Fluctuations, dissipation, and nonuniversal superfluid jumps in two-dimensional superconductors

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We report a comprehensive study of the complex ac conductivity of thin effectively two-dimensional amorphous superconducting InO_x films at zero applied field. Below a temperature scale T_{c0} where the superconducting order parameter amplitude becomes well defined, there is a temperature where both the generalized superfluid stiffness acquires a frequency dependence and the dc magnetoresistance becomes linear in field. We associate this with a transition of the Kosterlitz-Thouless-Berezinskii (KTB) type. At our measurement frequencies the superfluid stiffness at T_{KTB} is found to be larger than the universal value. Although this may be understood with a vortex dielectric constant of $\epsilon_v \approx 1.9$ within the usual KTB theory, this is a relatively large value and indicates that such a system may be out of the domain of applicability of the low-fugacity (low-vortex-density) KTB treatment. This opens up the possibility that at least some of the discrepancy from a nonuniversal magnitude is intrinsic. Our finite-frequency measurements allow us access to a number of other phenomena concerning the charge dynamics in superconducting thin films, including an enhanced conductivity near the amplitude fluctuation temperature T_{c0} and a finite dissipation at low temperature which appears to be a universal aspect of highly disordered superconducting films.

I. INTRODUCTION

Superconducting fluctuations have been an area of investigation for many years,[1,2] yet very many completely central issues even in conventional superconductors are not understood.[3,4] Recently, interest in superconducting fluctuation phenomena has been renewed and indeed been thrust under the *klieg* lights due to focus on the high-temperature superconductors, where many believe them to play a central role in the phenomenology of underdoped compounds.[5–7]

In the usual view, as one approaches the nominal superconducting transition temperature T_c from above, thermal fluctuations in the amplitude[8] of the complex superconducting order parameter (OP) $\Psi = \Delta e^{i\phi}$ occur. If the fluctuations are small, they can be expressed in terms of a Gaussian approximation within the Ginzburg-Landau (GL) theory. In such a treatment it is envisioned that the free energy fluctuations of the OP's amplitude inside a coherence volume ξ^3 are of order $k_B T$. In the context of microscopic theory, this physics can be found as the Aslamazov-Larkin (AL) contribution to the conductivity.[9,10]

In a typical good three-dimensional (3D) BCS superconductor such fluctuations occur only vanishingly close to T_c. However, the fluctuation regime can be significantly enhanced for a thin dirty short-coherence-length (ξ) superconductor (thickness d) where the coherence volume becomes $d\xi^2$.[2] In actuality true long-range order of a continuous OP cannot exist for $T > 0$ in a 2D system, but the expectation is that there is still a crossover temperature T_{c0} where the superconducting amplitude becomes relatively well defined, although the phase still fluctuates. It is believed an actual phase transition does not occur until a possibly much lower temperature where a transition to a topologically phase

ordered state can take place. Such a so-called Kosterlitz-Thouless-Berezinskii[12,13] (KTB) transition can be written formally in a system with 2D xy or U(1) symmetry as a vortex-antivortex unbinding transition from a state of matter with a generalized rigidity (the superfluid stiffness) and power-law correlations to one with no rigidity and only exponential correlations. Above the transition, the superconducting phase fluctuates via unbound vortex excitations. It is the finite superfluid stiffness at low temperature that allows the phase to assume a well-defined value. The superfluid stiffness is proportional to the superfluid density, and the expectation is that the KTB transition is typified by a discontinuous jump in this parameter, which in certain limits takes on a universal value given by the transition temperature itself, $N = k_B T_{KTB} 8m / \pi \hbar^2$, where m is the electron mass. It is relatively well established that a KTB transition with a universal jump occurs in thin superfluid He4 layers,[14] although the situation in thin homogeneously disordered superconductor films is far from clear.

For the case of superconductors, KTB physics has been mainly investigated via linear and nonlinear dc transport[15–20] where various predictions exist for the temperature-activated resistance and power law of the *I-V* characteristics. It has been found that the experimental situation is roughly consistent with theoretical predictions, with, for instance, the α exponent of $V \propto I^\alpha$ a decreasing function of temperature until dropping quickly to become ~ 3 at a temperature not so far from the expected T_{KTB}. Measurements of the frequency-dependent conductivity should, in principle, allow more specific tests of theories for various fluctuation regimes and crossovers between them due to the sensitivity of the probe to the time scale of the fluctuations themselves. Moreover, finite-frequency measurements lend themselves to the study of both real and imaginary components of the charge re-

sponse which gives more information and allows a more precise comparison to theory. In relatively thick films, a few groups have shown via infrared and microwave transmission in granular or low-disorder lead and aluminum[21–24] that there was a region around the bulk T_c that had superconducting fluctuations consistent with the dynamic AL amplitude fluctuation form. In high-disorder thin films, using much lower frequencies, a number of groups have shown a dependence roughly consistent with the predictions of the dynamic extension of the KTB formalism,[18,25–28] although certain discrepancies exist.

