange surface anisotropy.[21] Moreover, if surface anisotropy is neglected, only the mode with the lowest azimuthal dependence can be excited by a uniform microwave field, as it appears from the linear theory.[7] Nevertheless, we assume, thus, a completely unpinned surface in what follows. The inclusion of surface anisotropy requires a modification of the ansatz and is left for further investigation.

III. NONLINEAR ANALYSIS

A. Nonlinear Schrödinger equation

At order ε^2, the equations for the fundamental Fourier component $p=1$ give the solvability condition necessary to the determination of the velocity V. The justification is detailed in the AppendixA, together with some further relations of technical importance. In the frame of the reductive perturbation method for envelope evolution equations, V is the group velocity of the wave packet. The group velocity $v_g = d\Omega/dk$ is easily computed from Eq. (19). It is seen that $v_g=0$ at the limit $k=0$, this is due to the fact that Ω has a finite limit. Hence, $v=0$ coincides with the value of the group velocity in the long-wave limit.

Nonlinear terms are expected to appear at order ε^2, for the Fourier components $p=0$ and $p=2$. In fact, the only nonzero component is here

$$M_2^{2,z} = \frac{1}{2}m(\alpha+\beta\kappa^2)f^2(J_1(\kappa r))^2. \tag{22}$$

At order ε^3, for the fundamental Fourier component $p=1$, we get after some computation the following equation:

$$\left[-\Omega^2 + m^2\left(1+\alpha-\beta\left(\Delta_\perp - \frac{1}{r^2}\right)\right)\left(\alpha - \beta\left(\Delta_\perp - \frac{1}{r^2}\right)\right)\right]M_3^{1,\theta} = \mathcal{F}, \tag{23}$$

in which we have set

$$\mathcal{F} = 2\Omega^2\partial_\tau f J_1(\kappa r) - iC\partial_\zeta^2 f J_1(\kappa r) + \mathcal{G}, \tag{24}$$

$$C = -\Omega m^2\left(2\beta(\alpha+\beta\kappa^2) - \frac{\alpha}{\kappa^2}\right), \tag{25}$$

$$\mathcal{G} = \frac{-i}{2}\Omega m^2(\alpha+\beta\kappa^2)f|f|^2\mathcal{Q}, \tag{26}$$

and

$$\mathcal{Q} = -\left(1+\alpha-\beta\left(\Delta_\perp - \frac{1}{r^2}\right)\right)[\beta J_1^*\Delta_\perp J_1^2 + \beta\kappa^2 J_1|J_1|^2] + \beta(\alpha+\beta\kappa^2)J_1^*\Delta_\perp J_1^2 + (1+\beta\kappa^2)(\alpha+\beta\kappa^2)J_1|J_1|^2. \tag{27}$$

The key problem of the derivation is the reduction of Eq. (23). We consider the scalar product defined by

$$(\varphi|\psi) = \int_0^R \varphi^*(r)\psi(r) r dr \quad (28)$$

[κ and thus $J_1(\kappa r)$ are real; hence, the complex conjugate in (28) has no influence below]. Using the properties of the Bessel functions, we show that

$$(J_1(\kappa r)|J_1(qr)) = 0, \quad (29)$$

if $q \neq \kappa$, and, hence, the eigenmodes $J_1(qr)$, when the qR are the zeros of J_1', yield an orthogonal family. It is, thus, reasonable to assume that both $M_3^{1,\theta}$ and the right-hand side member $\mathcal{F}$ of (23) can be expanded on the $J_1(qr)$ as

$$M_3^{1,\theta} = \sum_q X_q J_1(qr), \quad \mathcal{F} = \sum_q \mathcal{F}_q J_1(qr). \quad (30)$$

Using this expansion, we can show that the solvability condition of Eq. (23) is

$$(J_1(\kappa r)|\mathcal{F}) = 0, \quad (31)$$

which yields

$$2i\Omega^2 \partial_\tau f + C\partial_\zeta^2 f + Df|f|^2 = 0. \quad (32)$$

The dispersion coefficient C is given by (25) and

$$D = \frac{1}{2}\Omega m^2(\alpha + \beta\kappa^2)\frac{(J_1(\kappa r)|\mathcal{Q})}{(J_1(\kappa r)|J_1(\kappa r))}. \quad (33)$$

Equation (32) is the NLS, which is well known to be completely integrable by means of the inverse scattering transform method.[22] Notice the unusual fact that the NLS equation is obtained here not as describing envelope solitons but within a long-wave approximation.

B. Lighthill criterion

Recall that the existence of soliton solutions of the NLS equation (32) is related to sign of the product CD of the coefficients, according to the so-called Lighthill criterion: the (bright) solitons exist if $CD > 0$. In this case, an input continuous wave suffers the modulational instability of Benjamin-Feir[23] type and breaks down into a train of solitons. On the other hand, if $CD < 0$, an input continuous wave is stable, while dark solitons can be formed.

