

Anisotropic current-induced spin accumulation in the two-dimensional electron gas with spin-orbit coupling

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(Received 19 January 2007; published 17 April 2007)

We investigate the magnetoelectric (or inverse spin-galvanic) effect in the two-dimensional electron gases with both Rashba and Dresselhaus spin-orbit couplings using an exact solution of the Boltzmann equation for electron spin and momentum. The spin response to an in-plane electric field turns out to be highly anisotropic, while the usual charge conductivity remains isotropic, contrary to earlier statements.

 DOI: [10.1103/PhysRevB.75.155323](https://doi.org/10.1103/PhysRevB.75.155323)

PACS number(s): 72.10.-d, 72.25.Dc

I. INTRODUCTION

Spin phenomena in semiconductor structures lie at the very heart of the emerging field of spintronics and are to a major and still growing direction of solid-state research. Among the plethora of concepts and ideas, the magnetoelectric effect in the two-dimensional electron gas (2DEG), sometimes also referred to as the inverse spin-galvanic effect, has attracted particular interest from both experimental¹⁻⁵ and theoretical⁶⁻¹⁰ points of view. This effect amounts in spin accumulation as a response to an applied in-plane electric field and is, therefore, a possible key ingredient toward all-electrical spin control in semiconductor structures, a major goal in today's spintronics research.

We find it important to emphasize that the phenomenon studied below is *not* the intrinsic spin Hall effect,^{11,12} though it would manifest in a very similar manner. The mechanisms involved in these two phenomena are, however, very different. In particular, the momentum relaxation due to the impurity scattering is necessary for the spin accumulation investigated in present paper, contrary to the intrinsic spin Hall effect where ballistic regime is preferred.^{11,12}

In this paper, we investigate the magnetoelectric effect in 2DEG's with both Rashba and Dresselhaus spin-orbit couplings. Our study is based on an exact analytical solution to the Boltzmann equation for electron spin and momentum in the presence of δ -function shape static impurities. Regarding the spin degree of freedom, our solution neglects off-diagonal elements of the semiclassical distribution matrix in the eigenbasis (or helicity basis) of the single-particle Hamiltonian, an approximation which is shown to be valid at sufficiently high temperatures common in experiments.

As a result, the magnetoelectrical effect, i.e., the magnetic response to an in-plane electric field, turns out to be highly anisotropic, whereas the usual charge anisotropy remains isotropic, contrary to earlier statements.¹³

The paper is organized as follows. First, we present an analytical solution of the semiclassical spin-incoherent Boltzmann equation. Second, we solve the spin-coherent kinetic equation. Finally, we discuss the plausibility of solutions obtained and apply them for investigations of the electric current and spin accumulation.

II. SOLUTION OF THE KINETIC EQUATION

We consider the Hamiltonian as a sum of the kinetic energy and two spin-orbit coupling terms: Rashba¹⁴ and

Dresselhaus.¹⁵ Then, the Hamiltonian takes the form

$$H = \frac{\hbar^2(k_x^2 + k_y^2)}{2m} + \alpha(\sigma_x k_y - \sigma_y k_x) + \beta(\sigma_x k_x - \sigma_y k_y). \quad (1)$$

Here, $\sigma_{x,y}$ are the Pauli matrices, $k_{x,y}$ are the electron wave vectors, and m is the effective electron mass. Introducing the angle γ_k , so that

$$\tan \gamma_k = \frac{\alpha k_x + \beta k_y}{\beta k_x + \alpha k_y}, \quad (2)$$

we obtain the following spinors as the eigenfunctions of the Hamiltonian H :

$$\Psi_{\pm}(x, y) = \frac{1}{\sqrt{2}} e^{ik_x x + ik_y y} \begin{pmatrix} 1 \\ \pm e^{-i\gamma_k} \end{pmatrix}. \quad (3)$$

