de Haas–van Alphen effect versus integer quantum Hall effect

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In the frame of a general statistical mechanics approach applied to a two-dimensional metal bar, we demonstrate the interrelationship between Landau diamagnetism, de Haas–van Alphen magnetization oscillations, and the integer quantum Hall effect.

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I. INTRODUCTION

The goal of this Brief Report is to give a simple physical description of the integer quantum Hall effect (IQME) in its relation to Landau diamagnetism and the de Haas–van Alphen effect. This does not mean that we can give an answer to all the questions concerning the problem. In some respects, we shall rather formulate the problems following from our treatment. With this purpose we begin with repetition of a very instructive derivation of the free electron gas diamagnetic moment due to Bloch,[1] based on the well-known Landau calculations[2] and taking into account the consideration given by Teller.[3] The important consequence of the Bloch approach is the possibility of dividing the electrons into two groups: (i) occupying the quantum states in the bulk and (ii) in the surface of the metal. Then we shall show that both groups are important in formation of the Landau diamagnetic moment. On the other hand, only bulk electrons produce the oscillating magnetization that is the de–Haas-van Alphen (dHvA) effect. Unlike the dHvA effect, only surface electrons are responsible for the quantized Hall resistivity plateau and the existence at the same time of dissipationless longitudinal currents.

In a real situation it would certainly be quite naive to divide a heterojunction into bulk and surface regions. The experiments clearly demonstrate that the currents in the dissipationless regime spread over the whole specimen (see Ref. 4 and references therein). Moreover, the integer quantum Hall effect has been observed in the absence of edges, that is, on samples of Corbino geometry.[5] In our opinion, however, this does not disprove the concept of division of electron states into bulk and edge states, but just demonstrates that the sample in the dissipationless regime is divided into many channels or rivers, each of which contains electronic states localized far from the river banks as well as bank states carrying persistent currents. So in our simplified treatment we shall work with a completely homogeneous one-channel model.

For brevity we shall work with spinless fermions. The spin degrees of freedom are easily included in the usual manner. We shall omit also the influence of disorder as unimportant for our model and leading just to the appearance of the Dingle factor in thermodynamic values.

II. ELECTRON GAS IN A MAGNETIC FIELD: ROLE OF BULK AND SURFACE ELECTRON STATES

We shall discuss the two-dimensional (2D) clean metal bar with length A in the x direction and width B in the y direction ($|y| \leqslant B/2$) under a perpendicular magnetic field $\mathbf{H}=(0,0,H)$. The thermodynamic potential of the electron gas is

$$\Omega = -T\sum_{\nu} \ln(1 + e^{(\mu-\varepsilon_\nu)/T}), \qquad (1)$$

where the electron energies ε_ν are determined as eigenvalues of the Schrödinger equation

$$\left(\frac{1}{2m}\left[\left(-i\frac{\partial}{\partial x} - eHy\right)^2 - \frac{\partial^2}{\partial y^2}\right] + V(y)\right)\psi_\nu = \varepsilon_\nu\psi_\nu. \qquad (2)$$

We put $\hbar=c=1$ and $e=|e|$ throughout the paper. The potential $V(y)$ is negligible everywhere except near the edges, where it increases from zero at $|y|=B/2-\Delta$ to infinity at $|y|=B/2$. The length Δ is chosen much larger than the magnetic length $\lambda_H=1/\sqrt{eH}$:

$$\frac{1}{(eH)^{1/2}} \ll \Delta \ll B. \qquad (3)$$

The search for a solution in the standard form

$$\psi_\nu = \exp(iqx)\varphi_\nu(y), \qquad (4)$$

where $q=2\pi Q/A$ and Q is an integer, leads us to the following eigenproblem:

$$\hat{H}\varphi_\nu = \varepsilon_\nu\varphi_\nu,$$

$$\hat{H} = \frac{1}{2m}\left[(q - eHy)^2 - \frac{\partial^2}{\partial y^2}\right] + V(y). \qquad (5)$$

