Quantitative aspects of entanglement in optically driven quantum dots

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We present an approach to look for the existence of maximum entanglement in a system of two identical quantum dots coupled by the Förster process and interacting with a classical laser field. Our approach is not only able to explain the existing treatments but also provides further detailed insights into the coupled dynamics of quantum-dot systems. The result demonstrates that there are two ways of generating maximum entangled states, one associated with far off-resonance interaction and the other associated with the weak-field limit. Moreover, it is shown that exciton decoherence results in the decay of entanglement.

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I. INTRODUCTION

Quantum dots are semiconductor structures containing a small number of electrons within a region of space with typical sizes in the submicrometer range.[1] Coupling of two quantum dots leads to double quantum dots, which, in analogy with atomic and molecular physics, is described as two-level systems with controllable level spacing and one additional transport electron.[2] This rather suggests the analogy with a simple model for an atom, in particular, if it comes to interact with external fields such as photons or phonons. Many properties of such systems can be investigated by transport, if the dots are fabricated between contacts acting as source and drain for electrons which can enter or leave the dot. The possibility of using pairs of quantum dots coupled by the dipole-dipole interaction as effective three-or four-level systems, whose transmission for an optical beam at some frequency may be switched on or off using a second optical beam, has been explored.[3] In contrast to real atoms, quantum dots are open systems with respect to the number of electrons which can easily be tuned with external parameters such as gate voltages or magnetic fields.[4,5]

The experimental realization of optically induced entanglement of excitons in a single quantum dot[6] and theoretical study on coupled quantum dots[7] have been reported recently. In those investigations, a classical laser field is applied to create the electron-hole pair in the dot(s). Several groups have performed transport experiments with double quantum dots, with lateral structures offering experimental advantages over vertical dots with respect to their tunability of parameters.[8] However, in contrast to atomic systems, carrier lifetimes in the solid state are much shorter because of the continuous density of states of charge excitations and stronger environment coupling.[9,10]

Recently, a major advancement in the field has come from different types of local optical experiments that allow the investigation of individual quantum dots, thus avoiding inhomogeneous broadening and simple coherent-carrier control in single dots.[11] Some quantum information processing schemes have been proposed exploiting exchange and/or direct Coulomb interactions between spatially separated excitonic qubits in coupled quantum-dot systems.[7,12–14] Using far-field light excitation to globally address two and three quantum dots in a spatially symmetric arrangement and preparations of both Bell and Greenberger-Horne-Zeilinger entangled states of excitons have been discussed.[12,13] Also, an alternative scheme for a three-qubit entangled state generation by nonlinear optical state truncation has been introduced.[15]

Based on adiabatic elimination treatment,[2] the creation of two-particle entangled states in a system of coupled quantum dots has been discussed. The authors of this study avoided dealing with the exact solution of the problem and employed the adiabatic elimination as an approximate treatment. However, one should be aware that its predictions for any coupled system need to be checked against the exact solution of the complete coupled equations. In order to avoid such limitations, one must begin by ignoring any approximation and try to find an analytical solution of the coupled equations that govern the system. At this point, there is no generally established approach that can provide a complete description of the dynamics for the system. It is the purpose of this paper to present such an approach with illustrative applications. The basic idea relies on the discretization of the coupled system, which is thus replaced in the formulation by only linear solvable equation. Related treatments based on either adiabatic elimination,[2,16,17] discussing entangled state generation conditions, or the coupled equations without the detuning dependence have been presented in the literature.[5]

What we study and present below is essentially the most general case of the complete equations of the two-quantum-dot system. Most interestingly, it is shown that features of the degree of entanglement are influenced significantly by different values of the involved parameters and exciton decoherence. With this approach, we could create a two-particle entangled state between the vacuum and biexciton states or single-exciton entangled state without using the approximation method adopted in previous studies.

The outline of this paper is arranged as follows: In Sec. II, we give the notation and definitions of the model. To reach our goal, an analytical approach for obtaining exact time-dependent expressions for the probability amplitudes is developed in Sec. II A and exciton decoherence is discussed in Sec. II B. Having obtained the solution, in Sec. III, we analyze the time evolution of the populations of the quantum levels for various values of the system parameters. In Sec. IV, we study the evolution of the degree of entanglement, measured by the negativity measure for the partial transpose density matrix. Finally, our conclusion is presented in Sec. V.

