Pair correlations and the survival of superconductivity in and around a superconducting impurity

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The problem of the survival of superconductivity in a small superconducting grain placed in a metal substrate is addressed. For this aim the pair correlations and superconducting gap around and inside a negative-U impurity in one and two dimensions are calculated in a discrete tight-binding model and a continuous model. Using a mean-field decomposition, it is shown that finite pairing in the grain develops when the system has a degeneracy between a successive number of electron pairs, and thus may oscillate as a function of the chemical potential. For finite pairing in the island, pair correlations in the normal part exhibit a crossover from being long ranged to exponentially decaying, depending on the strength of interaction in the grain. It is shown analytically that there is a minimal island size under which pairing vanishes, which is different from that given by Anderson's criterion [J. Chem. Phys. Solids **11**, 26 (1959)], and that it scales as a power law with island size, rather than exponentially as in isolated grains.

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I. INTRODUCTION

While superconductivity (SC) on the nanoscale has been a long-standing issue, dating back to the seminal work of Anderson,[1] only in the last decade has technological advancement enabled the realization of such systems in experiment.[2] Since then, manifestations of SC on the nanometer scale has been observed not only in isolated grains[3] or granule on insulating substrate,[4] but also in inhomogeneous superconducting thin films, where well-separated superconducting and normal regions have been observed.[5] In such hybrid systems, the proximity between the superconducting and normal phases gives rise to different effects, mainly manifested in the local density of states (LDOS), which may be directly measured using scanning tunneling microscopy.[6]

Encouraged by technological advancement, we ask the following question: what would be the properties of an ultrasmall SC grain placed on a *metallic* (or a doped semiconducting) substrate? In such a case, would the gap in the grain still obey Anderson's criteria,[1] or will the proximity effect yield a different criteria for the destruction of SC in such a grain, as seen in, e.g., thin superconducting layers attached to a normal layer?[7,8] Furthermore, in such systems one expects that the superconducting properties of the grain would be strongly affected by the properties of the surrounding metal, and that the proximity to a superconducting grain would generate pair correlations that would impinge on the local properties of the metallic area, such as its LDOS.[9] The above question is also interesting from a technological point of view, as Josephson arrays fabricated on metallic or semiconducting substrate seem to have a large technological potential as nanoelectrical devices.

In order to examine these issues, a minimal model of a single superconducting grain placed in a clean metal matrix is studied by means of a negative-U Hubbard Hamiltonian in which the interactions are confined to a small region in space (so-called negative-U impurity). Applying a Hartree-Fock-Gorkov mean-field decomposition[10] leads to the Bogoliubov–deGennes (BdG) Hamiltonian,[11] which serves as a starting point in the calculation.

The effect of the proximity between the superconducting grain and normal area is investigated by numerically solving a tight-binding BdG Hamiltonian in one and two dimensions. It is found that the pairing in the grain is strongly affected by the chemical potential (i.e., density) of the substrate, and that on the normal area pair correlations may either be suppressed exponentially away from the impurity or be long ranged, depending on the value of the attractive interaction in the grain.

The dependence of the gap size in the grain is studied using a continuous version of the BdG Hamiltonian. Solved analytically, the dependence of the gap on island size is found to diminish as a power law rather than exponentially (as in an isolated grain[13]), and the minimal island size under which SC vanishes in the grain[14] is evaluated and is found to depend on the properties of the substrate.

II. NEGATIVE-U IMPURITY IN THE TIGHT-BINDING MODEL

Let us start by examining the discrete tight-binding model for the negative-U impurity. The model Hamiltonian is

$$\mathcal{H} = -t\sum_{\langle ij\rangle\sigma} c_{i\sigma}^{\dagger}c_{j\sigma} - \mu\sum_{i\sigma} c_{i\sigma}^{\dagger}c_{i\sigma} - Uc_{0\uparrow}^{\dagger}c_{0\downarrow}^{\dagger}c_{0\downarrow}c_{0\uparrow}, \qquad (1)$$

