# Transverse acoustic wave in molecular magnets via electromagnetically induced transparency

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We have studied the propagation properties of transverse acoustic wave in molecular magnets in the presence of a strong ac magnetic field. Due to quantum interference introduced by the strong electromagnetic field, the acoustic wave can propagate without absorption under appropriate conditions, which is a direct result of electromagnetically induced transparency effect. Meanwhile, the nonlinear effects should be considered in the propagation of the acoustic wave because electromagnetic induction transparency effect can greatly enhance nonlinear effects. The transverse acoustic wave propagation equation including nonlinear effects in molecular magnets has been investigated and the corresponding accurate analytic solutions have been obtained. We believe that our research may promote the understanding of the acoustic wave's properties in molecular magnets.

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## I. INTRODUCTION

In recent years, molecular magnets have attracted considerable attention because the microscopic quantum effects, such as quantum tunneling of magnetization and quantum steps, can appear in the macroscopic properties of the system.[1–5] The synthesis technology of molecular magnets has solved the serious problem of the particle size and the size of molecular magnet can be exactly the same, which makes it possible to study extensively the nature of molecular magnets and impels more applied researches of molecular magnets. Molecular magnets have potential applications in magnetic memory and quantum computing.[2,6,7] Molecular magnets are also expected to be applied to form a powerful source of coherent microwave radiation.[3,8] As we known, acoustic waves can be generated by electromagnetic waves (including optical wave).[9–11] Therefore, it is important to thoroughly study the interactions of electromagnetic and acoustic waves in magnetic molecules.

The effect of electromagnetically induced transparency[12] (EIT) in the optically dense atomic medium has received a great deal of attention mainly in the past few years because it can significantly modify the optical properties of the atomic medium, such as enhancing certain nonlinearities and dispersion effects.[13–17] Quantum interference introduced by driving the upper two levels with a strong electromagnetic wave can eliminate the absorption of a weak (probe) electromagnetic wave under appropriate conditions, then the medium becomes effectively transparent without absorption for the probe electromagnetic wave.[18] This effect was named as electromagnetically induced transparency.[12,18] The effect of EIT has disclosed new possibilities for nonlinear optics and quantum information processing. The research of electromagnetic wave propagation properties in the ultracold atomic medium via EIT is an important topic. A novel class of ultraslow propagation phenomenon, such as ultraslow optical solitons[19,20] and two-color ultraslow optical solitons,[21] has been reported.

It is important to emphasize that the nonstationary behavior of a high-spin molecule in a bifrequency ac magnetic field[22] or in an acoustic wave and an ac magnetic field[23] was investigated theoretically. The parametric interaction of two acoustic waves in molecular magnets in the presence of a strong ac magnetic field was studied in Ref. 24. Recently, electromagnetically induced transparency in molecular magnets has been theoretically investigated and the absorption of a weak (probe) electromagnetic wave can be suppressed by a strong (coupling) electromagnetic wave because of quantum interference.[25] Analogous to the research of electromagnetic wave propagation properties in the ultracold atomic medium, the research of the propagation characters of acoustic or electromagnetic wave in molecular magnets via EIT also has important significance.

In fact, the nonlinear spin-phonon dynamics in molecular magnets has been studied and phonon laser effect has been reported.[26] The purpose of the present paper is to study the propagation properties of transverse acoustic wave in molecular magnets in the presence of a strong ac magnetic field. Any molecule is subject to a dc magnetic field perpendicular to the easy anisotropy axis of the molecule. The ac magnetic field is parallel to the easy anisotropy axis. We assume that all molecular magnets occupy the ground state, which means that the spin state of molecular magnets is saturation. The three lowest levels of the molecule are of interest to us. The frequency of the ac magnetic field (acoustic wave) is close to the transition frequency of the higher (lowest) state and highest state. Thus, just the three states are involved in the processes we discuss, and we can consider any magnetic molecule as a $\Lambda$-type three-level system. We find that, due to quantum interference introduced by the strong ac magnetic field, the acoustic wave can propagate without absorption under appropriate conditions. This is a direct result of electromagnetically induced transparency effect (certainly, we do not mean EIT at optical frequencies; typical transition frequencies in molecular magnets are much lower). Then, acoustic wave can exist in molecular magnets in the process of a strong ac magnetic field. Meanwhile, because electro-