So although various aspects of the commonly accepted picture have been reported, in-depth characterization of the finite frequency dynamics of both the Gaussian and vortex regimes has not been made; nor has a clear picture emerged as to how the system evolves from one regime of dominant fluctuations to the other, before ultimately entering the low-temperature phase. Moreover, it has never been shown whether or not these signatures of KTB physics actually occur at a superfluid stiffness given by the universal jump condition and by extension exactly what the nature of the low-temperature state is. In this paper we present a comprehensive study of the ac conductivity of highly disordered InO$_x$ films at GHz frequencies. We find that below a temperature T_{c0} where the superconducting amplitude becomes well defined, the films exhibit a temperature region where the generalized frequency-dependent superfluid stiffness acquires a frequency dependence. We observe that at essentially the same temperature the dc magnetoresistance becomes linear in applied field. This is the generic behavior expected near a KTB transition; however, in the present case it is exhibited at a stiffness well above the predicted universal value for the inferred T_{KTB}. Although it is possible to understand this within the usual KTB theory with a vortex dielectric constant of $\epsilon_v \approx 1.9$, this relatively large ϵ_v may indicate that the vortex fugacity is too large in such a system for the conventional low-vortex-density KTB treatment. This opens up the possibility that at least some of the nonuniversal value of the superfluid stiffness at T_{KTB} is intrinsic. At the lowest temperatures, we continue to observe a finite dissipation, which we correlate to an inhomogeneous superfluid density distribution. These results are put in the context of the considerable existing literature.

II. EXPERIMENTAL DETAILS

For these measurements, high-purity (99.999%) In$_2$O$_3$ was *e*-gun evaporated under high vacuum onto clean 1-mm-thick 19-mm-wide sapphire disks. Our synthesis methods are patterned off of the work of Ref. 29 where it was shown that amorphous InO$_x$ can be reproducibly made by a combination of *e*-beam evaporation of In$_2$O$_3$ and possibly annealing at low temperatures. This is unlike films prepared via other methods.[30,31] Essentially identical films have been used in a large number of recent studies of the 2D superconductor-insulator quantum phase transition.[32–38] Thin machined aluminum masks were used to pattern the films, creating a 200-Å-thick 3-mm-wide circular amorphous film centered on the disk. Sample deposition was well controlled

and for a certain conditions samples can be made reproducibly.[33,34] For structural characterization, we codeposit two more films along with the sample (i) onto a transmission electron microscope (TEM) grid for electron diffraction and (ii) for atomic force microscope (AFM) scans. We believe that the films are morphologically homogeneous with no crystalline inclusions or large-scale morphological disorder for the following reasons.

(i) The TEM-diffraction patterns are diffuse rings with no diffraction spots, suggesting amorphous films with no crystalline inclusions.

(ii) The AFM images show continuous films with no voids or cracks and in fact are completely featureless down to a scale of a few nanometers (the resolution of the AFM).

(iii) The R vs T curves when investigating the 2D superconductor-insulator transition[34,37] in these films are smooth with no reentrant behavior, which is the hallmark of gross inhomogeneity.

The ac conductivity was measured in a novel cryomagnetic resonant microwave cylindrical cavity system. The cavity diameter was optimized for performance in the 22-GHz ($\hbar\omega/k_B$=1.06 K) TE011 mode. A number of other discrete frequencies from 9 to 106 GHz were accessible by insertion of an additional sapphire puck (for the low frequencies) or use of a very short "pan"-shaped cavity (for greater than 100 GHz). Our highest operating frequency of 106 GHz corresponds in temperature units (using $\hbar\omega/k_B$) to 5.09 K, but as we will point out below, using the BCS relation $2\Delta/k_BT_c$=3.53 and the temperature scale where the amplitude becomes well defined (2.28 K) the threshold for above gap excitation is believed to be 158 GHz. The lack of sample heating was ensured by operating in a regime where the response was independent of input power. Relations between the resonances' frequency shift $\Delta\omega$ and change in quality factor $\Delta(1/Q)$ upon sample introduction to the complex conductivity are obtained by a cavity perturbation technique[39–42] (see Ref. 43 for a very thorough treatment). This standard experimental technique is based on an adiabatic modification of the electromagnetic fields in a cavity that arises from the introduction of a small sample to the interior of the cavity. Our data are analyzed in the depolarization regime in which the fields in the cavity penetrate the entire volume of the sample. It has been shown that for extremely thin films, only in-plane ac electric fields or out-of-plane ac magnetic fields at the sample position can affect an appreciable change in a cavity's resonance characteristics.[44] Samples were placed along the cavity's central axis, where due to symmetry considerations and depending on the particular TE mode being used, if the electric field is in plane, then there is a zero out-of-plane magnetic field and vice versa.

Modes with both of these field configurations were exploited in our setup and their analysis differed. The formulas used for analysis will be stated here for completeness in terms of the complex frequency $\hat{\omega} \equiv \omega_0 - i\omega_0/2Q$. For the case of an in-plane electric field, the complex frequency shift is

$$\frac{\delta\hat{\omega}}{\omega} = -\gamma\frac{4\pi\sigma^*/\omega}{1+n4\pi\sigma^*/\omega}, \tag{1}$$

where σ^* is the complex conjugate of the conductivity, n is the depolarization factor, $\gamma = \gamma_0 \frac{V_{sample}}{V_{cavity}}$ is a "filling factor," and

γ_0 is a constant that depends on the particular mode used. In the present case the sample is so thin that n is very small (3×10^{-6}) and so there is only very weak mixing between components of the complex frequency shift into the complex conductivity, so that the real part of the conductivity is largely proportional to the change in $\omega_0/2Q$ and the imaginary part of the conductivity is largely proportional to the shift in ω_0.

