Anisotropic spin transport in GaAs quantum wells in the presence of competing Dresselhaus and Rashba spin-orbit coupling

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Aiming at the optimization of the spin-diffusion length in (001) GaAs quantum wells, we explore the effect of the anisotropy of the spin-orbit coupling on the competition between the Rashba and the Dresselhaus spin-orbit couplings by solving the kinetic spin Bloch equations with the electron-phonon and the electron-electron scattering explicitly included. For identical strengths of the Rashba and the Dresselhaus spin-orbit couplings, the spin-diffusion length shows strong anisotropy not only for the spin-polarization direction but also for the spin-diffusion direction. Two special directions are used seeking for the large diffusion length: $(\bar{1}10)$ and (110). Without the cubic term of the Dresselhaus spin-orbit coupling and with the identical Dresselhaus and Rashba strengths, infinite diffusion lengths can be obtained *either* for the spin-diffusion/injection direction along $(\bar{1}10)$, regardless of the direction of spin polarization, *or* for the spin polarization along (110), regardless of the direction of the spin diffusion/injection. However, the cubic Dresselhaus term cannot be neglected, resulting in a finite spin-diffusion length which decreases with the temperature and the electron density. The anisotropy for the spin-diffusion direction and spin-polarization direction is maintained. For the spin-diffusion/injection direction along $(\bar{1}10)$, the spin-diffusion length increases first with the increase of the Rashba strength (from 0) which can be tuned by the external gate voltage; when the Rashba strength is slightly smaller than (instead of equal to) the Dresselhaus strength, the diffusion length reaches its maximum, followed by a decrease with further increase of the Rashba strength.

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I. INTRODUCTION

The control of the electron spin in nanoscale is an object of great interest in the field of spintronics.[1,2] The knowledge of how the spin evolves as the electron goes through a nanodevice is a key element for the purpose of using the spin degree of freedom as a mechanism for information transfer and processing. Randomization of the electron spin, however, occurs due to the spin-orbit coupling as the electron wave vector changes during and after scattering processes. Between scattering events, the procession frequency of the spin about the local magnetic field also changes from electron to electron depending on the wave vector. To overcome these detrimental effects, many efforts have been made to understand the spin diffusion and the spin relaxation,[3–16] in particular, after the proposal of the spin-field-effect transistor by Datta and Das more than 16 years ago.[17] In such a device, ferromagnetic material is used as source and drain for the injection and the detection of spin-polarized electrons. The electron spins, moving through a quasi-one-dimensional channel, process about an effective magnetic field tuned by a gate voltage. In *n*-type zinc-blende semiconductors, the local magnetic field gives origin to a Zeeman-like splitting which, combined with the scattering, can cause a spin relaxation/dephasing known as the D'yakonov-Perel' (DP) mechanism.[18] In GaAs, the Dresselhaus term[19] is dominant in the DP wave-vector-dependent magnetic field, giving rise to a spin splitting due to bulk inversion asymmetry. Furthermore, in low-dimensional semiconductor structures with asymmetric confining potential, another term known as the Rashba term[20] contributes to the DP mechanism, giving rise to a spin splitting based on the structure inversion asymmetry. While the Rashba term is linear, the Dresselhaus term is cubic in the wave-vector components.

A few years ago, Schliemann *et al.*[10] proposed a nonballistic spin-field-effect transistor based on the competition of the Dresselhaus and the Rashba terms. In such a transistor, a gate voltage is tuned to give equal strengths to both terms, leading to a very long spin dephasing time for the spin polarization along the (110) direction.[10] In their work, the cubic term is argued to be unimportant. Cheng and Wu studied the effect of the cubic term on the spin-relaxation time of spins along the (110) direction by solving the kinetic spin Bloch equations, and obtained a finite spin-relaxation time.[21] Actually, in spin transport, when spins are injected into a sample, in addition to the direction of spin polarization, the direction of spin diffusion/injection is also important. The spin relaxation in transport depends on both directions. In the present work, the anisotropy of the spin-orbit coupling is taken into account when we study the ideal balance between the Rashba and the Dresselhaus terms in a quasi-two-dimensional channel of a GaAs quantum well. Therefore, both the direction of the spin polarization and the direction of the spin injection are analyzed. We further show that the cubic term appearing in the Dresselhaus effective magnetic field cannot be neglected in the competition with the Rashba field. Our treatment goes beyond the single-particle method in that we use a fully microscopic many-body approach by solving the kinetic spin Bloch equations (KSBEs).[4] As shown by Weng and Wu, the correlations between the spin-up and spin-down states, i.e., the off-diagonal terms of the density matrix in spin space, play an essential role in the spin

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diffusion/transport.[5] Studying the spin polarizations and the oscillations in the direction of diffusion, one can calculate the injection length as a function of the direction of injection.

