Interface dependence of the Josephson-current fluctuations in short mesoscopic superconductor/normal-conductor/superconductor junctions

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We discuss the dependence of the Josephson-current correlations in mesoscopic superconductor/normal-conductor/superconductor (SNS) devices on the transparency of the superconductor/normal-conductor interfaces. Focusing on short junctions we apply the supersymmetry method to construct an effective field theory for mesoscopic SNS devices which is evaluated in the limit of highly and weakly transparent interfaces. We show that the two-point Josephson-current correlator differs by a universal factor of 2 in these two cases.

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I. INTRODUCTION

One of the most promising directions in the field of superconductivity in mesoscopic structures is provided by recent technological advances in hybrid superconductor/semiconductor technology. As an example we want to mention the very recently observed quantization of the critical current in superconducting quantum point contacts.[1] An important aspect of this development is that for hybrid semiconductor structures both the system geometry and the chemical potential can be varied flexibly, which greatly facilitates the observation of *mesoscopic fluctuations*. In contrast, for ordinary metal heterostructures only external magnetic fields effectively qualify as averaging parameters, implying that the dependence of fluctuation phenomena on the phase of the superconducting condensate is difficult to observe. Indeed, fabrication of such hybrid devices has advanced to the stage that fluctuation phenomena may be measured[2–7] in a wide range of geometries and conditions.

The observable behavior of a two-dimensional electron gas (N), as realized in a semiconductor heterostructure, in contact with a superconductor (S) is highly sensitive to the quality of the SN contact. In this work we discuss the SN interface dependence of the Josephson-current $J(\phi)$ through an electron gas, sandwiched between two superconductor terminals whose order parameters exhibit a phase difference ϕ. A measurement of the fluctuation behavior $\mathrm{var}J(\phi)$ of this quantity has been attempted already by Takayanagi *et al.*[2] As pointed out above, the use of a semiconductor heterostructure allows the use of an external gate voltage to tune fluctuations in the supercurrent, while keeping the phase difference *fixed*.[8] For typical experiments[2,5] transmission tends to be weak due to the presence of the Schottky barrier. However, as noted in Ref. 2, a theory of supercurrent fluctuations in the weak-transmission regime has so far been missing. In this work, we fill this gap and compare the behavior of the supercurrent for weakly and highly transparent interfaces. (We remark that an experimental realization of a crossover in the transition strength has already been attempted in Ref. 5.) Specifically, we will compute the mesoscopic fluctuations of the current, $\mathrm{var}J(\phi)$, and, more generally, its *correlation profile*

$$K(\phi_1,\phi_2) \equiv \langle J(\phi_1)J(\phi_2)\rangle - \langle J(\phi_1)\rangle\langle J(\phi_2)\rangle. \quad (1)$$

This latter function provides a wealth of information relating to the design of the system, and it is clear that its experimental determination—along the lines of Ref. 2—would provide a sensitive test of the theory of mesoscopic fluctuations in SN systems. However, before proceeding, let us briefly put our present analysis into the context of previous studies of the problem.

A pioneering work in the theory of supercurrent fluctuations is due to Altshuler and Spivak,[9] who calculated the supercurrent correlations by means of a diagrammatic perturbation theory. Their results assumed the limit of a long junction, so that $E_c \ll \Delta$, where E_c is the Thouless energy and Δ the superconducting order parameter. In this limit, the supercurrent fluctuations are not universal: instead, the supercurrent variance is proportional to $(eE_c/\hbar)^2$.

The applicability of the diagrammatic theory is limited, however, to conditions under which the fluctuations of the supercurrent exceed its average. While such a situation may be reached by application of a magnetic field or in the presence of a glassy structure in the N region, it clearly is rather specialized. Under more usual conditions, the essentially nonperturbative nature of the proximity effect leads to a proliferation of diagrams whose summation is practically impossible.[10]

A powerful alternative to the diagrammatic approach is provided by a multiple-scattering theory.[11,12] Here the property in question is related to the scattering matrix of the N region, whose statistical properties are known.[11] The variance of the supercurrent through the N region at fixed phase difference has been calculated by Beenakker[11,12] using the scattering approach. His results apply for a short junction $\Delta \ll E_c$ with highly transparent SN interfaces. In this case the fluctuations become universal: the supercurrent variance is proportional to $(e\Delta/\hbar)^2$.

