Quasiparticle states of electron systems out of equilibrium

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In a nonequilibrium system, quasiparticles are defined, consistently with the construction of quantum transport equations, as one-electron excitations whose propagator possesses a time-local optical potential. This potential is constructed as a solution of the phase equation, a self-consistency condition reducing the renormalized Dyson equation to a formerly proposed multiplicative composition rule for the propagators. The Dyson equation for the particle correlation function is transformed to a renormalized quasiparticle reconstruction equation, whose leading term represents the quasiparticle Kadanoff-Baym ansatz [Physica E **29**, 196 (2005)], an improvement over the standard generalized Kadanoff-Baym ansatz.

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I. INTRODUCTION

Quasiparticles enter in one form or another any treatment of an electron system, in which the nonequilibrium dynamics is represented as a transport problem. This has been postulated for the first time by Landau, who realized that transport in a Fermi liquid should be described in terms of the long-lived resonant states of one-electron excitations, dubbed quasiparticles, rather than in terms of the fermions themselves.[1,2] Linear transport, in particular, was described as a reshuffling of equilibrium quasiparticles, and the whole Landau theory could be rederived on the basis of equilibrium Green's functions.[1–4] Notably, the quasiparticle energies and renormalization constants were identified as the poles and their residuals the spectral representation of the one-electron propagators. This approach was extended to truly nonequilibrium situations by Kadanoff and Baym,[5] who employed the nonequilibrium Green's functions (NGF) in the Wigner representation to work with the time-dependent nonequilibrium spectral density functions and to eventually introduce the so-called Kadanoff-Baym *Ansatz*, reducing the nonequilibrium Green's function equations of motion to a quantum transport equation for the quasiparticle distribution function.[5–8] The sequel to this probably best-known method of deriving transport equations included various ways to the "extended quasiparticles" and is reviewed in Refs. 9–16.

The use of the Kadanoff-Baym *Ansatz* was restricted to processes "quasiclassical in time." An alternative *Ansatz* called the generalized Kadanoff-Baym *Ansatz* (GKBA) has been introduced in Refs. 17 and 18, refraining from the Wigner representation and leading to a quantum transport equation for the reduced density matrix of the true particles. This *Ansatz* with a strictly causal structure was applicable to rapid transients and seemed to circumvent the use of the nonequilibrium quasiparticles ill defined in such circumstances. Later, however, an alternative derivation of the GKBA (Refs. 14, 16, and 19) made apparent that the *Ansatz* is closely related to a factorization of the nonequilibrium propagator known as the semigroup rule (SGR). This factorization, in turn, was shown to be quantitatively unsatisfactory in interacting systems if the renormalizations during the particle correlation time after the injection of the excitation were pronounced.[20,21] This theoretical conclusion did not fit with the wholesale practical success of the generalized Kadanoff-Baym *Ansatz*, notably in the area of rapid optical transients in semiconductors.[6,7,22–24] For a reconciliation, it was proposed[16] that the renormalizations should be incorporated into the SGR factorization; the modified SGR appeared to be characteristic of the formation of nonequilibrium quasiparticles. In fact, the true nonequilibrium propagators entering the generalized Kadanoff-Baym *Ansatz* were, in practical computations, always replaced by suitable model approximations, achieving thus the nonequilibrium quasiparticle behavior[6,7] (for original work, see Ref. 25; for recent review, see Ref. 26).

The practice of generating the quantum transport equation from nonequilibrium Green's function has been established as follows: the *Ansatz* leads to a transport equation. This link is made on a relatively rigorous level. The whole procedure, however, stands on two legs. The transport equation leg is of iron, while the propagator (actually, quasiparticle) leg is of clay. At the same time, both aspects are interpenetrating and, in a consistent theory, should be treated on the same level. A corresponding program has been outlined in the review.[14–16] This paper is the initial step toward its implementation. In that, we will be guided by two considerations.

Firstly, any known reduction of the nonequilibrium Green's function (NGF) equations to a quantum transport equation, be it for the particles or for the quasiparticles, hinges upon the quasiparticle mode of the nonequilibrium propagators. More precisely, the quasiparticle mode is a necessary condition in line with the Bogolyubov principle, which, however, requires more: a similar behavior also for the particle correlation function $G^<$.

This mode appears to take place even under conditions beyond the adiabatic regime. Such nonstationary quasiparticles have to be described entirely in terms of time-dependent quantities because of the missing crutch of spectral representation. This is unusual, but not impossible. In fact, the time-dependent approach to quasiparticles in the special case of equilibrium has basically been worked out in the classical work of Wigner and Weisskopf.

Secondly, it should be stressed that in the preceding discussion, and in the whole paper, we use a formal definition of a quasiparticle, whose only indication is a specific time structure of its propagator. In equilibrium, in particular, the quasiparticles are slowly exponentially decaying resonances. On a phenomenological level, this behavior depends entirely on the properties of the retarded self-energy Σ^R. A detailed microscopic structure of the quasiparticles plays no role here. This is consistent with their specific use in developing the quantum transport equations. The quasiparticle behavior stems, of course, from the microscopic structure of the elementary excitations of the system. However, all these microscopic details are distilled into the self-energy kernel, which then is finally the quantity decisive for the existence of the quasiparticles.[27]

We start from equilibrium, for which the phenomenological characteristic in the time domain is well known: an equilibrium propagator $G^R(t,t')$ describes a quasiparticle if it has the form[3,16]

$$G^R(t,t') = \begin{cases} \text{QP formation process}, & t < t' + \tau_Q \\ ZG_W^R(t,t'), & t > t' + \tau_Q, \end{cases}$$

$$G_W^R(t,t') = -i\exp[-iz_W(t-t')]. \tag{1}$$

There are several parameters of the resonance: the quasiparticle energy $z_W = E_W - i(2\tau)^{-1}$, the renormalization constant Z, and the time of formation τ_Q. The quasiparticle condition, $\tau_Q \ll 2\tau$, between the quasiparticle time of formation and its lifetime is necessary to guarantee that the excitation will remain in the quasiparticle (Weisskopf-Wigner, hence the subscript W) mode G_W^R for a protracted period (we omit the long-time decoherence for simplicity).

These parameters are given by the self-energy Σ^R. In the spectral representation, z_W is the solution of the pole equation $[G^R(z_W)]^{-1} = z_W - \epsilon_{\mathrm{MF}} - \Sigma^R(z_W) = 0$, where ϵ_{MF} is the mean-field (free-particle) energy. Z is the related residuum. It will be important to write the condition for the pole in the time formalism, following Refs. 28 and 29 and, notably, Ref. 30:

$$z_W = \epsilon_{\mathrm{MF}} + \int_0^\infty d\bar{t}\Sigma^R(\bar{t})e^{iz_W\bar{t}}. \tag{2}$$

This integral is formally divergent, in agreement with the fact that the pole is located at the nonphysical sheet in the energy representation. This problem does not arise in the lowest-order iteration of Eq. (2),

$$z_W = \epsilon_{\mathrm{MF}} + \int_0^\infty d\bar{t}\Sigma^R(\bar{t})e^{i\epsilon_{\mathrm{MF}}\bar{t}} = \epsilon_{\mathrm{MF}} + \Sigma(\epsilon_{MF} + i0), \tag{3}$$

which represents the original Weisskopf-Wigner weak scattering result.[28,29]

It is less easy to formalize the definition of τ_Q, which measures in some sense the time spread of the self-energy. The inequality $\tau_Q \ll 2\tau$ thus compares the time spread and the reciprocal strength of the retarded self-energy Σ^R, in parallel with the Bogolyubov time hierarchy for the particle correlation time and the transport relaxation time, $\tau_c \ll \tau_r$, which is related to $\Sigma^<$.

