Lorentz covariance and the crossover of two-dimensional antiferromagnets

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(Received 23 November 2006; published 22 May 2007)

We derive a lattice β function for the two-dimensional antiferromagnetic Heisenberg model, which allows the lattice interaction couplings of the nonperturbative quantum Monte Carlo vacuum to be related directly to the zero-temperature fixed points of the nonlinear sigma model in the presence of strong interplanar and spin anisotropies. In addition to the usual renormalization of the gapful disordered state in the vicinity of the quantum critical point, we show that this leads to a chiral doubling of the spectra of excited states.

DOI: 10.1103/PhysRevB.75.174520 PACS number(s): 75.10.Jm, 75.40.Cx

I. INTRODUCTION

The two-dimensional (2D) antiferromagnetic Heisenberg model is a well studied system that has two renormalization-group (RG) fixed points that have been identified at zero temperature through various applications of the 2D nonlinear sigma model.[1,2] Depending on the relative anisotropy of the exchange coupling J between the two spatial directions of the system, the ground state of the 2D antiferromagnetic ground state is found to be either Néel ordered and gapless or a gapful quantum disordered state. Varying the anisotropy drives dynamical fluctuations causing a zero-temperature phase transition between the two ground states,[3] whereby the system effectively crosses over from two dimensions to three dimensions.[4] Without the inclusion of a θ term, the 2D nonlinear sigma model is used to give an effectively classical description of the Néel ordered ground state, and the quantum nature of the disordered state arises in some nontrivial way through the dynamical scale evolution of the system. It has been questioned whether the inclusion of an explicit source term for quantum fluctuations would be of relevance in the Néel phase[5,6] in order to understand the mechanism of symmetry breaking, but the inclusion of such a θ term provides an irrelevant perturbation. However, if no source term for quantum fluctuations is included, the renormalization scale of the system can only be defined through phenomenological input. This picture has been successfully verified in detail by comparing nonlinear sigma model predictions for the scaling of the correlation length in the Néel and quantum disordered regimes, with numerics obtained from quantum Monte Carlo (QMC) studies.[7,8]

Recently, a treatment of the reverse picture has been given via a conformal analysis of quantum spin chains.[9] Importantly, this identifies the effect of dynamical fluctuations on the θ term. In principle, this effect should simply be to rescale the couplings of the underlying sine-Gordon model by a dynamical factor. However, we have recently pointed out that the convergence of the perturbative deformation is not guaranteed and a nonperturbative renormalization prescription is required.[10] In this work, we extend this nonperturbative renormalization picture to give a description of the role of quantum effects in the dynamical fluctuations of the 2D antiferromagnetic Heisenberg model. Whether or not a θ term is relevant in the Néel phase,[11] the O(3) spin operators of the nonperturbative QMC method[12] have already been obtained through a form of topological dimensional reduction. Consequently, there are IR cutoff effects associated with the finite lattice system dynamics, and it is difficult to disentangle these effects from genuine mechanisms of symmetry breaking. Recent treatments based around the 2D O(3) model have included the effect of a marginally irrelevant topological term possessing a U(1) symmetry.[6] Our nonperturbative approach extends this picture by considering the effect of a finite IR lattice cutoff within the couplings of these terms.