The energy spectrum has the form

$$E_{\pm} = \frac{\hbar^2 k^2}{2m} \pm \sqrt{(\alpha k_x + \beta k_y)^2 + (\beta k_x + \alpha k_y)^2}, \quad (4)$$

where $k = \sqrt{k_x^2 + k_y^2}$. The velocity matrix in the helicity basis (3) is not diagonal, and its elements read

$$v_x^{11(22)} = \hbar k_x / m \pm (\beta \cos \gamma_k + \alpha \sin \gamma_k) / \hbar, \quad (5)$$

$$v_x^{12(21)} = \pm i(\beta \sin \gamma_k - \alpha \cos \gamma_k) / \hbar, \quad (6)$$

$$v_y^{11(22)} = \hbar k_y / m \pm (\alpha \cos \gamma_k + \beta \sin \gamma_k) / \hbar, \quad (7)$$

$$v_y^{12(21)} = \pm i(\alpha \sin \gamma_k - \beta \cos \gamma_k) / \hbar. \quad (8)$$

For the diagonal elements of the velocity matrix, we also use simplified notations $v^{11(22)} \equiv v_{\pm}$.

In the following, it will be convenient to use polar coordinates $k_x = k \cos \theta$, $k_y = k \sin \theta$. Then, the spectrum read $E_{\pm} = \hbar^2 k^2 / (2m) \pm |k| \kappa_{\theta}$, where

$$\kappa_{\theta} = \sqrt{\alpha^2 + \beta^2 + 4\alpha\beta \sin \theta \cos \theta}$$

is the generalized spin-orbit interaction constant for a given direction of motion. The wave vectors for a given energy read

$$k_{\pm} = \mp \frac{m}{\hbar^2} \kappa_{\theta} + \sqrt{\left(\frac{m}{\hbar^2}\right)^2 \kappa_{\theta}^2 + \frac{2mE}{\hbar^2}}, \quad (9)$$

and the expression for γ_k takes the form

$$\tan \gamma_k = \frac{\alpha \cos \theta + \beta \sin \theta}{\beta \cos \theta + \alpha \sin \theta}.$$

We model the influence of impurities using δ -potential scattering $V = \zeta \delta(x, y)$. The scattering probability between the states with k, s and k', s' then read

$$w(k_s; k' s') = \frac{\pi \hbar^2}{m \tau} [1 + s s' \cos(\gamma_k - \gamma_{k'})] \delta(E_{sk} - E_{s'k'}), \quad (10)$$

where τ is the relaxation time which relates to ζ as $\tau = \hbar^3 / (m \zeta^2)$. Note that the spin-dependent factor in Eq. (10) does not depend on the particular form of $V(x, y)$ in the Born approximation. We expect, therefore, that such a simple scattering model as δ -function shape static impurities is quite reliable for the description of spin-dependent transport phenomena.

A. Spin-incoherent kinetic equation

In general, the equation has the form

$$\frac{\partial f_s(\mathbf{k})}{\partial t} + \mathbf{v}_s \frac{\partial f_s(\mathbf{k})}{\partial \mathbf{r}} + (-e\mathbf{E}) \frac{\partial f_s(\mathbf{k})}{\hbar \partial \mathbf{k}} = \left(\frac{\partial f_s}{\partial t} \right)_{\text{coll}}. \quad (11)$$

Here, $s = \pm$ is the spin index, and \mathbf{v}_s are the diagonal elements (5) and (7) of the velocity matrices.

To solve Eq. (11), we follow the standard procedure widely spread in literature (see Ref. 16, or any textbook on solid state physics). We write down the distribution function as $f_s = f_s^0 + f_s^1 + f_s^2$, where f_s^0 is the Fermi distribution and $f_s^{1,2} \ll f_s^0$. Then, at zero temperature gradient and constant electric field in the linear-response regime, Eq. (11) takes the form

$$-e\mathbf{E} \mathbf{v}_s \left[-\frac{\partial f^0(E_{sk})}{\partial E_{sk}} \right] = \text{St}[f_s(\mathbf{k})]. \quad (12)$$

Assuming elastic scattering, fulfilling the microreversibility condition, the scattering operator can be written as

$$\text{St}[f_s(\mathbf{k})] = \sum_{s'} \int \frac{d^2 k'}{(2\pi)^2} \{w(\mathbf{k}_s; \mathbf{k}' s') [f_s^1(\mathbf{k}) + f_s^2(\mathbf{k}) - f_{s'}^1(\mathbf{k}') - f_{s'}^2(\mathbf{k}')] \}, \quad (13)$$

where $w(\mathbf{k}_s; \mathbf{k}' s')$ is given by Eq. (10).