It is straightforward that the dispersion parameter C vanishes for

$$\alpha = \frac{2\beta^2\kappa^2}{\kappa^{-2} - 2\beta}, \quad (34)$$

and is negative for $\alpha = 0$. The sign of the nonlinear coefficient D is the same as the one of the scalar product $(J_1(\kappa r)|\mathcal{Q})$. According to (27), the product $J_1(\kappa r)\mathcal{Q}$ is an algebraic expression involving the Bessel function $J_1(x)$ and its derivatives, to be integrated on the interval $[0, \kappa R]$, where κR is some zero of J_1'. The integrals can be computed numerically. For the first zero of J_1', we get

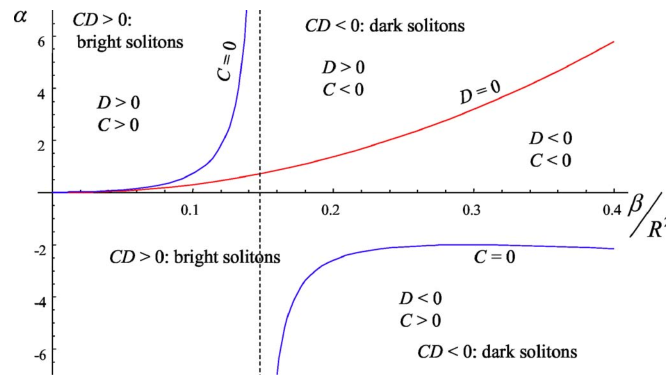

FIG. 1. (Color online) The sign of the coefficients of the NLS equations and the domains were either bright or dark solitons exist in the α vs β/R^2 plane. We consider here the fundamental mode only (hence, κR is the first zero of J_1').

$$(J_1(\kappa r)|\mathcal{Q}) = \frac{1}{\kappa}(0.079\,65\alpha + 0.020\,76\beta\kappa^2 - 0.2662\beta^2\kappa^4). \quad (35)$$

Hence, D vanishes for a strength of the external field characterized by

$$\alpha = 38.41\frac{\beta}{R^2}\left(\frac{\beta}{R^2} - 0.023\right). \quad (36)$$

The sign of the coefficients is shown on Fig. 1, which summarizes the results: it is seen that bright solitons exist for both the small and the large values of the ratio β/R^2. A domain of existence of dark solitons, and nonexistence of bright ones, appears for wires of small enough diameter, and a high enough applied field, with both signs of α. Recall that a negative α corresponds to a magnetization and field with opposite directions, which is an unstable configuration in the bulk but can be eventually stable in nanowires of diameter small enough with respect to the exchange length, i.e., β/R^2 large enough.

The same discussion can be done for all linear propagation modes. Recall that we only consider the modes with zero azimuthal dependence. Among them, each mode is characterized by a zero of J_1', and we will refer to the mode corresponding to the first of these zeros as the fundamental one. Figures 2 and 3 present the results of the discussion for modes 1–4. It is seen that, varying the diameter of the wire and the applied field strength, it is possible to select one or several propagation modes for the propagation of bright solitons. There are domains in which only the fundamental mode can support solitons. For thick nanowires (Fig. 2), it occurs at small applied field, α close to 0, and for $R/l = 15-20$, that is, e.g., in a nickel wire with diameter about 180–240 nm. It can also occur for thinner nanowires, with $R/l = 3-7$, i.e., a diameter about 15–40 nm (or $R/l = 5-7$ if negative α are not allowed), in a strong enough applied field (Fig. 3). Assume that only one mode, say, mode 1, can form bright solitons. The most general input wave packet will expand on several modes and then propagate. If we neglect the nonlinear interaction between modes, all modes except mode 1 suffer a

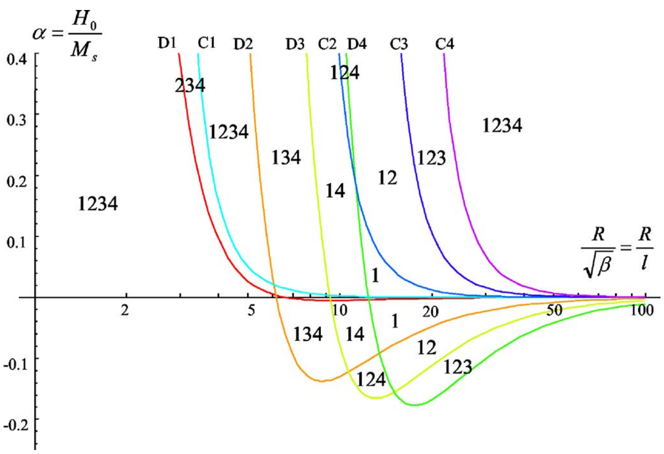

FIG. 2. (Color online) The domains where bright solitons exist in the α vs R/l_{exch} plane, for small applied fields. In each domain, the numbers indicate the modes which can support bright solitons. The curve $C=0$ and $D=0$ for the jth modes are labeled Cj and Dj, respectively.