We focus on the nonperturbative properties of the continuous-time QMC method. This scheme has the same basic transfer-matrix structure that is defined for the density-matrix renormalization-group (DMRG) method,[13,14] but the numerical basis of the lattice partition function for the continuous-time QMC method is defined through the application of classical loop-cluster methods. There are a number of closely related QMC schemes that can also be used to generate this lattice partition function, and these are discussed in detail in the review in Ref. 15. However, we are singling out the continuous-time scheme because the loop-cluster Monte Carlo updating process is slightly different. The closed loops in this scheme, which represent the trace of the lattice partition function, are generated through a sequence of successive local Monte Carlo updating decisions, and this is in contrast with other QMC schemes, closer to the original classical loop-cluster method, where the Monte Carlo updating decision is made by comparing the probabilistic weights of closed loops. All of the QMC schemes which use the DMRG form of the transfer matrix in Refs. 13 and 14 can be given the same interpretation as systems of O(3) spin vectors, but the continuous-time method has the additional feature that the lattice partition function is defined to be analytically continuous in Euclidean time over some finite interval. The reason why this is important is because, unlike the dynamical scaling relations of classical spin systems,[16] the dynamical critical exponent for the QMC Euclidean-time direction only relates to a topologically complete system in the large lattice volume and stochastic probability distribution limits of the QMC method.[17] Therefore, a complete understanding of numerical correction effects and a direct comparison with the couplings of the 2D nonlinear sigma model are difficult with the generic method because finite-size lattice systems only asymptotically approach these limits. Using the special continuous analytic property of the continuous-time methods, we are now able to define this limiting process in another way. The nonperturbative lattice renormalization-group equation for the 2D antiferromagnetic Heisenberg model that we now propose is useful as it can save computational effort in numerical studies by interpolation of the dynamical scaling.

II. QUANTUM MONTE CARLO METHOD

In order to understand the renormalization-group flow of the lattice couplings in the QMC method that are defined using the transfer matrices in Refs. 13 and 14 and how these relate to the 2D nonlinear sigma model, it is important to understand why these methods do not, in general, yield Lorentz invariant systems. This particular form of the QMC method has a deceptively simple overlap with the 2D O(3) model. Numerical loop evolution generates the lattice path integrals and partition functions in the generic approach. This loop evolution proceeds over both the spatial and Euclidean-time extents of the lattice on an equal footing through the evaluation of local Monte Carlo decisions. The three component O(3) quantum spin operators are represented by their S_z component as a discrete spin defined on the spatial lattice sites, and the S_x and S_y components of the spin operators are defined through the projection of the probability distribution of the loop evolution defined for the Euclidean-time extent.[18] Thus, we arrive at a very similar picture to the continuum 2D O(3) model with a θ term, but the crucial difference is that the topological term has a finite IR cutoff. Although for sufficiently large lattice volumes the distributions of the projections of the operators onto the spatial and Euclidean-time lattice directions do converge, these distributions are not constrained to be identical in the lattice ensemble through the definitions of the loop evolution.[19] Although the transfer matrix is isotropic in space and Euclidean time, the individual loops that are realized through probabilistic decisions are not. Spatial and Euclidean-time isotropies are only realized in the stochastic limit of the lattice ensemble. In practice, a model defined with isotropic interaction couplings can be realized as being locally anisotropic through the dynamics.

Our aim in this paper is to attempt to quantify the effect of this local anisotropy. In general, if we focus just on the spatial properties of the lattice ensemble, then the critical behavior, measured via the probability distribution of the S_z component of spin, can be defined as a function of the lattice interaction coupling J. This follows from the usual finite-size scaling (FSS) picture, which consists of asymptotic expansion about renormalization-group fixed points.[20] However, since the generic QMC spin operators are also defined through a form of topological dimensional reduction, this implies that the fixed point can also be approached smoothly in β (the inverse temperature and cutoff scale for Euclidean time). If this equivalence property is realized, the dynamical critical exponent is necessarily unity and the system is Lorentz invariant. However, in general, this scaling picture is disrupted by IR cutoff effects, and, consequently, the numerically realized system cannot be Wick rotated. Therefore, by defining an exact basis of dynamical fluctuations, the equivalence with the O(3) spin vectors is broken in the generic QMC method that is defined through the transfer matrix in Refs. 13 and 14.

What we will now therefore do is simply change the emphasis of the analysis. We will start with a Wick-rotated definition of the loop operators of the QMC method to make the O(3) equivalence exact and then consider the IR cutoff effects that arise in finite-size lattice system in this choice of basis. This will enable us to quantify the breakdown of Lorentz invariance through the emergence of dynamical scale effects and to quantify the effect of dynamical fluctuations on the irrelevant quantum perturbations of the 2D antiferromagnetic Heisenberg model through nonperturbative renormalization. We do this for the continuous-time QMC method in Ref. 12, because the lattice partition function is analytically continuous in Euclidean-time up to some finite scale.