We would like to emphasize that besides above-mentioned conventional assumptions, our solution is exact. It means that in contrast to a number of previous works, we do not make any further approximation from this point.

The solution of Eq. (12) has the form

$$f_s^1 = -\tau e \mathbf{E} \mathbf{v}_s \left[-\frac{\partial f^0(E_{sk})}{\partial E_{sk}} \right], \quad (14)$$

$$f_s^2 = -s \frac{\tau e \mathbf{E}}{\hbar \kappa_{\theta}} \left[-\frac{\partial f^0(E_{sk})}{\partial E_{sk}} \right] [(a_x \cos \theta + b_x \sin \theta) \mathbf{e}_x + (a_y \cos \theta + b_y \sin \theta) \mathbf{e}_y]. \quad (15)$$

Here, f_s^1 is the conventional solution of the Boltzmann equation, whereas an addition f_s^2 is usually discarded since it is much smaller than f_s^1 . However, this anisotropic addition plays an important role in the spin accumulation mechanism, as it is shown below.

In order to define the unknown coefficients $a_{x,y}$, $b_{x,y}$, we substitute $f_s^{1,2}$ into Eq. (12) (see Appendix for details) and get the following equations:

$$a_x \frac{\alpha^2 + \beta^2 - |\alpha^2 - \beta^2|}{4\alpha\beta} - \frac{b_x}{2} = \alpha \beta \left[1 + \frac{(\alpha^2 + \beta^2)^2}{4\alpha^2\beta^2} \left(\frac{|\alpha^2 - \beta^2|}{\alpha^2 + \beta^2} - 1 \right) \right], \quad (16)$$

$$b_x \frac{\alpha^2 + \beta^2 - |\alpha^2 - \beta^2|}{2\alpha\beta} - a_x = |\alpha^2 - \beta^2|. \quad (17)$$

The equations for a_y and b_y can be obtained from Eqs. (16) and (17) by the substitution $a_x \rightarrow b_y$, $b_x \rightarrow a_y$. From these equations, one can easily find that $a_x = b_y = -(\alpha^2 + \beta^2)$ and $a_y = b_x = -2\alpha\beta$.

Finally, we substitute (5) and (7) into Eq. (14), sum up f_s^1 and f_s^2 , and write down the solution of Eq. (12) in the following elegant form:

$$f_s = f_s^0 + (-e\mathbf{E}) \mathbf{k} \frac{\hbar \tau}{m} \left[-\frac{\partial f^0(E_{sk})}{\partial E_{sk}} \right]. \quad (18)$$

This solution should be compared with the expression in the absence of spin-orbit coupling which can be formulated in terms of the velocity as follows:

$$f_s = f_s^0 + (-e\mathbf{E}) \mathbf{v}_s \tau \left[-\frac{\partial f^0(E_{sk})}{\partial E_{sk}} \right], \quad (19)$$

where $\mathbf{v}_s = \hbar \mathbf{k} / m$ is the velocity. Note that this latter relation holds only in the absence of spin-orbit coupling.