For the case of the sample being in a perpendicular ac magnetic field the results are obtained following[40]

$$\frac{\delta\widetilde{\omega}}{\omega} = -\gamma\chi(\omega), \qquad (2)$$

where γ is again the "filling factor" and $\chi(\omega)$ is the susceptibility which includes not only the response from localized electric and magnetic dipoles, but also the response from induced ac currents. This is relevant for our analysis in a magnetic field antinode since the response from the local electric dipole moments will be zero (electric field node), the local magnetic moments are zero (nonmagnetic material), and so the cavity perturbation is due to the induced currents in the superconductor which has been fully penetrated by the external rf field. Brandt[39,45] has worked out the susceptibility for a thin superconducting disk in a perpendicular dc field due to in-plane ac currents from a superimposed ac field and found

$$\chi(\omega) = -\sum_{n=1}^{N} \frac{C_n/\Lambda_n^2}{\Omega^{-1}+\Lambda_n^{-1}} = -\frac{1}{\gamma}\frac{\delta\widetilde{\omega}}{\omega}, \qquad (3)$$

where the quantity we wish to extract—namely, the dynamic conductivity $\sigma(\omega)$—is contained in the normalized frequency $\Omega = i\omega\sigma(\omega)\frac{\mu_0 rd}{2\pi}$ where r is the radius and d the thickness of the sample. The coefficients C_n and Λ_n are given in Ref. 40, where it is found that to well within experimental accuracy, the series can be represented by a small number of terms. We use 30 elements—as in previous studies—but in reality many less are actually needed. With these relations in hand, the complex conductivity can be extracted from the complex frequency shift. Here also, due to the thinness of the samples, there is only weak mixing between real and imaginary frequencies into the components of the complex conductivity.

Although theoretical values exist for γ, in practice the conversions to complex conductivity were made by adjusting the free parameters for the two schemes (γ and n for the E-field case and γ for the B-field case) so that the ac resistance matched dc data at temperatures well above the occurrence of superconductivity and, using the expectation that the superfluid stiffness (defined below), was frequency *independent* at $H=0$ and low temperature. With a unique sample-dependent depolarization factor and γ's that depend on the particular mode being used a unique set of normalization factors could be found.

The dc resistance was measured on codeposited samples in a two-probe configuration by low-frequency ac lock-in techniques using excitation currents of approximately 10 nA. The probe's lead resistances, which have a negligible tem-
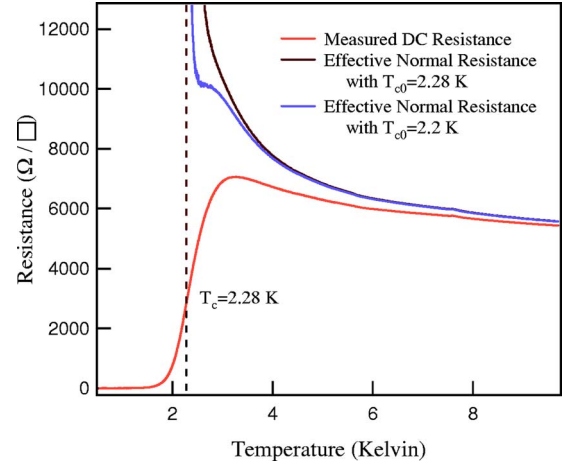

FIG. 1. (Color online) dc sheet resistance. Temperature dependence showing the broad resistive transition resulting from fluctuations. The normal-state resistance curves are generated using the procedure described in the text. T_{c0} is believed to be 2.28 K.

perature dependence in the displayed temperature range, were well characterized and have been subtracted from the displayed data.

III. RESULTS

We are interested in understanding the fluctuation behavior of the complex order parameter $\Delta e^{i\phi}$ as the temperature is lowered towards the superconducting state. A finite-frequency probe has access to this information because the complex electrodynamic response of a superconductor at experimental frequencies ω_{expt} below the gap 2Δ can be approximated as $\sigma_1 = (\pi n_s e^2/2m)\delta(\omega) + n_n e^2 \tau_n/m$ and $\sigma_2 = n_s e^2/m\omega$, where n_s (n_n) is the density of superconducting (normal) electrons, τ_n is the relaxation time for normal electrons, and e and m are the electron charge and mass, respectively. We note that, in fact, in a highly disordered superconductor the contribution from normal electrons is very small as the carrier lifetime is very short. In particular, the normal electron contribution to σ_2 is negligible when $\omega\tau \ll 1$ as certainly the case here in our highly disordered material. As will be shown below, also importantly for our study, the extremely short normal-state lifetime causes even the above gap contribution to σ_2 for $\hbar\omega > 2\Delta$ to be below detectable levels for our experimental frequencies. This is important and means that our σ_2 is still almost purely due to the superfluid response even at elevated temperatures when the superconducting gap begins to close.

