Angle-dependent optical transmission through a narrow slit in a thick metal film

R. Gordon*
Department of Electrical and Computer Engineering, University of Victoria, Victoria, British Columbia, Canada V8W 3P6
(Received 3 April 2007; published 4 May 2007)

The theory of optical transmission through a narrow slit in a thick metal screen was formulated previously for normal incidence [Y. Takakura, Phys. Rev. Lett. **86**, 5601 (2001)]. In this work, the theory is generalized to include the parallel angle dependence of the optical transmission for angles in the plane of the slit. A fully analytic formulation of the parallel angle-dependent reflection coefficient and transmission is provided. It is shown that the slit width has a strong influence on the angle tuning of the Fabry-Pérot resonances, which is caused by the angle dependence of the reflection coefficient. The theory agrees quantitatively with recent experiments and predicts angle-dependent variations that may readily be observed in future experiments over a wider angle and wavelength range.

DOI: 10.1103/PhysRevB.75.193401 PACS number(s): 78.20.Ci, 42.79.Gn, 42.82.Et

A slit in a metal screen is a fundamental system to demonstrate the rich physics of electromagnetics in subwavelength structures, including propagation and impedance mismatch, transmission resonances, optical vortex formation, surface wave excitation, transitions to the geometric limit of transmission, and angle tuning. In 2001, Takakura presented a theory which showed that the transmission resonances from lowest-order TM mode of subwavelength slit in a thick metal screen at normal incidence are shifted from the usual Fabry-Pérot resonances as the slit is narrowed.[1] The resonance shift was observed in the microwave regime[2] and further studies in the microwave regime showed that finite conductance influences the resonances.[3] In the optical regime, the metal properties influence not only the phase of reflection but also the propagation constant of light within the slit.[4–6] Furthermore, surface plasmons may be generated at the edge of a slit for the TM polarization.[7] For the TE polarization, enhanced transmission from waveguiding and the formation of optical vortices have been studied.[8]

The dependence of incidence angle has been studied theoretically for angles in the plane perpendicular to the slit.[9] That work showed how angle of incidence influences the coupling to the modes within the slit. The perpendicular angle, however, does not tune the Fabry-Pérot resonances, and so it is entirely distinct from the approach and the results presented here. For the parallel angle, there have been experimental results to show angle tuning of the resonances.[10] In that work, numerical methods showed an offset in the resonance frequency from the simple Fabry-Pérot resonances. The numerical calculations agreed well with the phase offset; however, no theory was given to explain the angle dependence. The parallel angle tuning of that work is well represented by the theory presented here. The small wavelength range and angle range used in that work did not show the strong parallel angle dependence that appears from large changes to the phase of reflection at large angles.

In this work, the angle tuning of the TM resonances of a slit is given for angles in the plane of the slit. This theory shows the effects that arise from the strong angle dependence of the phase of reflection within the slit. An analytic expression is found for the transmission and reflection coefficients as a function of the angle of incidence and slit width, which is a significant generalization of the normal incidence case that was treated previously.[1]

Figure 1 shows a schematic of the geometry under consideration; an infinitely long slit of width a in a metal screen of thickness l. The theoretical approach extends the analysis of resonances in a subwavelength slit, where the parallel components of the electric and magnetic fields are matched at the interface between the slit and free space. Considering only a single mode within the slit gives accurate results in the subwavelength regime (i.e., for a slit width significantly less than half the wavelength of light in a thick metal film).[1,7] The parallel angle, θ, dependence is included in the same manner that has been used previously for the analysis of surface-plasmon reflection.[11] A unique feature of the TM analysis is that there is no Fresnel refraction between the angle of incidence and the angle of transmission because the propagation constant of the lowest-order mode matches the free-space propagation constant.