B. Spin-coherent kinetic equation

Since we deal with the constant electric field only, the equation reads¹⁷

$$(-e\mathbf{E}) \frac{\partial \hat{f}(\mathbf{k})}{\hbar \partial \mathbf{k}} + \frac{i}{\hbar} [\hat{H}, \hat{f}(\mathbf{k})] = \left(\frac{\partial \hat{f}}{\partial t} \right)_{\text{coll}}. \quad (20)$$

Following Ref. 17, we rewrite Eq. (20) in the helicity basis where the Hamiltonian is diagonal, and the equation takes the form

$$\frac{e\mathbf{E}}{\hbar} \frac{\partial}{\partial \mathbf{k}} \begin{pmatrix} f_{11} & f_{12} \\ f_{21} & f_{22} \end{pmatrix} + \frac{ie\mathbf{E}}{2\hbar} \frac{\partial \gamma_k}{\partial \mathbf{k}} \begin{pmatrix} f_{21} - f_{12} & f_{22} - f_{11} \\ f_{11} - f_{22} & f_{12} - f_{21} \end{pmatrix} + \frac{i}{\hbar} \begin{pmatrix} 0 & f_{12}(E_+ - E_-) \\ f_{21}(E_- - E_+) & 0 \end{pmatrix} = \text{St}[\hat{f}(\mathbf{k})], \quad (21)$$

where

$$\frac{\partial \gamma_k}{\partial \mathbf{k}} = \frac{\alpha^2 - \beta^2}{k\kappa_\theta^2} (\mathbf{e}_x \sin \theta - \mathbf{e}_y \cos \theta). \quad (22)$$

The collision term reads¹⁷

$$\begin{aligned} \text{St}[\hat{f}(\mathbf{k})]_{ss_1} = & - \int \frac{d^2k'}{(2\pi)^2} \sum_{s',s'_1} \{ [\delta(E_{s'k'} - E_{sk}) + \delta(E_{s_1k'} \\ & - E_{sk})] K_{s's'_1}^{ss_1} f_{s's'_1}(k') - \delta(E_{s'k} - E_{s_1k'}) \\ & \times [K_{s_1s'_1}^{ss'} f_{s's'_1}(k) + K_{s's'_1}^{s's_1} f_{s's'_1}(k)] \}, \end{aligned} \quad (23)$$

where

$$\left(\begin{array}{c} e\mathbf{E}\mathbf{v}^{11} \frac{\partial f^0(E_{+k})}{\partial E_{+k}} \\ \frac{e\mathbf{E}\mathbf{v}^{21}}{2} \left[\frac{\partial f^0(E_{+k})}{\partial E_{+k}} + \frac{\partial f^0(E_{-k})}{\partial E_{-k}} \right] + \frac{i}{\hbar} f_{21}(E_- - E_+) \end{array} \quad \frac{e\mathbf{E}\mathbf{v}^{12}}{2} \left[\frac{\partial f^0(E_{+k})}{\partial E_{+k}} + \frac{\partial f^0(E_{-k})}{\partial E_{-k}} \right] + \frac{i}{\hbar} f_{12}(E_+ - E_-) \quad \begin{array}{c} e\mathbf{E}\mathbf{v}^{22} \frac{\partial f^0(E_{-k})}{\partial E_{-k}} \end{array} \right) = \text{St}[\hat{f}(\mathbf{k})]. \quad (25)$$

Though Eq. (25) is still quite cumbersome, it is easy to check that $f_{12}=f_{21}=0$ and $f_{11(22)}$ given by Eq. (18) represent the solution. The details of calculations can be found in Appendix. Thus, the solution for the spin-incoherent Boltzmann equation is the same as for the spin-coherent kinetic equation at high temperatures. This can be explained in what follows.

III. RESULTS AND DISCUSSION

Note that the off-diagonal elements of the distribution function have essentially quantum mechanical origin since they correspond to the off-diagonal elements of the density matrix. Classically, an electron can be in only one state of two, and, therefore, off-diagonal terms vanish here. In contrast, in the spin-coherent kinetic equation, the off-diagonal terms can be essential. However, in the real samples, the quantum effects are negligible at room temperatures because of the temperature smearing. Indeed, the spin-orbit coupling constant κ_θ is of the order of 10^{-11} eV m for typical InAs samples. To be specific, let us take n -type InAs quantum well containing the 2DEG used for photocurrent measurements at room temperature.¹⁸ The parameters are as follows: $\alpha/\beta = 2.15$, mobility is about 2×10^4 cm²/(V s), and free carrier density is 1.3×10^{12} cm⁻². The latter allows us to estimate characteristic Fermi wave vector $k_F = \sqrt{2\pi n_e} \approx 3 \times 10^6$ cm⁻¹. Thus, the spin-orbit splitting energy $k_F \kappa_\theta$ is about 3 meV, that is much smaller than $T_{\text{room}} = 25$ meV, and our solution is suitable for description of a large variety of experiments.