For a fluctuating superconductor one can define $\sigma_2 = n_s(\omega)e^2/m\omega$ where the fluctuation effects are captured by a frequency-dependent superfluid density $n_s(\omega)$.[37,46] A definition as such—or in terms of an equivalent frequency-dependent superfluid stiffness (defined below)—has been the usual treatment within the finite-frequency KTB theory,[46] where ac measurements allow one to probe the system on shorter length scales and reveal superconducting fluctuations even above T_{KTB}.

We begin by examining the dc data, as shown in Fig. 1.

We observe a broad region over which the superconducting transition occurs. The contribution of Gaussian amplitude fluctuations can be obtained by fitting to the Aslamazov-Larkin dc form. Using the procedure of Gantmakher and Golubkov,[38] a lower bound on T_{c0} can be estimated as the lowest temperature that does not cause an inflection point in the extracted effective normal-state resistance $R_N(T)$ as defined by the full expression for the Aslamazov-Larkin fluctuation resistivity:

$$R_{meas} = \frac{1}{\sigma_N d + \sigma_{2D}^{AL} d} = \frac{1}{1/R_N + \dfrac{e^2}{16\hbar} \dfrac{T_{c0}}{(T - T_{c0})}}. \quad (4)$$

Within this analysis 2.28 K is the best lower bound on T_{c0} and represents the temperature scale below which the superconducting amplitude is relatively well defined. Lower values of T_{c0} produce a kink in the extracted resistivity, where, for instance, a T_{c0} of 2.2 K is clearly too low as seen in Fig. 1.

As mentioned above, the imaginary conductivity σ_2 is proportional to the superfluid density, which then sets the scale for the superfluid stiffness—the energy scale to introduce phase slips in the superconducting order parameter. We extract this generalized superfluid stiffness T_θ (in degrees kelvin) from the conductivity via the relation $\sigma_2 = \sigma_Q \frac{k_B T_\theta}{\hbar\omega}$ (as done similarly recently),[7] where $\sigma_Q = \frac{4e^2}{hd}$ is the quantum of conductance for Cooper pairs divided by the sample thickness. We emphasize that in our notation T_θ *is not a temperature per se*, but is an energy scale expressed in temperature units. Our superfluid stiffness is essentially kinetic inductance expressed in temperature units. We could express this quantity in any one of a number of equivalents units[47] including superfluid density. We prefer this parametrization as it is unnecessary to make an estimate for quantities such as the Cooper pair mass. Moreover, with these units the universal jump condition for the KTB transition is of the particularly simple form $4T_{KTB} = T_\theta$. The fluctuations of the superconducting state can be captured by the frequency dependence of the stiffness, which would otherwise be frequency independent.

In the absence of fluctuations, we expect that an accurate description of the superfluid stiffness would be given by the dirty limit mean-field BCS theory, which predicts the onset of superconductivity at a temperature T_c, with a temperature dependence of $\sigma_2(T)$ given by[10,48]

$$\sigma_2(T) = \frac{\pi\Delta(T)}{\hbar\omega} \tanh\left[\frac{\Delta(T)}{2k_B T}\right], \quad (5)$$

where $\Delta(T) = \Delta(0)\sqrt{\cos\left[\frac{\pi}{2}\left(\frac{T}{T_c}\right)^2\right]}$ is the temperature-dependent gap function and $\Delta(0) = 1.76 k_B T_c$.[11]

In Fig. 2 we show the temperature dependence of the superfluid stiffness T_θ measured at various frequencies alongside the prediction of Eq. (5). The mean-field curve has been generated by specifying the conductivity in the normal state σ_n and then varying T_c to obtain the best fit to the data at the lowest temperature. We extract a transition temperature scale of 3.47 K which we associate with a microscopic scale
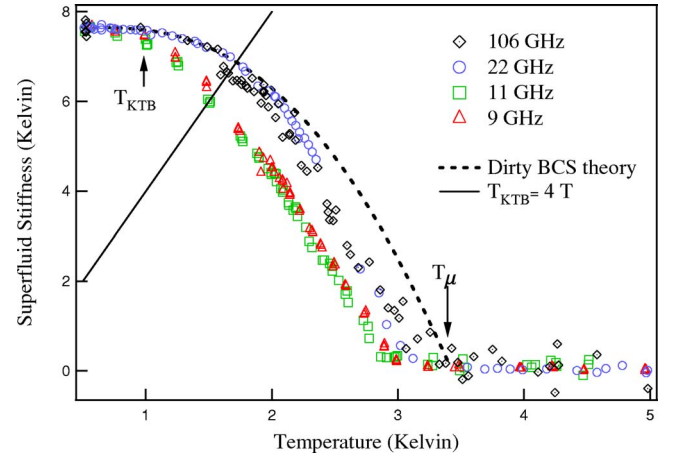

FIG. 2. (Color online) Temperature dependence of the superfluid stiffness T_θ. The prediction of the dirty BCS model is shown as a dashed line which goes to zero at T_μ. The lower-frequency data show a significant deviation from the mean-field behavior. The temperature where T_θ acquires a frequency dependence can be identified with T_{KTB}.