nonlinear dispersive effect, and are spread out, while mode 1 forms a bright soliton. Then the final wave packet entirely belongs to mode 1: the nonlinear effects transforms the multimode waveguide into an effectively monomode one.

In contrast to this situation, the nanowires of large diameter allow, in general, propagation of solitons for any modes. They are multimode even from the nonlinear point of view.

C. Wave profiles

The expression of the NLS soliton is

$$f(\zeta,\tau) = p\sqrt{\frac{2C}{D}}\,\text{sech}\,p\left(\zeta - k\frac{C}{\Omega^2}\tau\right)e^{i(k\zeta+(p^2-k^2)C/\Omega^2\tau+\varphi)}, \tag{37}$$

p, k and φ are arbitrary real parameters, which represent the inverse of the wave duration, proportional to its amplitude, a wave number, and a phase. For $k=0$, the modulus $|f|$ accounts for a sech-shaped deformation of the magnetization and magnetic field which does not propagate, but oscillates uniformly in time. The exponential factor in (37) corresponds to a frequency shift, such that $\omega=\Omega+C/(d^2\Omega^2)$, where $d=1/(\varepsilon p)$ is the pulse duration in the unit of z.

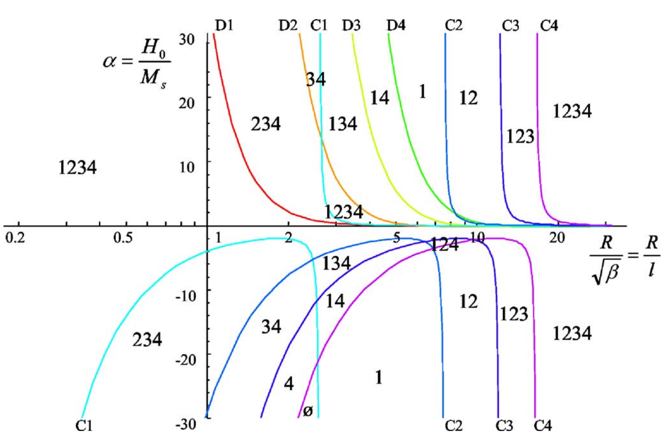

FIG. 3. (Color online) The same as Fig. 2, but for larger field strength α.

For nonzero λ, slow spatial oscillation adds to the sech profile, and the pulse propagates with velocity

$$\mathcal{V} = \frac{kC}{\Omega^2}. \tag{38}$$

However, the pulse is no merely a sech-shaped envelope modulating a fast carrier, since the carrier does not evolve spatially. The wave magnetization can be defined as $\mathbf{M}_w = \varepsilon\mathbf{M}_1 e^{i\Omega t} + \text{c.c.}$. Its r component, e.g., is

$$M_w^r = -\varepsilon m(\alpha+\beta\kappa^2)J_1(\kappa r)f(\zeta,\tau)e^{i\Omega t} + \text{c.c.}, \tag{39}$$

according to (37), it can be written as

$$M_w^r = AJ_1(\kappa r)\text{sech}\,p\zeta'\cos(k\zeta' + \psi(t,\tau)) \tag{40}$$

where A is the amplitude, ζ' the slow spatial coordinate in a frame at the wave velocity, and $\psi(t,\tau)$ a fast varying phase. Figure 4 shows the ζ dependency of the wave profile during one period of $\psi(t,\tau)$, for a few values of k.

IV. CONCLUSIONS

Propagation of spin waves in ferromagnetic nanowires has been studied, neglecting damping, conductivity, and surface anisotropy, by means of a long-wave approximation. For the purely radial modes, an asymptotic model of NLS type has been derived by means of the reductive perturbation method. From the point of view of asymptotic methods, this result is unusual since the NLS equation is typically related to the slowly varying envelope approximation, while the asymptotic models relevant for long waves are typically of the same type as the Korteweg–de Vries equation. The existence of soliton solutions to the NLS is determined by means of the Lighthill criterion. It depends on two parameters, which are the radius of the nanowire and the strength of the external field. We specify the values of these quantities with respect to the exchange length of the ferromagnetic material and its saturation magnetization, respectively. Domains in the space of these parameters where solitons can be formed have been characterized, for the few first linear guided modes. Therefore, the mode(s) for which solitons can be formed can be controlled by means of the two parameters. Some domains exist where only the fundamental mode allows soliton propagation. In this case, the nanowire would act nonlinearly as a monomode waveguide, although it is strongly multimode from the linear point of view. The wave profile corresponding to the solitons has also been described; it can be a single hump, or a few-cycle pulse relatively to the space variable, which oscillates quickly in time.