III. WICK ROTATION

The approach we take is to identify an analytic analog of the QMC partition function, which is Wick rotated. We express the O(3) spin operators using a formalism developed to treat the probabilistic dynamics of lattice systems analytically.[21] This gives an exact description of the lattice ensemble vacuum in the vicinity of the large volume stochastic limit, i.e., in the UV. We then expand this description toward the IR. The asymptotic freedom of the 2D O(3) model has been known for some time.[22] It is also known that the IR fixed point associated with the Berry phase can be treated via perturbative deformations.[23] What we treat here is the most general form of deformation of the irrelevant Berry term, via couplings which are modified by the dynamical fluctuations, where the convergence of the perturbative expansion would not then be guaranteed.

In general, a nonperturbative lattice ensemble is only known through the expectation of its numerical matrix elements λ. These matrix elements can be defined in terms of the generalized spin vectors $\boldsymbol{n}$ that describe the state of the spins on the lattice. For the QMC method, these matrix elements are defined in terms of both S_z components, defined on the spatial lattice sites, and also S_x and S_y components, defined through the projection of the probability distribution onto the Euclidean-time extent of the lattice. In the stochastic limit, we expect the system to be Lorentz invariant; therefore, we can define the Wick rotation of the lattice system through the analytic continuation $J \equiv i\theta$. The spin vectors defined on the spatial lattice sites will then represent the orientation of the S_x and S_y components of spin, and the spin vectors defined on the Euclidean-time direction will represent the projection of the probability distribution defined for the S_z component of the spins. The reason for doing this is that in the continuous-time QMC method,[12] the number of lattice sites in the Euclidean-time direction is variable. Therefore, if we Wick rotate the definitions, we are able to consider the FSS of the topological terms and the IR cutoff effects in quantifiable lattice units.

The transfer matrix of the 2D antiferromagnets is an 8×8 matrix,[24] and so we define the lattice matrix elements through three projection indices: onto the Euclidean-time direction and onto the two separate spatial directions. The implicit isotropy between the S_x and S_y components of spin defines a locally conserved current θ associated with each spatial lattice site. We therefore define two separate matrix elements for the two different spatial directions on the lattice,

$$A(\boldsymbol{n}) \equiv \sum_{(s,s',s'')}^{T\otimes\Theta_1\otimes\Theta_2} \sum_{\sigma\in G} \lambda_{ss's''\sigma}(\boldsymbol{n}) \frac{\langle \boldsymbol{n} \oplus \mathbf{1}_{s\sigma} \oplus \mathbf{1}_{s'\sigma} \oplus \mathbf{1}_{s''\sigma} | \theta_1 \rangle}{\langle \boldsymbol{n} | \theta_1 \rangle}, \tag{1}$$

$$B(\boldsymbol{n}) \equiv \sum_{(s,s',s'')}^{T\otimes\Theta_1\otimes\Theta_2} \sum_{\sigma\in G} \lambda_{ss's''\sigma}(\boldsymbol{n}) \frac{\langle \boldsymbol{n} \oplus \mathbf{1}_{s\sigma} \oplus \mathbf{1}_{s'\sigma} \oplus \mathbf{1}_{s''\sigma} | \theta_2 \rangle}{\langle \boldsymbol{n} | \theta_2 \rangle}, \tag{2}$$

where G is the discrete Z(2) algebra of the S_z spins, σ is an element of G, Θ_1, $\Theta_2 \equiv L$, and T is the Trotter number of the Euclidean-time extent of the lattice. The partition function for the 2D antiferromagnetic Heisenberg model is then given in terms of these matrix elements as a path integral over these two conserved currents,

$$\mathcal{Z} = \int d[\theta_1]d[\theta_2]\exp\left[\int_0^\beta A(\boldsymbol{n}_s)i\theta_1 + B(\boldsymbol{n}_s)i\theta_2 - V(\boldsymbol{n}_s)ds\right], \tag{3}$$

where V comprises the off-diagonal contributions to the spin operator basis, which is simply a general matrix element on the $T\otimes\Theta_1\otimes\Theta_2$ lattice with no special symmetries,