where t is the hopping element, μ is the chemical potential, and $U>0$ is the attractive interaction, which is only present in a single site at the origin (the negative-U impurity). By applying the Hartree-Fock-Gorkov decomposition,[10] the BdG mean-field Hamiltonian[11] is obtained,

$$\mathcal{H} = -t\sum_{\langle ij\rangle\sigma} c_{i\sigma}^{\dagger}c_{j\sigma} + \sum_{i\sigma}(\epsilon_i - \mu)c_{i\sigma}^{\dagger}c_{i\sigma} + (\Delta c_{0\uparrow}^{\dagger}c_{0\downarrow}^{\dagger} + \text{H.c.}), \qquad (2)$$

where $\Delta = -U\langle c_{0\uparrow}^{\dagger}c_{0\downarrow}^{\dagger}\rangle$ is the pairing potential and $\epsilon_i = \delta_{i0}\Sigma_{\sigma}|U|\langle c_{0\sigma}^{\dagger}c_{0\sigma}\rangle/2 = \delta_{i0}|U|\langle n_0\rangle/2$ is the Hartree shift. This mean-field approach is justified by noting that little is known about this system, and hence a preliminary mean-field treat-

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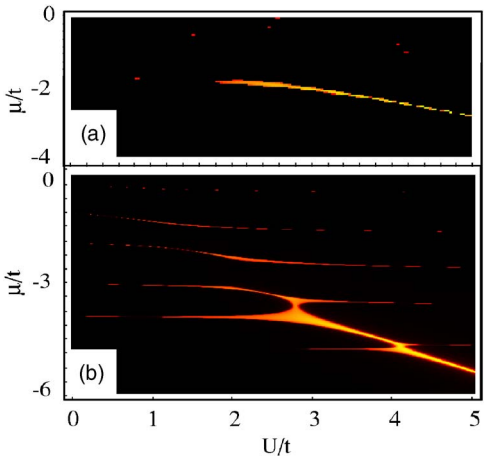

FIG. 1. (Color online) Color plot of the pairing amplitude in the impurity, f_0, as a function of chemical potential μ and interaction strength U for (a) one dimension and (b) two dimensions. The bright area indicates a large pairing amplitude.

ment is in place. The importance of maintaining the Hartree shift term, which naturally appears from the derivation of the mean-field Hamiltonian, has been discussed and demonstrated in Ref. 12, where a similar decomposition was used to study the properties of a disordered superconducting sample.

Introducing a Bogoliubov transformation, one obtains from the Hamiltonian of Eq. (2) the BdG equations[11] for the quasiparticle (QP) $u(\mathbf{r}_i)$ and quasihole excitations $v(\mathbf{r}_i)$,

$$\begin{pmatrix} \hat{\xi} + (\epsilon_i - \mu) & \Delta\delta_{i0} \\ \Delta^*\delta_{i0} & -\hat{\xi} - (\epsilon_i - \mu) \end{pmatrix} \begin{pmatrix} u_k(\mathbf{r}_i) \\ v_k(\mathbf{r}_i) \end{pmatrix} = E_k \begin{pmatrix} u_k(\mathbf{r}_i) \\ v_k(\mathbf{r}_i) \end{pmatrix}. \tag{3}$$

In Eq. (3), $\hat{\xi}u_k(\mathbf{r}_i) = -t\Sigma_{\hat{\delta}}u_k(\mathbf{r}_i + \hat{\delta})$, where $\hat{\delta} = \pm\hat{\mathbf{x}}, \pm\hat{\mathbf{y}}$ and similarly for $v_k(\mathbf{r}_i)$, and the energies are the QP excitation energies $E_k \geqslant 0$. The pairing potential Δ and the electron density per site n_i are to be determined self-consistently in terms of the QP amplitudes $u(\mathbf{r}_i)$ and $v(\mathbf{r}_i)$,

$$\Delta = |U|\sum_k u_k(0)v_k^*(0), \quad \langle n_i \rangle = 2\sum_k |v_k(\mathbf{r}_i)|^2. \tag{4}$$

The pairing amplitude Δ is finite only on the negative-U impurity. However, the proximity to the impurity induces pair correlations $f_i = \langle c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger \rangle$ even for $i \neq 0$, that is outside the impurity.

