

## Hall effect in strongly correlated low-dimensional systems

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We investigate the Hall effect in a quasi-one-dimensional system made of weakly coupled Luttinger liquids at half filling. Using a memory function approach, we compute the Hall coefficient as a function of temperature and frequency in the presence of umklapp scattering. We find a power-law correction to the free-fermion value (band value), with an exponent depending on the Luttinger parameter  $K_\rho$ . At sufficiently high temperature or frequency, the Hall coefficient approaches the band value.

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### I. INTRODUCTION

The Hall effect has been continuously playing an important role in experimental condensed-matter research, mostly because the interpretation of Hall measurements is rather simple in classical Fermi systems.<sup>1</sup> In such materials, the Hall coefficient is a remarkably robust property, which is unaffected by interactions and only depends upon the shape of the Fermi surface and the sign of the charge carriers. Deviations from this simple behavior are generally taken as evidence for the onset of strong correlations and a failure of the Fermi-liquid (FL) paradigm.<sup>2</sup> Several authors have investigated the Hall effect in three- and two-dimensional FLs,<sup>3–5</sup> but the question of the role of correlations in the Hall effect for low-dimensional systems remains largely unexplored.

In most three-dimensional systems, the interactions play a secondary role and the FL picture is appropriate.<sup>6</sup> However, the prominence of interactions increases as the dimensionality of the systems decreases, and the FL theory is believed to break down for many two-dimensional systems such as, e.g., the high- $T_c$  cuprate superconductors.<sup>7</sup> In one-dimensional (1D) systems, interactions are dominant, and the FL description must be replaced by the Luttinger liquid (LL) theory.<sup>8,9</sup> This theory predicts a rich variety of physical phenomena, such as spin-charge separation or nonuniversal temperature dependence of the transport properties,<sup>10</sup> many of which have been observed experimentally. Therefore, large deviations from the classical Hall effect are expected to occur in *quasi-one-dimensional* systems.

Among the various experimental realizations of low-dimensional systems (organic conductors,<sup>11</sup> carbon nanotubes,<sup>12</sup> ultracold atomic gases,<sup>13</sup> etc.), the organic conductors are good realizations of quasi-1D materials. Studies of the longitudinal transport have successfully revealed signatures of LL properties.<sup>11,14,15</sup> Transport transverse to the chains has given access to the dimensional crossover between a pure 1D behavior and a more conventional high-dimensional one.<sup>10,11,16,17</sup> To probe further the consequences of correlations in these compounds, several groups have undertaken the challenging measurement of the Hall coefficient  $R_H(T)$ .<sup>18–21</sup> The results, different depending on the direction of the applied magnetic field, proved difficult to interpret due to a lack of theoretical understanding of this problem. This prompted for a detailed theoretical analysis of the Hall effect in quasi-1D systems. A first move in this direction was re-

ported in Ref. 22, where the Hall coefficient of dissipationless weakly coupled 1D interacting chains was computed and found to be  $T$  independent and equal to the band value. This surprising result shows that in this case,  $R_H$ , unlike other transport properties, is insensitive to interactions. However, the assumption of dissipationless chains is clearly too crude to be compared with realistic systems for which a finite resistivity is induced by the umklapp interactions.<sup>23</sup>

In this work, we examine the effect of umklapp scattering on the  $T$  dependence of the Hall coefficient in quasi-1D conductors. We consider 1/2-filled 1D chains and compute  $R_H(T)$  to leading order in the umklapp scattering using the memory function approach.<sup>24,25</sup> We find that the umklapp processes induce a  $T$ -dependent correction to the free-fermion value, and this correction decreases with increasing temperature as a power law with an exponent depending on interactions (Fig. 2). We discuss the implications for quasi-1D compounds.

### II. MODEL AND METHOD

Our model is sketched in Fig. 1. We consider 1D chains coupled by a hopping amplitude  $t_\perp$  supposedly small compared to the in-chain kinetic energy. The usual LL model of the 1D chains assumes that the electrons have a linear dispersion with a velocity  $v_F$ . For a strictly linear band, however, the Hall coefficient vanishes identically, owing to particle-hole symmetry. A band curvature close to the Fermi momenta  $\pm k_F$  is thus necessary to get a finite  $R_H$ . We therefore take for the 1D chains of Fig. 1 the dispersion

$$\xi_\pm(k) = \pm v_F(k \mp k_F) + \alpha(k \mp k_F)^2. \quad (1)$$

The upper (lower) sign corresponds to right- (left-) moving electrons. Equation (1) can be regarded as the minimal

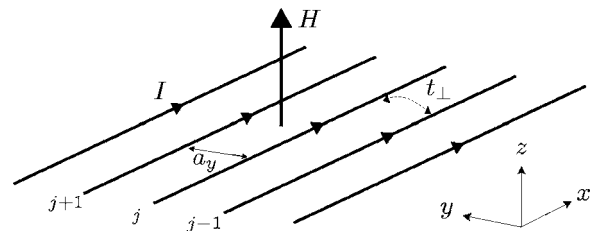


FIG. 1. Schematics of the model. The chains and the current  $I$  go along the  $x$  axis, the magnetic field  $H$  is applied along the  $z$  axis, and the Hall voltage is measured along the  $y$  axis.

model which gives rise to a Hall effect, while retaining most of the formal simplicity of the original LL theory, and its wide domain of validity. In particular, this model is clearly sufficient at low temperatures (compared to the electron bandwidth) since then only electrons close to the Fermi points contribute to the conductivities.