$$V(\boldsymbol{n}) \equiv \sum_{(s,s',s'')}^{T\otimes\Theta_1\otimes\Theta_2} \sum_{\sigma\in G} \lambda_{ss's''\sigma}(\boldsymbol{n}) \frac{\langle \boldsymbol{n} \oplus \mathbf{1}_{s\sigma} \oplus \mathbf{1}_{s'\sigma} \oplus \mathbf{1}_{s''\sigma} | \boldsymbol{n} \rangle}{\langle \boldsymbol{n} | \boldsymbol{n} \rangle}. \tag{4}$$

Instead of a single compact Berry phase term, in this nonperturbatively motivated operator formalism, we have two separate source terms in θ, one for each of the two anisotropic spatial directions. There is also an implicit cross term in $\theta_1\otimes\theta_2$, which comes from V. From this latter term, we are able to generate local states such as the plaquette ordered ground states considered in Ref. 25 because of the fourfold symmetry coming from the relative signs of θ_1 and θ_2. However, we can also consider what happens if we are unable to continue either θ_1 or θ_2 through 2π because of the finite IR cutoff. There will still be an equal number of instantons and anti-instantons within the vacuum, conserving the total topological charge, but if the rotational symmetry about the S_x and S_y spin component plane is lost, then the fourfold parity symmetry is broken. This symmetry is broken down to a twofold symmetry such that the instantons are no longer symmetric under reflection about the S_z component of spin and their chirality is lost. In the 2D O(3) model, the Néel phase is defined by having a larger correlation length than that of the dynamical fluctuations. The same is true for the ground state realized by the generic QMC method, but at finite Trotter number, the same is not necessarily true for correlations in the 4π rotational symmetry of the instantons away from the limit in which the lattice ensemble is Lorentz invariant.

To simplify the cross term of $\theta_1\otimes\theta_2$, we can define the projection of the θ_1 component of V,

$$\begin{aligned} C(\boldsymbol{n}) &\equiv \langle \boldsymbol{n}|V^2(\boldsymbol{n}_{s'})|\boldsymbol{n}\rangle \\ &= \sum_{(s,s',s'')}^{T\otimes\Theta_1\otimes\Theta_2} \sum_{\sigma\in G} \lambda_{ss's''\sigma}(\boldsymbol{n})\langle \mathbf{1}_{\sigma s'}|\boldsymbol{n}\rangle \sum_{(s,s',s'')}^{T\otimes\Theta_1\otimes\Theta_2} \sum_{\sigma\in G} \lambda_{ss's''\sigma}(\boldsymbol{n}) \\ &\quad \times \langle \boldsymbol{n} \oplus \mathbf{1}_{s\sigma} \oplus \mathbf{1}_{s'\sigma} \oplus \mathbf{1}_{\sigma s''}|\boldsymbol{n}_{s'}\rangle. \end{aligned} \tag{5}$$

Substituting, this then yields the partition function

$$\begin{aligned} \mathcal{Z} = \int d[\theta_1]d[\theta_2]\exp\Bigg[\int_0^\beta A(\boldsymbol{n}_s)i\theta_1 + \sqrt{C(\boldsymbol{n}_s)}i\theta_1 \\ + B(\boldsymbol{n}_s)i\theta_2 - V'(\boldsymbol{n}_s)ds\Bigg], \end{aligned} \tag{6}$$

where $V'(\boldsymbol{n}_s) \equiv V(\boldsymbol{n}_s) - C(\boldsymbol{n}_s)$.