 

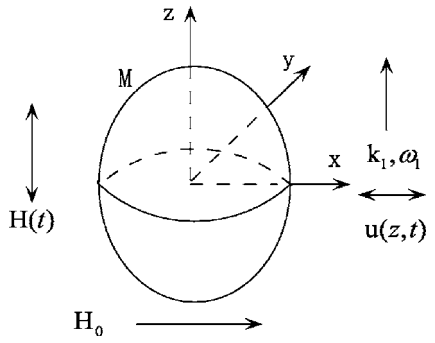


FIG. 1. A magnetic molecule ($M$) with the easy anisotropy axis $z$ in a dc magnetic field $H_0$ parallel to the $x$ axis. The vertical arrow with two arrowhead depicts an ac magnetic field $H(t)=H\sin(\omega t)$. The horizontal arrow with two arrowheads depicts the lattice displacement $u(z,t)$ parallel to the $x$ axis. The vertical arrows are the wave vectors $k_1$ of the transverse propagating acoustic wave whose frequency is $\omega_1$. Our scheme is similar to the scheme in Ref. 24.

magnetic induction transparency can greatly enhance nonlinear effects, the nonlinear effects should be considered in the propagation of the acoustic wave. In this paper, we study the transverse acoustic wave propagation equation including nonlinear effects in molecular magnets and get the accurate analytic solutions.

## II. THEORETICAL MODEL

We consider a crystal of noninteracting molecular magnets (for example, $Mn_{12}$ acetate or $Fe_8$) subjects to dc and ac magnetic fields and an acoustic wave simultaneously. The dc magnetic field is perpendicular to the easy anisotropy axis of the molecule. The ac magnetic field is parallel to this axis (see Fig. 1). First of all, we analyze the behavior of one magnetic molecule as Ref. 24. The Hamiltonian can be written as[24]

$$\hat{H} = -D\hat{S}_z^2 + \hat{H}_{tr} - g\mu_B\hat{S}_x H_0 - g\mu_B\hat{S}_z H\sin(\omega t) + \hat{H}_{me}. \quad (1)$$

Here, $z$ is the easy anisotropy axis, $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$ are the $x$, $y$, and $z$ projections of the spin operator, $\hat{H}_{tr}$ is the operator of the transverse anisotropy energy, $D$, $g$, and $\mu_B$ are the longitudinal anisotropy energy constant, the Landé factor, and the Bohr magneton, respectively, $H_0$ is the dc magnetic field and coincide with the $x$, $H$, and $\omega$ are the amplitude and angular frequency of the the ac magnetic field, and $\hat{H}_{me}$ is the magnetoelastic coupling.

In general, the transverse anisotropy is much weaker than the longitudinal one in magnetic molecules,[27,28] so it can be regarded as a perturbation. The molecule levels are formed doublets split due to the transverse anisotropy, which is not dependent on the transverse anisotropy's form. All molecular magnets are assumed in the ground state. The three lowest levels of the molecule are of interest to us. We denotes them as $\epsilon_0$, $\epsilon_1$, and $\epsilon_2$ ($\epsilon_0<\epsilon_1<\epsilon_2$). The eigenfunctions corresponding to $\epsilon_0$, $\epsilon_1$, and $\epsilon_2$ are denoted as $|0\rangle$, $|1\rangle$, and $|2\rangle$, respectively. The eigenvalues and eigenfunctions in the representation of operator $\hat{S}_z$ can be found numerically. We suppose that the ac magnetic field frequency $\omega$ and acoustic wave frequency $\omega_1$ are close to the transition frequencies $\omega_{21}=(\epsilon_2-\epsilon_1)/\hbar$ and $\omega_{20}=(\epsilon_2-\epsilon_0)/\hbar$, i.e., $|\omega-\omega_{21}|/\omega_{21}\ll 1$ and $|\omega_1-\omega_{20}|/\omega_{20}\ll 1$, respectively.