T_μ where the superconducting transition *would* occur in the absence of fluctuations. This very large temperature scale is over 1.6 times the temperature T_{c0} where the amplitude becomes well defined and over 3 times the scale of the onset of phase coherence (defined below) and shows the paramount role that superconducting fluctuations play in such a material. We also see that the superfluid stiffness curves progressively approach the mean-field curve at higher frequencies. However, because of the enhanced role of fluctuations, all of the curves approach zero at a temperature below T_μ.

The experimental curves in Fig. 2 show a broad temperature region characterized by a gentle roll-off of the superfluid stiffness with increasing temperature. We observe a distinct temperature where the superfluid stiffness acquires a frequency dependence. Within the standard theory, such an occurrence is indicative of the approach to a KTB transition. In this model, the *zero-frequency* superfluid stiffness is renormalized discontinuously to zero at a temperature T_{KTB} set by the superfluid stiffness itself at this temperature. Above T_{KTB} the system still appears superconducting on short length scales set by the separation between thermally generated free vortices, and therefore at finite frequencies we expect the superfluid stiffness to approach zero *continuously*. As T_{KTB} is the temperature where vortices proliferate, in at least moderate fugacity (a quantity related to the vortex core potential and defined below) superconductors, the temperature where the superfluid stiffness curves measured at different frequencies deviate from each other can be identified with the approach to T_{KTB}. As mentioned above, with our units of superfluid stiffness the predicted transition temperature is $T_{KTB} = T_\theta/4$, which is shown as a solid line in the figure. We point out here that although a frequency dependence as such is seen in Fig. 2, the stiffness where T_θ acquires its frequency dependence is well above the stiffness predicted to be the critical value. This is presumably due to the finite measurement frequency. We will expand on this point below.

Using the BCS relation $2\Delta/k_B T_{c0} = 3.53$ and the measured $T_{c0} = 2.16$ K, all operating frequencies are below the BCS

expectation of 158 GHz for above threshold gap excitation. Although at the lowest temperatures our highest frequency is well below the superconducting gap, in principle, there can be a normal electron contribution to σ_2 from both thermally excited quasiparticles at $T \neq 0$ and from above gap excitations when $\hbar\omega > 2\Delta$ on the approach to T_c. In practice, however, these give a negligible contribution to the response in a highly disordered superconductor. We can give a rough estimate of this contribution by a very approximate calculation. Magnetoresistance measurements on similar samples have shown that the coherence length is approximately 6 nm.[33] As such highly disordered superconductors have their coherence length set by the carrier mean free path, we can take this value to be of the order of the normal electron scattering length. With a reasonable estimate for the Fermi velocity ($c/200$), this gives the very large effective scattering rate ($1/\tau$) of approximately 200–300 THz. This is reasonable based on the high disorder and large normal-state resistivity of our sample. An estimate based on the normal-state Drude conductivity and using a free electron mass and carrier density estimated from Hall measurements on similar samples ($10^{21}/cm^3$) (Ref. [49]) gives a magnitude of the same order.

Using this scattering rate, the Drude relations, our definition of T_θ, and its measured low-temperature value (7.66 K) we can estimate the maximum possible *contamination* contribution to the measured stiffness from "normal" electrons if all the spectral weight in the superconducting δ function became subject to normal-state scattering as

$$T_{\theta,cont} = T_\theta[T \to 0] \frac{(\omega\tau)^2}{1 + (\omega\tau)^2}. \qquad (6)$$

At even our highest frequency, this represents a total contribution of less than 2×10^{-6} K, which is far smaller than our sensitivity. We note that even if we overestimated that effective scattering rate by even a factor of 100, the contamination contribution would only be 0.02 K which is still well below our sensitivity. Hence, the contribution from "normal" electrons give a completely insignificant contribution to σ_2 and hence T_θ.

As an aside we will note that an order-of-magnitude estimate of the effective superconducting electron density can be found via the definitions given above using the free electron mass and the $T \to 0$ limit of T_θ=7.66 K gives an approximate value of $3 \times 10^{17}/cm^3$ which can be compared to the Hall effect derived number of $10^{21}/cm^3$.[49] The effects of disorder are significant in reducing the number of charges that participate in pairing.

Figure 3 shows the low-field magnetoresistance data. We notice in the isotherms of magnetoresistance that there is a range of intermediate temperatures above the low-field saturation regime where the resistance changes from concave down to concave up with magnetic field. The condition $R(T) \propto B$ for low fields has previously been associated with the Kosterlitz-Thouless-Berezinskii temperature T_{KTB} (Ref. [50]) based on the Minnhagen criterion[51] and was applied[52] to similar samples of InO_x.