IV. LATTICE SPACING

Our motivation for making a Wick rotation is to quantify the effect of dynamical fluctuations on the irrelevant Berry term in the 2D antiferromagnetic Heisenberg model. In the above Wick-rotated operator definitions, θ_1 and θ_2 are noncompact Abelian variables, and the Euclidean-time extent is used to represent the multiplicity of these phases. Consequently, each of the discrete intervals of the Euclidean-time extent now corresponds to a different θ vacuum. To relate this formalism to the usual compact definition of topological charge [given for the 2D O(3) model], we must somehow select one of these θ vacua over the others to be our reference compact sector. Conveniently, this choice of θ vacuum naturally arises from the nonperturbative dynamics in the form of the numerical expectation value that is realized by the projection of the probability distribution defined for Euclidean time.[10] Stating this simply, each spin site on the $T\otimes\Theta_1\otimes\Theta_2$ lattice has an associated matrix element λ, and the Euclidean-time projection of λ has a larger value in one of the discrete intervals of the Euclidean-time extent than in the others. This is, however, a spatial site-specific result. In principle, each spatial site can have the maxima of this projection arise in a different Euclidean-time interval. Different local sites can, therefore, correspond to different θ vacua, which presents a source of nonintegrable singularity in analytically continuing the lattice interaction coupling J.[26] These singularities, though, are precisely the form of dynamical fluctuation-induced IR cutoff effect that we are aiming to quantify.

To quantify these properties, we introduce local measures of lattice spacing a and b defined in units of θ and β, respectively. These are given as the difference between the projection of the matrix element realized on a given lattice site and the expectation of the projection averaged over the corresponding lattice extent,

$$\theta_1 a_{s'} \equiv A(\boldsymbol{n}_{s'}) - \langle A(\boldsymbol{n}_{s'})\rangle_{\Theta_1},$$

$$\beta a_s \equiv A(\boldsymbol{n}_s) - \langle A(\boldsymbol{n}_s)\rangle_T,$$

$$\theta_2 b_{s''} \equiv B(\boldsymbol{n}_{s''}) - \langle B(\boldsymbol{n}_{s''})\rangle_{\Theta_2},$$

$$\beta b_s \equiv B(\boldsymbol{n}_s) - \langle B(\boldsymbol{n}_s)\rangle_T. \quad (7)$$

Practically, $A(\boldsymbol{n}_s)$ and $B(\boldsymbol{n}_s)$ can be found directly as they correspond to the diagonal entries of the numerical transfer matrix of the continuous-time QMC method. Similarly, the two operator projections $A(\boldsymbol{n}_{s'})$ and $B(\boldsymbol{n}_{s''})$ can be calculated by summing the local continuous-time QMC transfer-matrix entries over the spatial sites rather than Euclidean time. Thus, in a formal mathematical sense, these lattice spacing definitions define the support of the matrix elements found by integrating the local $T\otimes\Theta_1\otimes\Theta_2$ matrix elements over a given lattice direction.

A related quantity is the parallel transport between neighboring lattice sites for which we can define the following local derivative operators:

$$\partial_s A(\boldsymbol{n}_s) \equiv A(\boldsymbol{n}_{s+1}) - A(\boldsymbol{n}_s) = i\theta_1 A(\boldsymbol{n}_s),$$

$$\partial_s B(\boldsymbol{n}_s) \equiv B(\boldsymbol{n}_{s+1}) - B(\boldsymbol{n}_s) = i\theta_2 B(\boldsymbol{n}_s). \quad (8)$$

V. LATTICE β FUNCTION

To identify another nonperturbative lattice β function, we define an effective action for the partition function of Eq. (6), which is given as an expansion in the self-energy terms of the dynamical basis. A general loop generated by the Monte Carlo process is of the form $T\otimes\Theta_1\otimes\Theta_2$ and is given by the action of $V(\boldsymbol{n})$ operators on the vacuum. Similarly, the action of the $A(\boldsymbol{n})$ operators generates loops of the form $T\otimes\Theta_1$, and the $B(\boldsymbol{n})$ operators generate loops of the form $T\otimes\Theta_2$. Thus, the self-energy terms of the Wick-rotated dynamical basis we have defined, which describe the projection of the A and B operators onto V, are of the form Θ_1 and Θ_2 and correspond to the θ-symmetry breaking component of the vacuum. The effective action is of the form

$$S = S_1 + S_2 + S_{12}, \quad (9)$$

where

$$S_1 = \int_0^\beta ds\,\partial_s A(\boldsymbol{n}_s) - \partial_s[V'(\boldsymbol{n}_s)A(\boldsymbol{n}_s)] - V'(\boldsymbol{n}_s),$$