The magnetoelastic coupling in $Mn_{12}$ acetate can be obtained as follows:[3,29]

$$\hat{H}_{me} = D(\varepsilon_{xx}-\varepsilon_{yy})(\hat{S}_x^2-\hat{S}_y^2) + D\varepsilon_{xy}\{\hat{S}_x,\hat{S}_y\} + D(\varepsilon_{xz}\{\hat{S}_x,\hat{S}_z\} + \varepsilon_{yz}\{\hat{S}_y,\hat{S}_z\}) + D(\nu_{xz}\{\hat{S}_x,\hat{S}_z\} + \nu_{yz}\{\hat{S}_y,\hat{S}_z\}), \quad (2)$$

where

$$\varepsilon_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial u_\alpha}{\partial x_\beta} + \frac{\partial u_\beta}{\partial x_\alpha}\right), \quad \nu_{\alpha\beta} = \frac{1}{2}\left(\frac{\partial u_\alpha}{\partial x_\beta} - \frac{\partial u_\beta}{\partial x_\alpha}\right) \quad (3)$$

are linear strain tensors at the point where the molecule is located, $\mathbf{u}$ is the lattice displacement, $\alpha$, $\beta=x,y,z$, and $\{\hat{S}_\alpha,\hat{S}_\beta\}$ is the anticommutator. We suppose that the transverse monochromatic acoustic wave polarized along the $x$ axis propagates along the $z$ axis (see Fig. 1), i.e., $\mathbf{u}=\{u_x,0,0\}$, which can be given as

$$u_x = \frac{1}{2}u(z,t)\exp[-i(\omega_1 t - k_1 z)] + \text{c.c.}, \quad (4)$$

where $u(z,t)$, $\omega_1$, $k_1=\omega_1/v$, and $v$ are the amplitude, angular frequency, wave number, and velocity of the acoustic waves. Here, $v$ can be interpreted as phase velocity, similar to optical wave. Generally speaking, energy transmission velocity (group velocity) $v_g$ is unequal to $v$ and is determined by dispersion relation. As we know, group velocity is a very important and significant physical parameter for nonmonochromatic wave and pulse propagation. The dispersion relation of the transverse acoustic wave in molecular magnets will be discussed in the latter part of our paper. For the lattice displacement [Eq. (4)], the magnetoelastic coupling described by Eq. (2) reduces to

$$\hat{H}'_{me} = D\frac{\partial u_x}{\partial z}\{\hat{S}_x,\hat{S}_z\}. \quad (5)$$

Then, the interaction Hamiltonian of this system can be predigested as follows:

$$\hat{H}_{int} = -g\mu_B\hat{S}_z H\sin(\omega t) + D\frac{\partial u_x}{\partial z}\{\hat{S}_x,\hat{S}_z\}. \quad (6)$$

Because the frequencies of the ac magnetic field $\omega$ and acoustic wave $\omega_1$ are comparatively close to the transition frequencies $\omega_{21}$ and $\omega_{20}$, respectively, we can consider a magnetic molecule as a $\Lambda$-type three-level system (see Fig. 2). At low temperature, the assumption that the magnetic molecules occupy just the three lowest levels is reasonable. In the following, we will study the kinetics of such a three-level system in the presence of a strong ac magnetic field and a transverse acoustic wave. We assume a solution of the Schrödinger equation $\left(i\hbar\frac{\partial}{\partial t}|\Psi\rangle = \hat{H}_{int}|\Psi\rangle\right)$ in the form

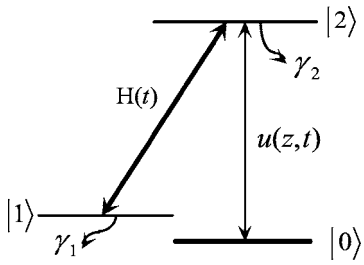


FIG. 2. Λ-type three-level system interacts with the ac magnetic field $H(t)$ and the acoustic wave $u(z,t)$. $\gamma_{1,2}$ is the decay rate of states $|1\rangle$ and $|2\rangle$.