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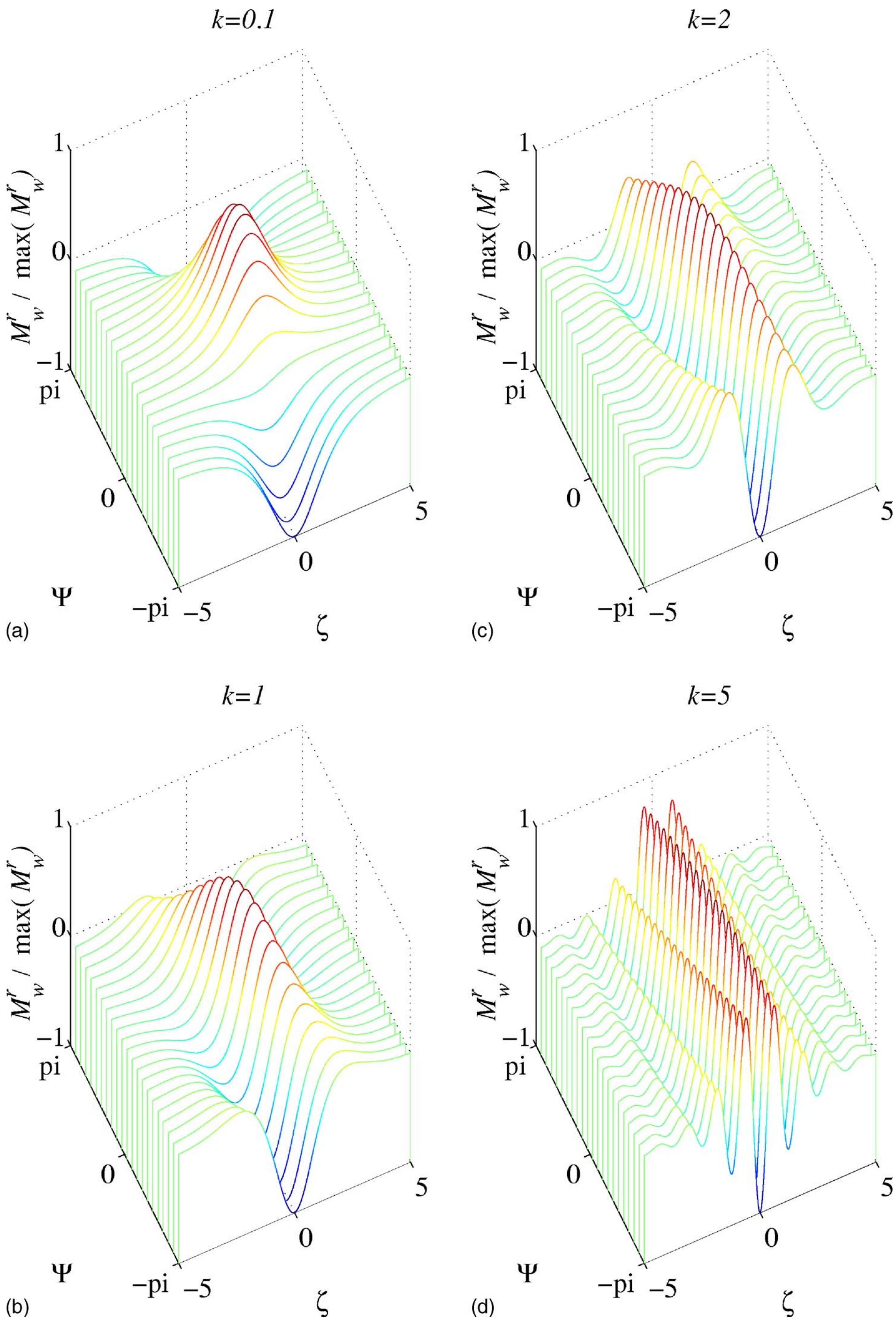

FIG. 4. (Color online) The wave profile corresponding to the fundamental soliton, for fixed r, as a function of ζ, for one period of the fast oscillating phase ψ, and for some value of the wave number k.