Using this method of analysis, for an incident plane wave of unity electric-field strength, gives the transmission into the lowest-order TM mode in the slit:

$$t(\theta) = \frac{2\cos\theta}{\cos\theta + I(\theta)}, \tag{1}$$

and the reflection of the lowest-order TM mode in the slit at the free-space interface:

$$r(\theta) = \frac{\cos\theta - I(\theta)}{\cos\theta + I(\theta)}, \tag{2}$$

where θ is the angle of incidence and $I(\theta)$ is given by

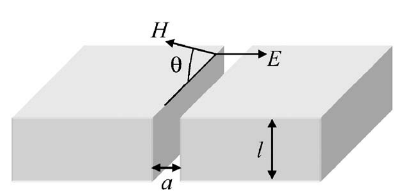

FIG. 1. Schematic geometry of TM transmission through a subwavelength slit in a thick perfect electric conductor for angled incidence.

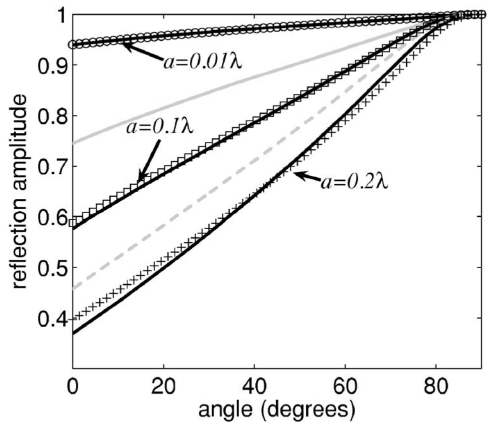

FIG. 2. Reflection coefficient of the fundamental TM mode within a slit for different angles of incidence and various slit widths. Analytic results are shown with lines and numerical integration results are shown with circles, squares, and crosses for slit widths of $a=0.01\lambda$, 0.1λ, and 0.2λ. The gray solid and dashed lines are for slit widths $a=0.05\lambda$ and 0.15λ.

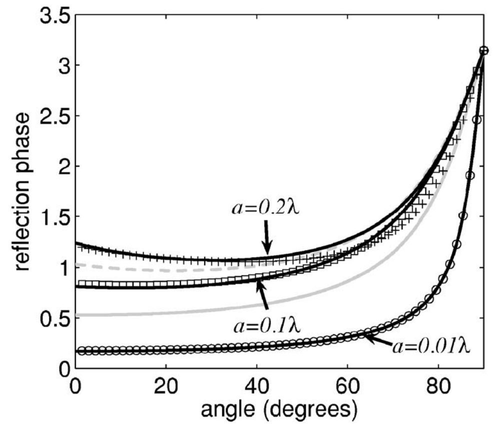

FIG. 3. Phase of reflection of the fundamental TM mode within a slit for different angles of incidence and various slit widths. The same labeling is used as in Fig. 2.

$$I(\theta) = \int_{-\infty}^{\infty} \frac{\sqrt{\cos^2\theta - u^2}}{1-u^2}\frac{\sin^2(\pi u a)}{\pi^2 u^2 a}du, \tag{3}$$

with slit width, a, normalized to the wavelength of the incident light. The total transmission through the slit is given by

$$T(\theta) = \frac{|t(\theta)|^2(1-|r(\theta)|^2)}{|1-r(\theta)^2\exp(i4\pi l\cos\theta)|^2}, \tag{4}$$

where l is the thickness of the metal normalized to the wavelength of light.

While $I(\theta)$ may be integrated numerically with standard techniques, an accurate analytical approximation may be formulated in the subwavelength regime:

$$I(\theta) \simeq a\pi(1-\sin\theta) + 2ai\left[\frac{\ln(\cos\theta)}{\sqrt{\cos\left(\frac{\pi}{2}\cos\theta\right)}} + \gamma\sin\theta + \ln\left(\frac{2\pi a}{\cos\theta}\right) - \frac{3}{2}\right], \tag{5}$$

where γ is the Euler-Mascheroni constant. This expression is found by noting the even symmetry of the integral in Eq. (3) and splitting it into three domains along the positive axis: $[0,\cos\theta]$, $[\cos\theta,(\cos\theta)^{-1}]$, and $[(\cos\theta)^{-1},\infty]$. The integration over the first and second domains may be performed directly in the subwavelength limit $a \ll \lambda$, where the wavelength dependence is included explicitly for general use; however, to compare with the above analysis, set $\lambda=1$. The integration over the third domain is given by a truncated series expansion, which agrees to within 5% of numerical integration results for both the real and imaginary parts when $a<0.1\lambda$. Due to the truncation in the series expansion in the third domain of the integral, it is most accurate for the angles where $\pi a < \cos\theta$. As required, this formulation reduces to the normal incidence result for $\theta=0$.[1]