Power-law fits to the low-field data (well above the saturation regime) of the magnetoresistance ($R \propto B^\alpha$) curves give
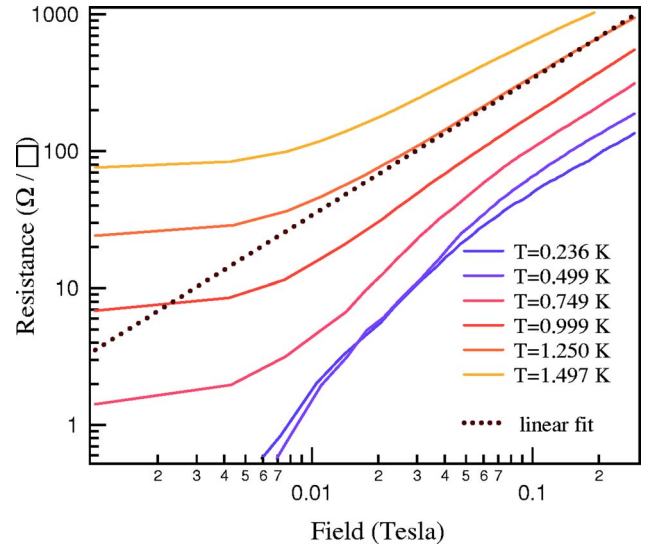

FIG. 3. (Color online) The dc magnetoresistance isotherms showing a power-law dependence on the applied field for temperature T=0.236, 0.499, 0.749, 0.999, 1.250, and 1.497 K. The dashed line is a linear fit.

a temperature-dependent exponent in this range. An interpolation of the data gives a temperature of $\approx$1.20 K where $\alpha = 1$. Interestingly, this temperature is to within the experimental uncertainty of where the superfluid stiffness acquires a frequency dependence in Fig. 2. On the basis of the above, we associate a temperature in the range of $\approx$1.15 K with a transition of the KTB type involving vortex unbinding and proliferation at a nonuniversal value of the superfluid stiffness.

We also note that the zero-field resistance is finite just below our inferred T_{KTB}. Such an observation is not surprising within the usual KTB theory with our non-negligible excitation current (10 nA) and the possibility of small residual magnetic fields, as the critical current and field of a 2D superconductor are zero. Finite resistance below the inferred T_{KTB} has been observed before,[16,20] but is not a universal observation.[25]

Within the simplest conception of the KTB theory the transition occurs at a value of the superfluid stiffness given by the above universal jump condition, where the superfluid density is driven to zero at T_{KTB} by a cascade of vortex proliferation. As noted above, our frequency dependence is acquired in T_θ at a stiffness that is in excess of the universal prediction. It is important to realize that this prediction is for the zero-frequency stiffness (longest length scales) and higher-frequency probes can reveal larger values that do not take into account long-wavelength renormalizations. Hence, transitions at nonuniversal values of the bare (shortest length scales) superfluid stiffness can occur where a large superfluid stiffness is renormalized downward even below T_{KTB} by thermally excited vortices if the vortex potential for their activation is low enough. Our finite-frequency measurements allow us to probe the system on intermediate length scales that are between these two limits. Within the usual KTB theory, the transition will occur at a universal value of the *renormalized* superfluid stiffness where the renormalization factor is given

by a vortex dielectric constant $1/\epsilon_v = 1 - 2\pi y_0$ which incorporates the screening effects of intervening thermally excited vortex-antivortex pairs where $y_0 = e^{-\mu_c/k_B T}$ is the so-called vortex fugacity (related to the thermal excitation probability) and μ_c is the vortex core potential. In order that the frequency dependence is observed at a T_θ of the universal value at T_{KTB}, one must be specifically in the low-vortex-density (low-fugacity) limit. It is not clear that thin-film superconductors are generically in this limit.

We can estimate ϵ_v if we associate our extracted mean-field superfluid stiffness as the unrenormalized bare superfluid value and take as the fully renormalized number the KTB prediction at the temperature at which the frequency dependence in the superfluid stiffness is acquired. With this procedure we find a ϵ_v of 1.9. This can be compared to a value of 1.3 for both simulations of the 2D *xy* model[53] as well as experiments on thin He4 films.[14,54] According to the above fugacity relations this gives a core potential μ_c of 2.97 K. This core potential is a sum of core energy and entropy terms $\mu_c = E_c + k_B T \ln N_0$ where E_c is the core energy and N_0 represents the number of statistically independent sites that two vortices can occupy within a coherence area and is of the order of $\frac{1}{2\pi}$. Using this estimate we get a core energy of 0.85 K, which is of the expected magnitude.[55] An understanding of our results is not impossible within the usual KTB theory; however, this large ϵ_v is indicative of a relatively large fugacity. As the usual KTB relations are derived in the limit of low fugacity and hence low vortex density our results may mean superconducting films as such are out of the domain of applicability of the low-fugacity theory.

A number of workers have raised the possibility that if the fundamental assumption of low fugacity is violated, intrinsically different physics may result, which may also be consistent with our observations. For instance, Minnhagen and co-workers have postulated an expanded set of KTB renormalization equations valid at larger fugacities and shown that out of the low-fugacity limit and above a critical value of $\epsilon_v \approx 1.74$ the transition may exhibit different behavior, including a nonuniversal jump of the renormalized superfluid stiffness and perhaps even a first-order transition.[56,57] Such considerations may be relevant in our case with our $\epsilon_v \approx 1.9$. Additionally it has been proposed[58,59] that at large fugacities vortices may not form bound vortex-antivortex pairs below T_{KTB} and, instead, their density is high enough that they crystallize into a vortex ionic crystal. Superconducting phase coherence is disrupted by the melting of this crystal, as opposed to the unbinding of vortex-antivortex pairs. Some evidence for such lattice formation and subsequent melting exists in numerical simulations.[60]

Irrespective of its origin, we associate this temperature determined by both magnetoresistance and ac conductivity to be the temperature scale where superconducting phase coherence is destroyed. Our analysis above has been made possible by the fact that our highest measurement frequencies are large enough to be close to the high-frequency limit (as shown by the negligible frequency dependence between 22 GHz and 106 GHz curves and their relative closeness to the mean-field curve), and thereby we can characterize the unrenormalized superfluid stiffness.