This paper is organized as follows: In Sec. II, we describe the Rashba and the Dresselhaus spin-orbit couplings and study the effective magnetic field under different injection directions in the quantum well. In Sec. III, we construct the KSBE and we discuss how they change with the injection direction. In Sec. IV, we apply the KSBE to our model and present the results for different parameters. We conclude in Sec. V.

II. SPIN-ORBIT COUPLING

The DP mechanism introduces a spin-orbit term in the Hamiltonian which can be expressed by a Zeeman-like term with a wave-vector-dependent effective magnetic field

$$H_{so} = \mathbf{\Omega}(\mathbf{k}) \cdot \boldsymbol{\sigma}. \tag{1}$$

For convenience, the effective g factor and the Bohr magneton are absorbed into the definition of $\mathbf{\Omega}(\mathbf{k})$. For GaAs, which lacks an inversion symmetry center, the effective magnetic field contains the Dresselhaus term. For electrons in infinite-barrier-height quantum wells with a small well width a in the Γ_6 band, this term is[22]

$$\mathbf{\Omega}_D(\mathbf{k}) = \gamma \begin{pmatrix} k_x(k_y^2 - \langle k_z^2 \rangle) \\ k_y(\langle k_z^2 \rangle - k_x^2) \\ 0 \end{pmatrix}, \tag{2}$$

where γ represents the spin splitting parameter and $\langle k_z^2 \rangle = (\pi/a)^2$.

If a space gradient is applied to the sample, another effect shows up, the Rashba effect. The Rashba term in the Hamiltonian for electrons in the Γ_6 band is

$$H_R = r_{41}^{6c6c} \boldsymbol{\sigma} \cdot \mathbf{k} \times \boldsymbol{\varepsilon}, \tag{3}$$

where r_{41}^{6c6c} is a material parameter, and $\boldsymbol{\varepsilon}$ is the electric field determined by the asymmetry.[22]

Taking the z axis in the (001) direction (z will be considered throughout this paper as the growth direction of the quantum well) and $\boldsymbol{\varepsilon} = \varepsilon \hat{\mathbf{z}}$, the contribution of the Rashba term to the effective magnetic field may be written as

$$\mathbf{\Omega}_R(\mathbf{k}) = \alpha \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix}, \tag{4}$$

where α is the Rashba parameter. Next, we consider the influence on the effective magnetic field when we change the injection direction to $(\cos\theta, \sin\theta, 0)$ and choose this direction as the x axis (a schematic of the configuration is shown in Fig. 1). The effective magnetic field changes according to $\mathbf{\Omega}'(\mathbf{k}) = U^{-1}\mathbf{\Omega}(U\mathbf{k})$, with

$$U = \begin{pmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{5}$$

Therefore, we obtain for the Dresselhaus field

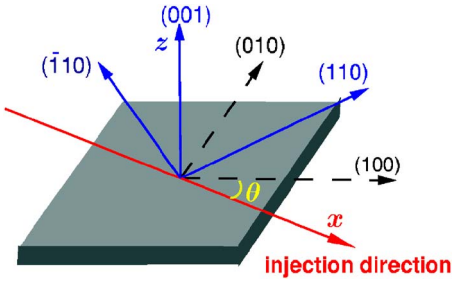

FIG. 1. (Color online) Schematic of the different directions considered for the spin polarizations [(110), (1̄10), and (001) axes] and spin diffusion/injection (x axis).

$$\mathbf{\Omega}'_D(\mathbf{k}) = \gamma\langle k_z^2 \rangle \begin{pmatrix} -k_x \cos 2\theta + k_y \sin 2\theta \\ k_x \sin 2\theta + k_y \cos 2\theta \\ 0 \end{pmatrix} + \gamma \left(\frac{k_x^2 - k_y^2}{2} \sin 2\theta + k_x k_y \cos 2\theta \right) \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix}, \tag{6}$$

where the wave-vector components are written in the rotated coordinate system. For the Rashba field, we obtain

$$\mathbf{\Omega}'_R(\mathbf{k}) = \alpha \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix}. \tag{7}$$

Notice that the Rashba field is expressed in the new coordinates in the same way as in Eq. (6).