$$|\Psi\rangle = C_0(z,t)|0\rangle + C_1(z,t)\exp[-i(\omega_1 t - \omega t - k_1 z)]|1\rangle + C_2(z,t)\exp[-i(\omega_1 t - k_1 z)]|2\rangle. \quad (7)$$

Substituting expression (7) into the Schrödinger equation and using the rotating wave approximation,[30] the motion equations of $C_1$ and $C_2$ can be obtained,

$$\frac{\partial}{\partial t}C_1 = i(\Delta_1 + i\gamma_1)C_1 + ig\mu_B H(\hat{S}_z)_{12}C_2/(2\hbar), \quad (8a)$$

$$\frac{\partial}{\partial t}C_2 = i(\Delta_2 + i\gamma_2)C_2 + Dk_1 u_x\{\hat{S}_x,\hat{S}_z\}_{20}C_0/\hbar + ig\mu_B H(\hat{S}_z)_{21}C_1/(2\hbar), \quad (8b)$$

where the matrix elements $(\hat{S}_z)_{21}=\langle 2|\hat{S}_z|1\rangle$ and $\{\hat{S}_x,\hat{S}_z\}_{20}=\langle 2|\{\hat{S}_x,\hat{S}_z\}|0\rangle$ are real. $C_0$ is determined by the relation $|C_0|^2+|C_1|^2+|C_2|^2=1$. In deriving Eq. (8), we define one- and two-photon detunings as $\Delta_2=\omega_1-\omega_{20}$ and $\Delta_1=\omega_1-\omega-\omega_{20}+\omega_{21}$. Due to the symmetry properties of the wave functions $|0\rangle$, $|1\rangle$, and $|2\rangle$, the matrix elements $\{\hat{S}_x,\hat{S}_z\}_{jj}=0$ $(j=0,1,2)$ are equal to zero. Meanwhile, we assume that $u(z,t)$ is the slow function of $z$, namely, $\partial u_x/\partial z \approx ik_1 u_x$, which have been taken into account. $\gamma_{1,2}$ is the decay rate of the states $|1\rangle$ and $|2\rangle$. Comparing Eq. (8) with the atomic equations of motion for the three-level system,[19] we can find that the solutions of Eq. (8) are analogous to them in Ref. 19. Assuming that $C_j=\Sigma_k C_j^{(k)}$ [$C_j^{(k)}$ is the $k$th-order part of $C_j$, $j=0,1,2$], $C_j^{(0)}=\delta_{j0}$ and $C_0^{(1)}=0$, we can obtain that[19,20]

$$C_1 \simeq C_1^{(1)} = \frac{i\Omega_m G}{|\Omega_m|^2 - (\Delta_1 + i\gamma_1)(\Delta_2 + i\gamma_2)}u, \quad (9a)$$

$$C_2 \simeq C_2^{(1)} = \frac{-i(\Delta_1 + i\gamma_1)G}{|\Omega_m|^2 - (\Delta_1 + i\gamma_1)(\Delta_2 + i\gamma_2)}u. \quad (9b)$$

Here, $\Omega_m=\frac{g\mu_B H(\hat{S}_z)_{12}}{2\hbar}$ and $G=\frac{Dk_1\{\hat{S}_x,\hat{S}_z\}_{20}}{\hbar}$. $\Omega_m$ presents the Rabi oscillation induced by the interaction of ac magnetic wave and molecular magnets. $G$ can be interpreted as coupling constant of acoustic wave and molecular magnets.