Our purpose is to treat the umklapp term perturbatively. We express the Hamiltonian as  $\mathcal{H}_0 + \mathcal{H}_u$ , where  $\mathcal{H}_u$  is the umklapp scattering term and  $\mathcal{H}_0$  reads

$$\begin{aligned} \mathcal{H}_0 = & \int dx \sum_{j\sigma} [v_F \psi_{j\sigma}^\dagger \tau_3 (-i\partial_x) \psi_{j\sigma} - \alpha \psi_{j\sigma}^\dagger \partial_x^2 \psi_{j\sigma} \\ & + g_2 \psi_{j\sigma R}^\dagger \psi_{j\sigma R} \psi_{j\sigma L}^\dagger \psi_{j\sigma L} - t_\perp (\psi_{j\sigma}^\dagger \psi_{j+1,\sigma} e^{-ieA_{j,j+1}} + \text{H.c.})]. \end{aligned} \quad (2)$$

In Eq. (2),  $j$  is the chain index,  $\tau_3$  is a Pauli matrix, and  $A_{j,j'} = \int_j^{j'} \mathbf{A} \cdot d\mathbf{l}$ . We choose the Landau gauge  $A_y = Hx$ , such that  $A_{j,j+1} = Hxa_y$ , with  $a_y$  the interchain spacing.  $\psi^\dagger = (\psi_R^\dagger, \psi_L^\dagger)$  is a two-component vector composed of right- and left-moving electrons. The second term in Eq. (2) is the band curvature, the third term is the forward scattering, and the last term corresponds to the coupling between the chains. In Eq. (2), we have omitted the backscattering terms ( $g_1$  processes) which are, for spin rotationally invariant systems, marginally irrelevant.<sup>9</sup> We therefore take  $g_{1\perp} = g_{1\parallel} = 0$ . At 1/2 filling, the umklapp term reads

$$\mathcal{H}_u = \frac{g_3}{2} \int dx \sum_{j\sigma} (\psi_{j\sigma R}^\dagger \psi_{j,-\sigma R}^\dagger \psi_{j\sigma L} \psi_{j,-\sigma L} + \text{H.c.}). \quad (3)$$

It corresponds to a process where two electrons with opposite spins change direction by absorbing a momentum  $4k_F$  from the lattice.

The Hall resistivity  $\rho_{yx}$  relates to the conductivity tensor  $\sigma_{\mu\nu}$  through

$$\rho_{yx} = \frac{\sigma_{xy}}{\sigma_{xx}\sigma_{yy} + \sigma_{xy}^2}. \quad (4)$$

Here, we calculate  $\rho_{yx}$  using a memory function approach.<sup>24</sup> One rewrites the conductivity tensor in terms of a memory matrix  $\mathbf{M}(\omega)$  as

$$\boldsymbol{\sigma}^T(\omega) = i\{\omega\mathbf{1} + \boldsymbol{\chi}(0)[\boldsymbol{\Omega} + i\mathbf{M}(\omega)]\boldsymbol{\chi}^{-1}(0)\}^{-1}\boldsymbol{\chi}(0), \quad (5)$$

where  $\boldsymbol{\sigma}^T$  denotes the transpose of  $\boldsymbol{\sigma}$ . The advantage provided by the memory function is the possibility to make finite-order perturbation expansions which are singular for the conductivities due to their resonance structure.<sup>24</sup> This formalism is especially useful for LL (Ref. 23) and was also used to estimate the Hall coefficient in the two-dimensional Hubbard model.<sup>5</sup>  $\boldsymbol{\chi}(0)$  is a diagonal matrix composed of the diamagnetic susceptibilities in each direction,

$$\boldsymbol{\chi}(0) = \begin{pmatrix} \chi_x(0) & 0 \\ 0 & \chi_y(0) \end{pmatrix},$$

with

$$\chi_\mu(0) = - \int dx \sum_j \left\langle \frac{\partial^2 \mathcal{H}}{\partial A_\mu^2(x)} \right\rangle \Big|_{A^{\text{el}}=0}. \quad (6)$$

The thermodynamic average  $\langle \dots \rangle_0$  is taken with respect to  $\mathcal{H}_0$  and  $\mathcal{H}$  is the Hamiltonian of Eq. (2) in the presence of electric and magnetic fields,  $\mathbf{A} = \mathbf{A}^{\text{el}} + \mathbf{A}^{\text{mag}}$ . The frequency matrix  $\boldsymbol{\Omega}$  in Eq. (5) is defined in terms of the equal-time current-current correlators as<sup>5</sup>

$$\Omega_{\mu\nu} = \frac{1}{\chi_\mu(0)} \langle [J_\mu, J_\nu] \rangle. \quad (7)$$