APPENDIX: MULTIPLE SCALE ANALYSIS

1. Coordinates and boundary conditions

The cylindrical coordinates (r,θ,z) are defined by $x=r\cos\theta$, $y=r\sin\theta$, and $z=z$. We denote by u^r, u^θ the radial and orthogonal components of a vector $\mathbf{u}$. Then the magnetostatic boundary conditions in at $r=R$, where R is the radius of the nanowire, can be given as

$$(H^r+M^r)_{(r=R-0)}=(H^r)_{(r=R+0)}, \tag{A1}$$

$$(H^\theta)_{(r=R-0)}=(H^\theta)_{(r=R+0)}, \tag{A2}$$

$$(H^z)_{(r=R-0)}=(H^z)_{(r=R+0)}. \tag{A3}$$

We now substitute the expansions (11) and (12) in the basic equations (1), (2), and (4), and collect the terms proportional to different powers of ε and solve the resultant equations. At order ε^0, we find that both the uniform fields are collinear to each other.

2. Solutions at order ε^1

At the next order ε^1, we obtain the following equations from Eqs. (1) and (2), respectively:

$$i\Omega \mathbf{M}_1^1 = -\mathbf{m}\wedge(-\nabla_\perp \Phi_1 + \beta\Delta_\perp \mathbf{M}_1) - \mathbf{M}_1 \wedge (\alpha \mathbf{m}), \quad (A4)$$

and

$$-\Delta_\perp \Phi_1 + \nabla_\perp \cdot \mathbf{M}_1 = 0. \quad (A5)$$

In order to solve the equations, we assume that the fields are uniform about z axis and so the operators in Eqs. (A4) and (A5) take the following form for a given scalar f and a vector $\mathbf{u}$ as

$$\nabla_\perp f = \begin{pmatrix} \partial_r f \\ \frac{1}{r}\partial_\theta f \\ 0 \end{pmatrix}, \quad (A6)$$

$$\Delta_\perp \mathbf{u} = \begin{pmatrix} \left(\Delta_\perp - \frac{1}{r^2}\right)u^r - \frac{2}{r^2}\partial_\theta u^\theta \\ \left(\Delta_\perp - \frac{1}{r^2}\right)u^\theta + \frac{2}{r^2}\partial_\theta u^r \\ \Delta_\perp u^z \end{pmatrix}, \quad (A7)$$

$$\nabla_\perp \cdot \mathbf{u} = \partial_r u^r + \frac{1}{r}u^r + \frac{1}{r}\partial_\theta u^\theta, \quad (A8)$$

and $\Delta_\perp = \partial_r^2 + \frac{1}{r}\partial_r + \frac{1}{r^2}\partial_\theta^2$.

Assuming azimuthal invariance Eq. (A4) reduces to $M_1^z = 0$ and Eqs. (13) and (14). The latter are solved to yield the linear modes, Eqs. (16) and (17), and the dispersion relation (18) arises as a solvability condition.

Again with the azimuthal invariance, we find that Eq. (A5) is satisfied if

$$\partial_r \Phi_1 = M_1^r. \quad (A9)$$

In order to complete our analysis at the first order, we consider the usual magnetostatic boundary conditions for the present geometry given in Eqs. (A1)–(A3), (5), and (6). We find that inside the wire $H_1^\theta = 0$ because of the azimuthal invariance, $H_1^z = 0$, and from Eq. (A3) it can be shown that $H_1^r + M_1^r = 0$, which shows that the magnetostatic boundary conditions are satisfied at this order of analysis, with a magnetic field $H \equiv \alpha m$ outside the nanowire.

3. Solutions at order ε^2

At order ε^2, for any Fourier component p, we obtain the following equations from Eqs. (1) and (2):

$$i\Omega \mathbf{M}_2^p - V\partial_\zeta \mathbf{M}_1^p + \partial_\tau \mathbf{M}_0^p = -\mathbf{m}\wedge(-\partial_\zeta \Phi_1^p \mathbf{e}_z - \nabla_\perp \Phi_2^p + \beta\Delta_\perp \mathbf{M}_2^p) - \sum_{q+s=p} \mathbf{M}_1^q \wedge (-\nabla_\perp \Phi_1^s + \beta\Delta_\perp \mathbf{M}_1^s) - \mathbf{M}_2^p \wedge (\alpha \mathbf{m}), \quad (A10)$$

in which we have set $\mathbf{M}_1^1 = \mathbf{M}_1^{-1^*} = \mathbf{M}_1$, $\Phi_1^1 = \Phi_1^{-1^*} = \Phi_1$, and $\mathbf{M}_1^p = \Phi_1^p = 0$ for $p \neq \pm 1$, and

$$-\Delta_\perp \Phi_2^p + \partial_\zeta M_1^{p,z} + \nabla_\perp \cdot \mathbf{M}_2^p = 0. \quad (A11)$$

a. Fourier component $p=1$: Determination of V. Now we collect the coefficients of the fundamental Fourier component $p=1$ at the same order ε^2. From Eq. (A10), we find that $M_2^{1,z} = 0$ and

$$i\Omega M_2^{1,r} - V\partial_\zeta M_1^r = -m\left(\alpha - \beta\left(\Delta_\perp - \frac{1}{r^2}\right)\right)M_2^{1,\theta}, \quad (A12)$$

$$i\Omega M_2^{1,\theta} - V\partial_\zeta M_1^\theta = m\left(1 + \alpha - \beta\left(\Delta_\perp - \frac{1}{r^2}\right)\right)M_2^{1,r}. \quad (A13)$$