Following Ref. 24, we can obtain the wave equation of the acoustic waves in magnetic molecules,

$$\frac{\partial^2 u_x}{\partial t^2} - v^2\frac{\partial^2 u_x}{\partial z^2} = \frac{2ND}{\rho}\{\hat{S}_x,\hat{S}_z\}_{20}\frac{\partial}{\partial z}\{C_0^* C_2 \exp[-i(\omega_1 t - k_1 z)]\}, \quad (10)$$

where $\rho$ is the crystal density and $N$ is the concentration of magnetic molecules. In the Eq. (10), we have considered the acoustic wave's decay because the right formula is a complex number. Because electromagnetic induction transparency can greatly enhance nonlinear effects, the nonlinear effects should be considered in the propagation of the acoustic wave. $C_0^* C_2$ can be expressed as[19]

$$C_0^* C_2 \approx C_2^{(1)}[C_0^{(0)}]^* + C_2^{(3)}[C_0^{(0)}]^* + C_2^{(1)}[C_0^{(2)}]^* = C_2^{(1)}[C_0^{(0)}]^* - C_2^{(1)}[|C_1^{(1)}|^2 + |C_2^{(1)}|^2]. \quad (11)$$

In above expression, solution of $C_0^* C_2$ is retained to third-order part. With the help of Eqs. (9)–(11), we can note that the linear polarization term and the lowest-order nonlinear polarization term have been considered. In general, the high-order nonlinear polarization is very small, so the lowest-order nonlinear polarization is sufficient to describe the nonlinear effects. Equation (10) can be rewritten as follows:

$$\frac{\partial^2 u_x}{\partial t^2} - v^2\frac{\partial^2 u_x}{\partial z^2} = \frac{\partial}{\partial z}[(A + B|u_x|^2)u_x]. \quad (12)$$

Hear, $A$ and $B$ are given by

$$A = \frac{-i2ND\{\hat{S}_x,\hat{S}_z\}_{20}(\Delta_1 + i\gamma_1)G}{\rho[|\Omega_m|^2 - (\Delta_1 + i\gamma_1)(\Delta_2 + i\gamma_2)]},$$

$$B = \frac{-A(|\Omega_m G|^2 + |(\Delta_1 + i\gamma_1)G|^2)}{|[|\Omega_m|^2 - (\Delta_1 + i\gamma_1)(\Delta_2 + i\gamma_2)]|^2}.$$

Utilizing Eq. (12) and Fourier transformation relations, i.e., $\frac{\partial}{\partial t}\leftrightarrow -i\omega_1$ and $\frac{\partial}{\partial z}\leftrightarrow ik$, we can get the dispersion relation of transverse acoustic wave in molecular magnets,

$$k^2 = \frac{\omega_1^2}{v^2 + iA/k_1} \quad (13a)$$

or

$$k = \sqrt{\frac{\omega_1^2}{v^2 + \frac{2ND^2\{\hat{S}_x,\hat{S}_z\}_{20}^2(\Delta_1 + i\gamma_1)}{\hbar\rho[|\Omega_m|^2 - (\Delta_1 + i\gamma_1)(\Delta_2 + i\gamma_2)]}}}. \quad (13b)$$

We can notice that the wave number $k$ is a complex number and is unequal to $k_1$, which is aroused by media polarization when the acoustic wave and ac magnetic field propagate in the molecular magnets. The imaginary part of $k$ presents the absorption coefficient. Now we will inspect $k$ with different parameters. For an illustration, in Fig. 3, the shapes of the real and imaginary parts of $\Delta k=(k-k_1)/k_1$ with parameter $\Delta_2/\gamma_2$ are given. According to Refs. 25, 31, and 32, for $Mn_{12}$ molecules at $H_0=60.6$ kOe, $S=10$, $D=0.68$ K, $g=1.9$, $\hat{H}_{tr}=-B(\hat{S}_+^4+\hat{S}_-^4)$, $\omega_{10}=(\epsilon_1-\epsilon_0)/\hbar \simeq 4.267\times 10^9$ s$^{-1}$, and $\omega_{20}\simeq 1.12\times 10^{12}$ s$^{-1}$. In the process of numerical analysis, we assume that $\gamma_2=1\times 10^8$ s$^{-1}$, $\gamma_1=10^{-2}\gamma_2$, and $\frac{2ND^2\{\hat{S}_x,\hat{S}_z\}_{20}^2}{v^2\hbar\rho\gamma_2}$