From Eq. (5), one can directly express the memory matrix  $\mathbf{M}$  in terms of the conductivity tensor. In the following, we will only need the off-diagonal term  $M_{xy}$  given by

$$iM_{xy}(\omega) = \frac{i\chi_y(0)\sigma_{xy}(\omega)}{\sigma_{xx}(\omega)\sigma_{yy}(\omega) + \sigma_{xy}^2(\omega)} - \Omega_{xy}. \quad (8)$$

It is then straightforward to rewrite the Hall coefficient  $R_H = \rho_{yx}/H$  as

$$R_H(\omega) = \frac{1}{i\chi_y(0)} \lim_{H \rightarrow 0} \frac{\Omega_{xy} + iM_{xy}(\omega)}{H}. \quad (9)$$

### III. RESULTS

From Eqs. (2) and (6), we obtain the longitudinal and transverse diamagnetic terms as

$$\chi_x(0) = - \frac{2e^2 v_F}{\pi a_y}, \quad (10a)$$

$$\chi_y(0) = - 2e^2 t_\perp a_y^2 \int dx \langle \psi_{0\uparrow}^\dagger(x) \psi_{1\uparrow}(x) e^{-ieHa_y x} + \text{H.c.} \rangle \quad (10b)$$

For evaluating the frequency matrix, we write down the current operators

$$J_x = e \int dx \sum_{j\sigma} \psi_{j\sigma}^\dagger(x) [v_F \tau_3 + 2\alpha(-i\partial_x) \tau_1] \psi_{j\sigma}(x), \quad (11a)$$

$$J_y = -iet_\perp a_y \int dx \sum_{j\sigma} (\psi_{j\sigma}^\dagger \psi_{j+1,\sigma} e^{-ieA_{j,j+1}} - \text{H.c.}). \quad (11b)$$

The expression resulting from Eq. (7) for the frequency matrix is then

$$\Omega_{xy} = -i \frac{2\pi e a t_\perp a_y^3 H}{v_F} \int dx \langle \psi_{0\uparrow}^\dagger(x) \psi_{1\uparrow}(x) e^{-ieHa_y x} + \text{H.c.} \rangle. \quad (12)$$

At this stage, we can already evaluate the high-frequency limit of  $R_H$  because the memory matrix vanishes as  $1/\omega^2$  (Refs. 5 and 24) and thus  $M_{xy}$  drops from Eq. (9) if  $\omega \rightarrow \infty$ .

The effects of the umklapp disappear at high frequency, and in this limit one recovers from Eqs. (9)–(12) the result obtained for dissipationless chains,<sup>22</sup> namely, that the Hall coefficient equals the band value  $R_H^0$ ,

$$R_H(\infty) = R_H^0 = \frac{\pi\alpha a_y}{e v_F}. \quad (13)$$

In the definition of the memory matrix, Eq. (8), we can ignore the terms of order  $H^2$  which do not contribute to  $R_H$  in Eq. (9). Furthermore we express the conductivities in terms of current susceptibilities as  $\sigma_{\mu\nu} = \frac{i}{\omega} [\chi_{\mu\nu} - \delta_{\mu\nu} \chi_\mu(0)]$ , which leads to

$$iM_{xy}(\omega) = \frac{\omega \chi_y(0) \chi_{xy}(\omega)}{[\chi_x(0) - \chi_{xx}(\omega)][\chi_y(0) - \chi_{yy}(\omega)]} - \Omega_{xy}. \quad (14)$$

We rewrite this formula at intermediate frequencies, such that  $|\chi_{\mu\mu}(\omega)/\chi_\mu(0)|$  is small. In this expansion we use the equation of motion of the susceptibilities<sup>24</sup> as well as the relation  $[\mathcal{H}_0, J_\mu] = -\Omega_{\nu\mu} J_\nu$ . For  $\mu=x$ , the latter equation is exactly satisfied in our model, while for  $\mu=y$ , it is only verified in the continuum limit  $a_y \rightarrow 0$ . The resulting expression of the memory matrix is

$$iM_{xy}(\omega) \approx -\frac{1}{\chi_x(0)} \frac{\langle K_x; K_y \rangle - \langle K_x; K_y \rangle_{\omega=0}}{\omega}, \quad (15)$$

where  $K_\mu$  are the *residual force* operators defined as the part of the Hamiltonian which in the absence of magnetic field does not commute with the currents, i.e.,  $K_\mu = [\mathcal{H}_0, J_\mu]$ , and  $\langle K_x; K_y \rangle$  stands for the retarded correlation function of the operators  $K_\mu$ . The terms omitted in Eq. (15) are either of second order in  $|\chi_{\mu\mu}(\omega)/\chi_\mu(0)|$ , of second order in  $H$ , or vanish in the continuum limit  $a_y \rightarrow 0$ . Using Eqs. (3) and (11), we find

$$K_x = 2e v_F g_3 \int dx \sum_{j\sigma} (\psi_{j\sigma R}^\dagger \psi_{j-\sigma R}^\dagger \psi_{j-\sigma L} \psi_{j\sigma L} - \text{H.c.}), \quad (16a)$$

$$K_y = iet_\perp g_3 a_y \int dx \sum_{j\sigma} \sum_{b=L,R} [e^{-ieA_{j,j+1}} (\psi_{j\sigma b}^\dagger \psi_{j-\sigma b}^\dagger \psi_{j-\sigma,-b} \psi_{j\sigma,-b} \\ \times \psi_{j+1,\sigma,-b} - \psi_{j-1,\sigma b}^\dagger \psi_{j-\sigma b}^\dagger \psi_{j-\sigma,-b} \psi_{j\sigma,-b}) + \text{H.c.}]. \quad (16b)$$