The descriptive quasiparticle picture Eq. (1), peculiar for equilibrium, can be turned into a few functional relations which are suited for generalization to nonequilibrium. First, there hold two multiplicative composition rules resulting from the functional equation of an exponential function: The quasiparticle propagator obeys the exact SGR:

$$G_W^R(t,t') = iG_W^R(t,\check{t})G_W^R(\check{t},t'), \quad t > \check{t} > t'. \tag{4}$$

For the full propagator G^R, the SGR is not correct. Instead, the multiplication rule which we will call the *quasiparticle composition rule* (QCR)

$$G^R(t,t') = iG_W^R(t,\check{t})G^R(\check{t},t') \tag{5}$$

is seen to hold for any intermediate time $\check{t}$ obeying

$$t > \check{t} > t' + \tau_Q > t'. \tag{6}$$

Another defining property of the exponential function is that its logarithmic derivative is constant. Thus, $i\partial_t G_W^R/G_W^R = z_W$. For G^R, we have, in the ideal case,

$$i\partial_t G^R/G^R = z_W, \quad t > t' + \tau_Q. \tag{7}$$

The logarithmic derivative of G^R need not be constant, in general, so that it provides a sensitive test of the time range where this quasiparticle property is valid.

While a phenomenological view of quasiparticles is commonplace in equilibrium, its truly broad generalization to nonequilibrium seems to be missing in the literature, and, in fact, a universally conceived definition of the nonequilibrium quasiparticle which can be found concerns slowly varying systems with small damping, in which a spectral function peaked at the quasiparticle resonance forms as a function of time, position, and momentum. An intuitive use of the concept has been connected with some simple particular situations.[6,7] In view of the importance ascribed to deriving quantum transport equations under the conditions departing from those limiting cases, a properly introduced concept of the nonequilibrium quasiparticle appears as a basic prerequisite.

The *basic goal of the paper* is to generalize the phenomenological view of quasiparticles to nonequilibrium by investigating under which conditions a construct $G_W^R(t,t')$ obeying an extended rule (4) can be associated with the true propagator $G^R(t,t')$ such that the composition rule (5) holds to a good approximation, at least in a certain time region.[16,31] Notice the very reserved formulation of the aims of the whole task appropriate out of equilibrium. These weak properties of G^R are sufficient, however, to set the stage for obtaining the related reduction of $G^<$, called quasiparticle Kadanoff-Baym *Ansatz* (QKBA), which in turn opens the path to a quantum transport equation.

A brief outline of the paper follows. In Sec. II, our attempt of building up the quasiparticle counterpart G_W^R to G^R, proceeds in two stages. In the first stage, Sec. II A, we define the class of quasiparticle propagators as those obeying the SGR, or, equivalently, having a time-local optical potential $\sigma_W(t)$.

In the second stage, we impose the conditions attributing a specific G_W^R to the full propagator in question. In contrast to equilibrium, the result depends on the time range selected. We proceed in steps. First, in Sec. II B, an extended "quasiparticle" composition rule is postulated as the basic defining property of the quasiparticle mode and this is reformulated in terms of the optical potential.

In Sec. II C, the existence of a finite formation time is assumed and the fundamental phase equation (24) for $\sigma_W(t)$ is derived for long times. This equation has to be modified so as to account for the influence of the past on the optical potential. In general, this is only possible as a compromise based on an average over initial times. This leads to the complete phase equation (26). The discussion of the phase equation in Sec. III A concentrates on the importance of its time semilocality for the properties of the optical potential. Section III B is devoted to deriving an integral equation (29) representing the renormalized counterpart of the intuitively postulated quasiparticle composition rule (13). The complete phase equation is rederived in Sec. III C in a systematic way: it is interpreted as a self-consistency condition leading to the vanishing of the vertex part of Eq. (29), which thus reduces to the QCR (13). The final brief Sec. IV compares the derivation of the standard GKBA Eq. (37) starting from an assumed SGR for propagators with the modified QKBA [Eq. (39)] resulting from a similar use of the QCR. The *Ansatz* may be judged by comparison with the exact renormalized reconstruction equation (41).

II. NONEQUILIBRIUM QUASIPARTICLES

Under nonstationary conditions, the Green's functions and other quantities will depend on two independent time arguments, as the nonequilibrium system will not be homogeneous in time. No privileged representation will be assumed, so that all Green functions and self-energies will be operators or matrices. The nonequilibrium propagator $G^R(t,t')$ will be assumed as known in a triangular time range,

$$t_M > t > t' > t_m. \tag{8}$$

The propagator specifies uniquely the self-energy in the range (8)

$$[G^R(t,t')]^{-1} = \{i\partial_t - H_{\mathrm{MF}}(t)\}\delta(t-t') - \Sigma^R(t,t'). \tag{9}$$

In H_{MF}, we lump together the free-particle Hamiltonian, the external fields, and the mean field, that is, the time-local ("singular") part of the self-energy. What remains is the true double-time self-energy kernel Σ^R. It contains precisely the information about the system needed for our study and represents the basic input for the construction of the quasiparticles.

We assume very little about Σ^R, but it is natural to restrict our study to the case when a nonequilibrium extension of the quasiparticle formation time entering Eq. (1) exists, at least in the time range (8) we consider. Specifically, we assume the existence of a characteristic time $\tau_Q(t)$ such that the self-energy is zero outside a double-time region, which has the general shape of a (uneven) strip adjoining the time diagonal $t=t'$ within the triangle (8),

$$\Sigma^R(t,t') \neq 0 \quad \text{only for } t > t' > t - \tau_Q(t). \tag{10}$$

A. Nonequilibrium quasiparticle propagators

The nonequilibrium quasiparticle propagators are specified and will be needed only in the region (8). It will be convenient to rewrite the quasiparticle propagator similarly to Eq. (9):

$$[G_W^R(t,t')]^{-1} = \{i\partial_t - H_{\mathrm{MF}}(t)\}\delta(t-t') - \Sigma_W^R(t,t'). \tag{11}$$

The class of quasiparticle propagators is formed by those obeying the SGR, a generalization of Eq. (4). This is equivalent with the requirement that the quasiparticle propagator be specified by a time-local optical potential $\sigma_W(t)$. The two definitions are bridged by the logarithmic derivative of the QCR. It is easy to see that the following four statements are equivalent:

$$G_W^R(t,t') = iG_W^R(t,\check{t})G_W^R(\check{t},t'), \quad t > \check{t} > t', \tag{12a}$$

$$\partial_t G_W^R(t,t')\frac{1}{G_W^R(t,t')} \quad \text{independent of } t', \tag{12b}$$

$$\Sigma_W^R \text{ time diagonal}: \Sigma_W^R(t,t') = \sigma_W(t)\,\delta(t-t'+i0), \tag{12c}$$

$$G_W^R(t,t') = -iT\exp\left[-i\int_{t'}^{t} du\{H_{\mathrm{MF}}(u) + \sigma_W(u)\}\right]. \tag{12d}$$

By the fraction symbol in Eq. (12b), we denote a plain operator inversion with time arguments fixed, the self-energy in Eq. (12c) is infinitesimally retarded, T appearing in Eq. (12d) is the time-ordering operator.