III. RESULTS IN ONE AND TWO DIMENSIONS

The pair correlations are investigated by solving Eq. (3) numerically and self-consistently for a one-dimensional (1D) and a two-dimensional (2D) metallic substrate, from which both the densities and pair correlations may be calculated. In Fig. 1 the pairing amplitude f_0 (bright points in Fig. 1) is plotted as a function of μ and U for a 1D lattice of size $L = 49$ and a 2D lattice of size 7×7 [Figs. 1(a) and 1(b), respectively]. In the calculation, hard-wall boundary conditions were taken, but similar calculation using periodic boundary conditions showed no qualitative change in the results. In one dimension, finite pairing is only visible along the line of constant density $\langle n \rangle \approx 0$. In two dimensions, stripes of finite pairing appear on lines of constant density, which are the boundary lines between sequential even (mean) number of electrons, N, in the system. It is clear that the phase space available for finite pairing is much larger in two dimensions than in one dimension.

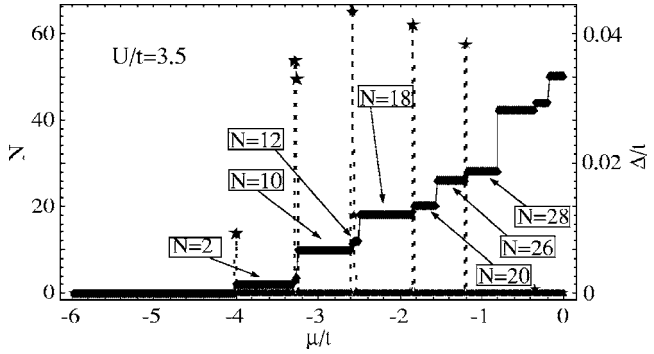

FIG. 2. The pairing potential Δ (stars) and mean total electron number N as a function of chemical potential for a 13×13 system with $U/t = 3.5$. The pairing potential (stars, right axes) is finite only at the transition between plateaus of constant even electron number N (diamonds, left axes).

In Fig. 2 the number of electrons N and the pairing potential Δ are plotted as a function of μ for a 13×13 system with $U/t = 3.5$. As seen, the pairing potential (stars, right axes) is finite only at the transition between plateaus of constant even electron number N (diamonds, left axes).

The reason for this behavior of the pairing is that the Hamiltonian of Eq. (2) couples between states with no electrons and two electrons at the impurity. Thus, the self-consistent pairing is finite only when states with N and $N+2$ electrons are degenerate at the Fermi energy. In one dimension, this only happens when the occupation changes from $N=0$ to $N=2$. In two dimensions, however, this degeneracy is much more common, resulting in many regions of finite pairing in μ-U phase space, and hence in the oscillatory behavior shown in Fig. 2. We note that while for three dimensions the computation is numerically demanding and will not be presented here, we expect similar behavior, with even more phase space available for SC in the grain. Such a case may be more relevant from the experimental side.

Further insight may be gained by studying the dependence of Δ on the interaction strength U. Although from Eq. (4) it would seem that the two are linearly dependent, this is not the case. Due to the Hartree term, U affects both the occupation of the impurity and the energy levels of the system, pushing the system in and out of the N-$N+2$ degeneracy required for finite pairing. This is demonstrated in Fig. 3, where Δ (stars, right axes) and N (diamonds, left axes) are plotted as a function of U for a 9×9 system at $\mu/t = -2.4$. This chemical potential corresponds to a low (≈ 0.25) electron filling, which is relevant for a semiconducting substrate. $\Delta(U)$ is a nonlinear (and nonmonotonic) function, only finite above a certain critical interaction $U_c(\mu)$, in the transition