To study the spin accumulation, we calculate the net spin density, whose x, y components read

$$K_{s's'_1}^{ss_1} = \frac{\pi \hbar^2}{4m\tau} [1 + ss's_1s'_1 + ss'e^{i(\gamma_k - \gamma_{k'})} + s_1s'_1e^{-i(\gamma_k - \gamma_{k'})}]. \quad (24)$$

Equation (24) was derived for δ -potential scattering introduced above, and each $K_{s's'_1}^{ss_1}$ contains either $\sin(\gamma_k - \gamma_{k'})$ or $1 \pm \cos(\gamma_k - \gamma_{k'})$, depending on the product of $ss's_1s'_1$.

In order to simplify Eq. (21), we assume $k\kappa_\theta \ll T$, where T is the temperature. This assumption will be discussed later in detail. Now, we expand the Fermi distribution function $f^0(E_{sk})$ in terms of $k\kappa_\theta/T$, so that Eq. (21) in the linear-response regime takes the form

$$\langle S_{x,y} \rangle = \sum_s \int \frac{d^2k}{(2\pi)^2} S_{x,y}(k, s) f_s(\mathbf{k}), \quad (26)$$

where $S_x(k, s) = \frac{s}{2} \cos \gamma_k$ and $S_y = -\frac{s}{2} \sin \gamma_k$ are the spin expectation values. (We do not consider S_z component, since it is zero.)

The integral over \mathbf{k} can be taken easily, making the substitution $\varepsilon = E(s, k)$ and assuming that $-\partial f^0(\varepsilon)/\partial \varepsilon = \delta(E_F - \varepsilon)$. This assumption is reasonable with respect to the system studied in Ref. 18 from which one can deduce a Fermi energy of the order of 100 meV, which is clearly larger than room temperature. Since our solution is valid for temperatures much higher than the spin-orbit splitting energy $k_F \kappa_\theta$, the inequality describing the applicability of our results obtained below reads

$$k_F \kappa_\theta \ll T \ll E_F.$$

The rest integrals over the polar angle can be taken analytically. After some algebra, we have

$$\langle \mathbf{S} \rangle = \frac{em\tau}{2\pi\hbar^3} \begin{pmatrix} \beta & \alpha \\ -\alpha & -\beta \end{pmatrix} \mathbf{E}. \quad (27)$$

Then, the magnitude of the spin accumulation $\langle S \rangle = \sqrt{\langle S_x \rangle^2 + \langle S_y \rangle^2}$ is given by

$$\langle S \rangle = \frac{eEm\tau}{2\pi\hbar^3} \sqrt{\alpha^2 + \beta^2 + 2\alpha\beta \sin(2\widehat{\mathbf{E}}\mathbf{e}_x)}. \quad (28)$$

It is interesting to note that $\langle S \rangle$ depends on the direction of the electric field (see Fig. 1), i.e., the spin accumulation is anisotropic. To our knowledge, this interesting feature was not noticed in the literature so far. If $\beta=0$, then $\langle S \rangle = eEm\tau\alpha/(2\pi\hbar^3)$ that is in agreement with Refs. 6 and 8. If

the electric field is applied along the x direction (the case studied in Ref. 9), then the spin density is $\langle S \rangle = eEm\tau\sqrt{\alpha^2 + \beta^2}/(2\pi\hbar^3)$. This result contradicts to Ref. 9, where the spin density has some strange kinks as a function of $m\alpha^2/\hbar^2$ and $m\beta^2/\hbar^2$. However, one can see from Eq. (28) that the kinks in the dependencies of $\langle S \rangle$ on α (or β) could not take place.