In the following, we discuss the diffusion in two different ways. (i) We take the identical strengths of $\alpha = \beta$ with $\beta \equiv \gamma\langle k_z^2 \rangle$ to study the diffusion direction dependence and the spin-polarization dependence of the diffusion. In this case, the spin-orbit coupling $\mathbf{\Omega}' = \mathbf{\Omega}'_D + \mathbf{\Omega}'_R$ is written as

$$\mathbf{\Omega}'(\mathbf{k}) = 2\beta \left[\sin\left(\theta - \frac{\pi}{4}\right) k_x + \cos\left(\theta - \frac{\pi}{4}\right) k_y \right] \hat{\mathbf{n}}_0 + \gamma \left(\frac{k_x^2 - k_y^2}{2} \sin 2\theta + k_x k_y \cos 2\theta \right) \begin{pmatrix} k_y \\ -k_x \\ 0 \end{pmatrix}, \tag{8}$$

with the special direction

$$\hat{\mathbf{n}}_0 = \begin{pmatrix} \cos(\pi/4 - \theta) \\ \sin(\pi/4 - \theta) \\ 0 \end{pmatrix}$$

representing the crystal direction (110) in the rotated coordinates. (ii) We study the effect of the gate voltage on the spin diffusion. We treat this issue in Sec. IV C.

III. KSBE AND INHOMOGENEOUS BROADENING

Our fully microscopic treatment concerns the calculation of the spin-density matrix for the electron with momentum $\mathbf{k}$ at position $\mathbf{r} = (x, y)$:

$$\rho_{\mathbf{k}}(\mathbf{r},t)=\begin{pmatrix} f_{\mathbf{k}\uparrow} & \rho_{\mathbf{k}\uparrow\downarrow} \\ \rho_{\mathbf{k}\downarrow\uparrow} & f_{\mathbf{k}\downarrow} \end{pmatrix}. \tag{9}$$

The diagonal elements $f_{\mathbf{k}\sigma}$ stand for the electron distribution functions of spin σ, whereas the off-diagonal elements $\rho_{\mathbf{k}\uparrow\downarrow}=\rho^{*}_{\mathbf{k}\downarrow\uparrow}$ represent the correlations between the spin-up and -down states.

By using the nonequilibrium Green's-function method with gradient expansion as well as the generalized Kadanoff-Baym ansatz,[23] we construct the KSBE as follows,[5] assuming the diffusion to take place in the x direction:

$$\hbar\frac{\partial\rho_{\mathbf{k}}(x,t)}{\partial t}+e\frac{\partial\Psi(x,t)}{\partial x}\frac{\partial\rho_{\mathbf{k}}(x,t)}{\partial k_x}+\frac{\hbar^2k_x}{m^*}\frac{\partial\rho_{\mathbf{k}}(x,t)}{\partial x}+i\left[[g\mu_B\mathbf{B}+\mathbf{\Omega}'(\mathbf{k})]\cdot\frac{\boldsymbol{\sigma}}{2}+\mathcal{E}_{\mathrm{HF}}(x,t),\rho_{\mathbf{k}}(x,t)\right]=\hbar\left.\frac{\partial\rho_{\mathbf{k}}(x,t)}{\partial t}\right|_s. \tag{10}$$

Here, $\Psi(x,t)$ is the electric potential satisfying the Poisson equation

$$\nabla^2_{\mathbf{r}}\Psi(\mathbf{r})=e[n(\mathbf{r})-N_0(\mathbf{r})]/(a\kappa_0), \tag{11}$$

with $n(\mathbf{r})$ standing for the electron density at position $\mathbf{r}$, $N_0(\mathbf{r})$ representing the background positive charge density, and κ_0 being the static dielectric function. $\mathcal{E}_{\mathrm{HF}}(x,t)$ is the Hartree-Fock term from the Coulomb interaction. The scattering terms $\left.\frac{\partial\rho_{\mathbf{k}}(x,t)}{\partial t}\right|_{\mathbf{s}}$ include the electron-electron and electron-phonon scattering. Their expressions can be found in Ref. 8. Details of the numerical scheme as well as the material parameters are laid out in Ref. 24. It has been shown that Coulomb scattering plays a very important role in spin dephasing[8,21,25,26] and spin diffusion/transport[24] and therefore cannot be neglected in calculating the spin-diffusion length.

When the system reaches its steady state, in the absence of an external field, it can be described by the equation

$$\frac{\hbar^2k_x}{m^*}\frac{\partial\rho_{\mathbf{k}}(x,t)}{\partial x}+i\left[\mathbf{\Omega}'(\mathbf{k})\cdot\frac{\boldsymbol{\sigma}}{2}+\mathcal{E}_{\mathrm{HF}}(x,t),\rho_{\mathbf{k}}(x,t)\right]=\hbar\left.\frac{\partial\rho_{\mathbf{k}}(x,t)}{\partial t}\right|_s. \tag{12}$$

By neglecting the Hartree-Fock and the scattering terms, Eq. (12) can be solved analytically.[24] After dividing both sides of the equation by k_x, we see that along with the diffusion, the spin polarization for each wave vector $\mathbf{k}$ processes along $\boldsymbol{\omega}_{\mathbf{k}}=\frac{m^*}{2\hbar^2k_x}\mathbf{\Omega}'(\mathbf{k})$. The fact that spins with different momentums process with different frequencies is referred to as inhomogeneous broadening.[27,28] With any spin-conserving scattering included, the inhomogeneous broadening results in spin dephasing.[5,27]