The present definition of the nonequilibrium quasiparticle propagator has the desired physical meaning. It is formally similar to the mean-field propagator, which describes a unitary evolution of independent effective particles moving in the average external and inner fields. In G_W^R, the optical potential σ_W enters in addition. The evolution is not unitary anymore, but it still is governed by an effective single-particle Hamiltonian, albeit typically non-Hermitian. Hence, it may correspond to a flight of independent quasiparticles. Equations. (12a)–(12d) or the Dyson Eq. (11) express this in various formal ways.

Any of the properties Eqs. (12a)–(12d) can be used interchangeably. In particular, because the quasiparticle self-energy is time diagonal, the task of finding G_W^R is reduced to specifying $\sigma_W(t)$. This will lead us to the nonequilibrium generalization of Eq. (2), which we will call the *phase equation*.

B. From the quasiparticle composition rule to the optical potential

Now, we proceed to the second stage outlined at the beginning of this section. We have to select among all G_W^R

obeying Eqs. (12a)–(12d) that one which will come closest to satisfying also the quasiparticle composition rule, which is specific for a given G^R. In this section, we start from an ideal case, extending the simple equilibrium Eq. (5). We postulate the functional equation expressing the *quasiparticle composition rule*

$$G^R(t,t') = iG_W^R(t,\check{t})G^R(\check{t},t'). \tag{13}$$

Visually, both Eqs. (5) and (13) look the same. The Green's functions are operators/matrices now, however, and they are true double-time quantities out of equilibrium. Thirdly, we cannot expect the rule (13) to be true for all times under nonstationary conditions.

Without a specific model, we simply assume that, to a fixed initial time t', at least one intermediate time exists such that Eq. (13) holds. Then, the validity of the same rule follows for a whole range of the two time variables t, $\check{t}$:

$$t_M > t > \check{t} > \underbrace{\min_{\mathring{t}} \check{t}}_{\mathring{t}} > t' > t_m. \tag{14}$$

While the QCR has a fundamental character, it is preferable to use once more its logarithmic derivative. Using repeatedly the QCR (13) and the definition (12c), we have

$$\begin{aligned} i\partial_t G^R(t,t')\frac{1}{G^R(t,t')} &= (i)^2\partial_t G_W^R(t,\check{t})G^R(\check{t},t')\frac{1}{G^R(t,t')} \\ &= \{H_{\mathrm{MF}}(t) + \sigma_W(t)\}iG_W^R(t,\check{t})G^R(\check{t},t') \\ &\quad \times \frac{1}{G^R(t,t')} \\ &= H_{\mathrm{MF}}(t) + \sigma_W(t). \end{aligned} \tag{15}$$

The QCR (13) itself, combined with Eq. (12d), assumes the form

$$G^R(t,t') = i\cdot\underbrace{(-i)T\exp\left[-i\int_{\check{t}}^{t} du\{H_{\mathrm{MF}}(u) + \sigma_W(u)\}\right]}_{G_W^R(t,\check{t})}G^R(\check{t},t'). \tag{16}$$

The quasiparticles thus exist in the time range $t > \mathring{t}$ if the optical potential exists in the same interval.

The procedure leading to Eq. (16) may be paraphrased without an *a priori* quasiparticle assumption. A number of relations may be written which have the outward look of the Weisskopf-Wigner results, but whose meaning is purely formal. A direct evaluation of the logarithmic derivative from the Dyson eq. (9) yields a generally valid identity defining an effective "time-local self-energy:"

$$i\partial_t G^R(t,t')\frac{1}{G^R(t,t')} = H_{\mathrm{MF}}(t) + \sigma_{t'}(t), \tag{17}$$

$$\sigma_{t'}(t) = \int_{t'}^{t} d\bar{t}\Sigma^R(t,\bar{t})G^R(\bar{t},t')\frac{1}{G^R(t,t')}. \tag{18}$$

This might suggest an obvious identification $\sigma_W = \sigma_{t'}$. However, as explicitly indicated, and as is apparent from Eq. (18), the latter quantity depends on t', in contrast to the Weisskopf-Wigner self-energy, whose independence of the initial time t' is its basic constitutive property.

The definition (18) of $\sigma_{t'}$ can be viewed as a differential equation $i\partial_t G^R(t,t') = [H_{\mathrm{MF}}(t) + \sigma_{t'}(t)]G^R(t,t')$, whose solution can be written in the form resembling Eq. (12d), but with $\sigma_{t'}(t)$ replacing $\sigma_W(t)$:

$$G^R(t,t') = -iT\exp\left[-i\int_{t'}^{t} du\{H_{\mathrm{MF}}(u) + \sigma_{t'}(u)\}\right]. \tag{19}$$

Because we aim at a Weisskopf-Wigner propagator, which should be free of t', it is important to fix a splitting time $\mathring{t}$, common to many t', such that the times $\check{t} \geqslant \mathring{t}$ may be tested as promising intermediate times for the QCR. For an arbitrary intermediate $\check{t}$ whose choice is given by purely formal considerations, the above differential equation has a solution representing a formal parallel to the QCR (16):

$$G^R(t,t') = i\cdot(-i)T\exp\left[-i\int_{\check{t}}^{t} du\{H_{\mathrm{MF}}(u) + \sigma_{t'}(u)\}\right]G^R(\check{t},t'). \tag{20}$$

The quasiparticle scheme will then hold in the time domain in which it will be possible to neglect the t' dependence of $\sigma_{t'}$. For a fixed dividing time $\mathring{t}$, this domain will be a subset of the whole range $(\mathring{t},t_M)\otimes(t_m,\mathring{t})$. Typically, the lower time t' will not be allowed to come close to $\mathring{t}$. In other words, to a given $\mathring{t}$, some $t'_{\max}$ will exist such that the quasiparticles will exist for times t, t' given by

$$t_M > t > \mathring{t},$$

$$\mathring{t} > t'_{\max}(\mathring{t}) > t' > t_m. \tag{21}$$

C. Phase equation for the optical potential

Introducing Eq. (19) into Eq. (18), we get the "basic tautology"

$$\sigma_{t'}(t) = \int_{t'}^{t} d\bar{t}\Sigma^R(t,\bar{t})\tilde{T}\exp\left[-i\int_{t}^{\bar{t}} du\{H_{\mathrm{MF}}(u)+\sigma_{t'}(u)\}\right]. \tag{22}$$