Having established the temperature scales of the various fluctuation phenomena in the problem we now discuss the
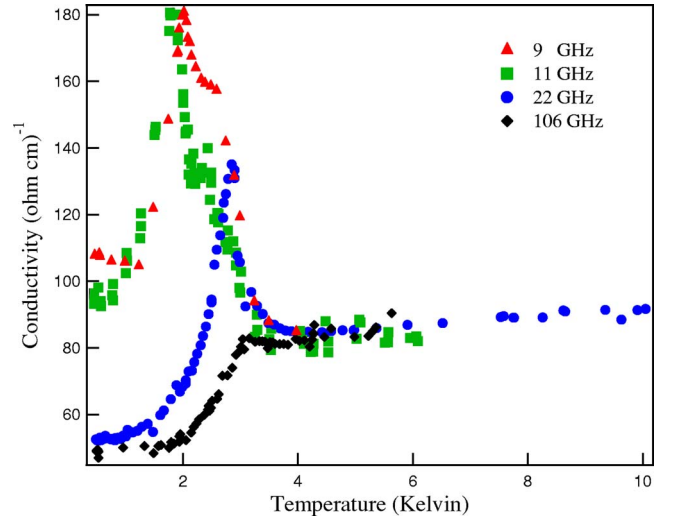

FIG. 4. (Color online) Temperature dependence of σ_1 at 9, 11, 22, and 106 GHz. Note the shift in the peak to higher temperatures and the decrease in its height with increasing frequency.

dissipative response of the real part of the conductivity σ_1 (Fig. 4), focusing our attention on the peak as well as the low-temperature absorption. We interpret this dissipation peak in σ_1 as arising from a partially coherent superfluid, as was conjectured previously in Ref. 7. We observe that this peak, found in the temperature region of amplitude fluctuations, shifts to higher temperatures and its amplitude decreases as the frequency increases. Such an effect is not predicted within the finite-frequency Gaussian AL theory where the peak in σ_1 is always at T_c.[2,10]

This behavior may be understood by comparing the length scale set by the fluctuations and those set by the frequency of the measurement probe. At very high temperatures, well above the fluctuation regime, there are no superconducting fluctuations. As the temperature is lowered the characteristic length scale for these fluctuations increases. Higher frequencies probe the system on shorter length scales and therefore are able to measure the onset of fluctuations at higher temperatures. We would also like to understand why there is a difference in the height of the different dissipation peaks at different frequencies in Fig. 4. This may be understood heuristically by returning to the "dirty-BCS-theory" mean-field curve in Fig. 2 and noticing that as the temperature decreases the superfluid stiffness is a rapidly increasing function of temperature in this range. For a given measurement frequency, the dissipation peak occurs when the time scale of the measurement probe matches the time scale of the superconducting fluctuations and the peak height is proportional to the superfluid at that temperature. Since this occurs at higher temperature for the higher-frequency probes and the superfluid stiffness is a strong function of temperature in this range, the peak height is expected to decrease with increasing frequency.

We can examine this further using a simple relaxation model[61] which attempts to incorporate the observed frequency dependence of $\sigma_1(T)$ in a simple way:
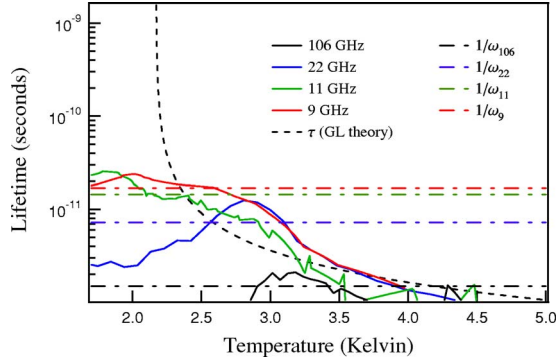
FIG. 5. (Color online) τ vs temperature at various given frequencies extracted via the simple relaxation model in Eq. (7). Also shown is the Ginzburg-Landau result using $T_{c0}=2.28$ K. The dashed horizontal lines mark the time scales set by the various probing frequencies, $1/\omega$. Notice how the τ levels off at lower temperatures, indicating the freezing out of amplitude fluctuations.

$$\tilde{\sigma}_{meas}(\omega,T) = \sigma_N^{dc}(T) + \frac{\sigma_{2D}^{AL}(T)}{1 + i\omega\tau}, \qquad (7)$$

where $\tilde{\sigma}_{meas}(\omega,T)$ is the measured finite-frequency conductivity, $\sigma_N^{dc}(T)$ is the normal dc conductivity (fit using a $1/T$ temperature dependent extrapolation)[61,62] combined with the dc Aslamazov-Larkin fluctuation conductivity from Eq. (4) which appears to be the substantial contribution to the resistivity in this temperature region. We can use this to parametrize the data in terms of τ, which can then be compared with the Ginzburg-Landau result $\tau_{GL}=\pi\hbar/8k_B(T-T_{c0})$. This is shown in Fig. 5.