We decompose $M_2^{1,r}$ and $M_2^{1,\theta}$ as

$$M_2^{1,r} = B^r(\zeta,\tau)J_1(\kappa r) + W^r(\zeta,\tau,r), \quad (A14)$$

$$M_2^{1,\theta} = B^\theta(\zeta,\tau)J_1(\kappa r) + W^\theta(\zeta,\tau,r), \quad (A15)$$

in which W^r and W^θ may have nonzero component on any characteristic or eigenspace of the operator $(\Delta_\perp - \frac{1}{r^2})$, except the one which corresponds to the eigenvalue $-\kappa^2$ of the selected mode. Substituting Eqs. (A14) and (A15) in Eqs. (A12) and (A3) and making use of Eqs. (16) and (17) we obtain, along this eigenmode $J_1(\kappa R)$,

$$i\Omega B^r + m(\alpha + \beta\kappa^2)V\partial_\zeta f = -m(\alpha + \beta\kappa^2)B^\theta, \quad (A16)$$

$$i\Omega B^\theta - i\Omega V\partial_\zeta f = m(1 + \alpha + \beta\kappa^2)B^\theta. \quad (A17)$$

Eliminating B^θ, and making use of Eq. (18), Eqs. (A16) and (A17) reduce to

$$-2i\Omega m(\alpha + \beta\kappa^2)V\partial_\zeta f = 0, \quad (A18)$$

and, hence, the velocity V is zero.

In the hyperplane supplementary to the eigenspace corresponding to $-\kappa^2$, the above substitution gives the equations

$$i\Omega W^r = -m\left(\alpha - \beta\left(\Delta_\perp - \frac{1}{r^2}\right)\right)W^\theta, \quad (A19)$$

$$i\Omega W^\theta = -m\left(1 + \alpha - \beta\left(\Delta_\perp - \frac{1}{r^2}\right)\right)W^r. \quad (A20)$$

It follows from Eqs. (A19) and (A20) that W^r and W^θ are eigenfunctions of $(\Delta_\perp - \frac{1}{r^2})$. Let us denote by $(-\kappa'^2)$ the eigenvalue

$$i\Omega W^r = -m(\alpha + \beta\kappa'^2)W^\theta, \quad (A21)$$

$$i\Omega W^\theta = -m(1 + \alpha + \beta\kappa'^2)W^r, \quad (A22)$$

and, hence, κ' must satisfy the same dispersion relation, Eq. (18), as κ, with the same frequency Ω. It follows that $\kappa' = \kappa$, which was excluded initially. We conclude that $W^r = W^\theta = 0$, and $M_2^{1,r}$ and $M_2^{1,\theta}$ belong to the same transverse mode as the first-order component $\mathbf{M}_1$.

Equations (A16) and (A17) are easily solved to yield

$$B^r(\zeta,\tau) = -g(\zeta,\tau)m(\alpha + \beta k^2), \quad (A23)$$

$$B^{\theta}(\zeta,\tau)=i\Omega g(\zeta,\tau), \tag{A24}$$

where $g(\zeta,\tau)$ is an arbitrary function. Then from Eqs. (A12) and (A13), we obtain

$$M_2^{1,r}=-m(\alpha+\beta\kappa^2)g(\zeta,\tau)J_1(\kappa r), \tag{A25}$$

$$M_2^{1,\theta}=i\Omega g(\zeta,\tau)J_1(\kappa r). \tag{A26}$$

Since $M_1^z=0$, from Eq. (A11) for the Fourier component $p=1$, we find that

$$\partial_r\Phi_2^1=M_2^{1,r}. \tag{A27}$$

Also using the same arguments as in the order ε, one can show that the boundary and the pinning conditions are satisfied at order ε^2 for $p=1$, with $H\equiv\alpha m$ outside the nanowire.