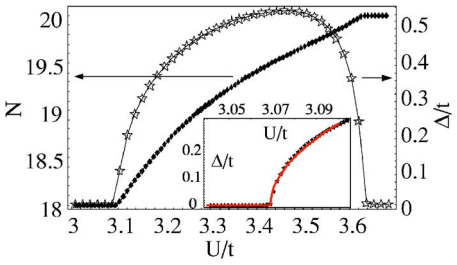

FIG. 3. (Color online) The pairing potential $\Delta(0)$ and mean total electron number N as a function of interaction strength U for a 9×9 lattice with $\mu/t=-2.4$, showing that $\Delta(U)$ is a nonmonotonic function, only finite above U_c when N fluctuates between $N=18$ and $N=20$. Inset: the numerical data (stars) in the region $U\simeq U_c$ is fitted with the function $\Delta(U)\propto(U/U_c-1)^x\Theta(U-U_c)$ (solid line), yielding $x\simeq1/2$.

region of N from $N=18$ to $N=20$. We note that by changing μ one may find finite pairing at higher electron densities. However, this is unlikely to occur above half-filling, as the Hartree term will suppress the pair function in that case.

In the region where the pairing in the superconducting island is finite, the proximity effect should yield pair correlations away from the impurity, $f_i=\langle c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger\rangle$. In Fig. 4 we plot f_i on a chain of length[15] $L=51$ (normalized to unity) for two values of interaction, $U/t=1.5$ (stars), within the energy band, and $U/t=2.5$ (squares), outside the band. The chemical potential is adjusted for each value of interaction in order to maintain finite pair correlation in the impurity. When U lies outside the band, it is found that the hole excitations $v(n)$ become localized, resulting in an exponential decay of the pair correlations. On the other hand, if U lies within the band, the hole excitations are periodic and generate long-range pair correlations. While this effect may be due to the finite size of the normal system, these long-ranged correlations may have a crucial effect[16] on the global behavior of a system with many negative-U impurities (so-called dilute negative-U model[14,17,18]), as they determine the effective Josephson coupling between the different impurities.

IV. TWO-DIMENSIONAL CONTINUOUS MODEL

The starting point for the following calculation is the continuous negative-U Hamiltonian (with $\hbar=1$):

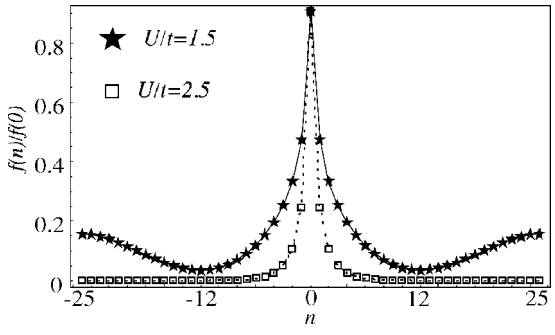

FIG. 4. Spatial structure of the pair correlations $f_i=\langle c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger\rangle$ in a linear chain of length $L=51$ for two values of interaction, $U/t=1.5$ (stars) and $U/t=2.5$ (squares). When U lies within the band there are long-range pair correlations, but an exponential decay of the correlations for U outside the energy band.

$$\mathcal{H}=\sum_\sigma\int d^2\mathbf{r}\Psi_\sigma^\dagger(\mathbf{r})\left(-\frac{1}{2m}\nabla^2-\mu\right)\Psi_\sigma(\mathbf{r})+\mathcal{H}_I, \tag{5}$$

where μ is the chemical potential, and

$$\mathcal{H}_I=\int d^2\mathbf{r}U(\mathbf{r})\Psi_\uparrow^\dagger(\mathbf{r})\Psi_\downarrow^\dagger(\mathbf{r})\Psi_\downarrow(\mathbf{r})\Psi_\uparrow(\mathbf{r}), \tag{6}$$

where $U(\mathbf{r})<0$ is a short-range attractive electron-electron interaction. The existence of a negative-U impurity is modeled by limiting the interaction to a finite islandlike region in space,[14] i.e.,

$$U(\mathbf{r})=\begin{cases}-|U|, & \mathbf{r}\in\mathbf{I},\\ 0, & \text{else},\end{cases} \tag{7}$$

where $\mathbf{I}$ is a disk with radius a around the origin.

