Influence of the single-particle Zeeman energy on the quantum Hall ferromagnet at high filling factors

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In a recent paper [B. A. Piot *et al.*, Phys. Rev. B **72**, 245325 (2005)], we have shown that the lifting of the electron spin degeneracy in the integer quantum Hall effect at high filling factors should be interpreted as a magnetic-field-induced Stoner transition. In this work, we extend the analysis to investigate the influence of the single-particle Zeeman energy on the quantum Hall ferromagnet at high filling factors. The single-particle Zeeman energy is tuned through the application of an additional in-plane magnetic field. Both the evolution of the spin polarization of the system and the critical magnetic field for spin splitting are well described as a function of the tilt angle of the sample in the magnetic field.

The integer quantum Hall effect has historically been described within the framework of a single electron picture. Electron-electron interactions are then introduced as a correction, leading to enhanced spin gaps at odd filling factors. Clearly, from a perturbation theory point of view, this approach is wrong, at least for the most widely investigated GaAs system, for which the energy scale of the electron-electron interactions ($e^2/4\pi\epsilon\ell_B$) is more than an order of magnitude larger than the single-particle Zeeman energy ($g^*\mu_B B$). At high magnetic field (low filling factors), this has wide ranging consequences, with the observation of the itinerant quantum Hall ferromagnet,[1] with spin wave[2] or spin texture excitations[3] at filling factor $\nu=1$.

The *collapse* of spin splitting at low magnetic fields (high filling factors) has been investigated experimentally[4–6] and theoretically.[7] Leadley *et al.*[6] showed that the critical filling factor for the *collapse* of spin splitting is found to increase with increasing tilt angle. The Zeeman energy, greatly enhanced at high tilt angles, favors the transition to a polarized state. This latter point is theoretically supported by the pioneering work of Fogler and Shklovskii,[7] who proposed an order parameter $\delta\nu$ to quantitatively characterize the *collapse* of spin splitting. This order parameter corresponds to the filling factor difference between two consecutive resistance maxima in $R_{xx}(B)$, related to spin-up and spin-down sublevels associated with a given Landau level. In the Fogler and Shklovskii model, the spin splitting ($\delta\nu$) collapses, when the disorder broadening of the Landau levels is comparable to the exchange enhanced spin gap.

In an equivalent, but intuitively different approach, we have recently shown[8] that the *appearance* of spin splitting results from a competition between the disorder-induced energy cost of flipping spins and the exchange energy gain associated with the polarized state. In this case, the Zeeman

energy plays no role, and the only effect of the magnetic field is to modify the density of states at the Fermi energy, essentially through the Landau level degeneracy eB/h. Here, we use the experimental behavior of the order parameter $\delta\nu$ to probe the appearance of spin splitting in Al$_x$Ga$_{1-x}$As/GaAs heterojunction (HJ) and quantum well (QW) structures. A large in-plane magnetic field is used to tune (enhance) the single particle Zeeman energy by more than an order of magnitude. Experimentally, this is achieved by rotating the sample in the magnetic field. We show that the behavior of the spin polarization as a function of the tilt angle can be quantitatively described within the framework of our previously developed approach for the appearance of spin splitting, with no free parameters.[8]

We briefly recall our simple model for the appearance of spin splitting in the highest occupied Landau level before introducing the effect of a nonzero Zeeman energy. In the limit of a zero Zeeman energy, we consider an unpolarized initial state in the Nth Landau level, with a total number of electrons $n_{tot}=eB/h$. In this situation, the Fermi level E_F lies in the center of the degenerate spin-up and spin-down sublevels and the filling factor of the system is odd. The development of a nonzero spin polarization requires that the "disorder" energy cost of populating higher energy levels by flipping spin should be less than the gain in exchange energy stabilizing the newly polarized state. The energy cost of flipping spin is inversely proportional to the density of states of one spin sublevel at Fermi level $D(E_F)$, and it can be shown[8] that it will be energetically favorable for spins to flip when

$$\frac{1}{D(E_F)} = X, \tag{1}$$

where X is the exchange energy between two spins, essentially depending only on the electron density n_s. This condi-

tion is nothing other than the well-known Stoner condition for ferromagnetism in metals.[9]