II. MODEL

The model under consideration consists of two identical quantum dots coupled by the Förster process.[18–21] This process originates from the Coulomb interaction whereby an exciton can hop between the two dots.[18] The quantum dots contain no net charge and interact with a high-frequency laser pulse. This in effect means that the present model has three processes: (i) the coupling of the carrier system with a classical laser field, (ii) the interdot Förster interaction, and (iii) the single-exciton, keeping in mind the fact that all constant energy terms may be ignored. The total Hamiltonian for the quantum-dot system is given by[22]

$$\hat{H} = \frac{\hbar}{2}\sum_{j=1,2}(\Omega\exp[-i\omega t + i\phi]\hat{e}_j^\dagger\hat{\psi}_j^\dagger + \Omega^*\exp[i\omega t - i\phi]\hat{\psi}_j\hat{e}_j) - \frac{1}{2}\sum_{j=1}^{2}\left[\sum_{k=1}^{2}\hbar\eta(\hat{e}_j^\dagger\hat{\psi}_k\hat{e}_k\hat{\psi}_j^\dagger + \hat{\psi}_j\hat{e}_k^\dagger\hat{\psi}_k^\dagger\hat{e}_j) - \varepsilon(\hat{e}_j^\dagger\hat{e}_j - \hat{\psi}_j\hat{\psi}_j^\dagger)\right], \tag{1}$$

where Ω represents the laser–quantum-dot coupling and $\hbar\Omega = \mu E$, where μ is the coupling strength and E is the laser field amplitude. The parameters ω and ϕ describe the angular frequency and phase of the laser field, respectively. The operator $\hat{e}_j$ ($\hat{\psi}_j$) is the electron (hole) annihilation operator and $\hat{e}_j^\dagger$ ($\hat{\psi}_j^\dagger$) is the electron (hole) creation operator in the jth quantum dot. We denote by ε the band-gap energy of the quantum dot and η the interdot process hopping rate.

For a coupled two-quantum-dot system, it is useful to write the Hamiltonian of Eq. (1) in the bases $|0\rangle = |0,0\rangle$, $|1\rangle = (|1,0\rangle + |0,1\rangle)/\sqrt{2}$, and $|2\rangle = |1,1\rangle$, which describe the vacuum state, the single-exciton state, and the biexciton state, respectively. In this system the antisymmetric state $|a\rangle = (|1,0\rangle - |0,1\rangle)/\sqrt{2}$ is completely decoupled from the remaining states. Then, the simple three-state representation of the two-quantum-dot system can be employed with $|0\rangle$, $|1\rangle$, and $|2\rangle$. Since the density matrix of the system is diagonal and the symmetric state $|1\rangle$ is a maximally entangled state, an entanglement can be produced in this model by a suitable population of the state $|1\rangle$.

Applying the rotating wave approximation and a unitary transformation, the resulting Hamiltonian may be written as

$$H = 2\hbar\Delta\hat{A}_{22} + \hbar(\Delta - \eta)\hat{A}_{11} + \frac{\hbar}{\sqrt{2}}[\Omega e^{i\phi}(\hat{A}_{01} + \hat{A}_{12}) + \text{H.c.}], \tag{2}$$

where $\hat{A}_{ij} = |i\rangle\langle j|$, related to the above states $|0\rangle$, $|1\rangle$, and $|2\rangle$. We denote by Δ the detuning of the laser frequency from exact resonance ($\hbar\Delta = \varepsilon - \hbar\omega$).

It is worth mentioning here that one can take advantage of the Förster interaction between two quantum dots and apply a finite rectangular pulse and subpicosecond duration to generate a Bell state such as $\alpha|00\rangle + \beta|11\rangle$, where $|11\rangle$ denotes the simultaneous presence of two excitons in a double dot structure. Also, formations of an entangled state between the vacuum and the exciton (or the biexciton) state have been discussed.[23,24]

A. An analytic solution

We devote the present section to find an explicit expression for the wave function in Schrödinger picture. We use an analytic approach that seeks to reduce the coupled equation system (probability amplitudes) to a solvable linear equation in order to study in detail the types of interaction that exist between them. To reach our goal, we assume that the wave function of the complete system may be expanded in terms of the well known eigenstates $|i\rangle$ ($i = 0,1,2$), namely,

$$|\Psi(t)\rangle = B_0(t)|0\rangle + B_1(t)|1\rangle + B_2(t)|2\rangle. \tag{3}$$