Figures 2 and 3 show the reflection amplitude and phase as a function of incidence angle for slit widths of $a=0.01\lambda$, 0.05λ, 0.1λ, 0.15λ, and 0.2λ. The results of numerical integration are presented for comparison for the cases of $a=0.01\lambda$, 0.1λ, and 0.2λ. The analytic results deviate from the numerical integration results at large angles for the phase of reflection $a=0.2\lambda$. It should be noted that all the results presented here, including the numerical integration, are accurate only in the subwavelength regime $a \ll \lambda$, so all cases where $a>0.1\lambda$ are expected to be less accurate.

Figure 4 shows the results for transmission through 80 μm slit in a 19.58 mm thick metal film for wavelengths between 5 and 7.5 mm. For this case, the slit width is $\leqslant 0.01\lambda$, clearly within the subwavelength regime. The ranges of angles and of wavelengths were chosen to be directly comparable with recent experiments.[10] The theoretical results obtained here are in good agreement with those experimental results. In particular, the angle tuning and the frequency offset of the Fabry-Pérot resonances agree well with those seen in Figs. 2 and 5(a) of that work.

To observe the strong angle-tuning dependence on the width of the slit, a wider range of the slit-width and incident angle parameters should be considered. Figures 5 and 6

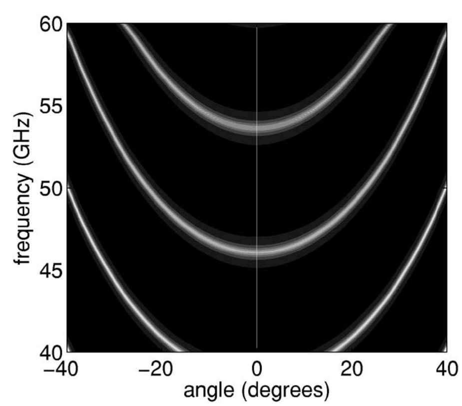

FIG. 4. Transmission through an 80 μm slit in a 19.58 mm thick metal film for wavelengths between 5 and 7.5 mm and angles between 0° and 40°. These results show the angle tuning and they are in good quantitative agreement with recent experiments (Ref. 10).

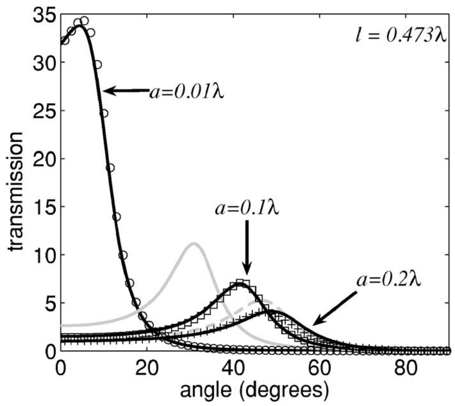

FIG. 5. Angle dependence of optical transmission through a metal film 0.473λ thick for various slit widths. The same labeling is the same as in Fig. 2.

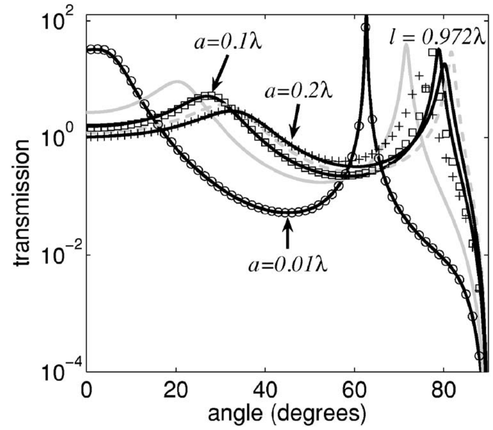

FIG. 6. Same as Fig. 5, but for a film thickness of 0.972λ. Logarithmic scale used for clarity.