Using the BdG mean-field decomposition,[11] we substitute $\mathcal{H}_I$ by

$$\mathcal{H}_\Delta=\int_{\mathbf{r}\in I}d^2\mathbf{r}(\Delta(\mathbf{r})\Psi_\uparrow^\dagger(\mathbf{r})\Psi_\downarrow^\dagger(\mathbf{r})+\text{H.c.}), \tag{8}$$

where $\Delta(\mathbf{r})=|U|\langle\Psi_\uparrow(\mathbf{r})\Psi_\downarrow(\mathbf{r})\rangle$ is the pairing potential. The Hamiltonian $\mathcal{H}_\Delta$ is now expanded with a being a small parameter (specifically $k_Fa\ll1$),

$$\mathcal{H}_\Delta\approx\left\{\Delta a^2\Psi_\uparrow^\dagger(0)\Psi_\downarrow^\dagger(0)+\text{H.c.}+\frac{1}{4!}\Delta^2a^4\nabla^2\left[\Psi_\uparrow^\dagger(\mathbf{r})\Psi_\downarrow^\dagger(\mathbf{r})+\text{H.c.}\right]\Big|_{\mathbf{r}=0}\right\}, \tag{9}$$

where $\Delta\equiv\Delta(0)$. Since $\nabla^2\psi_\uparrow^\dagger(\mathbf{r})\psi_\downarrow^\dagger(\mathbf{r})|_{\mathbf{r}=0}\sim k_F^2\Psi_\uparrow^\dagger(0)\Psi_\downarrow^\dagger(0)$, the second term in Eq. (10) is smaller than the first term by a factor of $(k_fa)^2$ and can be neglected, and we are left with the first term in Eq. (10) which can be treated as a local perturbation. Notice that Δ is not assumed to be small, as it might not be.

For typical metals, the condition $k_Fa\ll1$ yields an island size of a few nanometers, for which the continuum formulation is inappropriate. For semiconductors, where the Fermi wavelength may be a few orders of magnitude larger than in metals, the continuum limit will still be valid. In semiconductors which exhibit true superconductivity, due to the low carrier density, the key role is played by intervalley processes.[19] In the above model, on the other hand, attractive interactions take place only on the impurity and the sample as a whole need not become superconducting (on the contrary, it is assumed that it remains normal). Thus, local superconducting correlations will probably not be detected by conventional transport measurement. However, the manifestation of pairing correlations can still be detected via local measurements of, e.g., the LDOS, in which a minigap should appear [see Eq. (12) below].

As a first step, let us calculate the single particle LDOS in the presence of the impurity, given by $\rho(\mathbf{r},\omega)=-\frac{1}{\pi}\Im G^r(\mathbf{r},\mathbf{r},\omega)$, where $G^r(\mathbf{r},\mathbf{r}',\omega)$ is the retarded Green's function, which obeys the Dyson equation (depicted in Fig. 5),

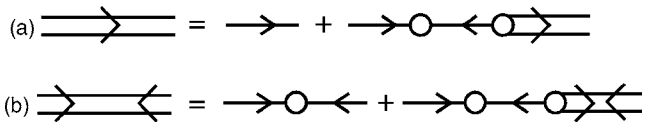

FIG. 5. (a) Dyson's equation for the single particle Green's function. (b) Dyson's equation for the anomalous Green's function.

$$G^r(\mathbf{r},\mathbf{r}',\omega) = g^r(\mathbf{r},\mathbf{r}',\omega) + \Delta^2 a^4 g^r(\mathbf{r},0,\omega) \times g^r(0,0,-\omega) G^r(0,\mathbf{r}',\omega), \tag{10}$$

where $g^r(\mathbf{r},\mathbf{r}',\omega) = -i\pi\Omega J_0[\sqrt{2m(\mu+\omega)}|\mathbf{r}-\mathbf{r}'|]$ is the Green's function for a clean 2D system, J_0 is the zero-order Bessel function, and Ω is the 2D density of states (DOS) at the Fermi energy. The solution for Eq. (10) is

$$G^r(\mathbf{r},\mathbf{r}',\omega) = g^r(\mathbf{r},\mathbf{r}',\omega) - \frac{i}{\pi\Omega}\frac{\gamma}{1+\gamma} g^r(\mathbf{r},0,\omega) g^r(0,\mathbf{r}',\omega), \tag{11}$$