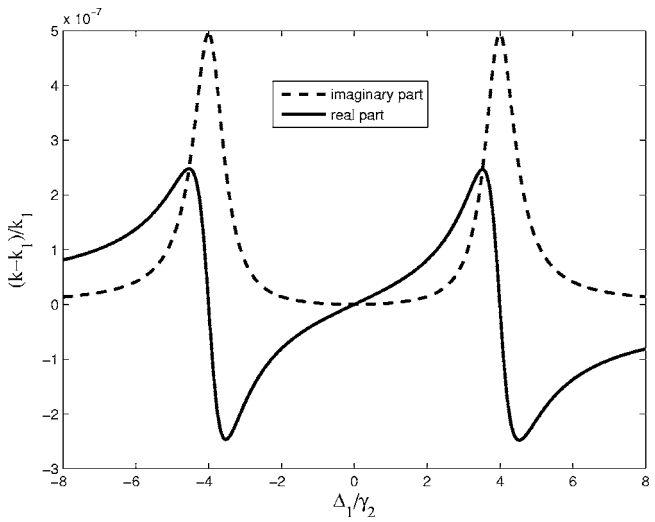


FIG. 3. The shapes of the real and imaginary parts of $\Delta k=(k-k_1)/k_1$ with parameter $\Delta_2/\gamma_2$. We have chosen the relative parameters $\Delta_1=\Delta_2$, $\gamma_1/\gamma_2=10^{-2}$, $\Omega_m/\gamma_2=4$, and $\frac{2ND^2\{\hat{S}_x,\hat{S}_z\}_{20}^2}{v^2\hbar\rho\gamma_2}=10^{-6}$.

$=10^{-6}$. Inspecting Fig. 3, the absorption of the acoustic wave can be controlled by choosing detuning $\Delta_2$ or frequency $\omega$ of the acoustic wave and can even be eliminated under certain conditions. Hereto we have obtained dispersion relation of transverse acoustic wave in molecular magnets and find that the acoustic wave can propagate without absorption under appropriate conditions.

If a reasonable and realistic set of parameters can be found so that, $A=A_r+iA_i\simeq iA_i$ and $B=B_r+iB_i\simeq iB_i$, then the equation of transverse acoustic wave's envelope $u(z,t)$ can be simplified as $(\partial u_x/\partial z\approx ik_1u_x)$

$$\frac{\partial^2 u}{\partial t^2}-2i\omega\frac{\partial u}{\partial t}-v^2\left(\frac{\partial^2 u}{\partial z^2}+2ik_1\frac{\partial u}{\partial z}\right)=-k_1(A_i+B_i|u|^2)u. \tag{14}$$

Equation (14) is the basis for our discussion of the propagation properties of transverse acoustic wave in molecular magnets.

## III. DISCUSSION AND NUMERICAL RESULTS

The subsequent task is to depict the stationary state with Eq. (12). It is fortunate that the current form of Eq. (12) can be solved analytically. As we know, the analytical and explicit solution has a great advantages to grasp the characters of the system over the slowly varying envelope approximation[24] or a numerical approach. The analytical expressions will be obtained and briefly discussed in the paper.

We seek transverse acoustic wave solutions of the following form: $u(z,t)=\tilde{u}(Z)\exp[ik'z]$ $(Z=z-v_gt)$. $k'$ is the meaning of the wave-number shift. $v_g=\frac{1}{\partial k/\partial\omega_1}$ is the group velocity of the acoustic wave, which is decided by the dispersion relation [Eq. (13)] of the acoustic wave propagating in molecular magnets. As we known, $v_g$ has its physical significance as the velocity of energy transport. Therefore, it is reasonable that we take the form of this solution. Then, the differential equation for $\tilde{u}(Z)$ can readily be cast into the following form:

$$(v_g^2-v^2)\frac{\partial^2\tilde{u}}{\partial Z^2}+2i[\omega v_g-v^2(k_1+k')]\frac{\partial\tilde{u}}{\partial Z}=[-k_1A_i-(k'^2+2k_1k')v^2]\tilde{u}-k_1B_i|\tilde{u}|^2\tilde{u}. \tag{15}$$

When $\omega v_g-v^2(k_1+k')=0$, we can simplify the above equation,

$$\frac{\partial^2\tilde{u}}{\partial Z^2}=c_2\tilde{u}+c_4\tilde{u}^3. \tag{16}$$

Here, $c_2=\frac{-k_1A_i-(k'^2+2k_1k')v^2}{v_g^2-v^2}$ and $c_4=\frac{-k_1B_i}{v_g^2-v^2}$. In deriving of Eq. (16), we utilize the condition that $\tilde{u}$ is a real number.