$$S_2 = \int_0^\beta ds\,\partial_s B(\boldsymbol{n}_s) - \partial_s[V'(\boldsymbol{n}_s)B(\boldsymbol{n}_s)] - V'(\boldsymbol{n}_s),$$

$$S_{12} = \int_0^\beta ds\,\partial_s\sqrt{C(\boldsymbol{n}_s)} - \partial_s[V'(\boldsymbol{n}_s)\sqrt{C(\boldsymbol{n}_s)}] + V'(\boldsymbol{n}_s). \quad (10)$$

The terms of the effective action are of the same form as the undeformed action in Eq. (3). These terms are defined through a form of topological dimensional reduction where an explicit integration is performed over β to project out the Euclidean-time dependence. Next, we integrate out the effective action terms in a second topological dimensional reduction step by integrating out the expansion over the compact sectors that define each lattice matrix element. The purpose of this second step is to make the effective action compact, which then allows us to make a direct comparison between our new RG flow and that of the 2D nonlinear sigma model. To do this, we introduce two variables; x which is a general position index on $T\otimes\Theta_1$ and y which is a general position index on $T\otimes\Theta_2$.

Our aim, following Ref. 9, is to treat the effect of irrelevant quantum fluctuations on the relevant dynamical fluctuations of the 2D nonlinear sigma model, given in Ref. 1. Our operator formalism is defined to be exact in the basis of these dynamical fluctuations from the properties of the continuous-time QMC method. In practice, the numerics suffer from IR cutoff effects but we have Wick rotated the operator definitions in order to quantify this effect on the topology of the O(3) spin vectors. In principle, the IR cutoff implies that the 2π-rotational symmetry of the S_x and S_y plane components is broken. However, we have the freedom, from the renormalization-group properties of the relevant dynamical fluctuations, to simply rescale the couplings of the operators such that the 2π-rotational symmetry is present. There is then a singularity that sits on the lattice vertices, which appears essentially by modifying a finite number of the poles of the lattice system to branch points.

The second topological dimensional reduction step is straightforward to evaluate for the first two terms in Eq. (9) using the local lattice spacing definitions given in Eq. (7),

$$S_1 = \int_0^\beta \int_{-\pi}^{\pi} d^2x a_{s'}\partial_x^2 A(\boldsymbol{n}) - a_s\partial_x^2 V'(\boldsymbol{n}) - a_s\wedge a_{s'}, \quad (11)$$

$$S_2 = \int_0^\beta \int_{-\pi}^{\pi} d^2y b_{s''}\partial_y^2 B(\boldsymbol{n}) - b_s\partial_y^2 V'(\boldsymbol{n}) - b_s\wedge b_{s''}. \quad (12)$$

These isotropic terms are of the form of the corresponding sine-Gordon model description of quantum spin chains. This is expected since the interaction between the two spatial lattice directions is ignored. These terms imply, following Ref. 9, that the effect of quantum fluctuations can be simply rescaled the into the existing couplings, which describe the dynamical fluctuations. The cross term, which links the two spatial lattice directions, is slightly more involved, however, since it involves a choice of branch,

$$\begin{aligned} S_{12} = \int_0^\beta \int_{-\pi}^{\pi} & d^2x - [1 - \langle\sqrt{C(\boldsymbol{n}_s)}\rangle_{\Theta_1}]\partial_x^2 V'(\boldsymbol{n}) \\ & + (1 + b_s\wedge b_{s''})[1 - \langle V'(\boldsymbol{n}_s)\rangle_{\Theta_1}]\partial_x^2[\sqrt{C(\boldsymbol{n})}] \\ & - \partial_x\langle\sqrt{C(\boldsymbol{n}_s)}\rangle_{\Theta_1}\partial_x\langle V'(\boldsymbol{n}_s)\rangle_{\Theta_1} \\ = & \int_0^\beta \int_{-\pi}^{\pi} d^2x a_{s'}\wedge(1 + b_s\wedge b_{s''})\partial_x^2[\sqrt{C(\boldsymbol{n})}] \\ & - a_s\wedge(b_s\wedge b_{s''})\partial_x^2 V'(\boldsymbol{n}) - (a_s\wedge a_{s'})\wedge(b_s\wedge b_{s''}). \end{aligned} \quad (13)$$