In this work, we apply the supersymmetry method[13] to construct an alternative approach to the problem. A general feature of our formulation is its close alliance to the quasiclassical approach,[10,14–16] itself a traditional means of describing the *mean* properties of SN systems. In particular, using the fact that the stationary phase configurations of the field theory of dirty[17] SN systems are determined by the quasiclassical Usadel equations,[10] one can benefit from the huge body of expertise on (ensemble averaged) Josephson currents in mesoscopic SN devices. The evaluation of fluctuations around the Usadel mean field then leads to results for the mesoscopic fluctuations of the current.

To be specific, we apply our approach to the analysis of short junctions with moderately weak coupling of the superconductor condensate to the normal-metal "quantum dot," which allows us to use random-matrix theory (RMT) to model the normal metal. That is, we assume the hierarchy of energy scales $\Delta \ll E_g \ll E_c$, where $E_g \equiv \bar{d} g$ is the inverse of the so-called dwell time,[18,19] $g \gg 1$ the normal-state conductance of the system, and $\bar{d}$ its mean level spacing. (However, the application of the method to other regimes requires only technical rather than conceptual modifications.) Here onwards we put $\hbar = 1$.

II. MAIN RESULT

Deferring an outline of the technicalities of the analysis to the last part of this work, we here merely anticipate that the correlator $K(\phi_1, \phi_2)$ is obtained by a second-order perturbative expansion around the Usadel functional free energy. In this way we find

$$K_\Gamma(\phi_1,\phi_2) = \frac{c_\Gamma e^2 \Delta^2}{\pi^2} \sin\phi_1 \sin\phi_2 \int_0^\infty dx_1 \int_0^\infty dx_2 \times \frac{\sqrt{x_1^2+1}\sqrt{x_2^2+1}}{\sqrt{x_1^2+\cos^2(\phi_1/2)}\sqrt{x_2^2+\cos^2(\phi_2/2)}} \times [\sqrt{x_1^2+1}\sqrt{x_2^2+\cos^2(\phi_2/2)} + \sqrt{x_2^2+1}\sqrt{x_1^2+\cos^2(\phi_1/2)}]^{-2}, \quad (2)$$

where

$$c_\Gamma = \begin{cases} 1/2 & \text{for highly transparent interfaces } (\Gamma = 1), \\ 1 & \text{for weakly transparent interfaces } (\Gamma \ll 1), \end{cases} \quad (3)$$

and Γ is the transmission coefficient characterizing the transparency of the SN interfaces; see below. While Eqs. (2) and (3) are a new result for weakly transparent interfaces, current fluctuations of the moderately weakly coupled quantum dot with highly transparent interfaces have been the subject of an earlier scattering theory analysis in Ref. 12. Within this approach both the diagonal contribution $\mathrm{var} J(\phi) = K(\phi,\phi)$ (Ref. 12) and the straightforward extension to the full correlation function $K(\phi_1,\phi_2)$ are represented as a double integral over eigenvalues of the transmission matrix, which despite its difference in form, is numerically identical. The agreement with these earlier results provides a check on the consistency of the field theory. Notice that the validity of the result is limited to values of the phase outside the domain[20] $|\phi - \pi| < g^{-1}$, a fact that was pointed out earlier in Ref. 12. The main result of this work is that the Josephson-current fluctuations for the case of weakly and highly transparent interfaces differ by a universal factor of 2. This factor of 2, in fact, can be anticipated by the observation that for small phase differences ϕ, one may establish contact to the theory of universal conductance fluctuations (UCFs) through a disordered quantum dot—i.e., for $\phi \ll 1$,

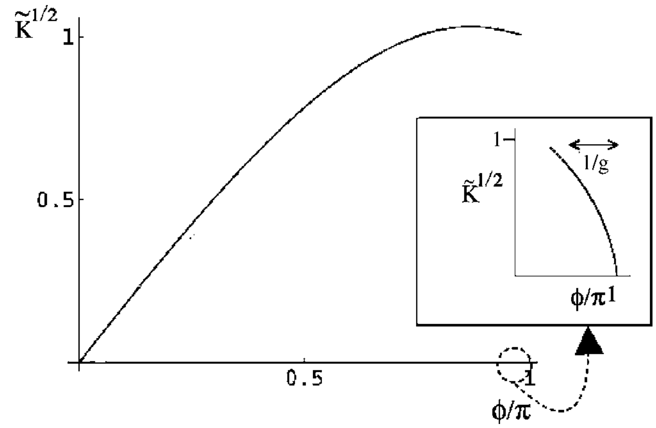

FIG. 1. Plot of $\sqrt{\tilde{K}(\phi,\phi)}$ against phase ϕ, where $\tilde{K} \equiv \pi^2/(c_\Gamma e^2 \Delta^2) K$. The inset shows how $\sqrt{\tilde{K}(\phi,\phi)}$ falls to zero within a narrow window around $\phi = \pi$.