Note the time ordering from t to $\bar{t}$, which is *anticausal*, as indicated by the $\tilde{T}$ chronological operator. This relation could serve as an equation for the determination of $\sigma_{t'}$ from a known self-energy Σ^R, as it incorporates the initial condition $\sigma_{t'}(t')=0$. More importantly, it is the departure point toward the phase equation for σ_W, if the self-energy possesses the "formation" time according to Eq. (10). Equation (22) then becomes t' independent in form for t large enough, $t-\tau_Q(t) > t_m$ and $t-\tau_Q(t) > t'$. Namely,

$$\sigma_{t'}(t) = \int_{t-\tau_Q(t)}^{t} d\bar{t}\Sigma^R(t,\bar{t})\tilde{T}\exp\left[-i\int_{t}^{\bar{t}} du\{H_{\mathrm{MF}}(u)+\sigma_{t'}(u)\}\right]. \tag{23}$$

The solution $\sigma_{t'}$ *depends* on t', however. This results from the fact that Eq. (23) may be viewed as an equation with a delayed time argument. For a t range bounded from below, say $t > \mathring{t}$, it is not closed. Indeed, it is solved by any $\sigma_{t'}$, and we have to specify the values of the solution in the lowest interval, $\mathring{t}+\tau_Q(\mathring{t}) \geqslant t > \mathring{t}$, as an *initial condition*.

A simple step leads to an equation for σ_W: if the identification $\sigma_{t'} \to \sigma_W$ is justified, the optical potential also satisfies Eq. (23), which will be called the phase equation in this case:

$$\sigma_W(t) = \int_{t-\tau_Q(t)}^{t} d\bar{t}\Sigma^R(t,\bar{t})\tilde{T}\exp\left[-i\int_{t}^{\bar{t}} du\{H_{\mathrm{MF}}(u)+\sigma_w(u)\}\right], \quad t-\tau_Q(t) > \mathring{t}. \tag{24}$$

It expresses the unknown Weisskopf-Wigner self-energy σ_W in terms of the full self-energy Σ^R and the mean-field propagator. This large t relation is a necessary condition for any use of the quasiparticle picture, and in this sense, it has a fundamental nature. We return to the detailed meaning of the phase equation below. This equation alone is not sufficient, however, as the initial condition for the phase equation has to be known in the interval $\mathring{t}+\tau_Q(\mathring{t}) \geqslant t > \mathring{t}$. Unfortunately, there is no natural choice for this initial condition in selecting the correct optical potential σ_W. In fact, it may be necessary to accept a compromise for its approximation.

To study this question, it is convenient to start from Eq. (20) rather than from Eq. (19). Suppose we want to construct σ_W for times $t > \mathring{t} > t_m$. The whole integral Eq. (18) can be split into two: $\int_{t'}^{t} = \int_{\mathring{t}}^{t} + \int_{t'}^{\mathring{t}}$. The identity (20) is applied to the propagators with time arguments separated by $\mathring{t}$:

$$\sigma_{t'}(t) = \int_{\mathring{t}}^{t} d\bar{t}\Sigma^R(t,\bar{t})\tilde{T}\exp\left[-i\int_{t}^{\bar{t}} du\{H_{\mathrm{MF}}(u)+\sigma_{t'}(u)\}\right] + \int_{t'}^{\mathring{t}} d\bar{t}\Sigma^R(t,\bar{t})G^R(\bar{t},t')\frac{1}{G^R(\mathring{t},t')} \times\tilde{T}\exp\left[-i\int_{t}^{\mathring{t}} du\{H_{\mathrm{MF}}(u)+\sigma_{t'}(u)\}\right]. \tag{25}$$

The formation time assumption [Eq. (10)] is naturally made again. Equation (25) reduces to Eq. (23) for $t-\tau_Q(t) > \mathring{t}$. The initial condition $\sigma_{t'}(t)$ with $t-\tau_Q(t) \leqslant \mathring{t}$ missing in Eq. (23) can be obtained from the full Eq. (25). The lower time t' enters this equation implicitly through the expected t' dependence of the solution $\sigma_{t'}(t)$ and explicitly. The sole source of all explicit t' dependence in Eq. (25) is the second integral. It is reasonable to restrict the initial times to those preceding the splitting time by more than the formation time, that is, $\mathring{t}-\tau_Q(\mathring{t}) > t'$. Then the lower integration limit is given by Σ^R and equals to $t-\tau_Q(t) \geqslant \mathring{t}-\tau_Q(\mathring{t}) > t'$. The dependence on t' is then contained in the operator ratio $G^R(\bar{t},t')\cdot[1/G^R(\mathring{t},t')]$. This ratio, if known in a limited range of times $\mathring{t} > \bar{t} \geqslant \mathring{t}-\tau_Q(\mathring{t}) > t'$, determines the same ratio in the whole time interval $t > \mathring{t}$, as follows from Eq. (20).

If the quasiparticle composition rule (13) were valid, this quantity would reduce to $1/G_W^R(\mathring{t},\bar{t})$, entirely independent of t'. The solution $\sigma_{t'}$ would then coincide with the optical potential σ_W. In general, we cannot expect this to be exactly true. If, however, the t' dependence of $\sigma_{t'}$ will not be overly strong in an interval $\mathcal{I}$ of the t' times, a solution σ_W of the phase equation (24) may be constructed with optimized initial conditions based on the whole bundle $\{\sigma_{t'}(t), t' \in \mathcal{I}\}$ of the $\sigma_{t'}$ functions. The optimization may be an average, or a selection of some special (median, extremal, etc.) member of the bundle. We will denote this operation by $(\cdots)_{\{t'\}}$. With this definition, Eq. (25) can be transformed: $\sigma_{t'}(t)$ with $t > \mathring{t}$ is replaced by $\sigma_W(t)$ everywhere. There remains a single average of the initial condition. The resulting *complete phase equation* is

$$\sigma_W(t) = \int_{\mathring{t}}^{t} d\bar{t}\Sigma^R(t,\bar{t})\tilde{T}\exp\left[-i\int_{t}^{\bar{t}} du\{H_{\mathrm{MF}}(u)+\sigma_w(u)\}\right] + \int_{\mathring{t}-\tau_Q(\mathring{t})}^{\mathring{t}} d\bar{t}\Sigma^R(t,\bar{t})\left(G^R(\bar{t},t')\frac{1}{G^R(\mathring{t},t')}\right)_{\{t'\}} \times\tilde{T}\exp\left[-i\int_{t}^{\mathring{t}} du\{H_{\mathrm{MF}}(u)+\sigma_w(u)\}\right], \quad t > \mathring{t}. \tag{26}$$

The averaged expression can be written in two equivalent forms,

$$\left(G^R(\bar{t},t')\frac{1}{G^R(\mathring{t},t')}\right)_{\{t'\}} = \left(\tilde{T}\exp\left[-i\int_{\mathring{t}}^{\bar{t}} du\{H_{\mathrm{MF}}(u)+\sigma_{t'}(u)\}\right]\right)_{\{t'\}}. \tag{27}$$

The first form derives from the use of Eq. (20), while the other one would replace it if Eq. (19) were used instead.