Note that each of the  $K$ 's is of first order in  $g_3$ ; hence,  $M_{xy}$  is of order  $g_3^2$ . The quantity  $\langle K_x; K_y \rangle$  entering Eq. (15) is the real-frequency, long-wavelength limit of the correlator, which we evaluate as

$$\langle K_x; K_y \rangle = - \int_0^\beta d\tau e^{i\Omega\tau} \langle T_\tau K_x(\tau) K_y(0) \rangle \Big|_{i\Omega \rightarrow \omega + i0^+}. \quad (17)$$

It is easy to prove that  $\langle K_x; K_y \rangle$  vanishes for  $H=0$  or  $\alpha=0$  by applying spatial inversion and particle-hole symmetry, respectively. Retaining only leading-order terms in  $t_\perp$  and  $\alpha$ , the first nonvanishing contribution in Eq. (17) is of order

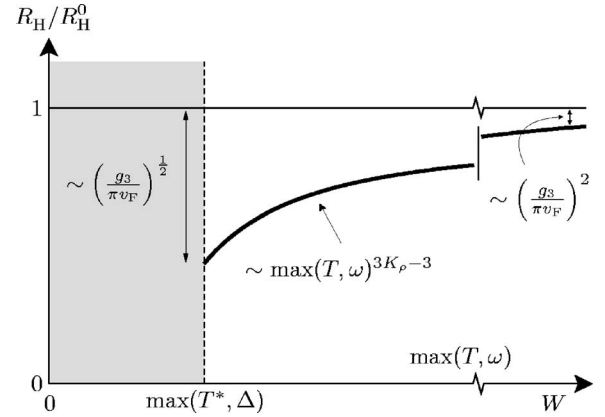


FIG. 2. Correction of the high-temperature/high-frequency Hall coefficient  $R_H$  by the umklapp scattering in weakly coupled Luttinger liquids.  $R_H^0$  is the value of the Hall coefficient in the absence of umklapp scattering, Eq. (13), and  $W$  is the electron bandwidth. Our approach breaks down below some crossover scale (dashed line, see text). In this figure, we have assumed that  $A$  in Eq. (19) is negative.

$\alpha t_\perp^2 g_3^2 H$  and involves three spatial and three time integrations, which we were not able to perform analytically. Based on a scaling analysis, we can nevertheless extract the temperature (or frequency) dependence of this contribution (see Appendix A), which yields

$$\frac{1}{i\chi_x(0)\chi_y(0)} \frac{\langle K_x; K_y \rangle}{\omega H} \sim \alpha g_3^2 \max(\omega, T)^{3K_\rho-3}, \quad (18)$$

where  $K_\rho$  is the LL parameter in the charge sector. In the absence of interactions, we have  $K_\rho=1$ , while  $K_\rho < 1$  ( $K_\rho > 1$ ) for repulsive (attractive) interactions. If the interactions are strong and repulsive ( $K_\rho \ll 1$ ), the exponent in Eq. (18) changes due to the contraction<sup>9</sup> of the operators in  $K_x$  and  $K_y$ , which gives the relevant power law in this case.

Together with Eqs. (15) and (9), Eq. (18) leads to our final expression for the Hall coefficient,

$$R_H = R_H^0 \left[ 1 + A \left( \frac{g_3}{\pi v_F} \right)^2 \left( \frac{T}{W} \right)^{3K_\rho-3} \right], \quad (19)$$

with  $W$  the electron bandwidth. Equation (19) shows that in  $1/2$ -filled quasi-1D systems, the umklapp scattering changes the absolute value of the Hall coefficient with respect to the band value, which is only recovered at high temperature or frequency. Note that Eq. (19) also describes the frequency dependence of  $R_H$ , provided that  $T$  is replaced by  $\omega$ . The backscattering term  $g_1$  (neglected here) could possibly give rise to multiplicative logarithmic corrections to the power law in Eq. (19).<sup>9</sup> The sign of the dimensionless prefactor  $A$  can only be determined through a complete evaluation of  $\langle K_x; K_y \rangle$  in Eq. (17) and is for the time being unknown. The available experimental data are consistent with Eq. (19) if one assumes that  $A$  is negative (see below), as illustrated in Fig. 2.