The basic assumption of this approach is that there is a contribution to $\sigma_1(\omega,T)$ in this temperature region which—at zero frequency and temperatures above T_{c0}—is described by the Gaussian approximation given by the dc Aslamazov-Larkin formula in Eq. (4). We then expect that the T_{c0} extracted from the dc data should appropriately capture the underlying physics. To account for the frequency dependence, we assume a relaxation model in which the evanescent pairs contribute to the conductivity for a time τ before decaying back into normal electrons. As shown in Fig. 5, such a modeling qualitatively accounts for the frequency dependence as evidenced by the relative collapse of the τ's extracted at different frequencies in Fig. 4. Importantly, we also see that this treatment gives τ a plateau near T_{c0}, indicating a slowing down of the fluctuation dynamics that we are sensitive to. If, as we believe, the dissipation in this temperature range is largely sensitive to the fluctuations of the superconducting amplitude, then the plateau is consistent with our interpretation that there is a temperature T_{c0} where the amplitude fluctuations become frozen out. Finally, it is important to note the temperature where the measured τ equals the inverse of the measurement frequency $1/\omega$ occurs at approximately the same temperature where we find a peak in the dissipative (σ_1) response in Fig. 4. This supports the notion that the peak in $\sigma_1(T)$ arises when the bandwidth of the probing frequency matches the lifetime of the fluctuations.

Finally, we would like to point out the finite dissipation (σ_1) in Fig. 4 in the limit $T \to 0$, which is not accounted for by the BCS derived theories of Mattis and Bardeen[10,63] or Leplae,[64] which predict that dissipation disappears for $\hbar\omega < 2\Delta$ as $T \to 0$. Anomalous absorption as such has been seen previously in other highly disordered superconducting films[65] as well as in Bi-based high-temperature superconductors[66] which may show nanoscale inhomogeneity.[67] The dissipation in the case of the Bi-based high-temperature superconductors was interpreted as a consequence of an inhomogeneous superfluid density distribution. Such a situation is certainly a possibility here in our highly disordered films. We reiterate, however, that our amorphous films are themselves morphologically homogeneous down to the lowest measured length scales (nanometers) as detailed above. It would be interesting to perform a scanning tunneling microscopy study to make a detailed comparison with the cuprate superconductors. That any inhomogeneity could arise out of films which are essentially homogeneous on all measurable length scales is the interesting and remarkable aspect. It is perhaps indicative of an exponential sensitivity to the disorder which inevitably exists at the shortest length scales (the materials are amorphous). This extreme sensitivity to disorder and dynamically created inhomogeneity is then interesting in its own right as it is an intrinsic part of the problem.[68,69] Similar conclusions about dynamically created inhomogeneity have been arrived at previously via dc transport-based probes in similar films.[29]

We note that such an interpretation need not change our assumption of a frequency-independent superfluid stiffness at $H=0$ and $T \to 0$. These other measurements that have seen low-T dissipation (for instance, Ref. 66) measure the superfluid stiffness's absolute value directly and find that it is frequency independent at low T. Additionally, we will point out that explicit theoretical models that treat such a scenario (within an effective medium granular scenario—for instance, Ref. 70) with intermixed superconducting and metallic grains finds that the σ_2 still has its characteristic $1/\omega$ dependence (and hence the superfluid stiffness in frequency independent) for globally superconducting systems. This derives from the δ function in σ_1 at $\omega=0$ which Kramers-Kronig constrains the frequency dependence of σ_2.

Finally, we point out that as an alternative path to describing this dissipation, it has been proposed that dissipative phenomena may be generic in the vicinity of quantum critical points.[71] In this regard, our system is not very far in parameter space to the 2D disorder-tuned superconductor-insulator quantum phase transition[34] and may be effected by this physics.

IV. CONCLUSION

In this work we have established various temperature scales associated with the crossover between Gaussian and KTB-vortex fluctuations. We have been able to independently establish the temperature of a KTB-like vortex unbinding transition by ac techniques which allowed us to determine where the superfluid stiffness acquired a frequency dependence, as well as by dc measurements using the

Minnhagen criterion. The magnitude of the superfluid stiffness disagrees with the prediction for the universal superfluid jump criterion at T_{KTB}. Although this can be understood by invoking a relatively large vortex dielectric constant $\epsilon_v \approx 1.9$, which can renormalize the stiffness from a bare value to the universal value, it is also indicative of a high fugacity, which may invalidate the low-fugacity assumptions under which the KTB theory is derived.

We have also shown how the peak in the dissipative piece of the complex response arises from superconducting fluctuations near T_{c0}. In this model, the peak arises when the time scale of the measurement matches the lifetime of the fluctuations, and we have given a simple model that causes a collapse in the relaxation time measured at different frequencies. We give a simple picture that describes this behavior and which shows how the dynamics slow upon passing be-

low the amplitude temperature scale. Finally, we have discussed how the dissipative piece of the complex conductivity can be understood as arising from an inhomogeneous superfluid.

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