b. Nonlinear term for Fourier components $p=0$ and $p=2$. For the Fourier component $p=0$, we find that Eq. (A10) reduces to

$$-m\left(\alpha-\beta\left(\Delta_\perp-\frac{1}{r^2}\right)\right)M_2^{0,\theta}=R_0^r, \tag{A28}$$

$$m\left(1+\alpha-\beta\left(\Delta_\perp-\frac{1}{r^2}\right)\right)M_2^{0,r}0=R_0^{\theta}, \tag{A29}$$

$$R_0^z=0. \tag{A30}$$

The nonlinear term is

$$\mathbf{R}_0=\mathbf{M}_1^*\wedge(-\nabla_\perp\Phi_1+\beta\Delta_\perp\mathbf{M}_1)+\text{c.c.}\,. \tag{A31}$$

Using Eqs. (13) and (14), it is seen that $\mathbf{R}_0=0$ and, hence, $M_2^{0,r}=M_2^{0,\theta}=0$, while $M_2^{0,z}$ remains free. We set $M_2^{0,z}=0$ for simplicity.

Similarly, for $p=2$, we find

$$2i\Omega M_2^{2,r}=-m\left(\alpha-\beta\left(\Delta_\perp-\frac{1}{r^2}\right)\right)M_2^{2,\theta}-R_2^r, \tag{A32}$$

$$2i\Omega M_2^{2,\theta}=m\left(1+\alpha-\beta\left(\Delta_\perp-\frac{1}{r^2}\right)\right)M_2^{2,r}-R_2^{\theta}, \tag{A33}$$

$$2i\Omega M_2^{2,z}=-R_2^z. \tag{A34}$$

The nonlinear term is

$$\mathbf{R}_2=\mathbf{M}_1\wedge(-\nabla_\perp\Phi_1+\beta\Delta_\perp\mathbf{M}_1), \tag{A35}$$

which reduces to

$$\mathbf{R}_2=-m(\alpha+\beta\kappa^2)i\Omega f^2(J_1(\kappa r))^2\mathbf{e}_z. \tag{A36}$$

Hence, Eq. (A32) and (A34) are homogeneous, and $M_2^{2,r}=M_2^{2,\theta}=0$. From Eqs. (A34) and (A36), we get the expression (22) of $M_2^{2,z}$.

The divergence equation (A11) yields

$$\partial_r\Phi_2^p=M_2^{p,r}, \tag{A37}$$

for $p=2$ and 0, and hence $\Phi_2^0=\Phi_2^2=0$. As the only nonzero component of $\mathbf{M}_2^2$ and $\mathbf{M}_2^0$ is parallel to the wire axis z, the boundary conditions do not produce a nonzero correction to the magnetic field outside the wire at this order.

4. Solutions at order ε^3

Collecting the coefficients of ε^3, we obtain the following equations from the Landau-Lifshitz equation (1) and the magnetostatic equation (2), respectively,

$$\begin{aligned}i\Omega\mathbf{M}_3^1-V\partial_\zeta\mathbf{M}_2^1+\partial_\tau\mathbf{M}_1=&-\mathbf{m}\wedge[-\partial_\zeta\Phi_2^1\mathbf{e}_z-\nabla_\perp\Phi_3^1\\&+\beta(\Delta_\perp\mathbf{M}_3^1+\partial_\zeta^2\mathbf{M}_1)]\\&-\sum_{q+s=p}\mathbf{M}_1^q\wedge(-\partial_\zeta\Phi_1^s\mathbf{e}_z-\nabla_\perp\Phi_2^s\\&+\beta\Delta_\perp\mathbf{M}_2^s)-\sum_{q+s=p}\mathbf{M}_2^q\wedge(-\nabla_\perp\Phi_1^s\\&+\beta\Delta_\perp\mathbf{M}_1^s)-\alpha\mathbf{M}_3^p\wedge\mathbf{m},\end{aligned} \tag{A38}$$

and

$$-\partial_\zeta^2\Phi_1-\Delta_\perp\Phi_3^1+\partial_\zeta M_2^{1,z}+\nabla_\perp\cdot\mathbf{M}_3^1=0. \tag{A39}$$

a. Nonlinear term for Fourier component $p=\pm1$. Using the expressions (16), (17), (A25), and (A26) of the components of $\mathbf{M}_1$ and $\mathbf{M}_2^1$ (and $M_1^z=M_2^{1,z}=0$), we can reduce Eq. (A38) for the fundamental Fourier component $p=1$ to the following set:

$$\begin{aligned}i\Omega M_3^{1,r}+\partial_\tau M_1^r=&\,m\beta\left(\Delta_\perp-\frac{1}{r^2}\right)M_3^{1,\theta}-m\alpha M_3^{1,\theta}\\&+m\beta\partial_\zeta^2M_1^{\theta}-N_3^{1,r},\end{aligned} \tag{A40}$$