$$J(\phi) \approx \frac{ge\Delta}{2}\phi, \quad (4)$$

where g is the (dimensionless) conductance through the disordered quantum dot. Using the fact that for the orthogonal ensemble[21]

$$\mathrm{var}\, g = \begin{cases} \frac{1}{4} & \text{for high barriers } (\Gamma \ll 1), \\ \frac{1}{8} & \text{for transparent contacts } (\Gamma = 1), \end{cases} \quad (5)$$

one is led to the expectation ($\phi \ll 1$)

$$\mathrm{var}\, J(\phi) = \begin{cases} \frac{e^2\Delta^2}{16}\phi^2 & \text{for high barriers } (\Gamma \ll 1), \\ \frac{e^2\Delta^2}{32}\phi^2 & \text{for transparent contacts } (\Gamma = 1), \end{cases} \quad (6)$$

in agreement with Eqs. (2) and (3).

Figure 1 shows the Josephson-current fluctuations $\sqrt{\mathrm{var} J(\phi)} = \sqrt{K(\phi,\phi)}$ as a function of ϕ. We observe that the maximum falls to a fixed value $\phi \sim 2.7$. The strong increase of the fluctuations is a manifestation of the fact that $\phi = \pi$ defines an instability of the Josephson action, in the vicinity of which minute changes of the disorder/boundary geometry trigger drastic changes in the configuration-specific $J(\phi)$.

III. FORMALISM

Having discussed our results for the SNS quantum dot, we next provide a brief outline of the formalism.

Assuming M channels coupling the disordered quantum dot to the superconductors (with each $M/2$ modes propagating to the left and to the right, respectively[22]) the Hamiltonian of the SNS junction is given by

$$H = H_{\mathrm{d}} + H_\Delta^1 + H_\Delta^2 + H_{\mathrm{c}}^1 + H_{\mathrm{c}}^2, \quad (7)$$

where $(i = 1, 2)$

$$H_{\mathrm{d}} = \sum_{\mu\nu} |\psi_\mu\rangle H_{\mathrm{dot}}^{\mu\nu}\langle\psi_\nu|, \tag{8}$$

$$H_\Delta^i = \sum_{aa'} \int dE |\chi_a^i(E)\rangle H_{\mathrm{BdG}}^{i,aa'}\langle\chi_{a'}^i(E)|, \tag{9}$$

$$H_{\mathrm{c}}^i = \sum_{\mu a} \int dE[|\psi_\mu\rangle W_i^{\mu a}\langle\chi_a^i(E)| + |\chi_a^i(E)\rangle W_i^{\mu a}\langle\psi_\mu|]. \tag{10}$$

$\{|\psi_\mu\rangle\}$ denotes a set of dot states (for further convenience we use the Nambu formalism; i.e., the states $|\psi_\mu\rangle$ comprise particle and hole degrees of freedom in a single object) and $\{|\chi_a^i(E)\rangle\}$ a set of scattering states into superconductor i. The dot Hamiltonian is of the form $H_{\mathrm{dot}}=\sigma_3^{\mathrm{ph}}\otimes h$ where h is drawn from the orthogonal Gaussian ensemble with the variance set by the spectrum width λ, H_{BdG}^i is the Boguliubov–de Gennes Hamiltonian of superconductor i with order parameters $\Delta_i=\Delta e^{i\phi_i}$, and the coupling matrices W^i are characterized by the transmission coefficients[23] Γ, $W_iW_i^t=\frac{f(\Gamma)}{\pi}\lambda\delta_{\mathrm{ch},i}^{\mathrm{op}}$, where $f(\Gamma)=\frac{2}{\Gamma}-1-\frac{2}{\Gamma}\sqrt{1-\Gamma}$ and $\delta_{\mathrm{ch},i}^{\mathrm{op}}$ describes a diagonal matrix with entries 1 for the $M/2$ open channels connecting the dot to superconductor i and 0 otherwise (We assume that the sets of dot states coupling to superconductors 1 and 2 are disjoint.) The capacitance of the dot is assumed to be sufficiently large that the Coulomb blockade can be ignored.