The time dependence of the amplitudes in Eq. (3) is governed by the Schrödinger equation with the Hamiltonian given by Eq. (2); therefore, we obtain

$$i\frac{\partial B_j(t)}{\partial t} = \sum_{k=0}^{2}\xi_{jk}B_k(t), \tag{4}$$

where $\xi_{jk} = \langle j|\hat{H}|k\rangle$. In this case and using Eqs. (2) and (3), we obtain $\xi_{01} = (\xi_{10})^* = \xi_{12} = (\xi_{21})^* = \Omega' e^{-i\phi}$, $\xi_{11} = \Delta - \eta$, $\xi_{22} = 2\Delta$, $\Omega' = \Omega/\sqrt{2}$; otherwise, $\xi_{ij} = 0$.

In order to solve Eq. (4), we introduce the following function:[25]

$$G(t) = B_0(t) + xB_1(t) + yB_2(t), \tag{5}$$

which leads to the equation

$$i\frac{dG(t)}{dt} = \beta\left(B_0(t) + \frac{\gamma_1}{\beta}B_1(t) + \frac{\gamma_2}{\beta}B_2(t)\right), \tag{6}$$

where $\beta = x\xi_{10}$, $\gamma_1 = \xi_{12} + x\xi_{11} + y\xi_{21}$, and $\gamma_2 = x\xi_{12} + y\xi_{22}$. Now, let us seek a solution of $G(t)$ such that $\dot{G}(t) = -izG(t)$. This holds if and only if $z = \beta$, $x = \gamma_1/\beta$, and $y = \gamma_2/\beta$.

After some minor algebra, this leads to a cubic equation which contains three eigenvalues (to determine the z_i) corresponding to the same number of the eigenfunctions $G_j(t) = G_j(0)\exp(-iz_jt)$. Using Eq. (5), one can write

$$G_j(t) = \sum_{l=1}^{3}O_{jl}\bar{B}_l(t), \tag{7}$$

where O_{jl} is a 3×3 matrix whose elements are $O_{j1} = 1$, $O_{j2} = x_j$, and $O_{j3} = y_j$ and $\bar{B}_l = B_{l+1}$.

Now, we can express the unperturbed state amplitude $\bar{B}_j(t)$ in terms of the dressed state amplitudes $G_j(t)$ in this form $\bar{B}_i(t) = \Sigma_{j=1}^{3}(O^{-1})_{ij}G_j(0)\exp(-iz_jt)$. Using the above equations, we have

$$\bar{B}_j(t) = \lambda_{j1}\exp(-iz_1t) + \lambda_{j2}\exp(-iz_2t) + \lambda_{j3}\exp(-iz_3t), \tag{8}$$

where

$$z_j = -\frac{\xi_{10}}{3}\Bigg\{\alpha_1 + 2(\alpha_1^2 - 3\alpha_2)^{1/2} \times \cos\left[\frac{1}{3}\arccos\left(\frac{9\alpha_1\alpha_2 - 2\alpha_1^3 - 27\alpha_3}{2(\alpha_1^2 - 3\alpha_2)^{3/2}}\right) + [j-1]/3\right]\Bigg\},$$

$$\alpha_1 = -\frac{\xi_{11} + \xi_{22}}{\xi_{01}^*}, \quad \alpha_2 = -\frac{\xi_{11}\xi_{22} + 2|\xi_{01}|^2}{(\xi_{01}^*)^2}, \quad \alpha_3 = -\frac{\xi_{22}}{\xi_{01}^*}.$$

The parameter $\lambda_{ij}=(O^{-1})_{ij}G_j(0)$, where $(O^{-1})_{ij}$ is the ij element of the matrix O^{-1} which is the inverse of the matrix O. We thus have completely determined an analytic solution of the coupled quantum-dot system in the presence of the detuning parameter and phase.