For $\alpha=\beta$, $\boldsymbol{\omega}_{\mathbf{k}}$ reads

$$\boldsymbol{\omega}_{\mathbf{k}}=\frac{m^*}{2\hbar^2}\left\{2\beta\left[\sin\left(\theta-\frac{\pi}{4}\right)+\cos\left(\theta-\frac{\pi}{4}\right)\frac{k_y}{k_x}\right]\hat{\mathbf{n}}_0+\gamma\left(\frac{k_x^2-k_y^2}{2}\sin 2\theta+k_xk_y\cos 2\theta\right)\begin{pmatrix} k_y/k_x \\ -1 \\ 0 \end{pmatrix}\right\}. \tag{13}$$

We find that $\boldsymbol{\omega}_{\mathbf{k}}$ includes two parts: the zeroth-order term (on k) which is always along the same direction $\hat{\mathbf{n}}_0$, and the second-order term. The zeroth-order term includes both the $\mathbf{k}$-dependent and the $\mathbf{k}$-independent terms, while the second-order term is always $\mathbf{k}$ dependent. According to the previous works,[5,6,24,27] the $\mathbf{k}$-dependent term contributes to the inhomogeneous broadening.

We first analyze the spin diffusion without the third-order term of the Dresselhaus term (and hence the second-order term of $\boldsymbol{\omega}_{\mathbf{k}}$). In this case, the effective magnetic field for each wave-vector $\mathbf{k}$ points to the same direction $\hat{\mathbf{n}}_0$. From the previous works,[10,21] we know that the spin dephasing in the time domain, in the spatial homogeneous case, shows strong anisotropy with respect to the direction of spin polarization: The spin polarization along the direction $\hat{\mathbf{n}}_0$ has an infinite dephasing time, while the ones vertical to this direction have a very short spin dephasing time due to the large spin-orbit coupling. This is because when the spin-polarization direction is the same as the direction of the effective magnetic field, the spin polarization cannot process and hence there is no inhomogeneous broadening. Consequently, the spin polarization has an infinite dephasing time. The same is true for the spin diffusion: When the spin polarization is along the same direction of $\boldsymbol{\omega}_{\mathbf{k}}$, i.e., $\hat{\mathbf{n}}_0$ [the crystal direction (110)], the spin polarization will not decay regardless of the direction of the *spin injection*, even in the presence of scattering, as there is no spin procession and hence no inhomogeneous broadening. However, when the spin polarization is perpendicular to $\hat{\mathbf{n}}_0$, the spin-diffusion length is very short due to the large inhomogeneous broadening.

Nevertheless, in contrast to the spin dephasing in a spatially homogeneous system, the spin diffusion has an extra degree of freedom, i.e., the spin-diffusion/injection direction (which should not be confused with the spin-polarization direction). This degree of freedom introduces an additional level of anisotropy to the spin-diffusion/transport. When the spin-diffusion/injection direction is along $\theta=3\pi/4$, i.e., $(\bar{1}10)$ direction, the procession frequency $\boldsymbol{\omega}_{\mathbf{k}}=\frac{m^*\beta}{\hbar^2}\hat{\mathbf{n}}_0$ becomes $\mathbf{k}$ independent if the cubic term of the Dresselhaus term is neglected. Moreover, even with the scattering included, Eq. (12) can also be satisfied by the following solution:

$$\rho_{\mathbf{k}}(x)=e^{im^*\gamma\langle k_z^2\rangle\sigma_y x/\hbar^2}\rho_{\mathbf{k}}(x=0)e^{-im^*\gamma\langle k_z^2\rangle\sigma_y x/\hbar^2}, \tag{14}$$

where $\rho_{\mathbf{k}}(x=0)$ is the Fermi distribution with the spin polarization at the left boundary. We see from Eq. (14) that in this special case, the oscillation period is the same for all $\mathbf{k}$'s, with the value $\frac{\pi\hbar^2}{m^*\gamma\langle k_z^2\rangle}$. So, there is no inhomogeneous broadening and the spin has an infinite diffusion length, regardless

of the direction of the *initial spin polarization*. This result is mainly induced by the peculiar procession frequency $\boldsymbol{\omega}_{\mathbf{k}}$ which is determined by the spin-orbit coupling and the diffusion velocity k_x, so it cannot be obtained in the spatially homogeneous case where the procession frequency is only determined by the spin-orbit coupling. Of course, when we include the cubic term in the spin-orbit coupling, the oscillation period becomes wave vector dependent. This inhomogeneous broadening, then, results in a finite diffusion length.