shows the transmission through slits for film thicknesses of 0.473λ and 0.972λ and for various slit widths. It is clear that the Fabry-Pérot resonance angles are shifted by changing the slit width. In Fig. 5, the shift in resonance angle is 40° when going from a slit width of 0.1λ to 0.01λ. While the change in this angle of resonance is the direct result of the variation of the phase of reflection, as shown in Fig. 3, it should be noted that this phase of reflection is strongly angle dependent too. The differential influence of the phase of reflection on the resonance angle is

$$\frac{d\theta}{d\phi} = \frac{1}{2\pi l \sin\theta}. \tag{6}$$

It is clear from this expression that the resonance shifts are more pronounced for small angles. A uniform phase shift would produce a 3.6 times larger tuning angle at 15° than at 70°. The observed angle tuning in about those angles in Fig. 6 is only 1.7 times as large because the phase-shift difference between the 0.01λ and the 0.1λ slits is half as big at 15° as it is at 70°. It is expected that this strong angle dependence may be observed under similar experimental conditions to those already presented in Ref. 10 by using a wider wavelength range and increasing the angle of incidence.

The width of the angle-tuned Fabry-Pérot resonance peaks decreases with increasing angle of incidence because of the increasing reflection amplitude, as shown in Fig. 2. There is also a contribution to the asymmetry and angular width of the resonance peaks from the phase of reflection, which is strongly angular dependent. The asymmetry of the resonance peaks is particularly apparent in Fig. 6 for angles close to 80°; the strong variation in the reflection phase (see Fig. 3) gives a steep edge to the large-angle side of the resonance.

This analysis may be extended to include angles perpendicular to the plane of the slit, which will modify only Eq. (1) by scaling factor, and will not change the wavelengths of the Fabry-Pérot resonances. Nevertheless, in the subwavelength regime, the modification to the transmission for perpendicular angles of incidence is negligible.[9] In addition, any TE component will contribute negligibly in the subwavelength regime for a thick metal, since all TE modes are cut off.

In conclusion, the theory of angle-dependent transmission through a subwavelength slit was presented. This work extends the past theory for normal incidence and it includes a fully analytic approximation of the parallel angle dependence. The theory predicts a large angle dependence on the amplitude and phase of reflection. The transmission results are in good quantitative agreement with recent experiments,[10] matching a resonance-frequency shift that was only verified by simulations in that work. The theory also predicts a significant angle-dependent deviation from the typical Fabry-Pérot angle tuning that should be easily verified with similar experiments to those already demonstrated.

R.G. acknowledges the research support of NSERC (Canada).

*Electronic address: rgordon@uvic.ca

[1] Y. Takakura, Phys. Rev. Lett. **86**, 5601 (2001).
[2] F. Yang and J. R. Sambles, Phys. Rev. Lett. **89**, 063901 (2002).
[3] J. R. Suckling, A. P. Hibbins, M. J. Lockyear, T. W. Preist, J. R. Sambles, and C. R. Lawrence, Phys. Rev. Lett. **92**, 147401 (2004).
[4] R. Gordon, Phys. Rev. B **73**, 153405 (2006).
[5] H. T. Miyazaki and Y. Kurokawa, Phys. Rev. Lett. **96**, 097401 (2006).
[6] K. Y. Kim, Y. K. Cho, H.-S. Tae, and J.-H. Lee, Opt. Express **14**, 320 (2006).
[7] P. Lalanne, J. P. Hugonin, and J. C. Rodier, Phys. Rev. Lett. **95**, 263902 (2005).
[8] H. F. Schouten, T. D. Visser, D. Lenstra, and H. Blok, Phys. Rev. E **67**, 036608 (2003).
[9] J. Bravo-Abad, L. Martín-Moreno, and F. J. García-Vidal, Phys. Rev. E **69**, 026601 (2004).
[10] J. R. Suckling, A. P. Hibbins, J. R. Sambles, and C. R. Lawrence, New J. Phys. **7**, 250 (2005).
[11] R. Gordon, Phys. Rev. B **74**, 153417 (2006).