B. Quantum decoherence

The original meaning of decoherence was specifically designated to describe the loss of coherence in the off-diagonal elements of the density operator in the energy eigenbasis.[26] Amongst the most crucial requirements for the implementation of quantum logic devices is a high degree of quantum coherence. Coherence is lost when a qubit interacts with other quantum degrees of freedom in its environment and becomes entangled with them. Exciton decoherence in semiconductor quantum dots is affected by many environmental effects; however, it is dominated by acoustic-phonon scattering at low temperatures.[27] The decoherence effects due to the exciton–acoustic-phonon coupling on the generation of an exciton maximally entangled state in quantum dots were studied in Ref. 28. This process is governed by the Hamiltonian[29]

$$\hat{H}_T = \hat{H} + \sum_k \omega_k a_k^\dagger a_k + \sum_k g_k J_z(a_k^\dagger + a_k), \qquad (9)$$

where $\hat{H}$ is given by Eq. (1) and $a_k^\dagger$ (a_k) stands for the creation (annihilation) operator of the acoustic phonon with wave vector k and g_k the coupling between the dots and the field. By the general procedure, we can deduce a master equation for the density operator $\rho(t)$ of the total system in the following form:

$$\frac{\partial}{\partial t}\rho(t) = J\rho(t) + \pounds_1\rho(t), \qquad (10)$$

with

$$\pounds_1\rho(t) = -i\Gamma[J_z,[J_z,\rho(t)]],$$

where the superoperator J is defined as $J\rho(t)=-i[\hat{H},\rho(t)]$ and Γ is the decoherence rate.[5] However, its dependence on the mode distribution of phonons as well as on a cutoff frequency is given by[30]

$$\Gamma = \int d\omega' \omega'^n \exp\left(\frac{-\omega'}{\omega_c}\right)[1 + 2N(\omega',T)], \qquad (11)$$

with n depending on the dimensionality of the phonon field, ω_c is a cutoff frequency, and $N(\omega',T)$ is the phonon occupation factor. Here, we consider pure decoherence effects that do not involve energy relaxation of excitons. The solution of Eq. (10) can be formally written as

$$\rho(t) = \exp[t(J + \pounds_1)]\rho(0). \qquad (12)$$

Here, $\rho(0)$ is the initial state of the system. The decoherence parameter Γ is temperature dependent and it amounts to 20–60 μeV for typical semiconductor quantum dots in a temperature range from 10 to 30 K.[27,30] The results of this analysis are more closely related to experimental situations, which are usually strongly affected by decoherence and relaxation. However, because time scales are very long, the relaxation processes are not considered here.

III. OCCUPATION PROBABILITIES

By making use of the theoretical treatment in the previous section, one can investigate the statistical properties of the system. Using the final state $|\Psi(t)\rangle$ or $\rho(t)$, all relevant quantities can be computed. In this section, our motivation is to investigate the occupation probabilities associated with two identical quantum dots coupled by the Förster process and interacting with a classical laser field. The expressions $\rho_{ii}(t)=|B_i(t)|^2$ $(i=0,1,2)$ represent the probabilities that at time t, the coupled quantum dots are in the state $|i\rangle$.

In Fig. 1, we plot the probability amplitudes as functions of the dimensionless parameters Ωt and Δ/Ω. The parameters used in these figures are $\phi=0$, $\eta/\Omega=0.1$, and $\Gamma=0$. It implies that the complete Rabi oscillations between the localized states occur. Once the detuning parameter is taken into account, the populations of biexciton state are decreased [see Fig. 1(c)]. On further increase of the detuning parameter Δ, one finds that the occupation probability of this level tends to zero, while the oscillations of the other levels show fast oscillations with small amplitudes (see Fig. 1). From this point of view, the transition can be considered as existing only between two states $|0\rangle$ and $|1\rangle$ for larger detuning, which means that the biexciton state is decoupled.

One, possibly not very surprising, principal observation is that the numerical calculations corresponding to the same parameters, which have been considered in Ref. 2 give nearly the same behavior, but with different scaled time. The important consequence of this observation is that using smaller values of the laser–quantum-dot coupling, the creation of a two-particle entangled state between the vacuum and the biexciton states or a two-particle entangled state between vacuum and single-exciton states depends essentially on controlling the detuning parameter.