In the presence of a nonzero Zeeman energy, there is an initial spin polarization of the system at odd filling factors. To include this effect, one has to consider the global spin polarization m in the Landau level N, resulting from the total spin gap induced by exchange and Zeeman energies. The latter can then be written as

$$\Delta_s = |g^*|\mu_B B + Xmn_{tot}, \tag{2}$$

where g^* is the effective bare g factor and $n_{tot} = eB/h$ and m are the occupancy and the spin polarization of the Nth Landau level, respectively. Here, mn_{tot} corresponds to the difference between the number of spin-up and spin-down electrons which is at the origin of the exchange gap. For simplicity, we define the zero of energy to be at the Fermi level ($E_F \equiv 0$). The spin polarization $m = (n_\uparrow - n_\downarrow)/n_{tot}$ resulting from the spin gap Δ_s can then be written as

$$m = \frac{1}{n_{tot}} \int_{-\infty}^{0} \left[D\left(E + \frac{\Delta_s}{2}\right) - D\left(E - \frac{\Delta_s}{2}\right) \right] dE, \tag{3}$$

where $D(x) = (1/\Gamma\sqrt{\pi})\exp(-x^2/\Gamma^2)$ for Gaussian broadened Landau levels of width Γ. Equation (2) and (3) need to be solved self-consistently to find m and Δ_s and are essentially equivalent to the solution proposed by Fogler and Shklovskii.[7] For Gaussian Landau levels,

$$m = \text{erf}\left[\frac{1}{2} \frac{(|g^*|\mu_B B + Xmn_{tot})}{\Gamma} \right], \tag{4}$$

where $\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$. We note that assuming an energy independent density of states around E_F (equivalent to assuming a "rectangular" Landau level), we obtain

$$m = \frac{(|g^*|\mu_B B + Xmn_{tot})D(E_F)}{n_{tot}}. \tag{5}$$

However, this is only a good approximation for small values of m, so the equation required to express the continuous evolution of the spin polarization as a function of the magnetic field is Eq. (4). Fogler and Shklovskii[7] have shown that there is a simple relation, $m = \delta\nu$, linking the polarization and the filling factor separation of the peaks in $R_{xx}(B)$, which allows a direct comparison of theory with experiment. In Fig. 1, we plot the magnetoresistance $R_{xx}(B)$ and the measured $\delta\nu$ for sample NRC0050 (the sample parameters are summarized in Table I). The evolution of $m(B)$ calculated using Eq. (4) with $g^* = -0.44$, the generally accepted value for bulk GaAs,[10] and $g^* = 0$ are also plotted in Fig. 1. The values for Γ and X used in the calculation have been independently determined for this sample, as explained in Ref. 8.

Equation (4) reproduces extremely well the collapse of the spin polarization observed in $\delta\nu$, especially considering that there are no fitting parameters. The effect of a nonzero Zeeman energy is similar to the one obtained in Ref. 7, shifting the phase transition to lower magnetic field. To evaluate quantitatively the Zeeman correction, we define a critical magnetic field B_{ss}, corresponding to a value of $\delta\nu = 0.5$, as already proposed in Refs. 6 and 7. B_{ss} can be extracted from
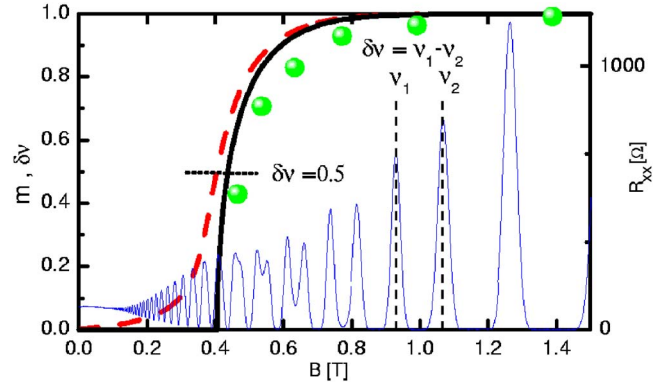

FIG. 1. (Color online) $R_{xx}(B)$ measured at $T = 50$ mK for sample NRC0050. The experimentally determined $\delta\nu$ are also plotted (closed circles). The evolution of the spin polarization $m(B)$ calculated using Eq. (4) in the presence of a Zeeman energy ($g^* = -0.44$) (dashed line), and without Zeeman energy ($g^* = 0$) (thick solid line) are also shown. The condition $\delta\nu = 0.5$ used to estimate B_{ss} is indicated by the dotted horizontal line.

the model by setting $m = \delta\nu = 0.5$ into Eq. (4), with and without the Zeeman correction. The difference between these two results is at most ~10% for $g^* = -0.44$, undiscernible within experimental error, confirming the negligible role of the Zeeman energy in the perpendicular configuration.