In brief, we stress the importance of the inhomogeneous broadening caused by the diffusion term as compared to the spin dephasing in the spatially homogeneous case. Consequently, the spin polarization shows strong anisotropy not only for the direction of the initial spin polarization but also for the direction of the spin diffusion. Moreover, the cubic Dresselhaus term cannot be neglected. In what follows, we solve the problem by using the fully microscopic many-body approach of the KSBE with the cubic Dresselhaus term and the scattering explicitly included.

IV. NUMERICAL RESULTS

The KSBE are solved numerically for (001) GaAs quantum wells of width $a=5$ nm including the electron-electron and the electron-LO-phonon scattering. The value of the Dresselhaus coefficient is as $\gamma=25$ eV Å^3.[29] The electron density and the temperature are taken as $N_e=4.0\times10^{11}$ cm^{-2} and $T=200$ K separately unless otherwise specified. All matrix elements of the interactions are given in Ref. 8. For the spin polarization along direction $\hat{\mathbf{n}}$, the boundary conditions are given by[21]

$$\rho_{\mathbf{k}}(x=0,t)|_{k_x>0}=\frac{F_{\mathbf{k},\uparrow}+F_{\mathbf{k},\downarrow}}{2}+\frac{F_{\mathbf{k},\uparrow}-F_{\mathbf{k},\downarrow}}{2}\hat{\mathbf{n}}\cdot\boldsymbol{\sigma},$$

$$\rho_{\mathbf{k}}(x=L,t)|_{k_x<0}=\frac{F_{\mathbf{k},\uparrow}+F_{\mathbf{k},\downarrow}}{2}, \tag{15}$$

with the Fermi distribution $F_{\mathbf{k},\sigma}=[e^{(k^2/2m^*-\mu_\sigma)/k_BT}+1]^{-1}$ ($\sigma=\uparrow\downarrow$) and the chemical potential μ_σ determined by the polarized electron density. The diffusion length and the oscillation period are extracted from the spatial evolution of the spin polarization along $\hat{\mathbf{n}}$ direction $\Delta N(x)=\Sigma_{\mathbf{k}}\,\mathrm{Tr}[\rho_{\mathbf{k}}(x)\hat{\mathbf{n}}\cdot\boldsymbol{\sigma}]$.

A. Spin-diffusion/injection-direction and spin-polarization-direction dependence at $\alpha=\beta$

We first fix the spin polarization along the (110) direction ($\hat{\mathbf{n}}_0$) and study the spin-diffusion length as a function of the spin injection direction in the presence of the cubic Dresselhaus term. For comparison, we also study the cases when the spin polarizations are perpendicular to the (110) direction, i.e., $\hat{\mathbf{z}}$ and $\hat{\mathbf{n}}_1=\hat{\mathbf{z}}\times\hat{\mathbf{n}}_0$ and show how the spin injection lengths change as a function of spin-diffusion/injection direction. Subsequently, we explore the special case when the spin-diffusion/injection direction is along $(\bar{1}10)$, i.e., $\theta=3\pi/4$ and show how the spin-injection length changes with the spin polarization in the presence of the cubic Dresselhaus term. The spin polarization calculated from the KSBE [Eq. (10)] and the Poisson [Eq. (11)] along these three spin polarizations can be well fitted by

$$\Delta N(x)=C\exp\left(-\frac{x}{L_d}\right)\cos\left(\frac{2\pi x}{L_0}+\phi\right), \tag{16}$$

where L_d is the spin-diffusion length and L_0 represents the oscillation period.

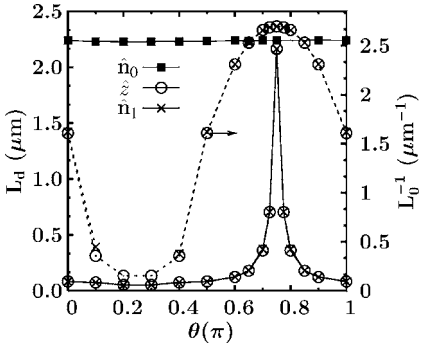

FIG. 2. Spin-diffusion length L_d (solid curves) and the inverse of the spin oscillation period L_0^{-1} (dashed curves) for $\alpha=\beta$ as functions of the injection direction for different spin-polarization directions $\hat{\mathbf{n}}_0$, $\hat{\mathbf{z}}$, and $\hat{\mathbf{n}}_1$ at $T=200$ K. It is noted that the scale of the spin oscillation period is on the right-hand side of the frame.