Apart from helping us to know different conditions for creating maximum entangled states, using our exact solution, that would otherwise be very hard to figure out, some of our testing results are of interest. A particularly nonintuitive one is the following: previously, we used to think that increasing the detuning parameter always decreases the populations of the single-exciton state or biexciton state depending on its relation with the laser-dot coupling parameter. In Fig. 2, even with fixing the detuning parameter to be $\Delta=0$, a maximum entangled state between the vacuum and biexciton states is obtained when η/Ω takes larger values (see Fig. 2). When we first encountered this result (quite rare depending on the

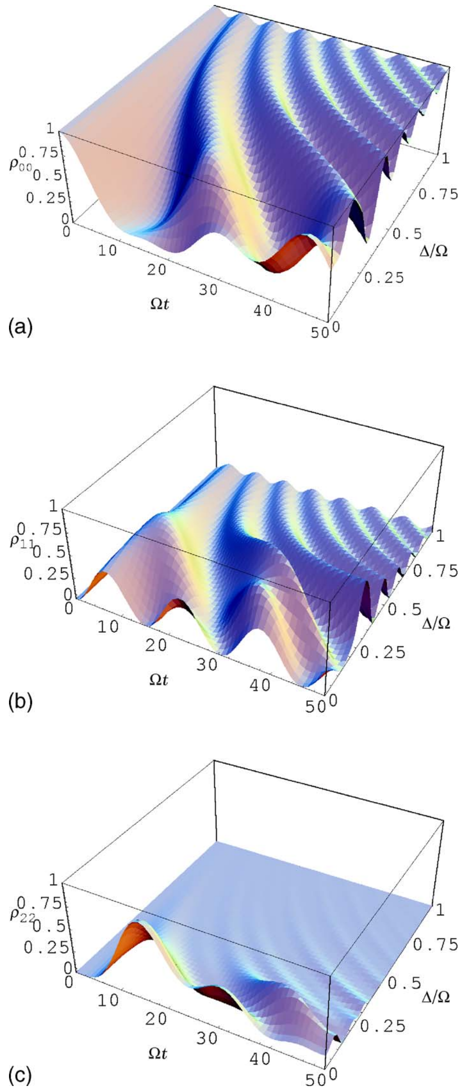

FIG. 1. (Color online) The probability amplitudes as functions of Ωt and Δ/Ω. The parameters used in these figures are $\phi=0$, $\eta/\Omega=0.1$, and $\Gamma=0$.

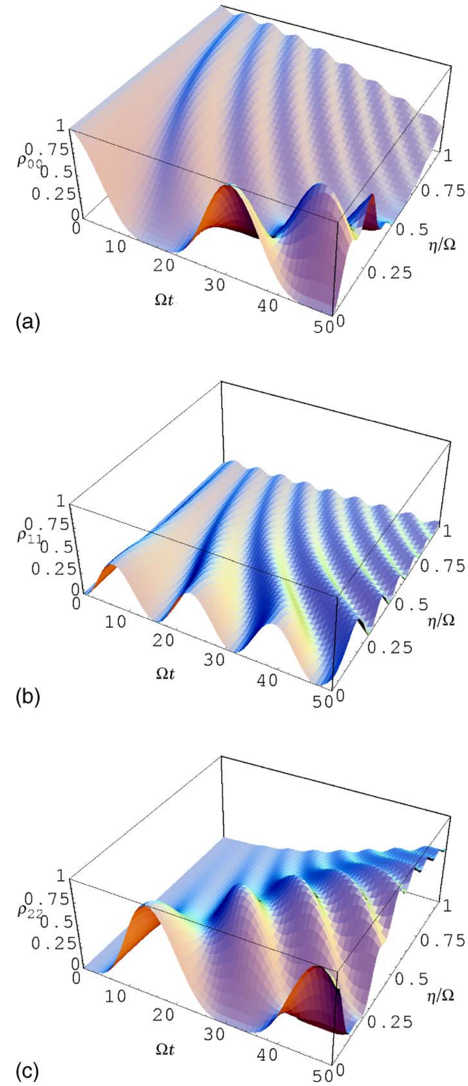

FIG. 2. (Color online) The probability amplitudes as functions of Ωt and η/Ω. Calculations assume that $\phi=0$, $\Delta/\Omega=0$, and $\Gamma=0$.

detuning), the problem becomes interesting and needs more investigations. Promisingly, we find that further small reductions in the laser-dot coupling parameter contribution will lead to significant improvements in entanglement.

Whereas the previous section dealt with the general behavior of the probability amplitudes and their indications to the entangled states generation, the next section introduces another view of the realization of the maximum entanglement.