Relying on the effect depicted in Fig. 1, the following spintronic device can be proposed. Let us attach two pairs of contacts to the 2DEG, so that the first pair provides the electric current along the crystallographic axis corresponding to the minimal spin accumulation (i.e., $[1\bar{1}0]$ axis for $[001]$ -grown InAs samples¹⁸), and the second one is connected along the perpendicular axis. Then, the spin accumulation depends strongly on which contacts the transport voltage is applied, and its anisotropic contribution can be extracted easily using optical methods²⁻⁵ or just measuring the magnetization. To give an example, applying the electric field of 20 V/m (which corresponds to the current density of 1 mA/cm), we obtain the magnetization difference of the order of $10^6 \mu_B/\text{cm}^2$, where μ_B is the Bohr magneton. This is comparable to the Pauli magnetization at the magnetic fields of a few gauss. It is also interesting to note the small characteristic switching time $\tau \approx 10^{-13}$ s of the device proposed. Therefore, besides the fundamental importance of such an experiment, our four terminal device could find some applications as a high-speed spin switch.

At the end of the discussion, let us turn to the charge current, whose density can be found as

$$\mathbf{j} = -e \sum_s \int \frac{d^2k}{(2\pi)^2} \mathbf{v}_s f_s(\mathbf{k}). \quad (29)$$

The integral over \mathbf{k} in (29) can be taken in the same manner as in (26), and, after some algebra, the conductivity tensor σ takes the form

$$\sigma_{yy(xx)} = \frac{e^2 \tau}{\pi \hbar^2} \left(\frac{m(\alpha^2 + \beta^2)}{\hbar^2} + E_F \right), \quad (30)$$

and, most surprisingly, $\sigma_{xy(yx)} = 0$. It is convenient to express $\sigma_{yy(xx)}$ via the electron concentration (31) in a 2DEG with spin-orbit interactions

$$n_e = \frac{mE_F}{\pi \hbar^2} + \left(\frac{m}{\hbar^2} \right)^2 \frac{\alpha^2 + \beta^2}{\pi}. \quad (31)$$

Then, the conductivity takes much simpler form, namely, $\sigma_{yy(xx)} = e^2 n_e \tau / m$. This is the Drude formula, i.e., the conductivity is just a number (not a tensor), though the distribution function (18) is anisotropic. This result is in contrast with the findings of Ref. 13, where a subtle approximation generated off-diagonal elements in the conductivity tensor.¹⁹ This artifact is absent in our truly exact solution: The electrical conductivity is isotropic for any relation between α and β . The latter simplifies essentially the relation between the spin and charge current densities, which takes the form

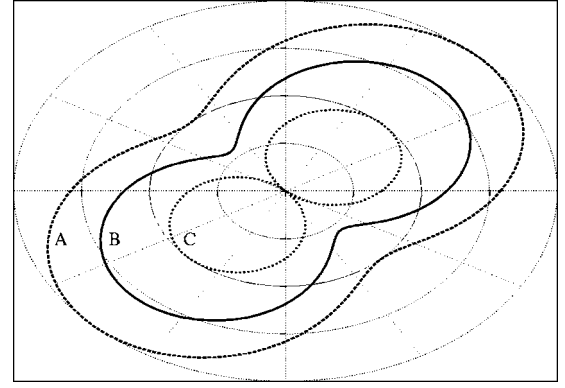


FIG. 1. Spin accumulation (in arbitrary units) vs direction of the electric field in polar coordinates for different Rashba and Dresselhaus constants: A, $\alpha = 3\beta$; B, $\alpha = 2.15\beta$; C, $\alpha = \beta$. Curve B corresponds to the real situation in n -type InAs (Ref. 18).