The cross-term contribution to the effective action, within the contour between π and $-\pi$, is of the same form as Eq. (11), but the contribution from the branch point on the boundary leads to an additional phase contribution to the action.[17] By comparing Eqs. (11) and (13), we find that the cross-term contribution is implicitly anisotropic and, therefore, is of the form of the double sine-Gordon model not the sine-Gordon model.[9] This means that although the effect of quantum fluctuations on the dynamical fluctuations is irrelevant (amounting simply to a change in the renormalization scale of the vacuum), the effect of quantum fluctuations is relevant for the Lorentz covariance scale of the dynamical fluctuations. The effect of having a branch point associated with parity in the cross term is that the parity of the vacuum (that allows an exact symmetry between the instantons and anti-instantons) can be explicitly broken via quantum fluctuations. It is only in the limit when $b_s \wedge b_{s''}=1$ that the effective action term in Eq. (13) is Lorentz invariant and this effect is vanishing.

What we should therefore find is that the spectrum of excited states exhibits a doubling due to the degeneracy of the broken chirality of the instanton–anti-instanton pairs, when we are in the region of the classical dimensional crossover of the 2D quantum antiferromagnet with the presence of strong anisotropies due to these quantum fluctuation effects.

The sine-Gordon renormalization-group equations for Eqs. (11) and (12) are then trivially modified in the presence of anisotropy.[28] However, this leads to the emergence of an unstable fixed point and finite renormalized region in the vicinity of the second- order quantum critical point,

$$\frac{da_s}{dl} = -\frac{1}{2}a_s^2\left(\frac{a_{s'} - b_s \wedge b_{s''}}{\pi}\right)^2,$$

$$\frac{da_{s'}}{dl} = (a_{s'} - b_s \wedge b_{s''})(2 - 2a_s). \tag{14}$$

The axis crossing in the usual hyperboloid scaling picture of the $(a_s, a_{s'})$ phase plane is simply shifted by the inclusion of branch singularities. This leads to a very simple analytic rescaling of the relevant couplings of the gap state, but it is one which is not easily quantified perturbatively in terms of a global cross term (as in Ref. 1) because of the local site dependence of the branch.[17] It was argued in Ref. 27 that such a contribution should be responsible for the appearance of deconfined spinons at a finite energy gap above the ordered ground state. What we should, therefore, find is that the spectrum of excited states exhibits a doubling due to the degeneracy of the broken chirality of the instanton–anti-instanton pairs when we are in the region of the classical dimensional crossover of the 2D quantum antiferromagnet through the Lorentz covariance of the irrelevant quantum fluctuations.

VI. SUMMARY

From early spin-wave analyses of the 2D quantum antiferromagnet, it was concluded that the role of quantum fluctuations is irrelevant to the stability of the Néel ordered ground state.[29] Subsequent refinement of these arguments has suggested that the crucial analytic property of the analysis, which enables this stability, is the continuity of generalized spin operators that describe the vacuum.[5,30] In this paper, we have now considered the role of dynamical cutoff effects on these quantum fluctuations in an exact basis of dynamical fluctuations following the nonlinear sigma model treatment given in Ref. 1. The crucial difference from previous studies is that we have now considered the role of anisotropy on modifying the renormalization cutoff scale on the phenomenological couplings of the ordered state. While it is known to high accuracy that the Néel ordered ground state closely follows spin-wave theory predictions,[31] considerably less is known about relevant renormalization of the dynamical couplings by quantum fluctuations when the Lorentz symmetry of the system is broken. This nonperturbative scaling treatment forms a more realistic picture of the numerical scaling of the dynamical basis defined by the generic QMC method, which is only truly Lorentz invariant asymptotically close to the stochastic limit. Similarly, the nonperturbative renormalization-group formalism provides a loop expansion formalism which is suitable for rigorously probing the conformal correspondence of experimental high-temperature superconductivity data at strong interaction couplings. Our main result from this analysis is that the effect of the branch points that describe the chirality of the instantons can be spontaneously broken through quantum fluctuations through the Lorentz covariance of the couplings. We have argued, following Ref. 27, that this should then lead to a doubling of the spectrum of excited states in the vicinity of the quantum critical point of the 2D quantum antiferromagnet when the dynamical couplings are strong.