IV. DEGREE OF ENTANGLEMENT

The characterization and classification of entanglement in quantum mechanics are two of the cornerstones of the emerging field of quantum information theory. Although an entangled two-qubit state is not equal to the product of the two single-qubit states contained in it, it may very well be a convex sum of such products. In general, it is known that microscopic entangled states are found to be very stable, for example, electron sharing in atomic bonding and two-qubit entangled photon states generated by parametric down-conversion.[19]

In this paper, we take the measure of negative eigenvalues for the partial transposition of the density operator as an entanglement measure. According to the Peres-Horodecki condition for separability,[31,32] a two-qubit state for the given set of parameter values is entangled if and only if its partial transpose is negative. The measure of entanglement can be defined in the following form:[33,34]

$$E_{\rho}(t) = \max\left(0, -2\sum_{i} \lambda_i\right), \tag{13}$$

where the sum is taken over the negative eigenvalues of the partial transposition of the density matrix ρ of the system. In the two-qubit system ($C^2 \otimes C^2$), it can be shown that the partial transpose of the density matrix can have at most one negative eigenvalue.[32]

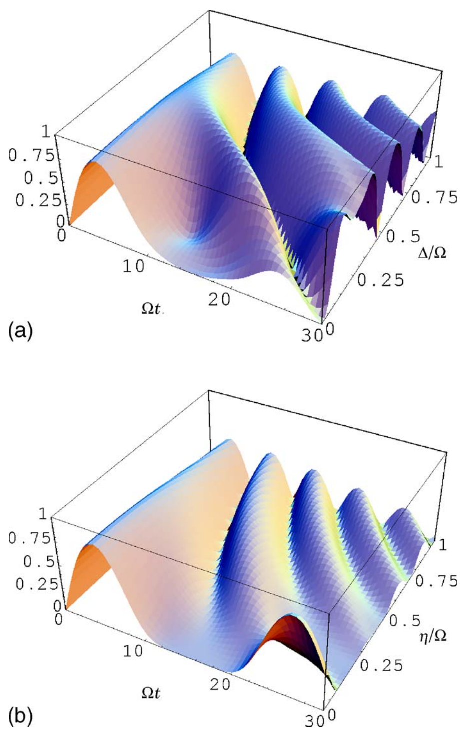

FIG. 3. (Color online) The entanglement $E_\rho(t)$ as a function of Ωt and Δ/Ω (η/Ω). The parameters used in these figures are (a) $\phi=0$, $\eta/\Omega=0.1$, and $\Gamma=0$ and (b) $\Delta/\Omega=0$ and $\Gamma=0$.

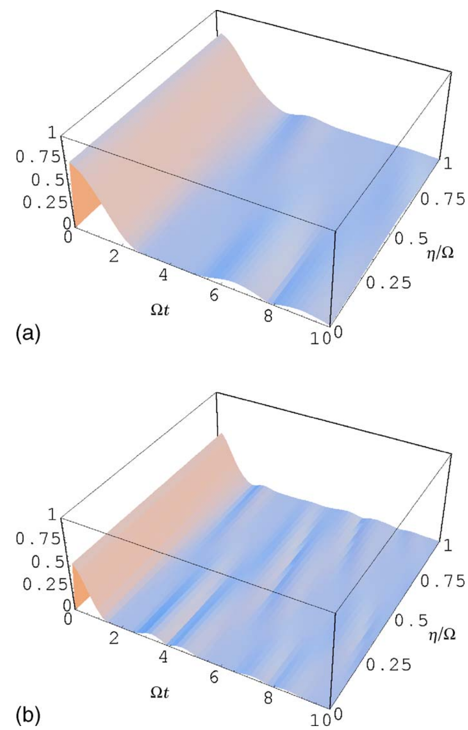

FIG. 4. (Color online) The entanglement $E_\rho(t)$ as a function of Ωt and η/Ω. The parameters used in these figures are (a) $\Delta/\Omega=0$ and $\Gamma/\Omega=0.01$ and (b) $\Delta/\Omega=0$ and $\Gamma/\Omega=0.05$.

The entanglement measure then ensures the scale between 0 and 1 and monotonously increases as entanglement grows. An important situation is that when $E_\rho(t)=0$, the two qubits are separable, and $E_\rho(t)=1$ indicates maximum entanglement between the two qubits. It was proven[35] that the negativity is an entanglement monotone and, hence, is a good entanglement measure.