$$\langle \mathbf{S} \rangle = \frac{m^2}{2\pi\hbar^3 e n_e} \begin{pmatrix} \beta & \alpha \\ -\alpha & -\beta \end{pmatrix} \mathbf{j}. \quad (32)$$

We find it useful to write down Eq. (32) in the basis $\frac{1}{\sqrt{2}}(1; 1)$, $\frac{1}{\sqrt{2}}(1; -1)$. Then the relation between $\langle \mathbf{S} \rangle$ and \mathbf{j} takes the simpler form

$$\langle \mathbf{S} \rangle = \frac{m^2}{2\pi\hbar^3 e n_e} \begin{pmatrix} 0 & \beta - \alpha \\ \beta + \alpha & 0 \end{pmatrix} \mathbf{j}. \quad (33)$$

From this equation, one can easily see that the spin accumulation is strongly anisotropic if the constants α and β are close to each other. The reason of such an anisotropy is the angular dependence of the dispersion law (4). Indeed, the spin-orbit splitting is different for different direction of the momentum. Therefore, the spin precession frequency depends essentially on the direction of the electron motion. This leads to the anisotropic spin relaxation times (see e.g., Ref. 20), and, thus, the anisotropy of the spin accumulation occurs. In particular, the electric current of arbitrary strength applied along the $[1\bar{1}0]$ crystallographic axis does not lead to any spin accumulation at $\alpha = \beta$. The latter is due to the vanishing spin splitting along the $[1\bar{1}0]$ axis (see e.g., Ref. 21).

Equations (32) and (33) are the main results of our work, which can be applied directly to experimental studies of current-induced spin accumulation in $[001]$ -grown InAs samples. The theory developed here is, of course, applicable to arbitrary oriented heterostructures after a few minor changes regarding the spin-orbit coupling terms in the Hamiltonian.

IV. CONCLUSIONS

In summary, we solved the semiclassical Boltzmann equation analytically for 2DEGs with arbitrary large spin-orbit interactions of both Rashba and Dresselhaus types. Moreover, we demonstrated that this solution is also suitable for the spin-coherent case at sufficiently high temperatures common to experiments. Using this solution, we discovered the anisotropy of the current-induced spin accumulation, though

the conductivity remains isotropic. Finally, our analytical study is expected to be a reliable starting point for further investigations of spin-dependent electron transport.

ACKNOWLEDGMENTS

We thank S. D. Ganichev and D. Loss for useful discussions and acknowledge financial support from Collaborative Research Center 689.

APPENDIX: THE COLLISION INTEGRAL CALCULATIONS

To take the integrals of the form $\int \frac{d^2k'}{(2\pi)^2} F(k', E_{s'k'})$, it is convenient to perform the substitution $E_{s'k'} = \varepsilon'$, where $F(k', E_{s'k'})$ is a given function, and $E_{s'k'}$ is the dispersion relation (4).

1. Spin-incoherent case

The first term of $\text{St}[f_s(\mathbf{k})]$ given by Eq. (13) reads

$$\begin{aligned} \sum_{s'} \int \frac{d^2k'}{(2\pi)^2} w(\mathbf{k}s; \mathbf{k}'s') [f_s^1(\mathbf{k}) + f_s^2(\mathbf{k})] &= [f_s^1(\mathbf{k}) + f_s^2(\mathbf{k})] \sum_{s'} \int_0^{2\pi} \frac{d\theta'}{(2\pi)^2} \int d\varepsilon' \frac{m}{\hbar^2} \left[1 - s' \frac{m\kappa_{\theta'}/\hbar^2}{\sqrt{\left(\frac{m}{\hbar^2}\right)^2 \kappa_{\theta'}^2 + \frac{2m\varepsilon'}{\hbar^2}}} \right] \frac{\pi\hbar^2}{m\tau} \delta(E_{sk} - \varepsilon') \\ &\times \left[1 + ss' \frac{(\alpha^2 + \beta^2)\cos(\theta - \theta') + 2\alpha\beta \sin(\theta + \theta')}{\kappa_{\theta}\kappa_{\theta'}} \right] = \frac{1}{\tau} [f_s^1(\mathbf{k}) + f_s^2(\mathbf{k})]. \end{aligned} \quad (\text{A1})$$