III. PHYSICAL PROPERTIES OF NONEQUILIBRIUM QUASIPARTICLES

A. Discussion of the phase equation

We gave the name of phase equation to Eq. (24) with reference to Eq. (12d), in which the single-time effective self-energy σ_W clearly plays the role of a generalized phase shift. The idea to construct and solve the equations for the phase shifts directly is similar to the Calogero equations in the scattering theory,[32] or to the linked cluster expansion for Green's functions[1,33]—quite close to the present approach. Representing the Green's function in terms of its logarithm has the primary advantage of going over from a rapidly oscillating quantity to its slowly changing phase. It also makes full use of the robust nature of the quasiparticle mode, which is entirely described by the effective Hamiltonian $H_{\mathrm{MF}}+\sigma_W$. This is clearer and more straightforward than the formulation in terms of G_W^R and G^R governed by the functional relations (12a) and (13). We will relate both languages in the next two sections.

The essential physical ingredient in deriving Eq. (24) was the assumption that a quasiparticle formation time τ_Q existed and the self-energy had a correspondingly restricted "support," the range (10) of nonzero values. Similar assumptions of a "dominant strip" for $\Sigma^<$ are well known from the transport theory.[14,16,31] Physically, they are equivalent to the Bogolyubov postulate of the existence of the particle correlation (collision duration) time τ_c, which is supposed to bound the time needed for a decay of initial correlations and establish the kinetic regime. Similarly, in our case, for times at least τ_Q beyond the boundary time $\mathring{t}$, the phase equation does not depend on t', the initial time of the underlying GF, and not even on $\mathring{t}$. In fact, the integration limits in Eq. (24) are floating with t and have the width τ_Q restricting the range of relevant times. In consequence, the phase equation turns out to be *semilocal* in time. The time locality of the "quasiparticle Hamiltonian" $H_{\mathrm{MF}}(t)+\sigma_W(t)$ is then but effective, valid in the holistic context of the evolution over the full time range of the quasiparticle mode.

The effective nature of the time locality of the quasiparticle picture is connected with two points deserving a comment. One is the appearance of the anticausal time ordering in Eq. (24) concerning the retarded quantities. In contradiction to the unidirectional character of the quasiparticle evolution expressed by Eqs. (12), notably by Eq. (12a) we encounter an "upstream" evolution here, from the future to the past. This is possible because of the existence of the quasiparticle trajectories permitting this inverse mapping, at least over the restricted time $\sim\tau_Q$. At the same time, it is enforced by the same reason: to obtain the local self-energy σ_W at t, we have to extrapolate somewhat to the future along the trajectories, from $\bar{t}$ to t, and then compensate for it. This is another instance of such "violation" of causality well known in similar circumstances.[10,14,15]

The other point, where the semilocality of the equations for σ_W is essential, concerns the unavoidable influence of the past transmitted into the quasiparticle domain; in contrast to the phase equation, there is no general argument that the phase σ_W itself is t' independent. The corresponding initial conditions are incorporated in the complete phase equation (26). While they are restricted to one τ_Q layer adjoining the splitting time $\mathring{t}$, they are decisive for all later times $t>\mathring{t}$. The basic concept of the quasiparticle composition rule (13) thus may have an approximate nature at best.

B. Integral equation relating G^R and G_W^R

The point of departure for deriving the phase equation (24) in Sec. II C was an assumed exact validity of the quasiparticle composition rule. That is, for many initial times t', a single G_W^R should have existed obeying Eq. (13). This appeared as not possible, because of the individual t'-dependent initial conditions. An optimized approximation for the optical potential σ_W was proposed in Eq. (26). To judge the properties of this approximation, we will now develop an integral equation relating G_W^R with G^R. By subtracting the full Dyson Eqs. (9) and (11) from each other, a renormalized ("relative") Dyson equation is obtained as

$$G^R(t,t') = G_W^R(t,t') + \int_{t'}^{t} d\bar{t}\int_{t'}^{\bar{t}} d\bar{\bar{t}}\, G_W^R(t,\bar{t})\{\Sigma^R - \Sigma_W^R\}(\bar{t},\bar{\bar{t}}) \times G^R(\bar{\bar{t}},t'). \tag{28}$$

The definition range of G_W^R is easily extended in a formal fashion to that of G^R.

For any intermediate time $\check{t}$, this Dyson equation can be rearranged (Appendix A) to the *renormalized* QCR

$$G^R(t,t') = iG_W^R(t,\check{t})G^R(\check{t},t') + \int_{\check{t}}^{t} d\bar{t}\int_{t'}^{\bar{t}} d\bar{\bar{t}}\, G_W^R(t,\bar{t}) \times\{\Sigma^R - \Sigma_W^R\}(\bar{t},\bar{\bar{t}})G^R(\bar{\bar{t}},t'), \tag{29}$$

which will be central for our considerations in refining the nonequilibrium quasiparticle concept. There occur two important changes in Eq. (29). The free term G_W^R is replaced by the operator product $iG_W^RG^R$ coinciding with the desired form Eq. (13). In the second term, which is, in fact, a vertex correction to the simple product, the outer integration range is narrowed upward to $\check{t}<\bar{t}<t$. Such integration limits guarantee that only $\sigma_W(t)$ for $t>\check{t}$ enters Eq. (29).

The double integral can be reduced to a simple one if the definition (18) of $\sigma_{t'}(\bar{t})$ is recalled. Introducing this into Eq. (29), we have

$$G^R(t,t') = iG_W^R(t,\check{t})G^R(\check{t},t') + \int_{\check{t}}^{t} d\bar{t}\, G_W^R(t,\bar{t})\{\sigma_{t'}(\bar{t}) - \sigma_W(\bar{t})\}G^R(\bar{t},t'). \tag{30}$$

This integral relation confirms the discussion following Eq. (18) and makes it quantitative. The QCR (13) will be more precise the smaller the difference $\sigma_{t'}-\sigma_W$. It can trivially be made zero for one initial time t', but, as we saw, the real task is to search for an optimized choice of σ_W minimizing the correction uniformly in t'.