It is clear that if the "equilibrium position" $y_0=q/eH$ is limited by

$$-\frac{B}{2} + \Delta \leqslant \frac{q}{eH} \leqslant \frac{B}{2} - \Delta, \qquad (6)$$

then to a quite good approximation the eigenfunctions of Eq. (5) are Landau wave functions

$$\varphi_{nq}(y) = \frac{1}{\sqrt{2^n n!\sqrt{\pi}\lambda_H}} \exp[-(y-y_0)^2/2\lambda_H^2] H_n[(y-y_o)/\lambda_H]$$

with q-independent eigenvalues

$$\varepsilon_n = \omega_c\left(n + \frac{1}{2}\right), \tag{7}$$

$\omega_c = eH/m$. If y_0 is out of the interval (6) then the eigenvalues $\varepsilon_\nu = \varepsilon_{qn}$ are not so simple: they are q dependent and tend to infinity when $|q| \to eHB/2$.

Applying the standard notations for quantum mechanical averages $\langle \hat{A} \rangle_\nu = \int dy (\varphi_\nu \hat{A} \varphi_\nu)$ we obtain the following equality:

$$-H\frac{\partial\varepsilon_{nq}}{\partial H} = q\frac{\partial\varepsilon_{nq}}{\partial q} - \frac{(eH)^2}{m}\langle (y-y_0)^2\rangle_\nu, \tag{8}$$

where

$$-\frac{\partial\varepsilon_{nq}}{\partial H} = -\left\langle \frac{\partial\hat{H}}{\partial H}\right\rangle_\nu = \langle \hat{M}_z\rangle_\nu \tag{9}$$

and

$$\frac{\partial\varepsilon_{nq}}{\partial q} = \left\langle \frac{\partial\hat{H}}{\partial q}\right\rangle_\nu = \langle \hat{v}_x\rangle_\nu. \tag{10}$$

For the equilibrium value of the system magnetic moment at given temperature we have

$$M = -\left(\frac{\partial\Omega}{\partial H}\right)_\mu = -\sum_{nQ}\frac{\partial\varepsilon_{nq}/\partial H}{e^{(\varepsilon_{nq}-\mu)/T}+1},$$

$$= -\frac{A}{2\pi}\int_{-eHB/2}^{eHB/2} dq \sum_{n=0}^{\infty}\frac{\partial\varepsilon_{nq}/\partial H}{e^{(\varepsilon_{nq}-\mu)/T}+1}. \tag{11}$$

The integral over q can be written as

$$\int_{-eHB/2}^{eHB/2} dq = \left(\int_{-eH(B/2-\Delta)}^{eH(B/2-\Delta)} + \int_{-\infty}^{-eH(B/2-\Delta)} + \int_{eH(B/2-\Delta)}^{\infty}\right) dq, \tag{12}$$

where the infinite limits are taken due to fast exponential convergency of the integral when $\varepsilon_{nq} \to \infty$ for q outside the interval (6). Correspondingly the magnetic moment is the sum of three terms

$$M = M_1 + M_2 + M_3. \tag{13}$$

For the first term the energy levels have the q-independent value (7); hence

$$M_1 = -\frac{eS}{2\pi}\sum_{n=0}^{\infty}\frac{\varepsilon_n}{e^{(\varepsilon_n-\mu)/T}+1}, \tag{14}$$

where $S = AB$ is the bar area. For the second term, by making use of the equality (8) and omitting the contribution proportional to $\langle (y-y_0)^2\rangle_\nu$, which is of the order of Δ/B in comparison with the other terms, we obtain

$$M_2 = \frac{A}{2\pi H}\int_{-\infty}^{-eH(B/2-\Delta)} q\, dq \sum_{n=0}^{\infty}\frac{\partial\varepsilon_{nq}/\partial q}{e^{(\varepsilon_{nq}-\mu)/T}+1}$$

$$= -\frac{eS}{4\pi}\int_{-\infty}^{-eH(B/2-\Delta)} dq \sum_{n=0}^{\infty}\frac{\partial\varepsilon_{nq}/\partial q}{e^{(\varepsilon_{nq}-\mu)/T}+1}$$

$$= \frac{eST}{4\pi}\sum_{n=0}^{\infty}\ln(1+e^{(\mu-\varepsilon_n)/T}). \tag{15}$$