ACKNOWLEDGMENT

This work was supported by the DFG under Project No. SFB668.

[1] S. Chakravarty, B. I. Halperin, and D. R. Nelson, Phys. Rev. B **39**, 2344 (1989).
[2] P. Kopietz and S. Chakravarty, Phys. Rev. B **40**, 4858 (1989).
[3] L. Wang, K. S. D. Beach, and A. W. Sandvik, Phys. Rev. B **73**, 014431 (2006).
[4] S. Chakravarty, Phys. Rev. Lett. **77**, 4446 (1996).
[5] F. D. M. Haldane, Phys. Rev. Lett. **61**, 1029 (1988).
[6] T. Senthil, L. Balents, S. Sachdev, A. Vishwanath, and M. P. A. Fisher, Phys. Rev. B **70**, 144407 (2004).
[7] J.-K. Kim and M. Troyer, Phys. Rev. Lett. **80**, 2705 (1998).
[8] M. Matsumoto, C. Yasuda, S. Todo, and H. Takayama, Phys. Rev. B **65**, 014407 (2002).

[9] D. Controzzi and G. Mussardo, Phys. Rev. Lett. **92**, 021601 (2004).
[10] P. R. Crompton, J. Stat. Mech.: Theory Exp. (2006) L002.
[11] M. Troyer, M. Imada, and K. Ueda, J. Phys. Soc. Jpn. **66**, 2957 (1997).
[12] B. B. Beard and U.-J. Wiese, Nucl. Phys. B, Proc. Suppl. **53**, 838 (1997).
[13] N. Shibata, J. Phys. A **36**, R381 (2000).
[14] J. V. Alvarez and C. Gros, Eur. Phys. J. B **15**, 641 (2000).
[15] H. G. Evertz, Adv. Phys. **52**, 1 (2003).
[16] P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 436 (1977).
[17] P. R. Crompton, Phys. Rev. D **72**, 076003 (2005).
[18] B. B. Beard, R. C. Brower, S. Chandrasekharan, D. Chen, A. Tsapalis, and U.-J. Weise, Nucl. Phys. B, Proc. Suppl. **63**, 775 (1998).
[19] R. H. Swendsen and J.-S Wang, Phys. Rev. Lett. **58**, 86 (1987).
[20] M. E. Fisher, Rep. Prog. Phys. **30**, 615 (1967).
[21] M. Ostilli and C. Presilla, J. Stat. Mech.: Theory Exp. (2005) P007.
[22] A. M. Polyakov, Phys. Lett. **82B**, 247 (1979).
[23] V. A. Fateev, E. Onofri, and A. B. Zamolodchikov, Nucl. Phys. B **406**, 521 (1993).
[24] N. Shibata, J. Phys. A **3**, R381 (2003).
[25] T. Senthil and M. P. A. Fisher, Phys. Rev. B **74**, 064405 (2006).
[26] P. R. Crompton, Phys. Rev. D **74**, 096001 (2006).
[27] G. Baskaran, Phys. Rev. B **68**, 212409 (2003).
[28] J. M. Kosterlitz, J. Phys. C **7**, 1046 (1974).
[29] P. W. Anderson, G. Baskaran, Z. Zou, and T. Hsu, Phys. Rev. Lett. **58**, 2790 (1987).
[30] I. E. Dzyaloshinskii, A. M. Polyakov, and P. B. Wiegmann, Phys. Lett. A **127**, 112 (1988).
[31] A. W. Sandvik, Phys. Rev. B **56**, 11678 (1997).