Equation (19) would imply that in the noninteracting limit  $K_\rho \rightarrow 1$  ( $g_2 \rightarrow 0$ ), the correction to the Hall coefficient behaves as  $\log(T/W)$ . In order to check this prediction, we

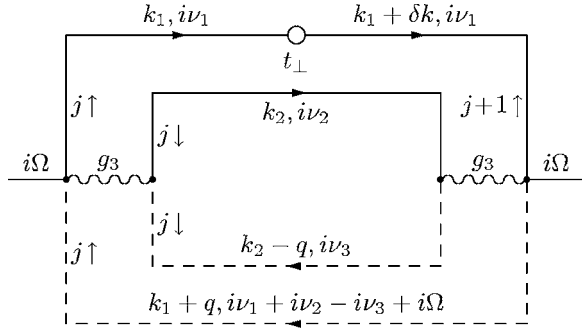


FIG. 3. Example of a diagram appearing in Eq. (17) at first order in  $t_{\perp}$  and for  $g_2=0$ . The full (dashed) lines correspond to free right-(left-) moving fermions,  $j$  is the chain index, and the arrows represent up and down spins. The magnetic field increases the momentum of the electron by  $\delta k = eHa_y$ .

have evaluated the correlator in Eq. (17) for  $g_2=0$ . The corresponding diagram sketched in Fig. 3 involves three frequency-momentum integrations, which in this case can be fully worked out analytically (see Appendix B). The resulting expression of  $R_H$  at zero frequency is ( $T \ll W$ )

$$R_H = R_H^0 \left[ 1 + \frac{1}{8} \left( \frac{g_3}{\pi v_F} \right)^2 \log \left( \frac{T}{W} \right) \right], \quad (20)$$

consistently with Eq. (19). For noninteracting electrons, though, we see that the relative correction induced by the 1/2-filling umklapp is positive at  $T < W$ . Since all properties are analytic in  $K_{\rho}$ , we can also deduce from Eqs. (19) and (20) that  $A$  tends to  $[24(1-K_{\rho})]^{-1}$  in the limit  $K_{\rho} \rightarrow 1$ . Note that Eq. (20) would also apply to models in which  $g_2 \sim g_3$ , such as the Hubbard model, while Eq. (19) is valid only when  $g_3 \ll g_2$ .

#### IV. DISCUSSION AND CONCLUSION

The result of Eq. (19) shows that in 1/2-filled quasi-1D systems, the umklapp processes induce a correction to the free-fermion value (band value) of the Hall coefficient  $R_H$ , which depends on temperature as a power law with an exponent depending on interactions. At high temperatures or frequencies,  $R_H$  approaches the band value, as shown in Fig. 2, implying that any fitting of experimental data must be done with respect to the value of  $R_H$  at high temperature or frequency.

To study the range of validity of our result, one must consider that at low temperature the quasi-1D systems generally enter either an insulating state characterized by a Mott gap  $\Delta$  or a coherent two- or three-dimensional phase below a temperature  $T^*$  controlled by  $t_{\perp}$  (Ref. 10); in either case, our model of weakly coupled LL is no longer valid, as illustrated in Fig. 2. The variations of  $R_H$  below  $\max(T^*, \Delta)$  can be very pronounced and depend strongly on the details of the materials. When the ground state is insulating, for instance,  $R_H(T)$  is expected to go through a minimum and diverge like  $e^{\Delta/T}$  as  $T \rightarrow 0$ , reflecting the exponentially small carrier density. Other behaviors, such as a change of sign due to the formation of an ordered state or nesting in the FL regime,<sup>4</sup> can also

occur. The validity of Eq. (19) is therefore limited to the LL domain:  $\max(T^*, \Delta) < \max(T, \omega) \ll W$ .

For the case  $\Delta > T^*$ , we can estimate the change of  $R_H$  with respect to  $R_H^0$  at the crossover scale  $\Delta$ , for a system with  $g_3 \ll U$ , where  $U$  is the Coulomb repulsion. The umklapp-induced Mott gap in 1/2-filled systems is given<sup>9</sup> by  $\Delta/W \sim [g_3/(\pi v_F)]^x$  with  $x = [2(1-K_{\rho})]^{-1}$ . We thus find that the largest correction is  $\sim [g_3/(\pi v_F)]^{1/2}$  and has a universal exponent. On the other hand,  $R_H$  approaches the asymptotic value  $R_H^0$  quite slowly, and according to Eq. (19) a correction of  $\sim [g_3/(\pi v_F)]^2$  still exists at temperatures comparable to the bandwidth.

The available Hall data in the TM family and in the geometry of the present analysis<sup>18,20</sup> show a weak correction to the free-fermion value which depends on temperature. Some attempts to fit this behavior to a power law have been reported.<sup>18</sup> However, the analysis was performed by fitting  $R_H(T)$  to a power law starting at zero temperature. As explained above, the proper way to analyze the Hall effect in such quasi-1D systems is to fit the deviations from the band value starting from the high-temperature limit. It would be interesting to check whether a new analysis of the data would provide good agreement with our results. However, in these compounds both 1/4-filling and 1/2-filling umklapp processes are present. For the longitudinal transport, the 1/4-filling contribution dominates.<sup>10</sup> For the Hall effect, the analysis in the presence of 1/4-filling umklapp is considerably more involved, but a crude evaluation of the scaling properties of the corresponding memory matrix gives also a weak power-law correction with an exponent of  $2 - 16K_{\rho} + (K_{\rho} + K_{\rho}^{-1})/2$ , and thus similar effects, regardless of the dominant umklapp. The observed data are thus consistent with the expected corrections coming from LL behavior. However, more work, both experimental and theoretical, is needed for the TM family because of this additional complication and to understand the data in a different geometry where no temperature dependence is observed.<sup>19</sup>

Our result [Eq. (19)] is, however, directly relevant for 1/2-filled organic conductors such as (TTM-TTP)I<sub>3</sub> and (DMTSA)BF<sub>4</sub>.<sup>26</sup> Hall measurements for these compounds still remain to be performed. Comparison of the Hall effect in these compounds with the one in 1/4-filled nondimerized systems<sup>11,27</sup> for which only 1/4-filling umklapp is present could also help in understanding the dominant processes for the TM family.