Taking into account that $M_2 = M_3$ finally we have

$$M = M_1 + 2M_2 = -\left(\frac{\partial\Omega}{\partial H}\right)_\mu, \tag{16}$$

where

$$\Omega = -\frac{eHST}{2\pi}\sum_{n=0}^{\infty}\ln(1+e^{(\mu-\varepsilon_n)/T}). \tag{17}$$

One can rewrite this result also as

$$M = M_1 + 2M_2 = \left(H\frac{\partial}{\partial H}+1\right)\left(-\frac{\Omega}{H}\right)_\mu$$

$$= \left(H\frac{\partial}{\partial H}+1\right)\frac{eST}{2\pi}\sum_{n=0}^{\infty}\ln(1+e^{(\mu-\varepsilon_n)/T}). \tag{18}$$

As follows from the derivation, the first term here, $M_1 = -H(\partial/\partial H)(\Omega/H)_\mu$, is caused by electrons occupying the Landau states situated in the bulk metal. The second term $2M_2 = -\Omega/H$ is due to the electrons filling the edge states. Let us look now at the roles these two groups of electrons play in observable physical effects.

III. LANDAU DIAMAGNETISM

In the low-field limit $\omega_c \ll T$, the application of the Euler-Maclaurin summation formula[6] yields

$$M = \left(H\frac{\partial}{\partial H}+1\right)\left(-\frac{\Omega_{H=0}}{H} + \frac{eST}{2\pi}\frac{eH}{24m}\partial\ln(1 + e^{(\mu-\varepsilon)/T})/\partial\varepsilon|_{\varepsilon=0}\right) = -\frac{e^2HS}{24\pi m}. \tag{19}$$

We see that both groups of electrons give equal contributions to the Landau diamagnetism. It is worth noting that half of this momentum is the sum of orbital magnetic moments of electronic states in the bulk material. The other half is associated with the persistent current carried by electrons occupying the orbits skipping along the specimen surface. This persistent current is similar to the persistent currents in mesoscopic rings (see, for instance, Ref. 7) and has pure single-particle nature, unlike superconducting or superfluid currents, which are persistent due to multiparticle coherence.

IV. DE HAAS–VAN ALPHEN EFFECT

In the high-field limit $\omega_c \gg T$, the application of the Poisson summation formula[6] yields

$$M = \left(H\frac{\partial}{\partial H}+1\right)\left(-\frac{\Omega_{H=0}}{H}+\frac{eST}{2\pi}2\,\mathrm{Re}\sum_{1}^{\infty}\int_0^{\infty}dx\,\ln(1 + e^{[\mu-\omega(x+1/2)]/T})e^{2\pi i l x}\right) = \left(H\frac{\partial}{\partial H}+1\right) \times\left(-\frac{eST}{2\pi}\sum_{l=1}^{\infty}\frac{(-1)^l}{l}\frac{\cos(2\pi l\mu/\omega_c)}{\sinh(2\pi^2 lT/\omega_c)}\right). \tag{20}$$

In the last line, it is written just $-\Omega_{osc}^{2D}/H$, where Ω_{osc}^{2D} coincides exactly with that found in Ref. 8, where also the spin splitting and the impurity scattering have been taken into account. To obtain the oscillating part of the magnetization, that is, the de Haas–van Alphen effect, one must differentiate the fast-oscillating $\cos(2\pi l\mu/\omega_c)$,

$$M_{osc} = \frac{eST\mu}{\omega_c}\sum_{l=1}^{\infty}(-1)^{l+1}\frac{\sin(2\pi l\mu/\omega_c)}{\sinh(2\pi^2 lT/\omega_c)}. \tag{21}$$

Hence, it is clear that, in contrast to Landau diamagnetism, only the bulk electrons are responsible for the de Haas–van Alphen signal.

Unlike in a 3D metal, the chemical potential in Eq. (21) is a strongly oscillating function of the magnetic field. To find it, following Refs. 8–10, we calculate the number of particles by means of the thermodynamic relation

$$N = -\left(\frac{\partial\Omega}{\partial\mu}\right)_T \tag{22}$$

and then resolve this equation with respect to the chemical potential. The result is as follows:

$$\mu = \varepsilon_F + \mu_{osc}, \tag{23}$$

where

$$\mu_{osc} = \frac{H}{N}M_{osc}. \tag{24}$$

So the oscillating behavior of the magnetization and the chemical potential is determined self-consistently in accordance with the equations written above (for more details, see Refs. 8–10). Experimental evidence for de Haas–van Alphen oscillations in 2D heterostructures has been established recently.[11] We should stress here that both the magnetization and the chemical potential oscillations are determined by bulk electron states.