It is, however, possible to increase the Zeeman energy by applying a strong in-plane magnetic field. Experimentally, this can be achieved by rotating the sample away from the B normal to the two-dimensional electron gas configuration. The Zeeman energy depends on the total magnetic field B, in contrast to orbital effects such as the Landau level degeneracy or the cyclotron energy which are only sensitive to the $B_\perp$ component of the field. Hence, rotation can be used to tune the Zeeman energy. For a given perpendicular magnetic field $B_\perp$, the strength of the Zeeman energy can be increased by orders of magnitude at high tilt angles. We therefore expect that rotating the sample should provide an incisive test of the zero-free parameter model developed to predict the appearance of spin splitting in Ref. 8.

The effect of the increased single-particle Zeeman gap is clearly visible in Fig. 2, in which we plot $R_{xx}(B_\perp)$ measured at $T = 30$ mK for a GaAs heterojunction at various tilt angles (θ). $R_{xx}(B_\perp)$ was measured using a standard low-frequency lock-in technique under magnetic fields up to 23 T in a dilution refrigerator equipped with an *in situ* rotating sample holder. As the tilt angle is increased, for a given $B_\perp$, the minima at odd filling factor strengthen, reflecting the in-

TABLE I. Parameters of the samples investigated.

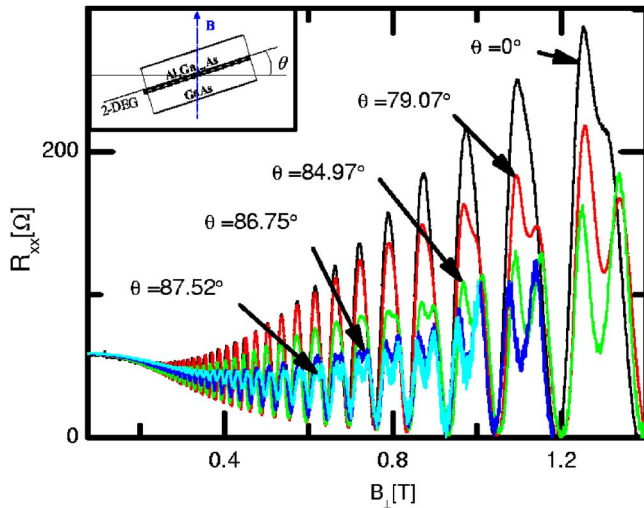| Sample	n_s (cm^{-2})	Γ (K)	Structure	g^*
NRC0050	1.7×10^{11}	1.8 ± 0.1	HJ	-0.44
LPN06	4.0×10^{11}	3.9 ± 0.6	HJ	-0.44
NU1783b	1.8×10^{11}	1.8 ± 0.2	HJ	-0.44
F1200	7.6×10^{11}	1.7 ± 0.1	QW	-0.1

FIG. 2. (Color online) $R_{xx}(B_\perp)$ for sample LPN06 measured at $T = 30$ mK for different tilt angles (θ).

crease of the spin gap. This effect can also be seen in the behavior of $\delta\nu$ as a function of $B_\perp$ [extracted from $R_{xx}(B_\perp)$] for different tilt angles, plotted for a GaAs heterojunction in Fig. 3. The appearance of spin splitting is clearly shifted to lower $B_\perp$ with increasing Zeeman energy at high tilt angles. Tilting the sample increases the Zeeman energy without affecting either the disorder or the exchange parameter. This is quite clear from Eq. (4) where the total magnetic field only enters through the Zeeman term, with n_{tot} involving only the perpendicular component. A similar approach was first proposed in Ref. 6 to extract the enhanced g factor from the coincidence method at high tilt angles. Practically, Eq. (4) can in this case be written as

$$m = \text{erf}\left\{ \frac{1}{2\Gamma} \left[|g^*| \mu_B \frac{B_\perp}{\cos(\theta)} + Xm \frac{eB_\perp}{h} \right] \right\}. \quad (6)$$