To compute Josephson-current correlations we introduce a supersymmetric field integral representation of the free energy:[13]

$$K(\phi_1,\phi_2) = \left(\frac{e}{\pi}\right)^2 \int d\omega_1 \int d\omega_2 \left.\frac{d^2\mathcal{F}(\hat\omega,\Phi)}{d\varphi_1 d\varphi_2}\right|_{\varphi_1=\varphi_2=0}, \tag{11}$$

where

$$\mathcal{F}(\hat\omega,\Phi) = \int \mathcal{D}Qe^{-S[Q]} \tag{12}$$

with

$$S[Q] = \frac{M}{2}\,\mathrm{Re\ str}\ \ln\left(1 + \frac{f(\Gamma)}{\sqrt{\omega^2+\Delta^2}}[\hat\omega\sigma_3^{\mathrm{ph}} + \Delta\sigma_2^{\mathrm{ph}}e^{i\Phi\sigma_3^{\mathrm{ph}}\sigma_3^{\mathrm{tr}}}]Q\right). \tag{13}$$

Here, Q is a 16-dimensional matrix acting on the product of particle-hole (ph) space, a two-component space (f) required to distinguish between the two supercurrents, a boson-fermion (bf) space implementing the supersymmetric structure of the theory, and a time reversal (tr) space required to correctly describe the behavior of the system under time-reversal. Further, the symbol "str" denotes the supersymmetric generalization of the matrix trace, and $\hat\omega=\mathrm{diag}(\omega_1,\omega_2)$ and $\Phi=\mathrm{diag}(\phi_1+\varphi_1,\phi_2+\varphi_2)$ are diagonal matrices in f space where $\omega_{1,2}$ and $\phi_{1,2}\propto\mathbb{1}^{\mathrm{bf}}$ are the two energy and phase arguments entering the integral representation of the supercurrent and the sources $\varphi_{1,2}=\mathrm{diag}(\varphi_{1,2},0)_{\mathrm{bf}}$ break supersymmetry.

Referring for a detailed discussion of the derivation of Eqs. (11)–(13) and of the internal structure of the matrix Q to Ref. 10, we here merely recapitulate that it originates from the coherent-state representation of the free energy along the standard procedure, including the generalization of the partition function to a supersymmetry formulation, RMT ensemble average, Hubbard-Stratonovich transformation, and expansion in Goldstone modes around the metallic saddle point σ_3^{ph}.

Notably, the nonlinear constraint $Q^2=\mathbb{1}$ indicates that the matrix Q represents a generalization of the Green function g central to the quasiclassical approach. Indeed, it is straightforward to see that a variation of Eq. (13) (under the restriction $Q^2=\mathbb{1}$) leads to

$$[\bar Q,\omega\sigma_3^{\mathrm{ph}} + \Delta\cos\Phi\sigma_2^{\mathrm{ph}}] = 0, \tag{14}$$

i.e., an equation which, upon identification of the matrix Q with the quasiclassical Green functions, immediately is recognized as the zero-dimensional limit of a Usadel equation (extended, however, to a larger structure accommodating more than one impurity averaged observable.) Void of elements coupling between the two observables (f space), Eq. (14) admits a block-diagonal solution $\bar Q=\mathrm{diag}(g_1,g_2)\otimes^{\mathrm{bf}}\equiv R\sigma_3^{\mathrm{ph}}R^{-1}$, where

$$g_i = \frac{1}{\sqrt{\omega_i^2+\Delta^2}}(\omega_i\sigma_3^{\mathrm{ph}} + \Delta\cos\Phi_i\sigma_2^{\mathrm{ph}}) \tag{15}$$

are the quasiclassical Green functions computed for phase difference $\Phi_{1,2}$ and the second representation expresses the solution as a rotation away from the metallic reference point, σ_3^{ph}. The ensemble-averaged Josephson current may now be calculated from the saddle-point action

$$S[\bar Q] = \frac{M}{4}\mathrm{str}\ \ln\left(1 + f^2(\Gamma) + 2f(\Gamma)\frac{\sqrt{\omega^2+\Delta^2\cos^2\Phi}}{\sqrt{\omega^2+\Delta^2}}\right), \tag{16}$$

and one obtains for highly transparent interfaces

$$J(\phi) = \frac{eM}{2\pi}\Delta\sin\phi \times \int_0^\infty dx\frac{1}{\sqrt{x^2+1}+\sqrt{x^2+\cos^2\phi/2}}\frac{1}{\sqrt{x^2+\cos^2\phi/2}}, \tag{17}$$

while for weakly transparent interfaces

$$J(\phi) = \frac{eM\Gamma}{4\pi}\Delta\sin\phi\int_0^\infty dx\frac{1}{\sqrt{x^2+\cos^2\phi/2}}\frac{1}{\sqrt{x^2+1}}. \tag{18}$$

Equations (17) and (18) coincide with quasiclassical results.[11,18,19,24]

Josephson-current correlations can now be explored by defining[10]

$$Q = RT\sigma_3^{\mathrm{ph}}T^{-1}R^{-1}, \tag{19}$$

where the generalized rotation matrix T describes fluctuations around the Usadel mean field and the free energy $\mathcal{F}(\hat{\omega},\Phi)=\int \mathcal{D}T \exp(-S[Q])$.