(1) *Homogeneous solution*. If the transverse acoustic wave is plane wave, Eq. (16) is reduced to

$$c_2\tilde{u}+c_4\tilde{u}^3=0. \tag{17}$$

The solutions can be readily obtained,

$$\tilde{u}=0,\quad \tilde{u}=\pm\sqrt{-c_2/c_4}\quad (\text{if } c_2/c_2<0). \tag{18}$$

(2) *Unhomogeneous solution*. Now we discuss the unhomogeneous solution of Eq. (16). Multiplying $\frac{\partial\tilde{u}}{\partial Z}$ in the both sides of Eq. (16), we can get the following function:

$$\left(\frac{\partial\tilde{u}}{\partial Z}\right)^2=c_0+c_2\tilde{u}^2+c_4\tilde{u}^4/2. \tag{19}$$

Here, $c_0$ is a integral constant. With the help of elliptic functions,[33,34] we can obtain the exact solution of Eq. (16) as follows:

$$\tilde{u}=\sqrt{\frac{-2c_2m^2}{c_4(m^2+1)}}\,\mathrm{sn}\left(\sqrt{-\frac{c_2}{m^2+1}}Z,m\right),\quad c_2<0,$$
$$c_4>0,\quad c_0=\frac{2c_2^2m^2}{c_4(m^2+1)^2} \tag{20a}$$

and

$$\tilde{u}=\sqrt{\frac{-2c_2}{c_4(2-m^2)}}\,\mathrm{dn}\left(\sqrt{\frac{c_2}{2-m^2}}Z,m\right),\quad c_2>0,$$
$$c_4<0,\quad c_0=\frac{2c_2^2(1-m^2)}{c_4(2-m^2)^2}, \tag{20b}$$

where $\mathrm{sn}(z,m)$ and $\mathrm{dn}(z,m)$ with the modulus $m$ are the Jacobi elliptic functions.[33] The Jacobi elliptic functions are doubly periodical and possess some special properties. When $m\to1$, the Jacobi functions degenerate to the hyperbolic functions. So we can get the corresponding solutions,

$$\tilde{u}=\sqrt{\frac{-c_2}{c_4}}\tanh\left(\sqrt{-\frac{c_2}{2}}Z\right),\quad c_2<0,$$
$$c_4>0,\quad c_0^2=\frac{c_2^2}{2c_4} \tag{21a}$$

and

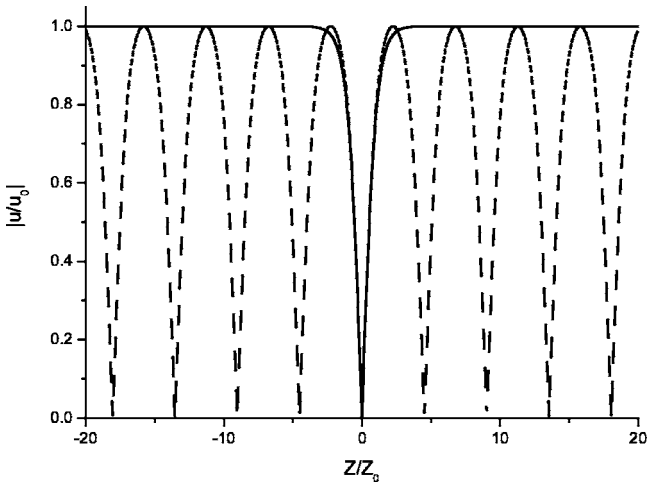


FIG. 4. The distribution of transverse acoustic wave amplitude for $c_2<0$, $c_4>0$. Here $u_0=\sqrt{\frac{-2c_2m^2}{c_4(m^2+1)}}$ and $Z_0=1/\sqrt{-\frac{c_2}{m^2+1}}$. We have chosen the relative parameters $m=0.8$ (dash) and $m=1.0$ (solid).