C. Complete phase equation vs the renormalized QCR

The renormalized quasiparticle composition rule (29) can be linked with the complete phase equation (26), giving it a deeper interpretation. In a preparatory step, Eq. (29) is transformed (Appendix B) to an equation valid for each value of t',

$$
\begin{aligned}
&T\exp\left[-i\int_{\check{t}}^{t}du\{H_{\mathrm{MF}}(u)+\sigma_{t'}(u)\}\right]\\
&\quad=iG_W^R(t,\check{t})+G_W^R(t,\check{t})\int_{\check{t}}^{t}d\bar{t}\\
&\qquad\times\tilde{T}\exp\left[-i\int_{\bar{t}}^{\check{t}}du\{H_{\mathrm{MF}}(u)+\sigma_W(u)\}\right]\\
&\qquad\times\left[\int_{\check{t}}^{\bar{t}}d\bar{\bar{t}}\Sigma^R(\bar{t},\bar{\bar{t}})T\exp\left[-i\int_{\check{t}}^{\bar{\bar{t}}}du\{H_{\mathrm{MF}}(u)+\sigma_{t'}(u)\}\right]\right.\\
&\qquad+\int_{\bar{t}-\tau_Q(\bar{t})}^{\check{t}}d\bar{\bar{t}}\Sigma^R(\bar{t},\bar{\bar{t}})G^R(\bar{\bar{t}},t')\frac{1}{G^R(\check{t},t')}\\
&\qquad\left.-\sigma_W(\bar{t})\cdot T\exp\left[-i\int_{\check{t}}^{\bar{t}}du\{H_{\mathrm{MF}}(u)+\sigma_{t'}(u)\}\right]\right].
\end{aligned}
\tag{31}
$$

Now comes the first basic step, to perform the "averaging" over t'. Its meaning is the same as in Sec. II C and we use the same symbol $(\cdots)_{\{t'\}}$. The average of the last equation is simply

$$
\begin{aligned}
&\left(T\exp\left[-i\int_{\check{t}}^{t}du\{H_{\mathrm{MF}}(u)+\sigma_{t'}(u)\}\right]\right)_{\{t'\}}\\
&\quad=iG_W^R(t,\check{t})+G_W^R(t,\check{t})\int_{\check{t}}^{t}d\bar{t}\tilde{T}\exp\left[-i\int_{\bar{t}}^{\check{t}}du\{H_{\mathrm{MF}}(u)\right.\\
&\qquad\left.+\sigma_W(u)\}\right]\left\{\int_{\check{t}}^{\bar{t}}d\bar{\bar{t}}\Sigma^R(\bar{t},\bar{\bar{t}})\left(T\exp\left[-i\int_{\check{t}}^{\bar{\bar{t}}}du\{H_{\mathrm{MF}}(u)\right.\right.\right.\\
&\qquad\left.\left.\left.+\sigma_{t'}(u)\}\right]\right)_{\{t'\}}+\int_{\bar{t}-\tau_Q(\bar{t})}^{\check{t}}d\bar{\bar{t}}\Sigma^R(\bar{t},\bar{\bar{t}})\right.\\
&\qquad\times\left(G^R(\bar{\bar{t}},t')\frac{1}{G^R(\check{t},t')}\right)_{\{t'\}}\\
&\qquad\left.-\sigma_W(\bar{t})\cdot\left(T\exp\left[-i\int_{\check{t}}^{\bar{t}}du\{H_{\mathrm{MF}}(u)+\sigma_{t'}(u)\}\right]\right)_{\{t'\}}\right\}.
\end{aligned}
\tag{32}
$$

Two closely related objects are averaged. First, the meaning of the average of the input quantity, the operator ratio $G^R(\bar{t},t')\cdot[1/G^R(\check{t},t')]$ with a limited range of times $\check{t}>\bar{t}\geqslant\check{t}-\tau_Q(\check{t})>t'$, has to be specified, in agreement with the discussion of Sec. II C. This defines a particular approximation for the quasiparticle picture. Second, the average extends over several phase factors. This average must be consistent with the averaged input, but here it will not be specified or performed separately.

Instead, we proceed to the second step and make the identification

$$
\begin{aligned}
&\left(T\exp\left[-i\int_{\bar{t}}^{t}du\{H_{\mathrm{MF}}(u)+\sigma_{t'}(u)\}\right]\right)_{\{t'\}}\\
&\quad=iG_W^R(t,\bar{t})=T\exp\left[-i\int_{\bar{t}}^{t}du\{H_{\mathrm{MF}}(u)+\sigma_w(u)\}\right]
\end{aligned}
\tag{33}
$$

be satisfied for any pair of times $t>\bar{t}>\check{t}$. This implies that the integral term in Eq. (32) must be zero. For that, the expression in the brackets must vanish:

$$
\{\cdots\}=0\quad\text{in Eq. (32)}.
\tag{34}
$$

This *self-consistency condition* becomes an explicit equation for σ_W whose solution satisfies the compatibility requirement Eq. (33), if all averaged phase factors are replaced accordingly. The resulting form of Eq. (34) is identical with the complete phase equation (26). This equation is thus derived in a systematic manner and it appears to guarantee that the vertex correcting the QCR in Eq. (29) vanishes on average. One advantage of this derivation is that it starts from a relation linear in the averaged expressions, while in the original method of Sec. II C, we made a hidden decoupling when averaging the third term of Eq. (25)

$$
\begin{aligned}
&\left(G^R(\bar{t},t')\frac{1}{G^R(\mathring{t},t')}\tilde{T}\exp\left[-i\int_{t}^{\mathring{t}}du\{H_{\mathrm{MF}}(u)+\sigma_{t'}(u)\}\right]\right)_{\{t'\}}\\
&\quad\approx\left(G^R(\bar{t},t')\frac{1}{G^R(\mathring{t},t')}\right)_{\{t'\}}\tilde{T}\exp\left[-i\int_{t}^{\mathring{t}}du\{H_{\mathrm{MF}}(u)\right.\\
&\qquad\left.+\sigma_w(u)\}\right].
\end{aligned}
\tag{35}
$$

Finally, we should comment on Eq. (33), which is a definition of σ_W, in fact. The present argument demonstrates that this nonlinear average is equivalent to the basic notion of the quasiparticle composition rule. This corroborates our comment at the beginning of Sec. III A, namely, that the optical potential plays the role of a generalized phase shift in the quasiparticle propagator and is related to its linked cluster form.

IV. QCR AND THE GKBA

The purpose of this brief section is to substantiate the claim made in Sec. I, namely, that the nonequilibrium quasiparticles are a necessary prerequisite for deriving a quantum transport equation, be it for the quasiparticle or for the particle distribution. More specifically, we will indicate, following Refs. 16, 19, and 31, how the QCR relates to the renormalized version of the GKBA called the quasiparticle Kadanoff-Baym *Ansatz*, which is the bridge between the full NGF formalism and its form reduced to a quantum transport equation. This will be done on two different levels. First, a heuristic motivation of both *Ansatze* will be compared. Second, a quasiparticle counterpart of the so-called reconstruc-

tion equations for $G^<$ will be derived. These equations are exact and the comparison will concern the conditions under which an approximation leading to one or the other *Ansatz* can be made.