The rest terms of $\text{St}[f_s(\mathbf{k})]$ containing f_s^1 and f_s^2 given by Eqs. (14) and (15), respectively, read

$$\begin{aligned} \sum_{s'} \int \frac{d^2k'}{(2\pi)^2} w(\mathbf{k}s; \mathbf{k}'s') [-f_{s'}^1(\mathbf{k}')] &= \left[-\frac{\partial f^0(E_{sk})}{\partial E_{sk}} \right] \times \sum_{x,y} \frac{-seE_{x,y}}{2\hbar\kappa_{\theta}} \left(\frac{|\alpha^4 - \beta^4|}{\alpha^2 + \beta^2} \begin{Bmatrix} \cos \theta \\ \sin \theta \end{Bmatrix} + 2\alpha\beta \begin{Bmatrix} \sin \theta \\ \cos \theta \end{Bmatrix} \right) \\ &\times \left[1 + \frac{(\alpha^2 + \beta^2)^2}{4\alpha^2\beta^2} \left(\frac{|\alpha^2 - \beta^2|}{\alpha^2 + \beta^2} - 1 \right) \right], \end{aligned} \quad (\text{A2})$$

$$\begin{aligned} \sum_{s'} \int \frac{d^2k'}{(2\pi)^2} w(\mathbf{k}s; \mathbf{k}'s') [-f_{s'}^2(\mathbf{k}')] &= \left[-\frac{\partial f^0(E_{sk})}{\partial E_{sk}} \right] \sum_{x,y} \frac{seE_{x,y}}{2\hbar\kappa_{\theta}} \left[\cos \theta \left(a_{x,y} + b_{x,y} \frac{\alpha^2 + \beta^2 - |\alpha^2 - \beta^2|}{2\alpha\beta} \right) \right. \\ &\left. + \sin \theta \left(b_{x,y} + a_{x,y} \frac{\alpha^2 + \beta^2 - |\alpha^2 - \beta^2|}{2\alpha\beta} \right) \right]. \end{aligned} \quad (\text{A3})$$

Substituting Eqs. (A1)–(A3) into the master equation (12), one can easily establish Eqs. (16) and (17) for unknown coefficients $a_{x,y}$, $b_{x,y}$.

2. Spin-coherent case

To prove the solution of Eq. (25), we take the integrals in its right-hand side in a similar way as before. Indeed, after the substitution of $f_{11(22)}$ given by Eq. (18) and $f_{12}=f_{21}=0$ into the collision term, we have

$$\text{St}[\hat{f}(\mathbf{k})]_{11(22)} = \frac{f_{11(22)}}{\tau} + \sum_{x,y} eE_{x,y} \left[-\frac{\partial f^0(E_{+(-)k})}{\partial E_{+(-)k}} \right] \frac{1}{\hbar\kappa_{\theta}} \begin{Bmatrix} (\alpha^2 + \beta^2)\cos \theta + 2\alpha\beta \sin \theta \\ (\alpha^2 + \beta^2)\sin \theta + 2\alpha\beta \sin \theta \cos \theta \end{Bmatrix}, \quad (\text{A4})$$

$$\text{St}[\hat{f}(\mathbf{k})]_{12(21)} = \pm \frac{i(\alpha^2 - \beta^2)}{2\hbar\kappa_{\theta}} \sum_{x,y} eE_{x,y} \begin{Bmatrix} \sin \theta \\ -\cos \theta \end{Bmatrix} \left[-\frac{\partial f^0(E_{+k})}{\partial E_{+k}} - \frac{\partial f^0(E_{-k})}{\partial E_{-k}} \right]. \quad (\text{A5})$$

Using Eqs. (A4) and (A5) and the velocity matrix elements (5)–(8), one can easily prove that the spin-incoherent solution (18) with $f_{12}=f_{21}=0$ satisfies Eq. (25).

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