V. INTEGER QUANTUM HALL EFFECT

In the presence of a current in the x direction there is a Lorentz force shifting the charge carriers in the y direction to the bar edges where extra (or a lack of) charge appears, leading to Lorenz force compensation by Coulomb interaction. As a result the values of the chemical potential at the opposite sides of the bar differ from each other by the Hall voltage,

$$\mu(B/2) - \mu(-B/2) = eU_H. \tag{25}$$

The local current density is

$$j_x = \frac{\partial \mathcal{M}_z}{\partial y}, \tag{26}$$

where $\mathcal{M}_z = M_z/S$ is the magnetic moment density. Hence the current is given by

$$J_x = \int_{-B/2}^{B/2} j_x dy = \mathcal{M}_z(B/2) - \mathcal{M}_z(-B/2), \tag{27}$$

and as the magnetic moment one must take the surface part of the magnetic moment. Thus

$$J_x = \frac{eT}{2\pi}\sum_{n=0}^{\infty}\left[\ln\left(1+\exp\frac{\mu(B/2)-\varepsilon_n}{T}\right) - \ln\left(1+\exp\frac{\mu(-B/2)-\varepsilon_n}{T}\right)\right]. \tag{28}$$

This current is not accompanied by a dissipation, because the edges of the specimen are at constant potential: the values of the chemical potential in Eq. (25) are not x dependent. So this current is quite similar to the persistent edge currents described above, responsible for Landau diamagnetism. Unlike the latter the currents at the opposite edges are not equal due to the chemical potential difference.

At small currents and, hence, at small Hall tensions, we have for the Hall conductance

$$G_{xy} = \frac{J_x}{U_H} \simeq \frac{e^2}{2\pi}\sum_{n=0}^{\infty}\frac{1}{e^{(\varepsilon_n-\mu_s)/T}+1}. \tag{29}$$

So, for negligibly small U_H, the Hall conductance at low temperatures has quantized values determined by the number of Landau levels below the chemical potential.

This property occurs if the surface chemical potential $\mu(x,\pm B/2)=\mu_s$ does not oscillate with magnetic field, unlike the chemical potential in the bulk, which is a strongly oscillating function according to Eqs. (23) and (24). Nonoscillating behavior of chemical potential is typical for 2D electron system connected with reservoir.[9] Here we can expect that the chemical potential of ensemble of the surface electronic states is maintained at the constant value by the reservoir of electronic states in the bulk. The equilibrium between the surface and the bulk electron subsystems is supported by means of the electric potential of the electron density, which changes in space.[12]

VI. CONCLUSION

The zero-temperature derivation of edge currents and the quantized Hall effect relationship were introduced in Ref. 13. Recently, general thermodynamic arguments were used for the description of the Hall effect in terms of diamagnetic currents.[14]

In addition to the statistical mechanics treatment of the IQHE as an equilibrium phenomenon at finite temperature and fixed number of particles, here we have pointed out the relationship between all types of oscillation phenomena in 2D metals. The difference between the surface and bulk chemical potentials comes out as an inevitable property of our approach. A similar physical conjecture has been put forward and qualitatively described by Egorov.[15]

The currents in the field interval of the Hall plateau are persistent currents similar to those responsible for Landau diamagnetism. Their stability is provided by energetic bariers preventing the dissipative electron density redistribution near filled Landau levels.[12] On the contrary, the dissipative regime arises in magnetic field intervals near half-filled Landau levels when the freedom for electron motion is not limited by the Pauli principle.

By a different approach formula (29) was derived recently by Champel and Florens.[16] We should stress, however, that these authors do not distinguish the surface and bulk chemical potential values. The absence of an oscillating part of the chemical potential is indicated by the presence of potential disorder that is smooth in space but strong in amplitude. However, the disorder potential at the specimen edges is taken as x-coordinate independent. As we already mentioned, experimental evidence of strong oscillations of the magnetic moment (related to bulk chemical potential oscillations) in high-mobility heterostructuters[11] has been reported recently.

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