where $\gamma = (\pi\Omega\Delta a^2)^2$. This yields for the DOS

$$\rho(\mathbf{r},\omega) \simeq \Omega\left(1 - \frac{\gamma}{1+\gamma} J_0[\sqrt{2m(\mu+\omega)}|\mathbf{r}|]^2\right), \tag{12}$$

which shows a decrease (the so-called minigap) in the LDOS at the location of the island, and Friedel-type oscillations from the island. The oscillations persist on a length scale $\sim\sqrt{\Omega/\mu}$, independent of the pairing potential in the island, which only affects the depth of the minigap. Due to these oscillations in the LDOS, one expects that the charge density will redistribute (and also exhibit oscillations) around the superconductor-semiconductor interface. One also expects that the density will redistribute within the superconducting island. However, this effect cannot be probed within the point-island limit [Eq. (10)], and will be discussed in a future study.

Next, we turn to a self-consistent calculation of Δ. Using the Dyson equation for the anomalous Green's function $F(x,x',t) = \langle \mathcal{T}\psi_\uparrow^\dagger(x,0)\psi_\downarrow^\dagger(x',t)\rangle$ [Fig. 5(b)], one finds

$$F(x,x',\omega) = \frac{\Delta a^2}{1+\gamma} g(x,0,\omega) g(0,x',-\omega). \tag{13}$$

The asymptotic properties of Bessel functions yield $F(x) \sim 1/x$ far from the impurity, independent from the value of interaction strength, as expected in a normal-SC interface. This is in contrast with the exponential decay in the tight-binding model.

The self-consistency equation for the pairing potential reads (see, e.g., Ref. 11)

$$\Delta(x) = -|U|\int_{-\omega_D}^{\omega_D} d\omega\, F(x,x,\omega), \tag{14}$$

where ω_D is the frequency cutoff of the interaction. Substituting Eq. (13) yields

$$\Delta(x) = -|U|\Delta a^2\left(\frac{\gamma}{1+\gamma}\right)\eta(x), \tag{15}$$

where $\eta(x) = \int_{-\omega_D}^{\omega_D} d\omega\, g(x,0,\omega) g(0,x,-\omega)$. At the center of the impurity, $\eta(0) = -2(\pi\Omega)^2\omega_D$. Inserting this into Eq. (15) results in an algebraic equation for $\Delta(0)$,

$$1 = 2(\pi\Omega)^2 a^2 |U|\omega_D\left(\frac{(\pi\Omega a^2)^2\Delta(0)^2}{1+(\pi\Omega a^2)^2\Delta(0)^2}\right), \tag{16}$$

which is easily solved to give

$$\Delta(0) = \frac{1}{\pi\Omega a^2}(2\pi^2\Omega^2|U|\omega_D a^2 - 1)^{1/2}. \tag{17}$$

This self-consistent solution vanishes when $a = a_I = (2\pi^2\Omega^2\omega_D|U|)^{-1/2}$, which is the minimal island area. Let us estimate the minimal island size, $\xi_a \sim a_I^{1/2}$ for a realistic system, composed of a Nb island embedded in a semiconducting quantum well made of Si or GaAs. Taking the effective masses $m^*/m = 0.98$ and $m^*/m = 0.063$ for Si and GaAs, respectively,[20] one can estimate the 2D-DOS in the quantum well. Taking for Nb $T_c = 9.26$ K and $\theta_D = 275$ K,[21] one finds that for the Nb/Si hybrid the minimal island radius is $\xi_I \sim 80$ nm, and for the Nb/GaAs system $\xi_I \sim 20$ nm. Both these lengths are still in the point-island regime, since the Fermi wavelength may be an order of magnitude larger for such quantum wells.

Equation (17) also supplies us with a dependence of Δ on the island size and interaction strength. In the inset of Fig. 3, we plot a fit of the numerical data in the region $U \simeq U_c$ (stars) to a function of the form $\Delta(U) \propto (U/U_c - 1)^x \Theta(U - U_c)$ (solid line), as in Eq. (17). The fit yields the exponent $x = 0.5002$, in good agreement with the continuous model.