To arrive at the GKBA, we follow the heuristic argument of Ref. 19 and start from the Dyson equation for the particle correlation function in the Keldysh form suitable for *uncorrelated initial conditions*[8,14]

$$G^<(t_1,t_2) = -iG^R(t_1,t_0)\rho(t_0)G^A(t_0,t_2) + \int_{t_0}^{t_1} dt_3 \int_{t_0}^{t_2} dt_4 G^R(t_1,t_3)\Sigma^<(t_3,t_4)G^A(t_4,t_2). \tag{36}$$

Let $t_1 > t_2 > t_0$. The formal integration region is a rectangle $\mathcal{R}=(t_0,t_1)\otimes(t_0,t_2)$. It can be split into two parts: $\mathcal{R}=\mathcal{R}_1 \oplus \mathcal{R}_2=(t_0,t_2)\otimes(t_0,t_2)\oplus(t_2,t_1)\otimes(t_0,t_2)$. It is assumed that the actual integration involves only a strip $\mathcal{S}$ of the width $2\tau_c$ along the time diagonal $t_1=t_2$, where the self-energy $\Sigma^<$ is significantly different from zero. This is the natural counterpart of a finite τ_Q assumed by Eq. (10).

If the "small" overlap area $\mathcal{R}_2 \cap \mathcal{S}$ is neglected, the integral extends only over the square $\mathcal{R}_1$. If, then, the semigroup multiplication rule is assumed to hold for the full propagators, $G^R(t_1,t_3)=iG^R(t_1,t_2)G^R(t_2,t_3)$, then Eq. (36) is reduced to the form of the GKBA

$$G^<(t_1,t_2) = -G^R(t_1,t_2)\rho(t_2), \quad t_1 > t_2, \tag{37}$$

where

$$\rho(t_2) = iG^<(t_2,t_2) = G^R(t_2,t_0)\rho(t_0)G^A(t_0,t_2) + i\int_{t_0}^{t_2} dt_3 \int_{t_0}^{t_2} dt_4 G^R(t_2,t_3)\Sigma^<(t_3,t_4)G^A(t_4,t_2) \tag{38}$$

as a special case of Eq. (36). [The other case, $t_2 > t_1 > t_0$, is treated in an analogous way and leads to $G^<(t_1,t_2) = \rho(t_1)G^A(t_1,t_2)$.]

In this derivation of the generalized Kadanoff-Baym *Ansatz*, three assumptions were made. Two have a general character: an uncorrelated initial condition and a uniformly small particle correlation time. The third one, the semigroup rule, is specific for this argument, demonstrating the link between the GKBA and the semigroup property of propagators. In interacting systems, the validity of this semigroup property is doubtful: at t_2, a spurious kink appears because of a repeated quasiparticle formation.[19–21] Instead, a modified multiplication rule and a related modification of the *Ansatz* can be attempted, however. In particular, the QCR (13) is valid anytime the quasiparticle picture works. Introducing Eq. (13), $G^R(t_1,t_3)=iG_W^R(t_1,t_2)G^R(t_2,t_3)$, into Eq. (36) and slightly modifying the considerations leading to Eq. (37), the so-called quasiparticle Kadanoff-Baym *Ansatz* is obtained in the form

$$G^<(t_1,t_2) = -G_W^R(t_1,t_2)\rho(t_2), \quad t_1 > t_2, \tag{39}$$

where ρ is given by the exact expression (38), just as before. This procedure was proposed in Ref. 19 at the time when the QKBA (39) still had not been proven. The present discussion indicates that the QKBA can be better justified than the GKBA proper once the quasiparticle behavior of the system is guaranteed.

A thorough study of the equations for $G^<$ renormalized in the way corresponding to one *Ansatz* of another, and the related possibility of a perturbation expansion, still has to be undertaken. Here, we confine ourselves to a basic comparison of the so-called reconstruction equation for $G^<$ (Refs. 14, 16–18, and 31) with its quasiparticle counterpart. The reconstruction equation, written for $t > t'$,

$$G^<(t,t') = -G^R(t,t')\rho(t') + \int_{t'}^{t} dt_1 \int_{t_0}^{t'} dt_2 G^R(t,t_1)\Sigma^<(t_1,t_2)G^A(t_2,t') + \int_{t'}^{t} dt_1 \int_{t_0}^{t'} dt_2 G^R(t,t_1)\Sigma^R(t_1,t_2)G^<(t_2,t') \tag{40}$$

is an integral equation whose source term coincides with the GKBA expression (37), while the integral terms link the past and the future with respect to t' and represent two different vertex corrections. The second one describes the single-particle ("polaron") renormalization on the out channel. The first one has an irreducible character and captures the two-particle correlations which surpass the *Ansatz* level.

The alternative rearrangement of Eq. (36) is obtained, again for $t > t'$, if the retarded propagator is rewritten with the use of Eqs. (28) and (29). After minor manipulations, the "quasiparticle reconstruction equation" results:

$$G^<(t,t') = -G_W^R(t,t')\rho(t') + \int_{t'}^{t} d\bar{t} \int_{t_0}^{t'} d\bar{\bar{t}} G^R(t,\bar{t})\Sigma^<(\bar{t},\bar{\bar{t}})G^A(\bar{\bar{t}},t') + \int_{t'}^{t} d\bar{t} \int_{t_0}^{t'} d\bar{\bar{t}} G_W^R(t,\bar{t})\Sigma^R(\bar{t},\bar{\bar{t}})G^<(\bar{\bar{t}},t') + \int_{t'}^{t} d\bar{t} \int_{t'}^{\bar{t}} d\bar{\bar{t}} G_W^R(t,\bar{t})[\Sigma^R(\bar{t},\bar{\bar{t}}) - \Sigma_W^R(\bar{t},\bar{\bar{t}})]G^<(\bar{\bar{t}},t'). \tag{41}$$

The true propagator G^R is replaced by the quasiparticle one in the source term, which is identical with the QKBA (39). Naturally, G_W^R also appears in the polaron terms in the last two lines. By contrast, the irreducible vertex correction is the same in Eq. (41) and in Eq. (40). This was the small integral over R_2 neglected in the derivation of Eqs. (37) and (39) above. Its form will be preserved and its proper inclusion will be necessary under any renormalization of the propagators. The advantage of the quasiparticle renormalization (41) concerns the polaron part of the vertex, that is, the last two lines. Just as in deriving the complete phase equation (26) in Sec. III C, the self-consistency requirement (34) may be shown to make these terms largely vanish. A refinement of this brief argument would require a closer look at the two

terms, the second of which violates the simple structure of integrals with ranges off-diagonal in the two times.

V. CONCLUSIONS

The results of this paper may be summarized in two parts.

Most of the work deals with the existence, nature, and properties of nonequilibrium quasiparticles as a specific mode of single-particle propagators:

(1) A consistent definition of single electron quasiparticles has been introduced for general nonequilibrium systems. It is purely phenomenological in that it is given in terms of properties of the one-electron propagators.

(2) The quasiparticle mode has two equivalent defining properties. Either it is required that the propagator satisfy the quasiparticle composition rule, or it should possess an optical potential dependent on a single time variable.

(3) The validity range of this definition depends on the retarded self-energy; in particular, it is contained in the time range for which the retarded self-energy exhibits a sufficiently short formation time.