#### ACKNOWLEDGMENT

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#### APPENDIX A: SCALING ANALYSIS

In order to establish Eq. (18), we evaluate the correlator  $\langle K_x; K_y \rangle$  to first order in  $\alpha$  and  $t_{\perp}$ . Let us denote by  $\mathcal{H}_{\alpha}$  the curvature [second term in Eq. (2)], by  $\mathcal{H}_{\perp}$  the interchain hopping [fourth term in Eq. (2)], and by  $\mathcal{H}_{1D}$  the remaining part of the Hamiltonian,  $\mathcal{H}_{1D} = \mathcal{H}_0 - \mathcal{H}_{\alpha} - \mathcal{H}_{\perp}$ . Standard perturbation theory yields



$$\begin{aligned} \langle K_x; K_y \rangle = & - \int d\tau e^{i\Omega\tau} \int d\tau_1 d\tau_2 \langle T_\tau K_x(\tau) K_y(0) \\ & \times \mathcal{H}_\perp(\tau_1) \mathcal{H}_\alpha(\tau_2) \rangle, \end{aligned} \quad (\text{A1})$$

where the average has to be taken with respect to  $\mathcal{H}_{1D}$ . The latter corresponds to a 1D chain and can be easily bosonized,<sup>9</sup>

$$\mathcal{H}_{1D} = \mathcal{H}_\rho + \mathcal{H}_\sigma, \quad (\text{A2})$$

$$\mathcal{H}_\nu = \int \frac{dx}{2\pi} \left\{ u_\nu K_\nu [\nabla \theta_\nu(x)]^2 + \frac{u_\nu}{K_\nu} [\nabla \phi_\nu(x)]^2 \right\}, \quad (\text{A3})$$

where  $\nu = \rho(\sigma)$  denotes the charge (spin) degrees of freedom,  $u_\nu$  is a velocity,  $K_\nu$  a dimensionless parameter, and  $\theta_\nu$  and  $\phi_\nu$  are bosonic fields. In our case, we have  $K_\rho < 1$  and  $K_\sigma = 1$ . The fields  $\theta_\nu$  and  $\phi_\nu$  and the fermionic fields  $\psi$  are related by

$$\psi_{\sigma b}(x) = \frac{e^{ibk_F x}}{\sqrt{2\pi a}} e^{-i(\sqrt{2})\{b\phi_\rho(x) - \theta_\rho(x) + \sigma[b\phi_\sigma(x) - \theta_\sigma(x)]\}} \quad (\text{A4})$$

with  $b = +1(-1)$  corresponding to right-(left-) moving fermions, and  $a$  a cutoff of the order of the in-chain lattice spacing. With the help of Eq. (A4), we bosonize each operator in Eq. (A1),

$$K_x = \frac{4iev_F g_3}{(2\pi a)^2} \int dx \sum_{j\sigma} \sin[\sqrt{8}\phi_\rho(x)]_j, \quad (\text{A5})$$

$$\begin{aligned} K_y = & \frac{iet_\perp g_3 a_y}{(2\pi a)^2} \sum_{\langle j, j' \rangle} \sum_{\sigma b} \int dx \epsilon_{jj'} \\ & \times (e^{-ieHa_y x} e^{i(\sqrt{2})\{3b\phi_\rho(x) - \epsilon_{jj'}\theta_\rho(x) - \sigma[b\phi_\sigma(x) + \epsilon_{jj'}\theta_\sigma(x)]\}}_j \\ & \times e^{i(\sqrt{2})\{b\phi_\rho(x) + \epsilon_{jj'}\theta_\rho(x) + \sigma[b\phi_\sigma(x) + \epsilon_{jj'}\theta_\sigma(x)]\}}_{j'} + \text{H.c.}), \end{aligned} \quad (\text{A6})$$

where  $j$  and  $j'$  are neighboring chains and  $\epsilon_{jj'} = (-1)^{j'-j}$ . For the hopping term, we have

$$\begin{aligned} \mathcal{H}_\perp = & - \frac{t_\perp}{2\pi a} \sum_{j\sigma b} \int dx \\ & \times (e^{-ieHa_y x} e^{i(\sqrt{2})\{b\phi_\rho(x) - \theta_\rho(x) + \sigma[b\phi_\sigma(x) - \theta_\sigma(x)]\}}_j \\ & \times e^{-i(\sqrt{2})\{b\phi_\rho(x) - \theta_\rho(x) + \sigma[b\phi_\sigma(x) - \theta_\sigma(x)]\}}_{j+1} + \text{H.c.}), \end{aligned} \quad (\text{A7})$$

and for the band curvature term, we take<sup>28</sup>

$$\mathcal{H}_\alpha = \frac{\alpha}{2\pi a} \int dx \frac{(\nabla \phi_\rho)^3}{2}. \quad (\text{A8})$$