In Fig. 2, the spin-diffusion length L_d and the inverse of the spin oscillation period L_0^{-1} are plotted against the spin-diffusion/injection angle θ for spin polarizations along $\hat{\mathbf{n}}_0$, $\hat{\mathbf{z}}$, and $\hat{\mathbf{n}}_1$, respectively. It is interesting to see that with the inclusion of the cubic Dresselhaus term, the spin-injection length becomes finite but still *independent* of the direction of the spin diffusion/injection if the spin polarization is along (110) (=$\hat{\mathbf{n}}_0$). This is in agreement with the case without the cubic term where L_d becomes infinite. At the same time, the spin polarization along this direction has an infinite oscillation period $L_0=\infty$. This can be understood because the spin polarization is in the same direction as the effective magnetic field given by the zeroth-order term in $\boldsymbol{\omega}_{\mathbf{k}}$ and, thus, results in no oscillations. It is noted that the effective magnetic field from the second-order term in $\boldsymbol{\omega}_{\mathbf{k}}$ is $\mathbf{k}$ dependent and therefore cannot lead to any oscillation at high temperature due to the scattering.[21,25,30] However, this second-order term causes an inhomogeneous broadening which leads to a finite spin-diffusion length. By rewriting the second-order term of $\boldsymbol{\omega}_{\mathbf{k}}$ into

$$\frac{m^*\gamma}{2\hbar^2}\frac{k^2}{2}\sin(2\theta_{\mathbf{k}}+2\theta)\begin{pmatrix} ky/kx \\ -1 \\ 0 \end{pmatrix}$$

with k and $\theta_{\mathbf{k}}$ denoting the magnitude and the direction of the wave vector $\mathbf{k}$ separately, one finds that the magnitude of the inhomogeneous broadening does not change with the spin-diffusion/injection direction θ. Consequently, the spin-diffusion length does not change with the spin-diffusion/injection direction.

Strong anisotropy is again found for spin polarizations along the directions perpendicular to $\hat{\mathbf{n}}_0$ except for the special case when $\theta=3\pi/4$. Moreover, it is found in the figure

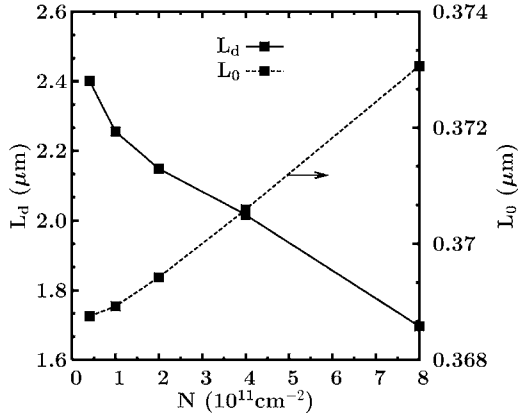

FIG. 3. Spin-diffusion length L_d and spin oscillation period L_0 at $\theta=3\pi/4$ and $T=200$ K as functions of the electron density. Note that the scale of the oscillation period is on the right-hand side of the frame.

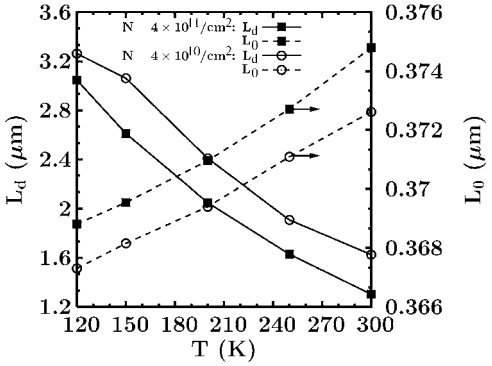

FIG. 4. Spin-diffusion length L_d and spin oscillation period L_0 at two different electron densities, 4×10^{11} and 4×10^{10} cm^{-2} versus the temperature. The spin-diffusion/injection direction $\theta=3\pi/4$. Note that the scale of the oscillation period is on the right-hand side of the frame.

that, for the perpendicular directions, both L_d and L_0 depend sensitively on the spin-diffusion/injection direction (additional anisotropy). It is further noticed that L_d and L_0 are almost identical for both the perpendicular directions $\hat{\mathbf{z}}$ and $\hat{\mathbf{n}}_1$. These results can be understood again from the inhomogeneous broadening induced by $\boldsymbol{\omega}_\mathbf{k}$. As γ in Eq. (13) is much smaller than β, the inhomogeneous broadening for spin polarizations $\hat{\mathbf{z}}$ and $\hat{\mathbf{n}}_1$ is therefore defined by the zeroth-order term of $\boldsymbol{\omega}_\mathbf{k}$, which is the same for both perpendicular directions and is spin-diffusion/injection direction θ sensitive. The oscillation period in the diffusion L_0 is determined by the $\mathbf{k}$-independent component of $\boldsymbol{\omega}_\mathbf{k}$,[24] i.e., $m^*\beta\sin(\theta-\pi/4)/\hbar^2$ which is in good agreement with the results in Fig. 2.