$$\begin{aligned}i\Omega M_3^{1,\theta}+\partial_\tau M_1^{\theta}=&\,m\partial\Phi_3^1-m\beta\left(\Delta_\perp-\frac{1}{r^2}\right)M_3^{1,r}+m\alpha M_3^{1,r}\\&-m\beta\partial_\zeta^2M_1^r-N_3^{1,\theta},\end{aligned} \tag{A41}$$

$$i\Omega M_3^{1,z}+\partial_\tau M_1^z=-N_3^{1,z}. \tag{A42}$$

The nonlinear term $\mathbf{N}_3^1$ is defined by

$$\begin{aligned}\mathbf{N}_3^1=&\,\mathbf{M}_1\wedge(-\nabla_\perp\Phi_2^0+\beta\Delta_\perp\mathbf{M}_2^0)+\mathbf{M}_1^*\wedge(-\partial_\zeta\Phi_1^2\mathbf{e}_z-\nabla_\perp\Phi_2^2\\&+\beta\Delta_\perp\mathbf{M}_2^2)+\mathbf{M}_2^0\wedge(-\nabla_\perp\Phi_1+\beta\Delta_\perp\mathbf{M}_1)\\&+\mathbf{M}_2^2\wedge(-\nabla_\perp\Phi_1^*+\beta\Delta_\perp\mathbf{M}_1^*).\end{aligned} \tag{A43}$$

Using the results of previous orders, it reduces to

$$\mathbf{N}_3^1=\frac{m}{2}(\alpha+\beta\kappa^2)f|f|^2[J_1^*\Delta_\perp J_1^2\mathbf{P}+J_1|J_1|^2\mathbf{T}], \tag{A44}$$

with

$$\mathbf{P}=\beta\begin{pmatrix}-i\Omega\\ m(\alpha+\beta\kappa^2)\\ 0\end{pmatrix}, \tag{A45}$$

$$\mathbf{T}=\begin{pmatrix}-i\Omega\beta\kappa^2\\ m(1+\beta\kappa^2)(\alpha+\beta\kappa^2)\\ 0\end{pmatrix}. \tag{A46}$$

Since $N_3^{1,z}=0$ and $M_1^z=0$, we obtain from Eq. (A42) that $M_3^{1,z}=0$. The divergence equation (A39) reduces to

$$-\partial_\zeta^2\Phi_1+\left(\partial_r+\frac{1}{r}\right)\partial_r\Phi_3^1+\left(\partial_r+\frac{1}{r}\right)M_3^{1,r}=0. \quad (A47)$$

We observe that

$$\Phi_1=\frac{-1}{\kappa^2}\left(\partial_r+\frac{1}{r}\right)M_1^r, \quad (A48)$$

which solves (A47) as

$$M_3^{1,r}=\partial_r\Phi_3^1-\frac{1}{\kappa^2}\partial_\zeta^2M_1^r. \quad (A49)$$

The remaining harmonics $p=0,2,3$ of order ε^3 can be useful to derive higher-order evolution equations, which can be expected in the present case to be higher-order NLS ones. However, such a derivation would necessitate the introduction of higher-order slow time variables, which are not present in Eqs. (8) and (9). Anyways, the computation of the higher harmonics do not yield additional conditions at the order considered.

5. Nonlinear Schrödinger equation

Substituting Eq. (A49) into Eq. (A41), we get Eq. (23), with the right-hand-side member:

$$\begin{aligned}\mathcal{F}=&-i\Omega\partial_\tau M_1^\theta+m\left(1+\alpha-\beta\left(\Delta_\perp-\frac{1}{r^2}\right)\right)\\&\times[-\partial_\tau M_1^r+m\beta\partial_\zeta^2M_1^\theta-N_1^{1,r}]\\&+i\Omega m\left(\frac{1}{\kappa^2}-\beta\right)\partial_\zeta^2M_1^r-i\Omega N_3^{1,\theta}.\end{aligned} \quad (A50)$$

Using expressions (16) and (17) of $\mathbf{M}_1$, the expression (A50) of $\mathcal{F}$ is reduced to formulas (24)–(27).

Then Eq. (23) is solved using expansion (30). We get

$$X_q=\frac{\mathcal{F}_q}{-\Omega^2+m^2(1+\alpha+\beta q^2)(\alpha+\beta q^2)}, \quad (A51)$$

for $q\neq\kappa$ and

$$0\,.\,X_\kappa=\mathcal{F}_\kappa. \quad (A52)$$

Since $\mathcal{F}_\kappa=(J_1(\kappa r)|\mathcal{F})$, condition (A52) is nothing else than Eq. (31).

To reduce the coefficients, the following formula is useful:

$$(J_1(\kappa r)|J_1(\kappa r))=\frac{R^2}{2}\left(1-\frac{1}{\kappa^2R^2}\right)(J_1(\kappa R))^2. \quad (A53)$$

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