(4) The full nonequilibrium retarded self-energy and the optical potential are linked by an integral relation—the phase equation. It can be solved for the optical potential if the initial conditions reflecting the period preceding the quasiparticle time range are incorporated, leading to the so-called complete phase equation.

(5) The QCR factorization of the propagator can be made the "free" term by a suitable transformation of its Dyson equation; the remaining integral term has the structure of a vertex correction. The condition for vanishing of this vertex is equivalent to the validity of the phase equation.

The quasiparticle properties of the propagator find their most important use in deriving the quantum transport equations by the technique of the Kadanoff-Baym *Ansatz* and its causal modifications, in particular, the causal version of the KBA called the generalized Kadanoff-Baym *Ansatz*. (Causal *Ansatzes* consist in a factorization of the particle correlation function $G^<$ into a propagator and the one-particle density matrix.) In this paper, we aimed at a derivation and justification of the physically well motivated variant, the quasiparticle Kadanoff-Baym *Ansatz*. The present results include the following:

(1) The QKBA has been derived from the Dyson equation for $G^<$ on the level of accuracy previously used to derive the standard GKBA.

(2) The Dyson equation for $G^<$ has been transformed to a coupled set of quasiparticle reconstruction equations, whose source term is $G^<$ factorized according to the QKBA. The validity of the *Ansatz* is thus shown to hinge on the possibility of neglecting the remaining three vertex correction terms. Two of the corrections depend entirely on the quasiparticle behavior of the propagators involved and can be suppressed similarly to the QCR. The task of deriving the QKBA comes down to a well controlled step, handling the irreducible vertex term. This cannot be done on the general level adopted here and a specific physical situation decisive for the properties of the vertex has to be invoked[3] The advantage of the present approach to the *Ansatz* concept is its incorporation into the systematic field theory permitting, for example, a properly renormalized perturbative treatment.

The outlook for a future work can again be divided into a possible further development of the nonequilibrium quasiparticle concept and into the implications for the quantum transport theory.

The work on the nonequilibrium quasiparticles should go beyond two limiting features of this paper: a wholly abstract look on the quasiparticles and the assumptions all too favorable for their formation. This leaves a number of topics for a more realistic treatment:

(1) To analyze conditions, under which the quasiparticle picture of this paper applies. Two general areas suggest themselves: (i) the weak scattering regime, when the Born approximation for the self-energy is valid, and (ii) the adiabatic region, when the external fields vary slowly on the scale of the formation time. This latter limit is much less restrictive than the true Kadanoff-Baym approach devised for a slow variation on the scale of the relaxation time.

(2) To single out some of the important cases when the straightforward quasiparticle picture of this paper is known to fail in advance but some of its extension still may be applicable: coherence induced by an external field, level crossovers due to the external fields (… laser pulses).

(3) To overcome the restrictive basic assumption of a cleanly defined formation time along several directions: a distinct formation period may occur even when the formation time is defined but vaguely; the formation and relaxation times may be comparable with one another and/or with the characteristic times of the external fields, like a pulse duration or Rabi period; the formation time may be nonuniform in the phase space.

In the direction toward the quantum transport equations it remains in particular:

(1) To use the quasiparticle Kadanoff-Baym *Ansatz* in the construction of a quantum transport equation and to explore its conserving properties.

(2) To put this transport equation to a numerical test similar to the extensive tests done for the GKBA related equations[34] and to compare the QKBA, the GKBA, and the direct solutions by the nonequilibrium Green's functions, all for the same task.

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APPENDIX A: DERIVATION OF EQUATION (29)

The integral in Eq. (28) is split into two:

$$G^R(t,t') = G_W^R(t,t') + \int_{t'}^{\check{t}} d\bar{t} \int_{t'}^{\bar{t}} d\bar{\bar{t}} G_W^R(t,\bar{t})\{\Sigma^R - \Sigma_W^R\}(\bar{t},\bar{\bar{t}}) \times G^R(\bar{\bar{t}},t') + \int_{\check{t}}^{t} d\bar{t} \int_{t'}^{\bar{t}} d\bar{\bar{t}} G_W^R(t,\bar{t})\{\Sigma^R - \Sigma_W^R\}(\bar{t},\bar{\bar{t}}) \times G^R(\bar{\bar{t}},t'). \tag{A1}$$

In the first two lines, the times are ordered as

$$t > \check{t} > t^*, \quad t^* = t' \text{ or } \bar{t},$$

so that the composition rule

$$G_W^R(t,t^*) = iG_W^R(t,\check{t})G_W^R(\check{t},t^*) \tag{A2}$$

can be applied. Reusing Eq. (28), the resulting two expressions can be joined to $iG_W^R(t,\check{t})G^R(\check{t},t')$. Equation (A1) then becomes identical to the relation (29) of the main text.

APPENDIX B: DERIVATION OF EQUATION (31)

Starting from the exact Eq. (29), a few rearrangements are made in steps. First, use is made of the properties (12a) and (12d) of G_W^R:

$$G^R(t,t')\frac{1}{G^R(\check{t},t')} = iG_W^R(t,\check{t}) + G_W^R(t,\check{t}) \times \int_{\check{t}}^{t} d\bar{t}\widetilde{T} \exp\left[-i\int_{\bar{t}}^{\check{t}} du\{H_{\mathrm{MF}}(u) + \sigma_W(u)\}\right] \times \int_{t'}^{\bar{t}} d\bar{\bar{t}}\{\Sigma^R - \Sigma_W^R\}(\bar{t},\bar{\bar{t}})G^R(\bar{\bar{t}},t')\frac{1}{G^R(\check{t},t')}. \tag{B1}$$

Next, the inner integral is split as in Eq. (23), the existence of τ_Q [Eq. (10)] is taken into account, and the identity (20) is used:

$$T\exp\left[-i\int_{\check{t}}^{t} du\{H_{\mathrm{MF}}(u) + \sigma_{t'}(u)\}\right] = iG_W^R(t,\check{t}) + G_W^R(t,\check{t})\int_{\check{t}}^{t} d\bar{t}\widetilde{T}\exp\left[-i\int_{\bar{t}}^{\check{t}} du\{H_{\mathrm{MF}}(u) + \sigma_W(u)\}\right]\left[\int_{\check{t}}^{\bar{t}} d\bar{\bar{t}}\{\Sigma^R - \Sigma_W^R\}(\bar{t},\bar{\bar{t}}) \times T\exp\left[-i\int_{\check{t}}^{\bar{\bar{t}}} du\{H_{\mathrm{MF}}(u) + \sigma_{t'}(u)\}\right] + \int_{\bar{t}-\tau_Q(\bar{t})}^{\check{t}} d\bar{\bar{t}}\Sigma^R(\bar{t},\bar{\bar{t}})G^R(\bar{\bar{t}},t')\frac{1}{G^R(\check{t},t')}\right]. \tag{B2}$$

Finally, the time diagonality of Σ_W^R [Eq. (12c)] is employed. This yields Eq. (31) of the main text.

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