Now, we turn to the case with special fixed spin-diffusion/injection direction $\theta=3\pi/4$ and investigate the spin-diffusion length with different spin polarizations. It is interesting to see from Fig. 2 that similar to the case without the cubic Dresselhaus term, the spin-diffusion lengths for the three spin polarizations are almost identical. Therefore, the anisotropy disappears for this particular spin-diffusion/injection direction. This is because when the spin-diffusion/injection direction is $\theta=3\pi/4$, the $\mathbf{k}$-dependent component in the zeroth-order term of $\boldsymbol{\omega}_\mathbf{k}$ disappears. This results in the same inhomogeneous broadening for any spin polarization.

B. Temperature and electron-density dependence with injection direction along $(\bar{1}10)$ and $\alpha=\beta$

Now, we investigate the temperature and electron-density dependence of the spin-diffusion length at the special spin-diffusion/injection direction $\theta=3\pi/4$ with $\alpha=\beta$. As shown in the previous section, the spin diffusion is isotropic with respect to the spin-polarization direction. Hence, we choose the spin polarization to be along the z axis. In Fig. 3, the oscillation period and the diffusion length are plotted as functions of the electron density. One finds that the diffusion length L_d decreases with the density, whereas the oscillation period L_0 increases with it. However, it is noted that the change observed in the oscillation period is almost negligible (1%) in contrast to the pronounced changes observed in the diffusion length (of the order of 40%). We further investigate the temperature dependence of the spin diffusion length and the oscillation period in Fig. 4 by taking two typical electron densities, i.e., $N=4\times10^{10}$ and 4×10^{11} cm^{-2}. Similar behavior is found here compared to the density dependence: the spin-diffusion length L_d decreases markedly with the temperature, whereas the oscillation period L_0 only increases slightly with it.

Both dependences above can be again understood by the inhomogeneous broadening. Unlike the results in Ref. 24 where only the Dresselhaus term is considered and a very weak temperature dependence is obtained, the diffusion length here has a strong temperature dependence. This can be understood by the difference in the behavior of the inhomogeneous broadening, which is determined by the anisotropy of the procession frequency $\boldsymbol{\omega}_\mathbf{k}$. In the previous case, the zeroth-order term dominates the inhomogeneous broadening which changes little with temperature and electron density. However, in the present case with $\alpha=\beta$ and $\theta=3\pi/4$, the $\mathbf{k}$-dependent zeroth-order term is always zero and the second-order term alone determines the inhomogeneous broadening. This term increases effectively with the temperature and with the electron density due to the increase of the average value of k^2. This is the reason for the obtained marked decrease of the spin-diffusion length. The oscillation period L_0 is determined by the $\mathbf{k}$-independent zeroth-order term of $\boldsymbol{\omega}_\mathbf{k}$, which is independent of the electron density and the temperature. Therefore, one observes only a slight change in the oscillation period, originating from the second-order term and the scattering.

C. Gate-voltage dependence with injection direction along $(\bar{1}10)$

We now tune the gate voltage in order to change the relative importance of the Dresselhaus and the Rashba terms. The spin-diffusion/injection direction is again fixed at $\theta=3\pi/4$. The calculated spin-diffusion length L_d and the spin oscillation length L_0 are plotted against α/β in Fig. 5 at $T=200$ K, with the spin polarizations being along $\hat{\mathbf{n}}_0$, $\hat{\mathbf{z}}$, and $\hat{\mathbf{n}}_1$, respectively. It is noted that since $L_0=\infty$ when the spin

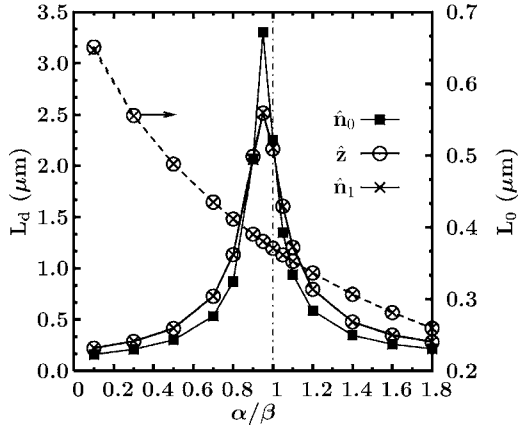

FIG. 5. Spin-diffusion length L_d (solid curves) and spin oscillation period L_0 (dashed curves) as functions of the ratio α/β with the injection direction $\theta=3\pi/4$ for different spin-polarization directions $\hat{\mathbf{n}}_0$, $\hat{\mathbf{z}}$, and $\hat{\mathbf{n}}_1$ at $T=200$ K. The chain line is used to guide the eyes. The scale of the oscillation period is on the right-hand side of the frame.

polarization is along $\hat{\mathbf{n}}_0$ [(110)], we only plot L_0 for the two perpendicular directions in the figure.