Going beyond the mean field level by substituting the generalized parametrization, Eq. (19), into the field integral, one finds that fluctuations around $T=\mathbb{1}$ are penalized by a parameter $\cos(\phi_i/2)g$ which is much larger than unity, *unless* $|\phi-\pi|<g^{-1}$ lies in the anomalous window discussed above. While in the latter case the correlator, Eq. (11), has to be evaluated by full integration over the manifold of T's (cf. the analogous situation with computing the fine structure of spectral correlation functions on energy scales smaller than the single particle level spacing[13]), in general it is sufficient to perturbatively expand the matrix $T=\mathbb{1}+W+\frac{1}{2}W^2+\cdots$ to second order around unity. Expansion of the logarithm then leads to the Gaussian actions for highly and weakly transparent interfaces,

$$S^{(2)}_{\Gamma\ll 1}[W] = \frac{\Gamma M}{4}\mathrm{str}[\Pi_{\hat{\omega}}(\Phi)W^2], \tag{20}$$

$$S^{(2)}_{\Gamma=1}[W] = \frac{M}{4}\mathrm{str}\left[W^2 + \left(\frac{1}{1+\Pi_{\hat{\omega}}(\Phi)}\frac{\Delta}{\sqrt{\hat{\omega}^2+\Delta^2}}\sin\Phi\sigma_2^{\mathrm{ph}}\sigma_3^{\mathrm{tr}}W\right)^2\right], \tag{21}$$

where we introduced $\Pi_{\hat{\omega}}(\Phi) := \frac{\sqrt{\hat{\omega}^2+\Delta^2\cos^2\Phi}}{\sqrt{\hat{\omega}^2+\Delta^2}}$. Performing the twofold derivative with respect to the sources followed by Gaussian integrals over the W's one obtains the final result, Eqs. (2) and (3).

IV. GENERALIZATION

While the outline above was specific to the case of a spatially structureless quantum dot, the generalization to more complex geometries is straightforward. Under conditions where the spatial variation of the quasiclassical Green function is no longer negligible, Eq. (13) generalizes to the action of a matrix *field* Q. For example, for an extended diffusive N region (i.e., for $E_c<\Delta,E_g$) and weakly transparent interfaces, $S[Q]=-(\pi D\nu/8)\int d^d r\mathrm{str}(\partial Q\partial Q)+S_\omega[Q]+S_c[Q]$ assumes the form of a generalized diffusive σ model,[10] where $S_\omega[Q]=-(\pi\nu/2)\int d^d r\mathrm{str}(Q\hat{\omega}\sigma_3^{\mathrm{ph}})$ and $S_c[Q]$ describes the coupling to the superconductor. Similarly, for a (nearly) clean system, $S[Q]=-v_F\int d^d r dn\mathrm{str}(T\sigma_3^{\mathrm{ph}}\mathbf{n}\cdot\partial T^{-1})+S_\omega[Q]+S_c[Q]$ becomes the free energy of the ballistic σ model.[25] In complete analogy to our discussion above, the variation of these actions leads to the general Usadel[14] and Eilenberger[15] equations, respectively. To compute Josephson-current correlations, one again employs the parametrization, Eq. (19), where, however, both R and T are field configurations with nonvanishing spatial variation. By evaluating the action of a generalized quadratic variation $T=+W+\frac{1}{2}W^2$ and subsequent computation of the correlator, Eq. (11), one can then, in principle, obtain the Josephson-current correlations of any SN system category (if amenable to the approximation schemes of quasiclassics.) However, in cases where the solution of the Usadel and Eilenberger equations displays a complex spatial profile, the concrete computation of the generalized Gaussian integrals over W can be cumbersome. Under these conditions one expects the result for the current correlations to be less universal (e.g., dependent on the system geometry, disorder concentration, etc.) than in the quantum dot case discussed above.

V. SUMMARY

In summary, we showed that the mesoscopic Josephson-current fluctuations through a weakly coupled disordered quantum dot differ by an universal factor of 2 for the cases of weakly and highly transparent interfaces. In our calculations we applied a supersymmetric field-theoretical approach. Although we concentrate on the case of short junctions (and use RMT to model the normal metal), this method allows for a microscopic derivation capable to cover a wide range of parameters characterizing the SNS junction. The observation of the Josephson-current correlations should be feasible using semiconductor technology.

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