In Fig. 3, we plot the entanglement degree $E_\rho(t)$ as a function of the dimensionless parameters Ωt and Δ/Ω (η/Ω). From Fig. 3(a), we see that the first maximum entanglement, as well as the disentanglement $[E_\rho(t)=0]$, occurs at earlier times when the detuning parameter is increased. These results are thus not in conflict with the well-established theory of adiabatic elimination. As soon as we take the detuning effects into consideration, it is easy to realize the decrease of the amplitudes of the oscillations with the increase of the value of the detuning parameter. Furthermore, if we take the parameter Δ large enough, then one can see that the entanglement degree tends to zero and the quantum dots become disentangled. This means that any change of the detuning parameter leads to change in the entanglement. It is also interesting to see that the number of oscillations increases with increasing detuning, however, with smaller amplitude. In all these cases, it should be noted that the entanglement vanishes for some periods of the interaction time (except for the case $\Delta=\eta$).

For a very small value of η (say, $\eta=0.01$), the situation becomes surprisingly interesting; at the initial period of the interaction time, the entanglement is strong between the two quantum dots, but as the time goes on, we have seen long survival of the disentanglement. This result indicates that the quantum dots will return to a pure state and completely disentangle from each other for a long period of the interaction time ($12\leqslant\Omega t\leqslant 19$). Finally, we may say that it is possible to obtain a long surviving disentanglement using small values of interdot process hopping rate, which means that the interdot process hopping rate plays an important role in the quantum entanglement.

An interesting question is whether or not the entanglement is affected by different values of the decoherence parameter Γ/Ω. Figure 4(a) displays the effect of Γ/Ω on the entanglement, where $\Gamma/\Omega=0.01$. Evidently, the evolution dynamics of $E_\rho(t)$ is sensitive to changes in the decoherence parameter Γ/Ω. In the long-time limit, the two quantum dots will be damped into their vacuum state due to the decoherence effect and entanglement decays to zero, which means that the decoherence plays an important role in the reduction of the degree of entanglement. If the decoherence parameter is increased further, for the system with fixed values of the interdot coupling, the decrease in the amount of entanglement is faster [see Fig. 4(b)]. It is likely that future source improvements will give values close to those expected for different initial states: the laser-dot coupling must be reduced to obtain sufficient entanglement to generate maximum entangled states. The oscillations in degree of entanglement between the quantum dots quickly damp out with an increase in Γ/Ω. The subsystems will disentangle from each other, and the steady state is reached at earlier interaction time. The change in (η/Ω) does not show much effect in the general structure of the degree of entanglement, in contrast to the case $\Gamma=0$.

It would be also worthwhile to use different initial-state settings, which would strongly help in creating the maximum entangled states. In a recent experimental work,[36] it has been demonstrated that quantum superpositions and entanglement can be surprisingly robust. This adds to the growing experimental evidence that robust manipulation of entanglement is feasible[38] with today's technology. Entangling many degrees of freedom, or equivalently many qubits (quantum dots), remains a challenge; however, these experimental results are encouraging.[36] Also, i n connection to the foundations of quantum theory, a deeper understanding of entanglement decoherence is expected to lead to new insights into the foundations of quantum mechanics.[37]

The remaining task is to identify and compare the results presented above for the entanglement degree with another accepted entanglement measure such as the concurrence.[39] One, possibly not very surprising, principal observation is that the numerical calculations corresponding to the same parameters, which have been considered above, give nearly the same behavior. This means that estimating the entanglement using either the negativity or concurrence measures gives qualitatively the same results.

V. CONCLUSION

We presented an analytical treatment for performing a maximum entangled states between two qubits in adjacent semiconductor quantum dots, formed through the interdot Förster interaction. The developed approach is capable of providing exact solutions to a class of problems which have only been treated approximately through previous studies. An important aspect is the insight gained by the possibility to combine the exact solution with the numerical treatments to generate maximum entangled states. More explicitly, in the exciton system, the large values of the detuning help in generating maximum entangled states. Nevertheless, the calculations indicate that the maximum entangled states can still exist, even for the resonant case, when the electron and hole are driven by a suitable laser field (weak-field limit).

We have extended our studies by giving an analysis and explanation of the predicted entanglement taking into account the decoherence effect. A remarkable property of the decoherence effect is that entanglement can fall abruptly to zero for a very long time and the entanglement will not be recovered, i.e., the state will stay in the disentanglement separable state. Needless to say, there is still much work to be done and technical and general questions to be addressed: in particular, sources of three and four entangled quanta have very recently been reported,[40] which in turn allows one to extend the question beyond two quantum dots to many-particle systems. This is also an exciting area for future study, both theoretically and experimentally.

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