Three main results are obtained. (i) Differing from the case without the cubic Dresselhaus term, the maximum diffusion length occurs interestingly at $\alpha/\beta\sim 0.95$ (instead of 1) for each spin-polarization direction. (ii) The isotropy of the spin-polarization direction at $\alpha/\beta\neq 1$ is suppressed except at $\alpha/\beta\sim 0.9$, and the diffusion lengths for the spin polarization along $\hat{\mathbf{z}}$ and $\hat{\mathbf{n}}_1$ are always identical. (iii) The spin oscillation period L_0 decreases with α/β. These results can also be understood from the inhomogeneous broadening. For $\theta=3\pi/4$, the spin procession frequency along the diffusion direction is given by

$$\boldsymbol{\omega}_{\mathbf{k}} = \left[\left(-\beta+\alpha+\frac{\gamma k^2}{2}\right)\frac{k_y}{k_x}-\gamma k_x k_y\right]\hat{\mathbf{n}}_1 - \left[\beta+\alpha-\frac{\gamma}{2}(k_x^2-k_y^2)\right]\hat{\mathbf{n}}_0. \quad (17)$$

At $T=200$ K, $\langle k^2\rangle/2\sim 0.11(\pi/a)^2$ with $\langle\cdot\rangle$ representing the average over the imbalance of the spin-up and -down electrons. Therefore, $\left(-\beta+\alpha+\frac{\gamma k^2}{2}\right)\frac{k_y}{k_x}\hat{\mathbf{n}}_1$ is almost zero at $\alpha/\beta\sim 0.95$ and does not contribute to the inhomogeneous broadening. This results in a maximum of the spin-diffusion length. It is further noted that the inhomogeneous broadening term $\frac{\gamma}{2}(k_x^2-k_y^2)\hat{\mathbf{n}}_0$ gives a larger effect when the spin polarization is along $\hat{\mathbf{z}}$ and $\hat{\mathbf{n}}_1$ than that when the spin polarization is along $\hat{\mathbf{n}}_0$ as the latter coincides with the procession axis. This explains why one obtains a larger diffusion length when the spin polarization is along $\hat{\mathbf{n}}_0$ at $\alpha/\beta\sim 0.95$. Finally, as the spin oscillation period L_0 is determined by the magnitude of the $\mathbf{k}$-independent term of $\boldsymbol{\omega}_{\mathbf{k}}$, i.e., $\beta(\alpha/\beta+1)\hat{\mathbf{n}}_0$, which increases with α/β. Therefore, L_0 decreases with α/β.

V. CONCLUSIONS

In conclusion, we study the spin diffusion in (001) GaAs quantum wells with competing Dresselhaus and Rashba spin-orbit coupling strengths by solving the KSBE with the electron-phonon and the electron-electron Coulomb scattering explicitly included. It is shown that unlike the spin dephasing in the time domain where strong anisotropy is determined by different spin polarization, for spin diffusion, it is also determined by the spin-diffusion/injection direction. By neglecting the cubic term of the Dresselhaus spin-orbit coupling and with $\alpha=\beta$, the ideal case of an infinite diffusion length is obtained *either* for the spin polarization along (110) regardless of the spin-diffusion direction *or* for the spin-diffusion direction along $(\bar{1}10)$ regardless of the spin-polarization direction. However, the cubic term cannot be neglected, resulting in a finite diffusion length, which, in fact, is small, about 2 μm at the temperature of 200 K when the electron density lies between 4×10^{10} and 4×10^{11} cm^{-2}. It is then shown that when $\alpha=\beta$, for the spin-polarization along (110), the spin-diffusion length changes little for the spin-diffusion direction, whereas for the other two perpendicular spin-polarization directions $(\bar{1}10)$ and (001), the spin-diffusion length shows strong anisotropy for the spin-diffusion/injection direction and has a peak when the spin-diffusion/injection direction is along $(\bar{1}10)$. When the spin-diffusion/injection direction is along $(\bar{1}10)$, the spin-diffusion length is isotropic for spin polarization. The electron density and temperature dependence of the spin diffusion is closely investigated.

We also tune the gate voltage to show the competing effect of the Dresselhaus and the Rashba strengths. It is found that with the cubic Dresselhaus term included, the maximum spin-diffusion length appears at $\alpha/\beta\sim 0.95$ instead of 1 where the cubic term is neglected. The spin-diffusion length decreases when the ratio of the Rashba and Dresselhaus strengths deviates from 0.95. These results can be well understood by the inhomogeneous broadening.

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