Next, we use the identity<sup>9</sup>

$$\begin{aligned} \left\langle \prod_n e^{i[A_n \phi(r_n) + B_n \theta(r_n)]} \right\rangle = & \exp \left[ -\frac{1}{2} \sum'_{n < m} - (A_n A_m K \right. \\ & + B_n B_m K^{-1}) F_1(r_n - r_m) \\ & \left. + (A_n B_m + B_n A_m) F_2(r_n - r_m) \right], \end{aligned} \quad (\text{A9})$$

where  $r \equiv (x, u\tau)$ , the notation  $\Sigma'$  means that the sum is restricted to those terms for which  $\sum_n A_n = \sum_n B_n = 0$ , and  $F_{1,2}$  are universal functions. The resulting expression for the correlator in Eq. (A1) is

$$\begin{aligned} \langle K_x; K_y \rangle \sim & H \int d^2 r d^2 r_1 d^2 r_2 e^{-3K_\rho F_1(r)} \\ & \times |r| e^{-K_\rho F_1(r-r_1)} e^{(1/2)(K_\rho - K_\rho^{-1}-2)F_1(r_1)} \frac{1}{|r_2|^3}. \end{aligned} \quad (\text{A10})$$

The factor  $|r|$  results from the linearization in  $H$ , and we have discarded all factors involving the  $F_2$  function, since they correspond to angular integrals of the  $r$  variables and therefore do not contribute to the scaling dimension. At distances much larger than the cutoff  $a$ , we have  $e^{-AF_1(r)} \sim (a/|r|)^A$ , and therefore we find the high temperature, high-frequency behavior as

$$\langle K_x; K_y \rangle \sim H \max(\omega, T)^{-3+4K_\rho - (1/2)(K_\rho - K_\rho^{-1})}. \quad (\text{A11})$$

We follow the same procedure for the diamagnetic term  $\chi_y(0)$ —however, at zeroth order in  $\alpha$  and  $H$ —and find

$$\chi_y(0) \sim \max(\omega, T)^{-1+(1/2)(K_\rho + K_\rho^{-1})}. \quad (\text{A12})$$

Combining these expressions and collecting the relevant prefactors, we deduce Eq. (18).

## APPENDIX B: HALL COEFFICIENT WITHOUT FORWARD SCATTERING

Here, we provide the derivation of Eq. (20), which gives  $R_H$  to leading order in  $g_3$  but in the absence of forward scattering ( $g_2=0$ ). Using Eqs. (9) and (13), we can express the zero-frequency Hall coefficient in terms of  $R_H^0$  and  $\text{Re } M_{xy}(i0^+)$ . We then perform a Kramers-Kronig transform, insert the free-fermion values of the diamagnetic susceptibilities,  $\chi_x(0) = -2e^2 v_F / (\pi a_y)$  and  $\chi_y(0) = -4e^2 t_\perp^2 a_y / (\pi v_F)$ , and use Eq. (15) to arrive at

$$\begin{aligned} R_H(0) = & R_H^0 \left[ 1 + \frac{v_F}{8e^3 \alpha t_\perp^2 a_y H} \right. \\ & \left. \times \int \frac{d\omega}{\omega^2} \text{Im}(i \langle K_x; K_y \rangle_0 |_{i\Omega \rightarrow \omega + i0^+}) \right], \end{aligned} \quad (\text{B1})$$

where  $\langle K_x; K_y \rangle_0$  is to be evaluated to first order in  $H$ . The  $\omega=0$  term in Eq. (15) disappears due to the principal part in the  $\omega$  integral in Eq. (B1). From Eq. (16), one sees that  $\langle K_x; K_y \rangle_0$  involves eight fermion fields and can be represented by diagrams like the one displayed in Fig. 3. There are 32 different diagrams, but all of them can be expressed in terms of only one function  $A(i\Omega, H)$ , whose expression is given by the diagram in Fig. 3. We thus obtain

$$R_H(0) = R_H^0 \left\{ 1 - \frac{4v_F^2 g_3^2}{e\alpha} \int \frac{d\omega}{\omega^2} \text{Im}[A'(\omega + i0^+) - A'(-\omega - i0^+)] \right\}, \quad (\text{B2})$$

where  $A'(i\Omega) = \partial A(i\Omega, H) / \partial H|_{H=0}$  and we have pulled all prefactors from Eq. (16), as well as a factor  $t_\perp$  from the diagram, out of the definition of function  $A$ . The explicit expression of  $A'$  is

$$A'(i\Omega) = \frac{e}{(2\pi)^3} \int dk_1 dk_2 dq \frac{d\xi_+(k_1)}{dk_1} \frac{1}{\beta^3} \sum_{\nu_1 \nu_2 \nu_3} \left[ \frac{1}{i\nu_1 - \xi_+(k_1)} \right]^3 \times \frac{1}{i\nu_2 - \xi_+(k_2)} \frac{1}{i\nu_3 - \xi_-(k_2 - q)} \times \frac{1}{i\nu_1 + i\nu_2 - i\nu_3 + i\Omega - \xi_-(k_1 + q)}. \quad (\text{B3})$$