The predicted behavior is shown in Fig. 3, calculated using Eq. (6), with the parameters Γ and $X(n_s)$, determined from an analysis of the oscillations in $R_{xx}(B)$ before spin splitting occurs, and the calculations of Attacalite *et al.*,[11] respectively, as detailed in Ref. 8. We impose here the generally accepted value of $g^* = -0.44$ for bulk GaAs. The effect of an increasing tilt angle on the collapse of $\delta\nu$ is well reproduced, considering the slight discrepancy between our model and experiment in the perpendicular configuration [see the curve for $\theta = 0°$ and the associated data (full circles)]. We stress that there are no free fitting parameters in our model which nevertheless provides a good quantitative description. For high tilt angles, the experimental collapse of $\delta\nu$ seems more pronounced than the predicted variation and a possible reason for this is proposed later.

It is important to mention here that the self-consistent nature of Eq. (6), which arises from the dependence of the exchange gap on the spin polarization, is essential in obtaining such good agreement. The reason for this is that even at fixed perpendicular magnetic field, the spin polarization at odd filling factor can be modified due to the change in the single-particle Zeeman energy, if the spin Landau levels
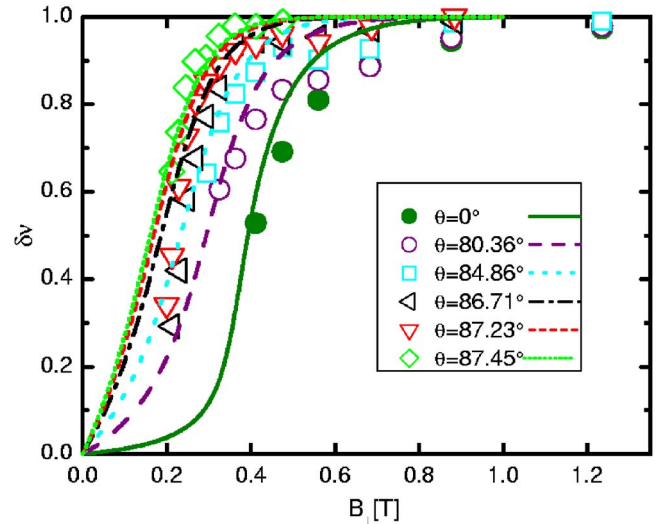

FIG. 3. (Color online) Parameter $\delta\nu(B_\perp)$ for sample NU1783b, extracted from measured $R_{xx}(B)$ at $T = 50$ mK for different tilt angles (symbols). Spin polarization m calculated using Eq. (6) (solid and broken lines).

overlap, leading to a modification of the exchange gap.

We now turn to the evolution of the critical magnetic field B_{ss} with tilt angle. From the curves of Fig. 3, we can extract the experimental magnetic field B_{ss} corresponding to the condition $\delta\nu = 0.5$. This quantity is shown for the three samples for which we have the rotation data in Fig. 4 as a function of the tilt angle θ. LPN06 and NU1783b are the two GaAs heterojunctions already presented and F1200 is a 13-nm-wide GaAs quantum well. In order to focus only on the effect of tilting, we plot the value $B_{ss}(\theta)$ normalized by its value in the perpendicular configuration, $B_{ss}(0)$. As expected, $B_{ss}(\theta)/B_{ss}(0)$ is greatly reduced at high tilt angles, when the Zeeman energy is increased by over an order of
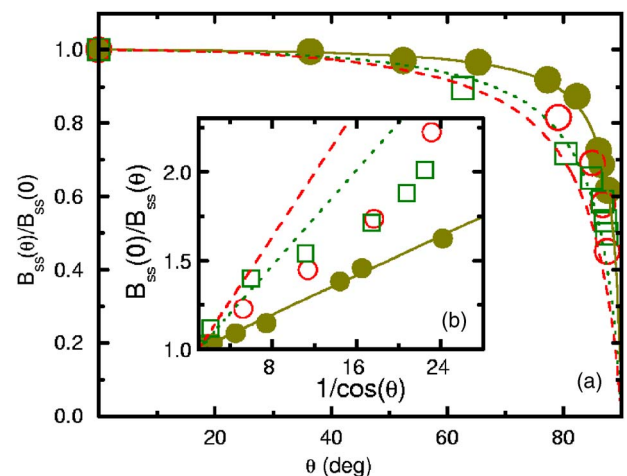