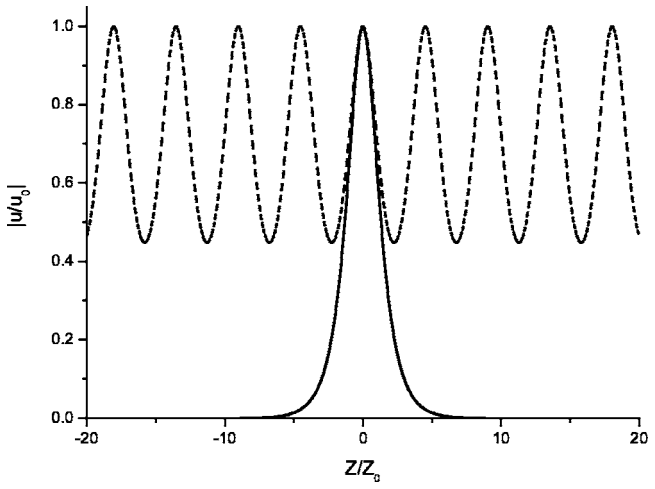


FIG. 5. The distribution of transverse acoustic wave amplitude for $c_2>0$, $c_4<0$. Here, $u_0=\sqrt{\frac{-2c_2}{c_4(2-m^2)}}$ and $Z_0=1/\sqrt{\frac{c_2}{2-m^2}}$. We have chosen the relative parameters $m=0.8$ (dash) and $m=1.0$ (solid).

$$\tilde{u}=\sqrt{\frac{-2c_2}{c_4}}\sinh(\sqrt{c_2}Z),\quad c_2>0,\quad c_4<0,\quad c_0=0. \tag{21b}$$

Above solutions mean that the transverse acoustic solitons can be formed in molecular magnets under certain conditions. When $m\to 0$, the corresponding solution is $|\tilde{u}|=0$, which is a mediocre solution. Solutions (20) and (21) are clearly illustrated in Figs. 4 and 5.

It is seen that, in the general case, the acoustic wave amplitude $|\tilde{u}|$ is a periodic function (see Figs. 4 and 5). The solutions obtained allow us to find the period of amplitude oscillations,

$$L_{Z1}=\frac{4K(m)}{\sqrt{-\frac{c_2}{m^2+1}}},\quad c_2<0 \tag{22a}$$

and

$$L_{Z2}=\frac{2K(m)}{\sqrt{\frac{c_2}{2-m^2}}},\quad c_2>0, \tag{22b}$$

where $K(m)=\int_0^{\pi/2}\frac{d\theta}{\sqrt{1-m^2\sin^2\theta}}$ is the complete elliptic integral. The physical interpretation of $L_{Z1}$ and $L_{Z2}$ are the period of amplitude oscillations in different conditions, respectively.

To sum up, we have obtained a set of analytical and explicit solutions of the transverse acoustic wave propagation equation in molecular magnets. All of them, both homogeneous and unhomogeneous, correspond to the stationary solutions, which is helpful to grasp the characters of transverse acoustic wave in molecular magnets.

## IV. CONCLUSIONS

In this paper, we have studied the propagation properties of transverse acoustic wave in molecular magnets in the presence of a strong ac magnetic field. In the processes we discuss, any magnetic molecule can be considered as a $\Lambda$-type three-level system. We find that the acoustic wave can propagate without absorption under appropriate conditions due to quantum interference introduced by the strong electromagnetic field. This is a direct result of electromagnetically induced transparency effect. Meanwhile, the nonlinear effects should not be ignored in the propagation of the acoustic wave because electromagnetic induction transparency effect can greatly enhance nonlinear effects. We investigate the transverse acoustic wave propagation equation including nonlinear effects in molecular magnets and give the corresponding accurate analytic solutions. We believe that our research may promote the understanding of the acoustic wave's properties in molecular magnets.

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