The frequency summations in Eq. (B3) are elementary, and the various momentum integrals can also be evaluated analytically to first order in  $\alpha$ , yielding

$$R_H(0) = R_H^0 \left[ 1 - \frac{1}{16} \left( \frac{g_3}{\pi v_F} \right)^2 \int \frac{d\omega}{\omega} \frac{(\beta\omega/4)^2 - \sinh^2(\beta\omega/4)}{\tanh(\beta\omega/4) \sinh^2(\beta\omega/4)} \right]. \quad (\text{B4})$$

The remaining energy integral is divergent and must be regularized. Cutting the integral at the bandwidth  $W$  and assuming  $T \ll W$ , we obtain the asymptotic behavior given in Eq. (20).

<sup>1</sup>N. Ashcroft and N. Mermin, *Solid State Physics* (Saunders, Philadelphia, 1976).

<sup>2</sup>P. W. Anderson, Phys. Rev. Lett. **67**, 3844 (1991).

<sup>3</sup>B. S. Shastry, B. I. Shraiman, and R. R. P. Singh, Phys. Rev. Lett. **70**, 2004 (1993).

<sup>4</sup>V. M. Yakovenko, Synth. Met. **103**, 2202 (1999).

<sup>5</sup>E. Lange, Phys. Rev. B **55**, 3907 (1997).

<sup>6</sup>R. Shankar, Rev. Mod. Phys. **66**, 129 (1994).

<sup>7</sup>P. W. Anderson, *The Theory of Superconductivity in the High- $T_c$  Cuprates* (Princeton University Press, Princeton, NJ, 1997).

<sup>8</sup>F. D. M. Haldane, Phys. Rev. Lett. **45**, 1358 (1980).

<sup>9</sup>T. Giamarchi, *Quantum Physics in One Dimension* (Oxford University Press, Oxford, 2004).

<sup>10</sup>T. Giamarchi, Chem. Rev. (Washington, D.C.) **104**, 5037 (2004).

<sup>11</sup>D. Jérôme, Chem. Rev. (Washington, D.C.) **104**, 5565 (2004).

<sup>12</sup>M. S. Dresselhaus, G. Dresselhaus, and P. C. Eklund, *Science of Fullerenes and Carbon Nanotubes* (Academic, San Diego, CA, 1995).

<sup>13</sup>T. Stöferle, H. Moritz, C. Schori, M. Köhl, and T. Esslinger, Phys. Rev. Lett. **92**, 130403 (2004).

<sup>14</sup>A. Schwartz, M. Dressel, G. Grüner, V. Vescoli, L. Degiorgi, and T. Giamarchi, Phys. Rev. B **58**, 1261 (1998).

<sup>15</sup>T. Giamarchi, Physica B **230-232**, 975 (1997).

<sup>16</sup>J. Moser, M. Gabay, P. Aubin-Senzier, D. Jérôme, K. Bechgaard,

and J. Fabre, Eur. Phys. J. B **1**, 39 (1998).

<sup>17</sup>W. Henderson, V. Vescoli, P. Tran, L. Degiorgi, and G. Grüner, Eur. Phys. J. B **11**, 365 (1999).

<sup>18</sup>J. Moser, J. R. Cooper, D. Jérôme, B. Alavi, S. E. Brown, and K. Bechgaard, Phys. Rev. Lett. **84**, 2674 (2000).

<sup>19</sup>G. Mihaly, I. Kezsmarky, F. Zamborsky, and L. Forro, Phys. Rev. Lett. **84**, 2670 (2000).

<sup>20</sup>B. Korin-Hamzić, E. Tafra, M. Basletić, A. Hamzić, G. Untereiner, and M. Dressel, Phys. Rev. B **67**, 014513 (2003).

<sup>21</sup>B. Korin-Hamzić, E. Tafra, M. Basletić, A. Hamzić, and M. Dressel, Phys. Rev. B **73**, 115102 (2006).

<sup>22</sup>A. Lopatin, A. Georges, and T. Giamarchi, Phys. Rev. B **63**, 075109 (2001). In this work  $R_H^0$  was calculated for spinless fermions, which explains the factor of 2 difference with respect to Eq. (13).

<sup>23</sup>T. Giamarchi, Phys. Rev. B **44**, 2905 (1991).

<sup>24</sup>W. Götze and P. Wölfle, Phys. Rev. B **6**, 1226 (1972).

<sup>25</sup>G. León and T. Giamarchi, J. Low Temp. Phys. **142**, 315 (2006).

<sup>26</sup>T. Mori, Chem. Rev. (Washington, D.C.) **104**, 4947 (2004).

<sup>27</sup>K. Heuzé, M. Fourmigué, P. Batail, C. Coulon, R. Clérac, E. Canadell, P. Auban-Senzier, S. Ravy, and D. Jérôme, Adv. Mater. (Weinheim, Ger.) **15**, 1251 (2003).

<sup>28</sup>F. D. M. Haldane, J. Phys. C **14**, 2585 (1981).