FIG. 4. (Color online) (a) $B_{ss}(\theta)/B_{ss}(0)$ as a function of θ for three different samples at $T = 50$ mK: F1200 (full circles), NU1783b (open squares) and LPN06 (open circles). Self-consistent solutions of Eq. (6), for the three samples, are also plotted (lines). (b) $B_{ss}(0)/B_{ss}(\theta)$ as a function of $1/\cos(\theta)$ for the three samples (symbols) together with the predictions of Eq. (6) (lines).

magnitude. Reductions of more than 50% are observed for angles approaching 90°. From a self-consistent solution of Eq. (6), we can obtain the predicted magnetic field B_{ss} as a function of the angle θ. As B_{ss} corresponds to the condition $\delta\nu=0.5$, the solution is obtained setting $m=0.5$ in Eq. (6). The predicted evolution of $B_{ss}(\theta)/B_{ss}(0)$ is plotted, as solid and broken lines, for the three samples in Fig. 4(a).

As before, Γ and X have been independently determined for each sample.[8] For the heterojunction samples, NU1783b and LPN06, good agreement with the data is obtained using the generally accepted value of $g^*=-0.44$ for bulk GaAs.[10] For the 13 nm quantum well sample, F1200, we expect a significantly lower value for the bare g factor owing to the confinement energy and the penetration of the wave function into the barriers.[12] In addition, nonparabolicity is expected to further reduce the g factor[13] as the electronic density (Fermi energy) is in this sample quite high ($n_s=7.5\times10^{11}$ cm^{-2}). Good agreement with the data can be obtained using $g^*=-0.1$ which is not unreasonable.

As can be seen in Fig. 4(a), generally good quantitative agreement is obtained confirming the relevance of the physical approach proposed. A slight discrepancy ($\sim20\%-30\%$) is observed at large angles for samples NU1783b and LPN06 between the prediction and the experimental B_{ss}, the latter being slightly larger. This can be seen in Fig. 4(b) in which we plot the experimental and predicted $B_{ss}(0)/B_{ss}(\theta)$ as a function of $1/\cos(\theta)$. Intuitively, the linear dependence, observed for the exact numerical solution of Eq. (6), can be understood from the approximate expression for m in Eq. (5), writing $B=B_\perp/\cos(\theta)$, $D(E_F)=eB_\perp/h\Gamma\sqrt{(\pi)}$, $n_{tot}=eB_\perp/h$, and $m=0.5$ which gives

$$\frac{B_{ss}(0)}{B_{ss}(\theta)} \approx \frac{1}{\cos(\theta)}\left(\frac{g^*\mu_B}{g^*\mu_B+Xe/2h}\right) + \left(\frac{Xe/2h}{g^*\mu_B+Xe/2h}\right).$$

Equation (6) provides a reasonable prediction for the slope of the $1/\cos(\theta)$ dependence, for the decrease in $B_{ss}(\theta)$ of less than $\sim50\%$. A deviation from theory is clearly visible at large θ [large $1/\cos(\theta)$] for samples NU1783b and LPN06. For F1200, however, a linear behavior in good agreement with the prediction is observed. A possible explanation for these discrepancies is that the large in-plane magnetic field, at large tilt angles, increases the Landau level width, which would shift the appearance of spin splitting to higher perpendicular magnetic field. At large tilt angles, the amplitude of the oscillations in $R_{xx}(B_\perp)$ decrease significantly for a given $B_\perp$ (see Fig. 2), revealing a modification of the scattering as the in-plane magnetic field increases. The effect of the in-plane magnetic field, which we have also observed in in-plane magnetoresistance measurements ($\theta=90°$), is in GaAs a complex interplay between spin and orbital effects[14,15] affecting both the quantum lifetime (Landau level width) and the effective mass. The orbital part of this effect is weaker in quantum wells in which the spacing between electronic subbands is larger than in heterojunctions, limiting the intersubband transitions induced by an in-plane magnetic field. This would be consistent with the fact that sample F1200, a 13 nm quantum well, is not affected by this process and shows no deviation from the predicted linear behavior.

In summary, we have extended our model for the quantum Hall ferromagnet at high filling factors to the case of a nonzero Zeeman energy. The Zeeman energy has been tuned via tilted field measurements. Our simple model, with no free fitting parameters, provides a reasonable quantitative description of the Zeeman energy dependence of spin polarization (spin splitting) at odd filling factors.

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