In the definition of the model [Eqs. (5)–(7)], we have neglected the boundary conditions on the normal-superconducting interface. Omitting the boundary effect is hard to justify *a priori*, especially when the size of the superconducting grain is smaller than the superconducting coherence length. The boundary should be accounted for by solving Eqs. (13)–(15) with an additional constraint $\Delta(a) = 0$. However, the comparison between the numerical calculation (in which the boundary conditions are inherently implemented) and the analytical result (inset of Fig. 3) shows a striking equivalence between them. This indicates that neglecting the boundary effect merely results in a quantitative modification. It does not change the qualitative behavior, which is mainly characterized by the power-law dependence specified in Eq. (17).

V. DISCUSSION

One main feature of the result shown in Eq. (17) is that, in contrast to previous works on the proximity effect,[7,8,14] the length scale is not the usual superconducting coherence length ξ, but rather a different length scale $a_I = (2\pi^2\Omega^2\omega_D|U|)^{-1/2}$. In order to understand the origin of this length scale, we cast the criterion for the vanishing of SC correlations in the island given by Eq. (17) to the form

$$\frac{1}{\sqrt{2}}\left(\frac{|U|}{\omega_D}\right)^{1/2} 2\pi a^2 \Omega \omega_D = 1. \quad (18)$$

The factor $2\pi a^2 \Omega \omega_D$ is nothing but the number of electrons with energy in the range ω_D within the island, and thus the condition turns out to be a continuous version of the Anderson criteria, which states that SC vanishes in a grain once the number of pairs, roughly given by Δ/δ, where δ is the level spacing, becomes less than unity. However, in the above model there is no discreteness of energy levels. Rather, the number of pairs is restricted due to the finite region to which the interaction is limited. It is also clear from this argument why ξ does not play a role in this system, as ξ indicates the existence of a region where superconductivity is developed to its bulk value, which is not the case here.

Yet another way of understanding the result of Eq. (17) is to note that in a superconductor-semiconductor junction, one expects that due to the low density on the normal side, the suppression of pair correlations in the superconductor (due to the proximity effect) will no longer be on a length scale ξ as in a superconductor-metal junction, but rather a different length scale, which in the point-island approximation corresponds to $\xi_I \approx a_I^{1/2}$. Thus, if the superconducting island is smaller than the length scale on which pair correlations are suppressed, SC in the island will vanish. One also expects that upon increasing the island size (beyond the point-island approximation), this length scale will change. This problem is beyond the scope of the present work and will be addressed in a future study.

In Eq. (17), Δ has a power-law dependence on the grain size. This is in contrast to the case of an isolated grain, where an exponential dependence on size is predicted.[13] Furthermore, due to the renormalization of electron number and the dependence of critical island size on the DOS, the critical size may be either larger or smaller than that given by Anderson's criteria. This may affect the possibility of fabricating devices made from superconducting grains embedded on a metallic matrix. This effect may be tested experimentally by, e.g., varying the DOS of the metallic substrate by changing its carrier density (by gating the sample, for instance).

More intuition on the existence of a critical island size or interaction strength may be obtained by noting the similarity between Eq. (17) and Anderson's criteria for the existence of a magnetic impurity in a metal.[22] This similarity, along with the Friedel-type oscillations of Eq. (12), implies that the negative-U impurity is screened by the free electron gas, in an analogous way to the screening of a magnetic impurity. It would thus be intriguing to investigate the possibility of the formation of an effect equivalent to the charge Kondo effect,[23] resulting from the presence of embedded superconducting grains.

We conclude by noting that these results may be tested experimentally by planting superconducting impurities on a metallic substrate and measuring the local gap, in a similar way to that of Ref. 4. Another system in which our results may be valid is a superconducting grain strongly coupled to matching leads (i.e., superconducting aluminum grain and normal aluminum leads), where the existence of SC may be verified as a function of grain size. While this may be experimentally challenging, it may be achieved by, for instance, planting magnetic impurities in the leads. The dependence of the SC gap on the properties of the substrate, as seen in Fig. 2 for instance, may be tested by changing the density on the metallic substrate.

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